Pion as a $q\bar{q}$ soliton bag: Dressing of the nucleon and Δ

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(Received 3 March 1986)

The pion cloud surrounding the nucleon and the Δ is studied in the soliton bag model. The quark-antiquark substructure of the pion is fully taken into account using generator-coordinate techniques. The one-gluon-exchange piece of the model Hamiltonian is responsible for creating the $q\bar{q}$ pair. The results we obtain for various hadronic properties are in good agreement with experimental data and qualitatively similar to those obtained in the cloudy bag model.

I. INTRODUCTION

The soliton bag model, introduced by Huang and Stump[1] and by Friedberg and Lee,[2,3] is a phenomenological attempt to bridge the gap between the fundamental quantum chromodynamics (QCD) theory of strong interactions and the description of observed hadronic properties at low energy in terms of quark and gluon degrees of freedom. This is achieved by assuming that the complicated nonperturbative features of QCD which yield color confinement can be approximated by a colorless, flavorless self-interacting scalar field: the soliton field $\sigma(x)$. The model Lagrangian density is written as

$$\mathscr{L}(x)=\mathscr{L}_q+\mathscr{L}_\sigma+\mathscr{L}_{q\sigma}+\mathscr{L}_G \ , \tag{1.1}$$

where

$$\mathscr{L}_q(x)=\overline{\Psi}(x)\not{p}\Psi(x) \tag{1.2}$$

is the free quark Lagrangian (Ψ is the quark-field operator);

$$\mathscr{L}_\sigma(x)=\tfrac{1}{2}(\partial_\mu\sigma)^2-U(\sigma) \tag{1.3}$$

is the self-interacting soliton field Lagrangian;

$$\mathscr{L}_{q\sigma}(x)=-g\overline{\Psi}(x)\sigma(x)\Psi(x) \tag{1.4}$$

represents the linear coupling between quark and soliton fields; and

$$\mathscr{L}_G=-\tfrac{1}{4}\kappa(\sigma)F^{\mu\nu l}F^l_{\mu\nu}-g_s\overline{\Psi}\gamma^\mu\frac{\lambda^l}{2}V^l_\mu\Psi \tag{1.5}$$

represents the gluons self-interactions and their coupling to the soliton field via the color-dielectric function $\kappa(\sigma)$ [constructed such that $\kappa(0)=1$ and $\kappa(\sigma_v)=0$] and to the quark field. In our notation, $V^l_\mu(x)$ is the gluon-field operator,

$$F^l_{\mu\nu}(x)=\partial_\mu V^l_\nu-\partial_\nu V^l_\mu+g_s f^{lmn}V^m_\mu V^n_\nu \ , \tag{1.6}$$

λ^l are the Gell-Mann matrices, generators of the color-SU(3) group, and f^{lmn} its structure constants. The QCD strong coupling constant g_s and the phenomenological quark-soliton coupling constant g are not directly related. The potential $U(\sigma)$ is taken as a polynomial in σ (limited to fourth order to ensure renormalizability):

$$U(\sigma)=\frac{a}{2}\sigma^2+\frac{b}{3!}\sigma^3+\frac{c}{4!}\sigma^4+p \ . \tag{1.7}$$

The model as presented here contains five parameters: a, b, c, g, and g_s [p is not a true parameter since it only contributes an overall constant to the Lagrangian density, and is determined by requiring $U(\sigma_v)=0$]. It has been extensively studied by the Seattle Nuclear Theory Group.[4–12] The general philosophy adopted in solving the Lagrangian density (1.1) has been to consider first only the quark and soliton pieces $\mathscr{L}_q$, $\mathscr{L}_\sigma$, and $\mathscr{L}_{q\sigma}$, neglecting the gluons, and subsequently to incorporate the gluons perturbatively. At every level of approximation, e.g., as one includes more gluons, the phenomenological parameters of the soliton bag model must be readjusted, and one expects the soliton field to play less and less of a role. In fact, in the limit where the gluons are treated exactly, the soliton field should decouple completely from the problem [that is, we expect $g\to 0$; $\kappa(\sigma)\to 1$]. In that ideal limit, the Lagrangian density (1.1) gives the exact QCD Lagrangian density:

$$\mathscr{L}_{\rm QCD}=-\tfrac{1}{4}F^{\mu\nu l}F^l_{\mu\nu}+i\overline{\Psi}\gamma^\mu\partial_\mu\Psi-g_s\overline{\Psi}\gamma^\mu\frac{\lambda^l}{2}V^l_\mu\Psi \ . \tag{1.8}$$

Considering the simplest version of the model for now ($\mathscr{L}_G$ neglected), one obtains the Hamiltonian operator in the following canonical way:

$$H_0=\int d^3r[\Psi^\dagger\boldsymbol{\alpha}\cdot\mathbf{p}\Psi+\tfrac{1}{2}(\pi^2+|\nabla\sigma|^2)+U(\sigma)+g\overline{\Psi}\sigma\Psi] \ , \tag{1.9}$$

where π is the momentum canonically conjugate to σ,

$$\pi=\frac{\partial\sigma}{\partial t} \ . \tag{1.10}$$

The fields obey the usual equal-time commutation and anticommutation relations

$$[\sigma(\mathbf{r},t),\pi(\mathbf{r}',t)]=i\delta^3(\mathbf{r}-\mathbf{r}') \ , \tag{1.11}$$

$$\{\Psi(\mathbf{r},t),\Psi^\dagger(\mathbf{r}',t)\}=\delta^3(\mathbf{r}-\mathbf{r}') \ . \tag{1.12}$$

