Analysis of radiative τ decays

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The similarity between the two-body τ decay into the pion and the neutrino and the pion decay into the muon and neutrino suggests the possibility of radiative τ decays ($\tau\to\pi\nu_\tau\gamma$ and $\tau\to\rho\nu_\tau\gamma$) analogous to radiative pion decay. These three-body decays can provide a means for determining the τ-neutrino mass. The decay $\tau\to\pi\nu_\tau\gamma$ is useful for determining the sign and magnitude of the structure parameter γ.

I. INTRODUCTION

All known properties of the τ particle[1] are consistent with the hypothesis that like the muon, it differs from an electron only in mass and in probably having its own associated neutrino, justifying its characterization as a third lepton. The radiative decay mode $\tau\to\pi\nu_\tau\gamma$ accompanying the decay $\tau\to\pi\nu_\tau$ [which has a branching ratio of 10.9% (Ref. 2)] is then closely related to $\tau\to\pi\nu_\tau\gamma$ and could be used as a further check of the e-μ-τ equivalence as well as to gain information on the structure-dependent terms in the radiative $\pi l\nu$ matrix element. The process could also be used to study the problem of the neutrino mass. Another important mode of the hadronic τ decays is into $\rho\nu_\tau$; therefore the rate for its accompanying radiative decay into $\rho\nu_\tau\gamma$ will be correspondingly frequent. The rate of this process can be calculated with reasonable assurance also.

Section II presents the calculation of the radiative decay process $\tau\to\pi\nu_\tau\gamma$. Previously published results contain errors and/or misprints for the related process $\pi\to l\nu_l\gamma$, which are noted and corrected. Section III presents a relatively unambiguous calculation of the decay $\tau\to\rho\nu_\tau\gamma$. A discussion of the numerical results of the two decay processes are presented in Sec. IV. The possibilities of neutrino-mass measurement from the decay are considered in Sec. V.

II. THE DECAY $\tau\to\pi\nu_\tau\gamma$

The matrix element for the radiative $\tau\to\pi\nu_\tau$ decay receives contributions from so-called inner bremsstrahlung (IB), in which the photon is emitted by one of the ingoing or outgoing particles, and the direct or structure-dependent (SD) terms, in which the photon is emitted by a virtual intermediate particle during the course of the decay process (see Fig. 1). The general form of the matrix element for the decay process

$$\tau^-(s)\to\pi^-(p)+\nu_\tau(q)+\gamma(k)\ ,$$

where the arguments in parentheses denote the corresponding four-momenta can be written as[3]

$$M=\frac{Ge}{\sqrt{2}}\left\{\cos\theta_V F^V(t)\epsilon_{\mu\nu\rho\sigma}J_l^{\mu}\epsilon^{\nu}k^{\rho}p^{\sigma}-i\bar{u}_\nu(q)(1+\gamma_5)\right.$$
$$\left.\times\left[\cos\theta_A f_\pi\left[1-\frac{m_\nu}{m_l}\right]\left[\frac{p\cdot\epsilon}{p\cdot k}-\frac{s\cdot\epsilon}{s\cdot k}-\frac{\not\epsilon\not k}{2s\cdot k}\right]+\cos\theta_A F^A(t)[(\not k)(\epsilon\cdot p)-(\not\epsilon)(p\cdot k)]v_l(s)\right]\right\}\ ,$$

where $J_l^m=\bar{u}_\nu(q)\gamma^\mu(1-\gamma_5)v_l(s)$ is the leptonic current. G is the universal weak constant, e is the electric charge of the proton, and ϵ^ν is the photon polarization. $F^V(t)$ and $F^A(t)$ are the vector and axial-vector form factors depending on the square of the momentum transfer, t, and are required to be real by time-reversal invariance. The differential decay rate with respect to the energies of the photon ($\zeta=2E\gamma/m_\tau$) and of the pion ($\eta=2E\pi/m_\tau$), is found to be

$$\frac{d^2\Gamma}{d\zeta\,d\eta}=\frac{\alpha\Gamma_{\pi\nu}}{2\pi(1-r)^2}\left[G_{\rm IB}(\zeta,\eta)+\frac{1}{4(1-2m_\nu/m_\tau)}\left[\frac{F^V}{f_\pi}\right]^2[(1+\gamma)^2G_{\rm SD^+}(\zeta,\eta)+(1-\gamma)^2G_{\rm SD^-}(\zeta,\eta)]\right.$$
$$\left.+\frac{1-m_\nu/m_\tau}{1-2m_\nu/m_\tau}\left[\frac{F^V}{f_\pi}\right][(1+\gamma)G_{\rm INT^+}(\zeta,\eta)+(1-\gamma)G_{\rm INT^-}(\zeta,\eta)]\right]\ ,$$

where $\Gamma_{\pi\nu}$ the total decay rate for the two-body decay process $\tau\to\pi\nu_\tau$ is[4]

$$\Gamma_{\pi\nu}=\frac{G^2f_\pi{}^2m_\tau{}^3\cos^2\theta}{16\pi}\left[1-\frac{\mu^2}{m_\tau{}^2}\right]\left[1-2\frac{m_\nu}{m_\tau}\right]\ .$$

The other quantities are given by

$$G_{\rm IB}(\zeta,\eta)=\frac{1}{\zeta^2(\zeta-A)}\left[A\zeta(\zeta-A)+(2-\eta-\zeta)\left[2\eta\zeta-2(\zeta-A)+\frac{2\zeta^2 r}{(\zeta-A)}+2A\beta\right]\right],$$

$$G_{\rm SD^+}(\zeta,\eta)=2\zeta B(\zeta-A)-\eta\zeta A ,$$

$$G_{\rm SD^-}(\zeta,\eta)=2\eta(\zeta-A)A-\eta\zeta A ,$$

$$G_{\rm INT^+}(\zeta,\eta)=\frac{A}{\zeta(\zeta-A)}\{(\zeta-A)[2(\zeta-A)+B+\eta]-2r\zeta\} ,$$

$$G_{\rm INT^-}(\zeta,\eta)=\frac{A}{\zeta(\zeta-A)}\{(\zeta-A)[2(\zeta-A)-B-\eta]+2r\zeta\} ,$$

$$r=\left[\frac{\mu}{m_\tau}\right]^2, \quad \mu=\text{pion mass} ,$$

with INT representing the interference terms, and

$$A=(1-\eta+r) ,$$

$$B=(1-\zeta-r) ,$$

$$\beta=(1-\zeta)(1-\zeta-\eta)+r .$$

The constants, multiplying the IB, $\mathrm{SD}^\pm$, and $\mathrm{INT}^\pm$ terms are

$$C_{\rm IB}=Cf_\pi{}^2\left[1-\frac{2m_\nu}{m_\tau}\right]m_\tau{}^3 ,$$

$$C_{\rm SD}=C\frac{m_\tau{}^7}{4}|F^V|^2 ,$$

$$C_{\rm INT}=C\frac{m_\tau{}^3}{2}\left[\frac{|F^V|}{f\pi}\right]\left[1-\left[\frac{m_\nu}{m_\tau}\right]\right]$$

with

$$C=\frac{G^2e^2\cos^2\theta}{128\pi^3} .$$

The vector form factor $F^V(0)$ has been obtained from the $\pi^0\to\gamma\gamma$ decay by replacing the photons with a lepton pair.[5] From the conserved-vector-current hypothesis,[6] one can relate the isovector part of the electromagnetic current and the strangeness-conserving weak vector hadronic current. So the vector form factor $F^V(0)$ is related to the $\pi^0\to\gamma\gamma$ decay amplitude d through the relation[5]

$$F^V(0)=-\frac{1}{\sqrt{2}}d$$

and so

$$|F^V(0)|=\frac{1}{\alpha}\left[\frac{2\Gamma(\pi^0\to\gamma\gamma)}{\pi m_{\pi^0}}\right]^{1/2} .$$

Using the 1980 value for the π^0 lifetime ($+0.828\times10^{-16}$ sec) (Ref. 7) the value of $|F^V(0)|=0.0265$. The above equation does not say anything about the sign of $F^V(0)$ and so it is undetermined. Likewise, the ratio of the axial-vector and vector form factors $\gamma=F^A/F^V$ (Ref. 8) is also undetermined in sign. The value of $f_\pi=0.945$ (Ref. 9). Unlike the case of the radiative decay process $\pi\to l\nu_l\gamma$, the maximum value of t can be large compared to the square of the ρ-meson mass, which presumably controls the variation of F^V given by

$$F^V\propto\left[\frac{m_\rho{}^2}{m_\rho{}^2-t}\right]$$

in the simple pole model. So the approximation $t=0$ is not strictly valid. However, since the precise dependence of the form factors on t is not known, an average value of $|F^V(t)|$ is assumed (~0.0265).

The partially integrated decay rates for the $\mathrm{SD}^\pm$ and $\mathrm{INT}^\pm$ terms are suitable for the study of the structure-dependent parameter γ. The photon energy spectra of the SD terms are (assuming a cut of $\eta=\eta_0$)

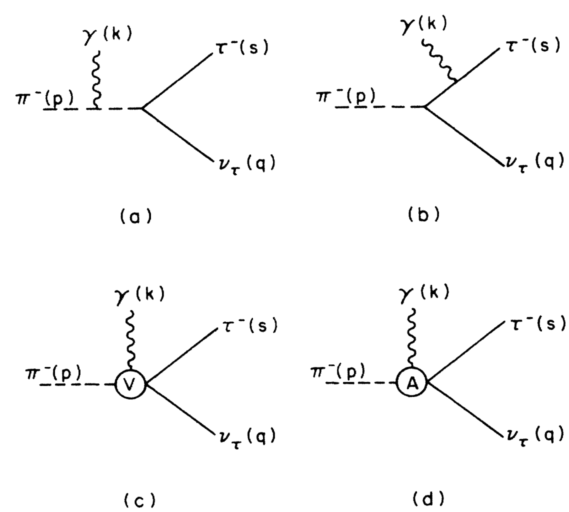

FIG. 1. Feynman diagrams for the process $\tau\to\pi\nu_\tau\gamma$. (a) and (b) represent the contributions to the inner bremsstrahlung. (c) and (d) represent the vector (V) and axial-vector (A) contributions, respectively.