Goldflam and Wilets[4] first extensively studied the Hamiltonian H_0 in the mean-field approximation (MFA), i.e., neglecting all quantum fluctuations of the soliton field. Using the Schrödinger picture, the quantum operator $\sigma(\mathbf{r})$ is replaced by a static c-number field $\sigma_0(\mathbf{r})$, while $\pi_0(\mathbf{r})$ vanishes identically. We expand the quark-field operator $\Psi(x)$ in a complete set of spinors,

$$\Psi(x)=\sum_k b_k\psi_k(\mathbf{r}) , \tag{1.13}$$

where $b_k^\dagger$ and b_k have the usual meaning of quark creation and annihilation operators. Neglecting vacuum-polarization effects one finds the following coupled equations of motion:

$$[\boldsymbol{\alpha}\cdot\mathbf{p}+g\beta\sigma_0(\mathbf{r})]\psi_k(\mathbf{r})=\epsilon_k\psi_k(\mathbf{r}) , \tag{1.14}$$

$$-\nabla^2\sigma_0+\frac{dU(\sigma_0)}{d\sigma_0}+g\sum_{k(\mathrm{val})}\bar{\psi}_k\psi_k=0 . \tag{1.15}$$

If the soliton potential $U(\sigma)$ has two minima (a local minimum at $\sigma=0$ and an absolute minimum for some large positive value $\sigma=\sigma_v$), as shown in Fig. 1, these mean-field equations admit baglike solutions: the radially symmetric soliton field is small and negative inside a cavity, and large, approaching σ_v outside the cavity with a smooth surface transition. As can be seen from the Dirac equation (1.14), to this configuration of $\sigma(\mathbf{r})$ there corresponds a whole spectrum of quark eigenstates, symmetric with respect to zero energy (positive- and negative-energy states). The lowest positive quark state (valence state) is the $1s_{1/2}$ state given by

$$\psi_{s,m}(\mathbf{r})=\begin{bmatrix} u(r) \\ iv(r)\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} \end{bmatrix}\chi_{s,m} \tag{1.16}$$

and corresponding to the eigenenergy ϵ_s. Here $\chi_{s,m}$ is the 12-component spinor carrying the spin-(2), flavor-(2), and color-(3) projection quantum numbers of the quark [we will limit the present discussion to two flavors (up and down) of quarks only]. Low-lying baryons and mesons can be constructed from these single-particle quark states (three quarks in the $1s_{1/2}$ state for baryons, one quark in the $1s_{1/2}$ state, and one hole in the $1\bar{s}_{1/2}$ negative-energy state for mesons) by coupling them to the correct total

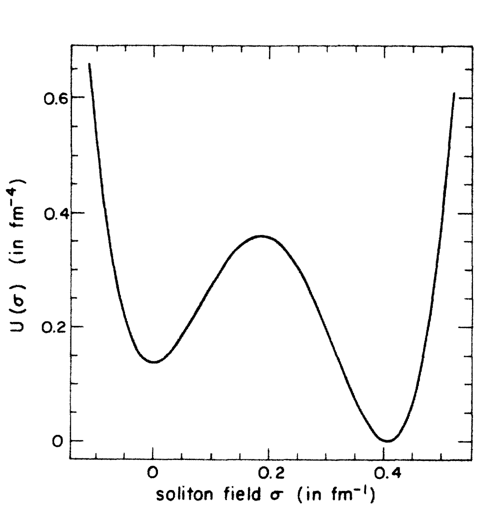

FIG. 1. The soliton field self-interacting potential $U(\sigma)$ for the following parameters: $a=51.6$, $b=-799.9$, and $c=4000$.

spin, total isospin, color-singlet state in the standard way. Static physical properties of low-lying hadrons can then be calculated and give reasonably good agreement with experiment.[4] The results obtained are shown to be qualitatively similar to those of the MIT bag model.[13]

This level of approximation of the soliton bag model (as well as of the MIT bag model) presents, however, two important and related shortcomings. First, it is in direct contradiction with some well-established experimental results of nucleon-nucleon scattering. Those data clearly show that at medium and long separation distances two nucleons interact by exchanging pions. A single nucleon should also be able to exchange pions with itself, or be dressed by a pion cloud. Neither the MIT nor the soliton bag in the MFA form allow for such pionic dressing. Furthermore, the model we have just presented grossly violates what is considered to be an important symmetry of nature: chiral symmetry. That symmetry is satisfied in QCD in the limit of massless quarks, but by introducing the soliton field $\sigma(x)$ to achieve confinement, we have given the quarks an effective mass $g\sigma_0(\mathbf{r})$, and by doing so we have violated chiral symmetry in the surface region of the cavity.

These two shortcomings (absence of a pion cloud and violation of chiral symmetry) are related through the Goldstone theorem: chiral symmetry is believed to be a hidden symmetry associated with the existence of a massless particle (the Goldstone boson). The abnormally low mass of the pion ($m_\pi=140$ MeV) compared to other hadron masses makes it a perfect candidate for that Goldstone boson. The fact that the pion is not absolutely massless indicates that chiral symmetry is only partially realized in nature. The corresponding conserved vector current, the axial-vector current in this case, is only partially conserved (PCAC).

These considerations have led a number of people to modify the traditional bag models in order to restore approximate chiral symmetry to those models.[14–18] All these approaches have in common the introduction of a new degree of freedom into the original Lagrangian: an elementary pion field. PCAC is then achieved in two steps: first, the massless pointlike pion is coupled to the quarks at the bag surface in just the right way so as to restore exact chiral invariance to the model Lagrangian; then that symmetry is broken by giving the pion its physical mass.

The approach adopted in this paper, however, treats the pion in a completely different way. Instead of modifying the original model Lagrangian, we investigate whether approximate chiral symmetry is restored dynamically. We believe that the soliton bag model should be able to adequately describe both baryons and mesons in terms of their quark substructure. Therefore, we treat both the nucleon and the pion surrounding it as bags containing quarks and antiquarks.

As we describe in the next section, the generator-coordinate method is used to handle the large amplitude dynamics of these two interacting bags. There we also show how the one-gluon-exchange (OGE) interaction can produce the quark-antiquark pair forming the pion. In Sec. III we give the formal solutions of the generator-coordinate equations with the correct treatment of total angular momentum, isospin, and color quantum numbers. The numerical method used to solve the radial part of these equations is discussed as well. Finally, Sec. IV contains a summary of the hadronic properties that can be calculated in this model and how well they compare to experiment.

II. PIONIC DRESSING OF N AND Δ

In this calculation, we choose to treat the gluon piece of the soliton bag model (1.5) at the level of one-gluon-exchange approximation. This choice corresponds to the following model Hamiltonian:

$$H=H_0+H' , \tag{2.1}$$

where H_0 is given by (1.9) and H' is the one-gluon-exchange interaction

$$H'=\tfrac{1}{2}\alpha_s \int d^3r \int d^3r' J_c^{\mu}(\mathbf{r})G_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)J_c^{\nu}(\mathbf{r}') . \tag{2.2}$$

Here $J_c^{\mu}(\mathbf{r})$ represents the quark-color current operator (color index c):

$$J_c^{\mu}(\mathbf{r})=\overline{\Psi}(\mathbf{r})\gamma^{\mu}\frac{\lambda^c}{2}\Psi(\mathbf{r}) , \tag{2.3}$$

α_s is the QCD strong coupling constant

$$\alpha_s=\frac{g_s{}^2}{4\pi} , \tag{2.4}$$

and $G_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)$ is the gluon propagator. In principle, it should be a confined propagator, calculated, for example, as a functional of the soliton field σ through the dielectric function $\kappa(\sigma)$. Such a propagator has been obtained by Bickeböller, Goldflam, and Wilets[7] and has been shown[6] to give an enhancement of the N-Δ mass splitting over the results obtained with a free propagator by a factor of about 2. However, in order to keep this calculation as simple as possible, we have opted for the use of a free gluon propagator, at least as a first step, hoping that most of the enhancement due to confining the gluons can be absorbed in the strong coupling constant α_s (which will be treated as a parameter in this approach).

Following Ref. 7 we write the free propagator $G^0_{\mu\nu}(\mathbf{r},\mathbf{r}',\omega)$ in the Coulomb (or transverse) gauge in terms of scalar and vector spherical harmonics:

$$G^0_{\mu\nu}=\begin{pmatrix} G & 0 & 0 & 0\\ 0 & & & \\ 0 & & G^{ii'} & \\ 0 & & & \end{pmatrix} , \tag{2.5}$$

where G and $G^{ii'}$ are given by

$$G(\mathbf{r},\mathbf{r}')=\frac{1}{|\mathbf{r}-\mathbf{r}'|}=\sum_l \frac{4\pi}{2l+1}\frac{r_<^l}{r_>^{l+1}}Y_{lm}(\hat{\mathbf{r}})Y^*_{lm}(\hat{\mathbf{r}}') , \tag{2.6}$$

$$\begin{aligned} G^{ii'}(\mathbf{r},\mathbf{r}';\omega)=4\pi\sum_l \Bigg[&-\omega j_l(\omega r_<)n_l(\omega r_>)[\mathcal{Y}_{llm}(\hat{\mathbf{r}}_<)]^{i_<}[\mathcal{Y}^*_{llm}(\hat{\mathbf{r}}_>)]^{i_>} \\ &-\frac{1}{\omega}[\nabla\times j_l(\omega r_<)\mathcal{Y}_{llm}(\hat{\mathbf{r}}_<)]^{i_<}[\nabla\times n_l(\omega r_>)\mathcal{Y}^*_{llm}(\hat{\mathbf{r}}_>)]^{i_>} \\ &-\frac{1}{\omega^2(2l+1)}[\nabla\times r_<^l\mathcal{Y}_{llm}(\hat{\mathbf{r}}_<)]^{i_<}\left[\nabla\times\frac{1}{r_>^{l+1}}\mathcal{Y}^*_{llm}(\hat{\mathbf{r}}_>)\right]^{i_>}\Bigg] . \end{aligned} \tag{2.7}$$

An even further simplification adopted here is to consider the static ($\omega=0$ limit) of the tensor propagator:

$$\begin{aligned} G^{ii'}(\mathbf{r},\mathbf{r}')=\sum_l \frac{4\pi}{2l+1}\Bigg[&\frac{r_<^l}{r_>^{l+1}}[\mathcal{Y}_{llm}(\hat{\mathbf{r}}_<)]^{i_<}[\mathcal{Y}^*_{llm}(\hat{\mathbf{r}}_>)]^{i_>}-\frac{1}{2(2l+3)}[\nabla\times r_<^{l+2}\mathcal{Y}_{llm}(\hat{\mathbf{r}}_<)]^{i_<}\left[\nabla\times\frac{1}{r_>^{l+1}}\mathcal{Y}^*_{llm}(\hat{\mathbf{r}}_>)\right]^{i_>} \\ &+\frac{1}{2(2l-1)}[\nabla\times r_<^l\mathcal{Y}_{llm}(\hat{\mathbf{r}}_<)]^{i_<}\left[\nabla\times\frac{1}{r_>^{l-1}}\mathcal{Y}^*_{llm}(\hat{\mathbf{r}}_>)\right]^{i_>}\Bigg] . \end{aligned} \tag{2.8}$$

We relax that approximation below by doing the calculation for different values of ω and show that a static limit makes sense.