$$\frac{d\Gamma_{SD+}}{d\zeta}\simeq R_{\rm SD}(1+\gamma)^2\left[2\zeta(1-\zeta)^2(\eta_0-1)+\zeta(1-\eta_0{}^2)(1-\zeta-\tfrac{1}{2})+\frac{\zeta}{3}(1-\eta_0{}^3)\right],$$

$$\frac{d\Gamma_{\rm SD-}}{d_\zeta}\simeq R_{\rm SD}(1-\gamma)^2\left[(\zeta-2)\left[\frac{1-\eta_0{}^2}{2}\right]+(5-2\zeta)\left[\frac{1-\eta_0{}^3}{3}\right]-\tfrac{1}{2}(1-\eta_0{}^4)\right].$$

The photon energy spectra for the interference terms are

$$\frac{d\Gamma_{\rm INT+}}{d\zeta}=R_{\rm INT}(1+\gamma)\left[\frac{1-\eta_0{}^2}{\zeta}-\frac{2}{3}\frac{1-\eta_0{}^3}{\zeta}\right]$$

and

$$\frac{d\Gamma_{\rm INT-}}{d\zeta}=R_{\rm INT}(1-\gamma)\left[\frac{2(\zeta-1)}{\zeta}(1-\eta_0)-\frac{\zeta-1}{\zeta}(1-\eta_0{}^2)\right].$$

Here $\zeta\geq(1-\eta_0)$. Likewise, the pion energy spectra for the different contributions are

$$\frac{d\Gamma_{\rm SD+}}{d\eta}=R_{\rm SD}(1+\gamma)^2\left[\tfrac{2}{3}(2-\eta)\eta(3-3\eta+\eta^2)-\eta(2-\eta)(1-\eta)\left[\frac{\eta}{2}+1\right]-2\eta+3\eta^2-2\eta^3+\frac{\eta^4}{2}\right],$$

$$\frac{d\Gamma_{\rm SD-}}{d\eta}=R_{\rm SD}(1-\gamma)^2\left[\eta^3(1-\eta)-\frac{\eta^2}{2}(1-\eta)(2-\eta)\right],$$

$$\frac{d\Gamma_{\rm INT+}}{d\eta}=2R_{\rm INT}(1+\gamma)\eta(1-\eta)\ln\left[\frac{1}{1-\eta}\right],$$

and

$$\frac{d\Gamma_{\rm INT-}}{d\eta}=2R_{\rm INT}(1-\gamma)\left[\eta(1-\eta)-(1-\eta)\ln\left[\frac{1}{1-\eta}\right]\right]$$

with $\eta\geq(1-\zeta_{\rm min})$ and

$$\frac{2\mu}{m_\tau}\leq\eta\leq1+\frac{\mu^2}{m_\tau{}^2},$$

$$\left[1-\frac{S}{2}-\frac{\eta}{2}\right]\leq\zeta\leq\left[1+\frac{S}{2}-\frac{\eta}{2}\right]$$

with

$$S=(\eta^2-4r)^{1/2}.$$

It may be mentioned here that there are some discrepancies between Refs. 3 and 10 regarding the expression for the differential decay rate for the accompanying radiation pion decay process $\pi\rightarrow l\nu_l\gamma$. In Ref. 3 a misprint was noted in the IB term of Ref. 10 but unfortunately, Ref. 3 itself incorrectly reported the SD contribution. The discrepancies are clarified and the new expression is quoted below:

$$\frac{d^2\Gamma}{dx\,dy}=C(A_{\rm SD}\{(|F^V|^2+|F^A|^2)a+2[\cos(\psi_A-\psi_v)|F^A|\,|F^V|]b\}+A_{\rm IB}c+A_{\rm INT}(\cos\psi_V|F^V|d+\cos\psi_A|F^A|e)),$$

where now

$$a=x(2-x-y)(x+y-1-r)+xy(1-y+r)$$
$$-2(1-y+r)(x+y-1-r),$$

$$b=x(1-x)\{[(1-y+r)+(x+y-1)]-\alpha\},$$

$$c=\frac{(1-y+r)}{x(x+y-1-r)}\times\left[x+2(1-r)(1-x)-\frac{2rx(1-r)}{(x+y-1-r)}\right],$$

$$d=\frac{x(1-y+r)}{(x+y-1-r)},$$

$$e=\frac{(1-y+r)}{x(x+y-1-r)}[2(1-x)(1-x-y)+2r-x],$$

$$F^V=e^{i\psi_V}|F^V|,\quad F^A=e^{i\psi_A}|F^A|,$$

$$C=\frac{\mu G^2e^2\cos^2\theta}{64\pi^3},\quad r=\frac{m_l{}^2}{\mu^2},$$

$$A_{\rm SD}=\mu^6/4,\quad A=m^2f_\pi{}^2,\quad A_{\rm INT}=m^2f_\pi\mu^2$$

with x=photon energy; y=lepton energy (both normalized to $\mu/2$, μ being the pion mass, and m being the lepton mass). The expressions for the SD terms differ from Ref. 3 in that the coefficients of b and a are interchanged and there is a disagreement in sign of a term in the expression for b.