In order to find the eigenstates (and especially the ground state) of the total Hamiltonian H, we use a perturbativelike treatment, building upon the MFA solutions of H_0 obtained by Goldflam and Wilets,[4] because they provide a good description of the low-lying hadron properties and because the one-gluon-exchange corrections are small effects (as demonstrated by the smallness of the N-Δ mass splitting compared to the nucleon or Δ mass). However, we also need to allow for large amplitude deformations such as the emission of a pion bag by the nucleon.

The generator-coordinate method (GCM) is well suited for these two purposes. A dynamical state is constructed from a superposition of static states each characterized by a parameter called the generator coordinate and weighted by a function (in general complex) of that parameter. That function is then determined by variation. Let $|B_i\rangle$ be a known approximate baryon eigenstate of H_0 obtained in the MFA. The index i indicates the spin (S, projection s), flavor (T, projection t), and radial excitation (n) quantum numbers:

$$|B_i\rangle = |S_i,s_i,T_i,t_i;n_i\rangle \ . \tag{2.9}$$

Our generator-coordinate ansatz for the dressed ground state of H will be

$$|B_i\rangle^d = C_i|B_i\rangle + \sum_j \int d^3\alpha\, \phi_{ij}(\boldsymbol{\alpha})\,|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle \ , \tag{2.10}$$

where the parameter $\boldsymbol{\alpha}$ is the generator coordinate and the state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ represents a baryon with quantum numbers j accompanied by a pion $|\underline{\pi}\rangle$ at a "distance" $\boldsymbol{\alpha}$, the baryon and pion isospins being coupled to give a total isospin of T_i, projection t_i. The weight factors C_i and $\phi_{ij}(\boldsymbol{\alpha})$ have to be determined variationally. The sum over j should be over all possible quantum numbers (spin, flavor, and radial excitation) of the baryon $|B_j\rangle$, but for simplicity we will restrict it to the lowest radial excitation (no sum over n). Furthermore, only two types of baryons, the nucleons ($S=\frac{1}{2}$, $T=\frac{1}{2}$) and the Δ's ($S=\frac{3}{2}$, $T=\frac{3}{2}$) will be considered.

The states $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ should be chosen very carefully and built from the MFA solutions if the model is to make any sense. For example, one feature we require is that for large baryon-pion separation distances ($\alpha\to\infty$), the state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ should go over to two separated bags:

$$|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle \to |B_j\rangle_0 \otimes |\underline{\pi}\rangle_{\boldsymbol{\alpha}} \quad \text{as } \alpha\to\infty \ . \tag{2.11}$$

Recoil and center-of-mass corrections are serious problems affecting all relativistic quark models. The MFA treatment described above is no exception. By "nailing down" the bag at the origin, one finds localized solutions that are *not* eigenstates of the total momentum operator. A thorough treatment of that problem in the framework of the soliton bag model has been obtained by Lübeck, Birse, Henley, and Wilets.[9] It involves variation after projection. We will not attempt to solve the delicate question of center-of-mass corrections in this calculation even though the treatment by Lübeck, Birse, Henley, and Wilets could in principle be applied here. The baryons $|B\rangle$ will be "nailed down" at the origins as in the MFA. Notice, however, that by considering pion states at different distances from the origin in the generator-coordinate state (2.10), we are in effect making center-of-mass corrections for the pion.

The first problem we address when working with the states $|B\rangle$ obtained from the MFA concerns the soliton field part of that state. The MFA is a classical approximation for σ and knowing the soliton mean field $\sigma_0(\mathbf{r})$ alone does not tell us what the quantum state is. A simple and elegant way of constructing a full quantum state from a classical field is the coherent-state approximation. First, let us expand the soliton field σ and its momentum conjugate π in plane waves, taking out the vacuum value σ_v for convenience:

$$\sigma(\mathbf{r}) = \sigma_v + \frac{1}{(2\pi)^{3/2}} \int d^3k \left[\frac{1}{2\omega_k}\right]^{1/2} (a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}} + a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}) \ , \tag{2.12}$$

$$\pi(\mathbf{r}) = \frac{i}{(2\pi)^{3/2}} \int d^3k \left[\frac{\omega_k}{2}\right]^{1/2} (a_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}} - a_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}) \ . \tag{2.13}$$

The operators $a_{\mathbf{k}}^\dagger$ and $a_{\mathbf{k}}$ have the usual meaning of creation and annihilation operators for the soliton field quanta, and the vacuum state is defined by

$$a_{\mathbf{k}}|\text{vac}\rangle = 0 \quad \text{for all } \mathbf{k} \ . \tag{2.14}$$

Starting from a given mean field $\sigma_0(\mathbf{r})$ (a c number) one first constructs $\tilde{f}(\mathbf{k})$, the Fourier transform of $f(\mathbf{r}) = \sigma_0(\mathbf{r}) - \sigma_0$, as

$$\tilde{f}(\mathbf{k}) = \frac{1}{(2\pi)^{3/2}} \int d^3r\, f(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \ . \tag{2.15}$$