A useful way of checking the validity of the SD terms of radiative pion decay is to see whether these terms have the physically required symmetry properties between the lepton and neutrino energy $z=2E_\nu/\mu=(2-x-y)$ (Ref. 11). In the electron mode, the term r can be neglected, and so the SD terms a and b become

$$a=xz(1-z)+xy(1-y)-2(1-y)(1-z)\ ,$$

$$b=x(1-x)[(1-y)+(1-z)]$$

which are symmetric in y and z.

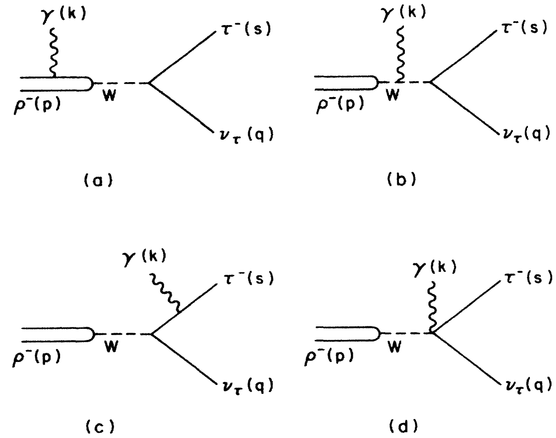

FIG. 2. Feynman diagrams for the process $\tau\to\rho\nu_\tau\gamma$. (a)—(c) are the contributions from the IB terms, (d) is the direct term.

III. THE DECAY $\tau\to\rho\nu_\tau\gamma$

The radiative decay process $\tau\to\rho\nu_\tau\gamma$ is represented by the four diagrams of Fig. 2. The first three diagrams constitute the IB contribution while the last term gives the structure-dependent contribution. The momentum of the ρ meson is very small compared to the W-boson mass. The matrix element for the process

$$\tau^-(s)\to\rho^-(p)+\nu_\tau(q)+\gamma(k)$$

is given by

$$\frac{Geg_\rho}{\sqrt{2}}\left[m_\rho^2\cos\theta_V\epsilon_{\mu\nu\rho\sigma}J_l^\mu\epsilon^\nu k^\rho\phi^\sigma -i\bar{u}_\nu(q)(1+\gamma_5)\left\{\cos\theta_V\left[1-\frac{m_\nu}{m_l}\right]\right\}\left[\left[\frac{p\cdot\epsilon}{p\cdot k}-\frac{s\cdot\epsilon}{s\cdot k}\right]-\frac{(\not\epsilon)(\not k)}{2s\cdot k}+\frac{(\phi\cdot\epsilon)(\not p)}{2p\cdot k}-\frac{(\phi\cdot\epsilon)(\not p)}{m_W^2}-\frac{(p\cdot\epsilon)}{m_W^2}\right]\right\}v_l(s)\right]$$

with g_ρ being a scalar parameter of dimension M^2 and ϕ^σ being the polarization vector of the ρ meson.

The differential decay rate of the process $\tau\to\rho\nu_\tau\gamma$ is given by

$$\frac{d^2\Gamma}{dX\,dY}=\frac{\alpha}{2\pi}\frac{\Gamma_{\rho\nu_\tau}}{(1-r)^2(1+2r)}\left[H_{\rm IB}(X,Y)+\frac{1}{4(1-2m_\nu/m_\tau)}H_{\rm SD}(X,Y)+\frac{(1-m_\nu/m_\tau)}{(1-2m_\nu/m_\tau)}H_{\rm INT}(X,Y)\right].$$

Here, X is the photon energy and Y is the ρ-meson energy (normalized to $m_\tau/2$) and

$$\text{direct term}=H_{\rm SD}(X,Y)=\frac{(X-A)(YA+XB)}{2}+rXA\ ,$$

$$\begin{aligned}H_{\rm IB}(X,Y)=&\frac{r}{X^2(X-A)}\{6XYZ+[2XA-4(Z+A-ZX)](X-A)+4BX-6BX^2\}\\&+\frac{1}{X^2(X-A)}[2X^2YB+2(AY-2B)(X-A)^2+BY(4-3Y)(X-A)]\\&+4r\left[XYZ+2BX+2XZ(X-A)-X(AY+BX)+\frac{rX^2(YB-ZR)}{(X-A)}\right],\end{aligned}$$