The coherent state corresponding to $\sigma_0(\mathbf{r})$ is then

$$|f\rangle \equiv \exp\left[\int d^3k \left[\frac{\omega_k}{2}\right]^{1/2} \tilde{f}(\mathbf{k}) a_{\mathbf{k}}^\dagger\right] |\text{vac}\rangle \ . \tag{2.16}$$

Such a state has the interesting property that the expectation value of any normal-ordered operator involving σ or π gives the mean-field result. More generally,

$$\frac{\langle f_1|:\sigma^n:|f_2\rangle}{\langle f_1|f_2\rangle} = \left[\sigma_v + \frac{f_1(\mathbf{r})+f_2(\mathbf{r})}{2}\right]^n = \left[\frac{\langle f_1|\sigma|f_2\rangle}{\langle f_1|f_2\rangle}\right]^n \ , \tag{2.17}$$

$$\frac{\langle f_1|:\pi^2:|f_2\rangle}{\langle f_1|f_2\rangle} = -\left[\frac{g_1(\mathbf{r})-g_2(\mathbf{r})}{2}\right]^2 \ , \tag{2.18}$$

where the function $g(\mathbf{r})$ is defined by

$$g(\mathbf{r}) \equiv \frac{1}{(2\pi)^{3/2}} \int d^3k\, \omega_k \tilde{f}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} \ . \tag{2.19}$$

Thus the state $|f\rangle$ is what we are looking for; it is a true quantum state and the expectation value of H_0 in that state gives the MFA (except for renormalization factors arising from normal ordering):

$$\frac{\langle f|:H_0:|f\rangle}{\langle f|f\rangle} = \int d^3r[\Psi^\dagger \boldsymbol{\alpha}\cdot\mathbf{p}\Psi + \tfrac{1}{2}(\pi^2 + |\nabla\sigma_0|^2) + U(\sigma_0) + g\bar{\Psi}\sigma\Psi] \ . \tag{2.20}$$

Since the state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ describes two possibly overlapping bags, we choose to describe the soliton part of that state by a coherent state constructed from the following mean field:

$$\sigma(\boldsymbol{\alpha};\mathbf{r})-\sigma_v=f(\boldsymbol{\alpha};\mathbf{r})=-\frac{\mu^2}{4\pi}\sigma_1\int d^3r'\Theta(\boldsymbol{\alpha};\mathbf{r}')\frac{e^{-\mu|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\ , \tag{2.21}$$

where the Θ function $\Theta(\boldsymbol{\alpha};\mathbf{r})$ is determined by the geometry of the system,

$$\Theta(\boldsymbol{\alpha};\mathbf{r})=\begin{cases}1 & \text{if } r\le R_B \text{ or } |\mathbf{r}-\boldsymbol{\alpha}|\le R_\pi,\\ 0 & \text{otherwise}\end{cases} \tag{2.22}$$

and is folded with a Yukawa function to give the approximate surface smoothness. At $\mathbf{r}\simeq 0$ or $\mathbf{r}\simeq\boldsymbol{\alpha}$ (in the center of either bag), $f\simeq-\sigma_1$ for large enough μ. So, this soliton mean field contains four parameters: R_B, R_π, μ, and σ_1, determined by least-square fits with the isolated MFA solutions for each individual bag. This choice was also the one adopted by Schuh, Pirner, and Wilets[8] in their nucleon-nucleon scattering calculations.

When it comes to choosing the quark part of the state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$, several options are available. One possibility is to keep the same quark wave functions as for the isolated system, regardless of whether the bags overlap or not. This can be called a sudden approximation for the quarks: as the separation distance α changes, they are not affected by the changing soliton field configurations. The single-particle baryon quark states are exactly the same as for an isolated baryon centered at the origin of the coordinate system, while the pion quark and antiquark wave functions are simply translated by $\boldsymbol{\alpha}$. On the other hand, one could also carry out the calculations for an adiabatic approximation where the quark wave functions would readjust instantly to the different soliton mean fields, by solving a different Dirac equation,

$$[\boldsymbol{\gamma}\cdot\mathbf{p}+g\sigma(\boldsymbol{\alpha};\mathbf{r})]\psi_k(\boldsymbol{\alpha};\mathbf{r})=\epsilon_k\beta\psi_k(\boldsymbol{\alpha};\mathbf{r})\ , \tag{2.23}$$

for different values of $\boldsymbol{\alpha}$. Better than either of these two extreme cases, a large set of single-particle quark states can be included in the ground state (2.10) by enlarging the sum over j.

In this calculation we have opted for the first choice (sudden approximation) for reasons of simplicitly. The quark-field operator is expanded in different orthonormal bases:

$$\Psi(x)=\sum_k b_k^B\psi_k^B(\mathbf{r})\ \text{ baryon expansion}\ , \tag{2.24}$$

$$\Psi(x)=\sum_k b_k^\alpha\psi_k^\pi(\mathbf{r}-\boldsymbol{\alpha})\ \text{ pion expansions}\ . \tag{2.25}$$

Neglecting vacuum-polarization effects arising from the transformation properties between these different bases, we write the quark part of the state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ as

$$|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle^{\text{quark}}=\sum_{t_j,m_q,m_{\bar{q}}}C_{t_j,m_q,m_{\bar{q}}}b^{\alpha}_{\bar{s},m_{\bar{q}}}(b^{\alpha}_{s,m_q})^\dagger|B_j\rangle\ , \tag{2.26}$$

where the coefficients $C_{t_j,m_q,m_{\bar{q}}}$ are the Clebsch-Gordan combinations needed in order to obtain the correct pseudoscalar, isovector, color-singlet properties of the pion and to couple the baryon and pion isospins to give the total isospin T_i, projection t_i.

The second quantized notation used here ensures the complete antisymmetry of the resulting state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ under permutation of any two quarks (Pauli principle). The Pauli principle was already built into the baryon $|B_j\rangle$, but now a new antisymmetrization arises when the quark in the pion is exchanged with one of the quarks in the baryon. Of course this type of effect is completely ignored in calculations treating the pion as an elementary field.

Now that we have specified the state $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ unambiguously, we determine the unknown coefficients C_i and $\phi_{ij}(\boldsymbol{\alpha})$ in (2.10) by invoking the variational principle

$$\frac{\delta}{\delta C_i^*}\left[\frac{{}^d\langle B_i|H|B_i\rangle^d}{{}^d\langle B_i|B_i\rangle^d}\right]=0\ , \tag{2.27}$$

$$\frac{\delta}{\delta\phi_{ij}^*(\boldsymbol{\alpha})}\left[\frac{{}^d\langle B_i|H|B_i\rangle^d}{{}^d\langle B_i|B_i\rangle^d}\right]=0\ \text{ for all } \boldsymbol{\alpha},j\ , \tag{2.28}$$

where H is the total Hamiltonian (2.1), including the one-gluon-exchange interaction. These equations give, respectively,

$$\langle B_i|H-E|B_i\rangle C_i+\sum_j\int d^3\alpha\langle B_i|H-E|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle\phi_{ij}(\boldsymbol{\alpha})=0\ , \tag{2.29}$$

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}|H-E|B_i\rangle C_i+\sum_{j'}\int d^3\alpha'\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}|H-E|B_{j'},\underline{\pi};T_i,t_i,\boldsymbol{\alpha}'\rangle\phi_{ij'}(\boldsymbol{\alpha}')=0\ , \tag{2.30}$$

where E is the total energy of the system in the state (2.10):

$$E=\frac{{}^d\langle B_i|H|B_i\rangle^d}{{}^d\langle B_i|B_i\rangle^d}\ . \tag{2.31}$$

Since we wish to study the bound-state problem of a baryon dressed by a pion, we regard these homogeneous coupled equations as an eigenvalue problem: the eigenvalue is the total energy E and the eigenvector is the set $[C_i,\phi_{ij}(\boldsymbol{\alpha})]$. The eigenvector can only be determined up to an arbitrary multiplicative constant corresponding to the arbitrary normalization of the state (2.10). Equations (2.29) and (2.30) yield a spectrum of discrete and continuous eigenvalues E. However, the same equations could also be used to study scattering problems (e.g., pion-nucleon elastic scattering). In that case, the energy E and the asymptotic form of $\phi_{ij}(\boldsymbol{\alpha})$ for large α are given and (2.29) and (2.30) are to be regarded as nonhomogeneous equations yielding $\phi_{ij}(\boldsymbol{\alpha})$ (for small α), phase shifts, and cross sections. We will not investigate scattering problems any further here, even though they are a nice application of the generator-coordinate formalism.

III. SOLUTIONS OF THE GENERATOR-COORDINATE EQUATIONS

In order to solve Eqs. (2.29) and (2.30) we must first evaluate the two nontrivial matrix elements

$$\langle B_j,\underline{\pi};T_i t_i,\boldsymbol{\alpha} \,|\, H-E \,|\, B_i\rangle$$

and

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha} \,|\, H-E \,|\, B_{j'},\underline{\pi};T_i,t_i,\boldsymbol{\alpha}'\rangle \,.$$

Since the states $|B_i\rangle$ and $|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle$ are orthogonal (because we neglect vacuum-polarization effects) and $|B_i\rangle$ is an eigenstate of H_0, the first of these two matrix elements only involves the OGE operator H':

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha} \,|\, H-E \,|\, B_i\rangle = \langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha} \,|\, H' \,|\, B_i\rangle \,. \tag{3.1}$$

This shows that the OGE interaction is entirely responsible for creating the pion in our approximation. After writing the OGE interaction H' explicitly in terms of the quark-field operator Ψ, as in (2.2), and expanding that quark field in the appropriate basis, Eq. (3.1) gives two terms:

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha} \,|\, H' \,|\, B_i\rangle = \mathscr{S}^d(\boldsymbol{\alpha})+\mathscr{S}^e(\boldsymbol{\alpha}) \,, \tag{3.2}$$

where

$$\mathscr{S}^d(\boldsymbol{\alpha})=\alpha_s \sum_{k,l} \langle B_j \,|\, (b_k^B)^\dagger b_l^B \,|\, B_i\rangle \times \sum_{t_j,m_q,m_{\bar q}} C^*_{t_j,m_q,m_{\bar q}} \int d^3r \int d^3r' \left[\bar\psi^{\pi}_{s,m_q}(\mathbf{r}-\boldsymbol{\alpha})\gamma^\mu \frac{\lambda^c}{2}\psi^B_l(\mathbf{r})\right] G^0_{\mu\nu}(\mathbf{r}-\mathbf{r}') \left[\bar\psi^B_k(\mathbf{r}')\gamma^\nu \frac{\lambda^c}{2}\psi^{\pi}_{\bar s,m_{\bar q}}(\mathbf{r}'\boldsymbol{\alpha})\right] \tag{3.3}$$