$$H_{\rm INT}(X,Y)=r\left[\frac{2X(BX-AY)+(X-A)(2A-ZX)}{(X-A)X}\right]+\frac{1}{2X}[(X-A)(2B-YZ)-Y(BX-AY)]$$

and $Z=(2-X-Y)$. The two-body decay rate is given by[12]

$$\Gamma_{\tau\to\rho\nu}=\left[1-\frac{2m_\nu}{m_\tau}\right]\frac{G^2m_\rho^2}{16\pi}\left[\frac{g_\rho^2}{m_\rho}\right]^2\times m_\tau^3\cos^2\theta(1-r)^2(1+2r)\ ,$$

where now

$$r=m_\rho^2/(m_\tau^2)\ .$$

IV. DISCUSSION OF RESULTS

In this section we briefly review the results of the calculations presented in the previous section.

TABLE I. Total widths (energy units dimensionless).

Process	Inner bremsstrahlung	Structure dependent	Interference
$\tau \to \pi \nu_\tau \gamma$	1.721×10^{-5}	$(1+\gamma)^2 \times 1.886 \times 10^{-9} + (1-\gamma)^2 \times 1.078 \times 10^{-9}$	$(1+\gamma) \times 5.302 \times 10^{-5} + (1-\gamma) \times 8.343 \times 10^{-6}$
	Inner bremsstrahlung	Direct	Interference
$\tau \to \rho \nu_\tau \gamma$	1.163×10^{-4}	1.205×10^{-5}	1.176×10^{-4}

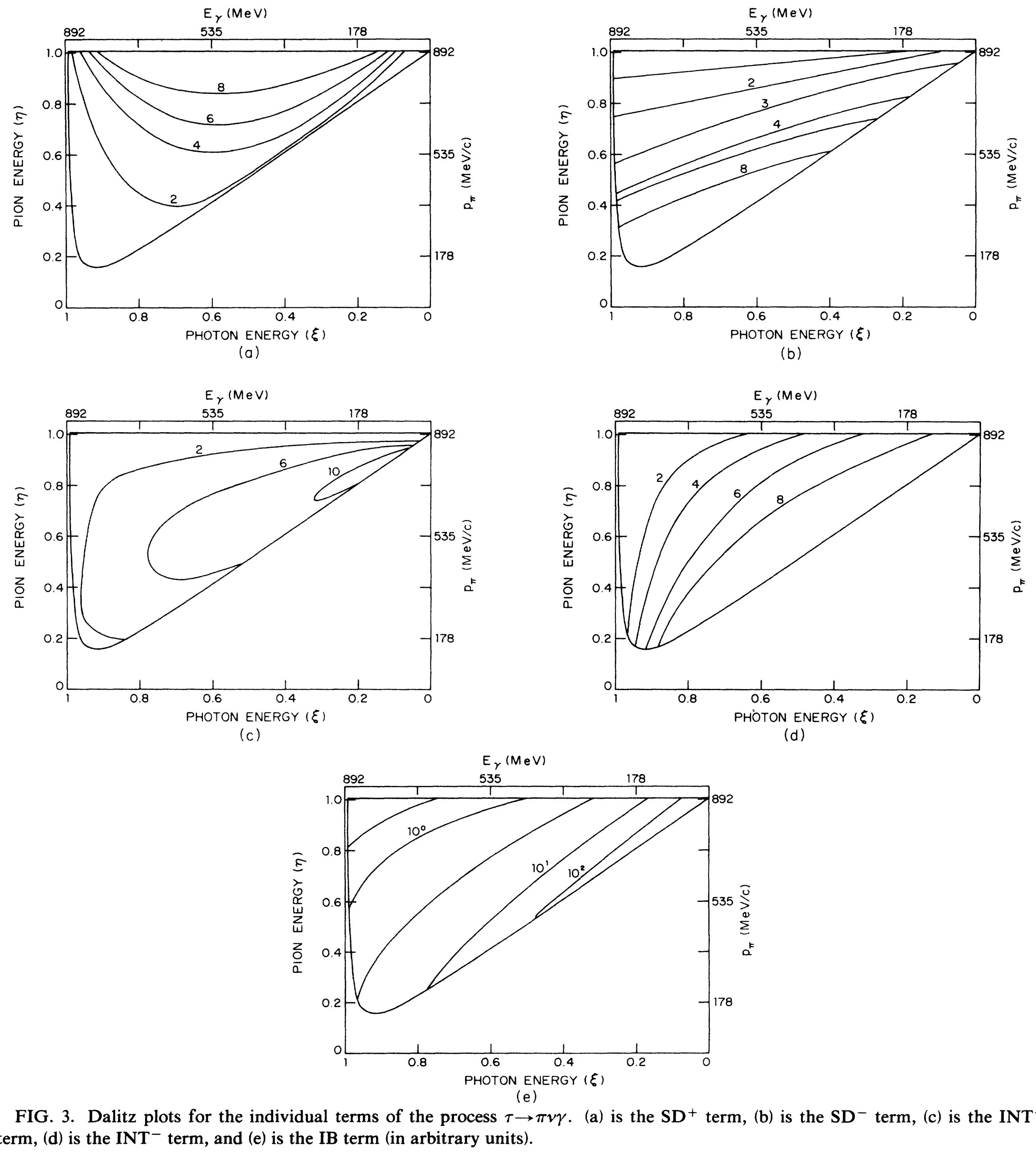

FIG. 3. Dalitz plots for the individual terms of the process $\tau \to \pi \nu \gamma$. (a) is the SD^+ term, (b) is the SD^- term, (c) is the INT^+ term, (d) is the INT^- term, and (e) is the IB term (in arbitrary units).