is represented by the direct diagram of Fig. 2, and

$$\begin{aligned}\mathscr{S}^e(\boldsymbol{\alpha})=\alpha_s &\sum_{k,l,k',l'} \langle B_j \,|\, (b_k^B)^\dagger (b_{k'}^B)^\dagger b_{l'}^B b_l^B \,|\, B_i\rangle \times \sum_{t_j,m_q,m_{\bar q}} C^*_{t_j,m_q,m_{\bar q}} \int d^3r \int d^3r' \int d^3r''[\psi^{\pi}_{s,m_q}{}^\dagger(\mathbf{r}'')\psi^B_l(\mathbf{r}''-\boldsymbol{\alpha})] \\ &\times \left[\bar\psi^B_k(\mathbf{r})\gamma^\mu \frac{\lambda^c}{2}\psi^{\pi}_{\bar s,m_{\bar q}}(\mathbf{r}-\boldsymbol{\alpha})\right] G^0_{\mu\nu}(\mathbf{r}-\mathbf{r}') \left[\bar\psi^B_{k'}(\mathbf{r}')\gamma^\nu \frac{\lambda^c}{2}\psi^B_{l'}(\mathbf{r}')\right]\end{aligned} \tag{3.4}$$

corresponds to the exchange diagram of Fig. 3, and results directly from the antisymmetrization discussed above. The evaluation of these two diagrams is lengthy and will not be reproduced here, but some general features can be obtained immediately using symmetry arguments. Since the operator H' is invariant under rotations in spin, isospin, and color spaces, and the pion is a pseudoscalar, isovector, color-singlet object, it is easy to see that $\mathscr{S}^d(\boldsymbol{\alpha})$ and $\mathscr{S}^e(\boldsymbol{\alpha})$ must have the general form

$$\mathscr{S}^d(\boldsymbol{\alpha})=R^d(\alpha)\sum_r \frac{\langle T^i 1 t_i r \,|\, T_j t_j\rangle}{(2T_j+1)^{1/2}} \left\langle B_j \left| \sum_k \tau_k^r \sigma_k\cdot\hat{\boldsymbol{\alpha}} \right| B_i\right\rangle , \tag{3.5}$$

$$\mathscr{S}^e(\boldsymbol{\alpha})=\sum_r \frac{\langle T^i 1 t_i r \,|\, T_j t_j\rangle}{(2T_j+1)^{1/2}} \left\langle B_j \left| \sum_{k\neq l} \tau_k^r \frac{\lambda_k^c}{2}\frac{\lambda_l^c}{2} \left[\sigma_k\cdot\hat{\boldsymbol{\alpha}} R_1^e(\alpha)+\sigma_l\cdot\hat{\boldsymbol{\alpha}} R_2^e(\alpha)+\frac{(\sigma_k\times\sigma_l)}{i}\cdot\hat{\boldsymbol{\alpha}} R_3^e(\alpha)\right] \right| B_i\right\rangle , \tag{3.6}$$

where the sums over k and l are over quarks and $R^d(\alpha)$, $R_1^e(\alpha)$, $R_2^e(\alpha)$, and $R_3^e(\alpha)$ are scalar functions of the magnitude of $\boldsymbol{\alpha}$ only. Using the result

$$\left\langle B_j \left| \frac{\lambda_k^c}{2}\frac{\lambda_l^c}{2} \right| B_i\right\rangle = -\tfrac{2}{3} \quad \text{for } k\neq l \tag{3.7}$$

and the Wigner-Eckart theorem, the matrix element $\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha} \,|\, H-E \,|\, B_i\rangle$ can be written as

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha} \,|\, H' \,|\, B_i\rangle = \alpha^{m*}\langle f^{\alpha}|f\rangle \frac{\langle S^i 1 s_i m \,|\, S_j s_j\rangle}{(2S_j+1)^{1/2}} S_{ij}(\alpha) \,, \tag{3.8}$$

where α^m is the m component of the unit vector $\hat{\boldsymbol{\alpha}}$ in the spherical basis, $\langle f^{\alpha}|f\rangle$ is a soliton overlap factor, and the functions $S_{ij}(\alpha)$ only depend on the scalar quantity α and on the total spins and isospins, not on the magnetic quantum numbers. There are only three different functions $S_{ij}(\alpha)$ in our case, given by

$$S_{NN}(\alpha)=10R^d(\alpha)-\tfrac{8}{3}[R_1^e(\alpha)-R_2^e(\alpha)] \,, \tag{3.9}$$

$$S_{N\Delta}(\alpha)=8\sqrt{2}R^d(\alpha)+\frac{16\sqrt{2}}{3} \times[-2R_1^e(\alpha)+R_2^e(\alpha)+3R_3^e(\alpha)] \,, \tag{3.10}$$

$$S_{\Delta\Delta}(\alpha)=20R^d(\alpha)-\tfrac{80}{3}[R_1^e(\alpha)+R_2^e(\alpha)] \,. \tag{3.11}$$

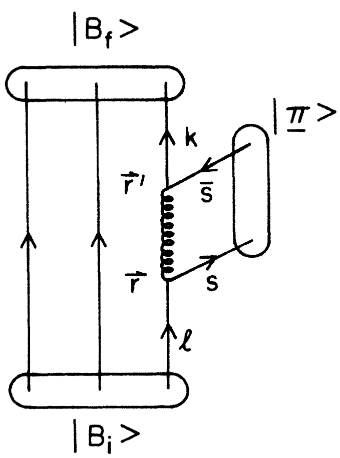

FIG. 2. The direct OGE diagram.