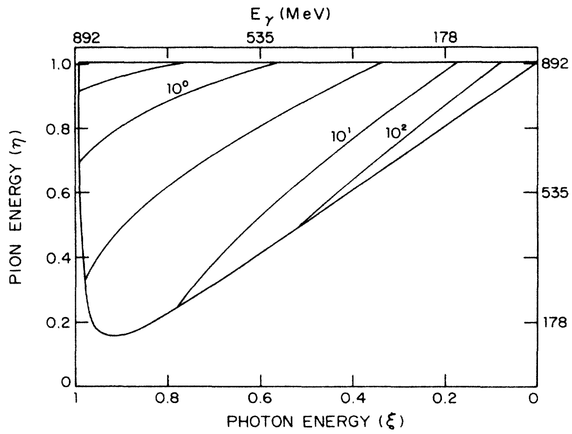

FIG. 4. Dalitz plot for total contribution in the process $\tau \to \pi\nu_\tau\gamma$ (in arbitrary units).

For the decay process $\tau \to \pi\nu_\tau\gamma$, the total rates of the individual contributions for photon and pion energies greater than 669 MeV ($\zeta_0 = \eta_0 = 0.75$) are given in Table I. The values of the multiplicative constants are

$$R_{\rm IB} = \frac{\alpha}{2\pi(1-r)^2}\Gamma_{\pi\nu_\tau} = 1.266 \times 10^{-4} ,$$

$$R_{\rm SD} = \frac{R_{\rm IB}}{4(1-2m_\nu/m_\tau)}\left[\frac{F^V}{f\pi}\right]^2 = 2.489 \times 10^{-8} ,$$

$$R_{\rm INT} = \frac{R_{\rm IB}(1-m_\nu/m_\tau)}{(1-2m_\nu/m_\tau)}\left[\frac{F^V}{f\pi}\right] = 3.550 \times 10^{-7}$$

with $\Gamma_{\pi\nu_\tau} = 10.9\%$ (Ref. 2).

The IB, SD$^\pm$, and INT$^\pm$ terms have very distinctive energy distributions. A comparison between Figs. 3(e) and 4 reveals that the IB contribution is significant throughout most of the kinematic region. Following well-known arguments by Low,[13] the inner bremsstrahlung contribution, which depends only on the nonradiative decay amplitude, exactly yields terms of order ζ^{-1} and ζ^0 and dominates at low photon energies.

The SD amplitudes, which are quadratic in γ and the INT amplitudes are relevant for the study of the structure-dependent parameter γ (Ref. 14). Because of their linear dependence on γ, the INT terms are useful for determining the sign of γ. The kinematic regions suitable for the study of the individual terms are as follows (Figs. 3 and 4).

IB: $540 <$ pion momentum $(p) < 800$ MeV/c.
SD: $625 <$ pion momentum < 800 MeV/c
500 MeV $<$ photon energy (E_γ).
SD: $170 <$ pion momentum < 530 MeV/c.
INT: $350 <$ pion momentum < 700 MeV/c.
INT: $170 <$ pion momentum < 500 MeV/c.

The photon and pion energy spectra for the individual contributions in the appropriate energy and momentum ranges are shown in Figs. 5 and 6.

Likewise, the total decay widths for the process $\tau \to \rho\nu_\tau\gamma$ are given in Table I for ρ-meson momenta greater than 892 MeV/c and photon energies greater than 535 MeV ($X_0 = 0.6$, $Y_0 = 1.10$). Here,

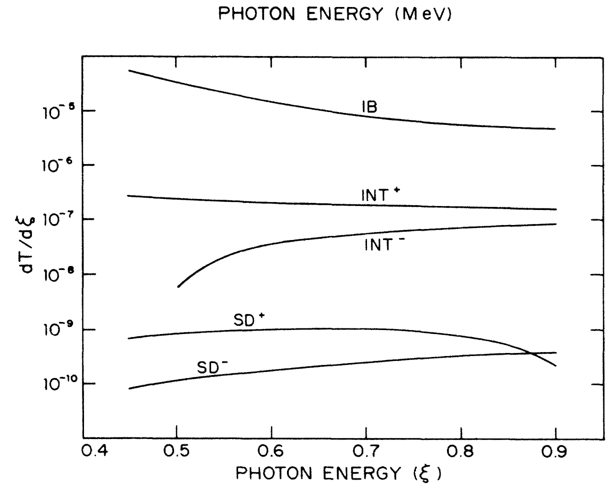

FIG. 5. Photon energy spectrum for $\tau \to \pi\nu_\tau\gamma$ in the photon energy range 350–900 MeV.

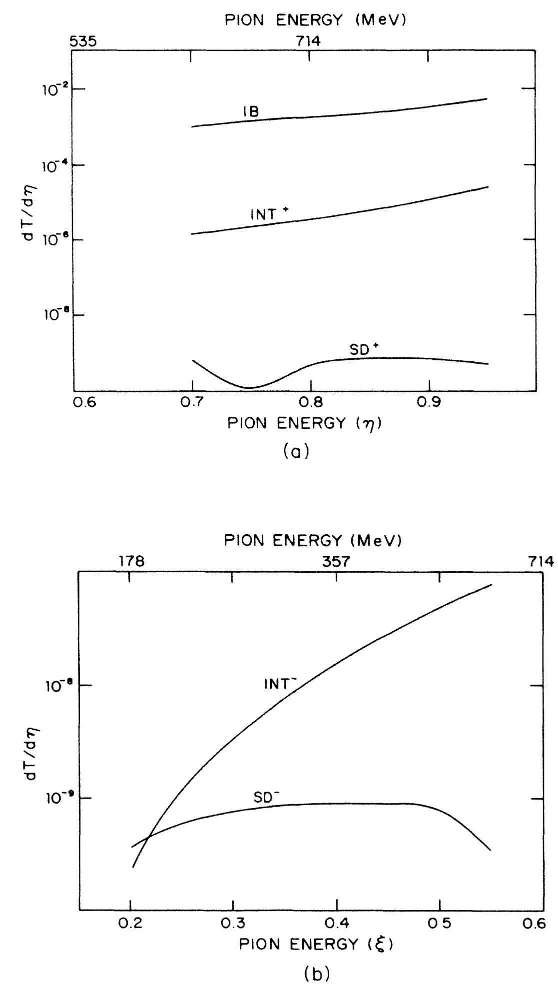

FIG. 6. Pion energy spectra for $\tau \to \pi\nu_\tau\gamma$. (a) Contributions for IB, INT$^+$, and SD$^+$ in the pion energy range 535–800 MeV/c. (b) Contributions from INT$^-$ and SD$^-$ in the pion momentum range 178–700 MeV/c.

$$(R_{\mathrm{IB}})_\rho = \frac{\alpha}{2\pi} \frac{\Gamma_{\rho\nu_\tau}}{(1-r)^2(1+2r)} = 2.555\times 10^{-4} ,$$

$$(R_{\mathrm{SD}})_\rho = \frac{(R_{\mathrm{IB}})_\rho}{4(1-2m_\nu/m_\tau)} = 7.596\times 10^{-5} ,$$

$$(R_{\mathrm{INT}})_\rho = (R_{\mathrm{IB}})_\rho \frac{(1-m_\nu/m_\tau)}{(1-2m_\nu/m_\tau)} = 2.797\times 10^{-4} .$$

The decay width $\Gamma_{\rho\nu_\tau}=22\%$ (Ref. 2). The Dalitz plots

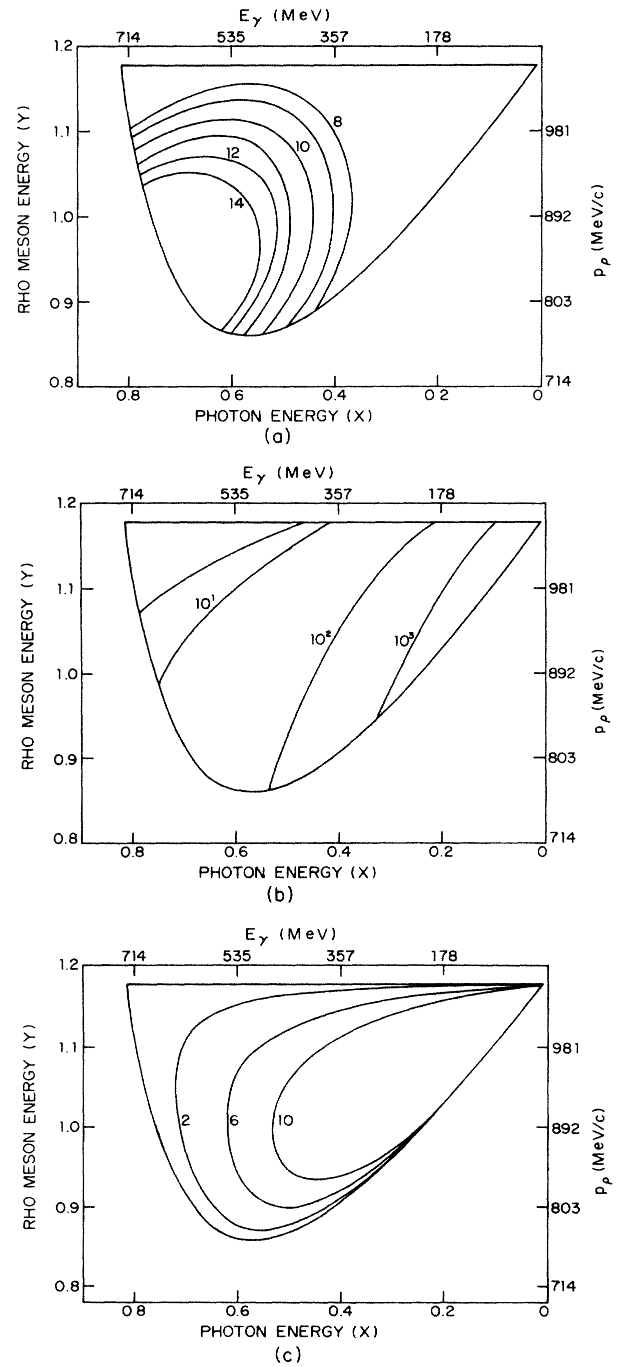

FIG. 7. Dalitz plots of the individual contributions for $\tau \to \rho\nu_\tau\gamma$ (a) is the direct term, (b) is the IB term, (c) is the INT term (in arbitrary units).

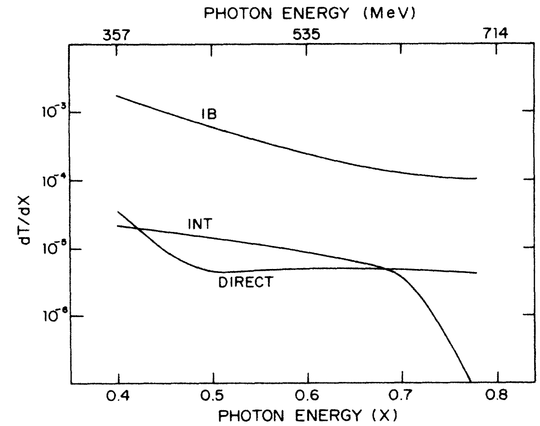

FIG. 8. Photon energy spectrum for $\tau \to \rho\nu_\tau\gamma$ in the photon energy range 350–700 MeV.

are shown in Fig. 7.

The photon and pion energy spectra are shown in Figs. 8 and 9.

V. REMARKS ON THE NEUTRINO MASS

Particle decays into three bodies including a massive neutrino are more suitable than the two-body case from a kinematic point of view. This is due to the fact that the neutrino can be produced with an arbitrarily small velocity. That is, the neutrino energy becomes a function of its mass rather than the square of the mass. Hence, measurable effects linear in the neutrino mass can be expected.

The maximal photon energy in these three-body decays is

$$\xi_{\max} = 1 - (\mu/m_1 + m_\nu/m_1)^2 .$$

The corresponding pion energy is

$$\eta_{\xi_{\max}} = \frac{1}{1+m_\tau/m_\nu}[1+(\mu/m_\tau+m_\nu/m_\tau)^2] .$$

The shift in the end-point energy of the photon depends

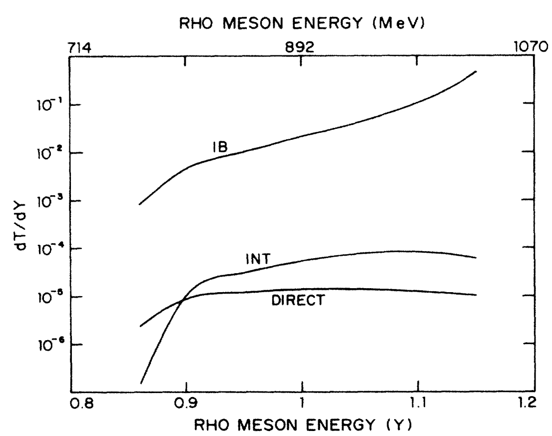

FIG. 9. ρ-meson energy spectrum in the ρ-momentum range 714–1070 MeV/c.

linearly on m:

$$(\Delta k) = k_{0_{\max}}(m \neq 0) - k_{0_{\max}}(m = 0)$$
$$= m_\nu(\mu/m_\tau)(1 + m_\nu/2\mu) .$$

For a τ-neutrino mass of 142 MeV/c^2 (Ref. 15), this corresponds to an energy shift of 16.73 MeV. A mass determination by establishing the exact boundary of the Dalitz plot from a measurement of the differential rates is possible although a determination of the partially integrated rates over the selected regions of the domain might also be used to determine the neutrino mass.

In conclusion, it may be remarked that a study of the radiative processes $\tau \to \pi\nu_\tau\gamma$ and $\tau \to \rho\nu_\tau\gamma$ not only consolidates the evidence for τ being a sequential lepton, but also provides a means of determining the upper limit of the τ-neutrino mass and the sign and magnitude of the parameter γ (the ratio of the pion form factors).

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