To evaluate the second matrix element $\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\,|\,H-E\,|\,B_{j'},\underline{\pi};T_i,t_i,\boldsymbol{\alpha}'\rangle$ we have shown that antisymmetrization terms of the kind discussed above are small and can be neglected. Their inclusion would complicate the algebra but not add any serious conceptual difficulties. In that approximation the matrix element in question has a very simple diagonal form

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\,|\,H-E\,|\,B_{j'},\underline{\pi};T_i,t_i,\boldsymbol{\alpha}'\rangle = \langle B_j\,|\,B_{j'}\rangle[\mathscr{H}^j(\boldsymbol{\alpha},\boldsymbol{\alpha}')-E\eta(\boldsymbol{\alpha},\boldsymbol{\alpha}')] \;. \tag{3.12}$$

The Hamiltonian piece depends on S_j and T_j only, through the OGE interaction (it splits the nucleon and Δ). Both functions $\mathscr{H}^j(\boldsymbol{\alpha},\boldsymbol{\alpha}')$ and $\eta(\boldsymbol{\alpha},\boldsymbol{\alpha}')$ actually depend only on the difference $\boldsymbol{\alpha}-\boldsymbol{\alpha}'$ or the magnitudes α and α' and the angle between $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$. Their angular dependence can be expanded in Legendre polynomials

$$\langle B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\,|\,H-E\,|\,B_{j'},\underline{\pi};T_i,t_i,\boldsymbol{\alpha}'\rangle = \langle B_j\,|\,B_{j'}\rangle\sum_{l=0}^{\infty}P_l(\hat{\boldsymbol{\alpha}}\cdot\hat{\boldsymbol{\alpha}}')[\mathscr{H}^j_l(\alpha,\alpha')-E\eta_l(\alpha,\alpha')] \;. \tag{3.13}$$

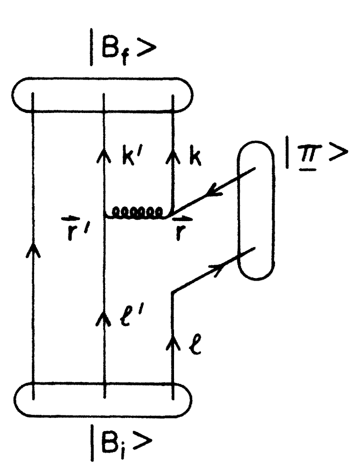

FIG. 3. The exchange OGE diagram.

Using the matrix elements [(3.8) and (3.13)] just derived, the second generator-coordinate equation (2.30) (also called the Hill-Wheeler equation) becomes

$$C_i\langle f^{\alpha}\,|\,f\rangle\frac{\langle S^i1s_im\,|\,S_js_j\rangle}{(2S_j+1)^{1/2}}S_{ij}(\alpha) + \int d^3\alpha'[\mathscr{H}^j_1(\alpha,\alpha')-E\eta_1(\alpha,\alpha')]\alpha'^m\phi_{ij}(\boldsymbol{\alpha}')=0 \;. \tag{3.14}$$

The solution of this integral equation has the general form

$$\phi_{ij}(\boldsymbol{\alpha})=\alpha^{m*}\frac{\langle S^i1s_im\,|\,S_js_j\rangle}{(2S_j+1)^{1/2}}F_{ij}(\alpha) \;, \tag{3.15}$$

where the radial function $F_{ij}(\alpha)$ depends only on the total spins and isospins, not on their projections [like $S_{ij}(\alpha)$]. We have four unknown functions $F_{NN}(\alpha)$, $F_{N\Delta}(\alpha)$, $F_{\Delta N}(\alpha)$, and $F_{\Delta\Delta}(\alpha)$. Note that $F_{ij}(\alpha)$ is *not* necessarily symmetric in i,j.

The only remaining unknown quantities C_i and $F_{ij}(\alpha)$ are solutions of the eigenvalue problem

$$C_i\langle B_i\,|\,H-E\,|\,B_i\rangle + \frac{1}{(2S_i+1)}\sum_j\frac{4\pi}{3}\int_0^{\infty}\alpha^2d\alpha\,F_{ij}(\alpha)S_{ij}(\alpha)=0 \;, \tag{3.16}$$

$$C_i\langle f^{\alpha}\,|\,f\rangle S_{ij}(\alpha) + \frac{4\pi}{3}\int_0^{\infty}\alpha'^2d\alpha'[\mathscr{H}^j_1(\alpha,\alpha')-E\eta_1(\alpha,\alpha')]F_{ij}(\alpha')=0 \;. \tag{3.17}$$

Solving these equations numerically in a straightforward way, i.e., by discretizing the α and α' axes and using an ordinary integration scheme, leads to numerical instability. Such problems are characteristic of Fredholm integral equations of the first kind, of which (3.17) is an example. They are caused by the fact that any arbitrarily large, high-frequency function is killed by the smooth kernel after integration. The instabilities build up in the solution in the form of large oscillations with a higher and higher frequency as the grid size becomes finer and finer. An elegant solution to this numerical problem was proposed by Phillips[19] and Tikhonov.[20] In matrix notation (3.17) has the general form of a linear system

$$Kf=g \;, \tag{3.18}$$

where f is the unknown function. This equation can be derived from the variational principle

$$\frac{\delta}{\delta f}||Kf-g||^2=0 \;, \tag{3.19}$$

where the norm $||\;||$ is defined as

$$||F||^2=\int d\alpha\int d\alpha' F(\alpha)F(\alpha') \;. \tag{3.20}$$

Phillips and Tikhonov proposed to regularize this equation by replacing it by the more stable

$$\frac{\delta}{\delta f}[\,||Kf-g||^2+\lambda\mathscr{L}(f)]=0 \;, \tag{3.21}$$

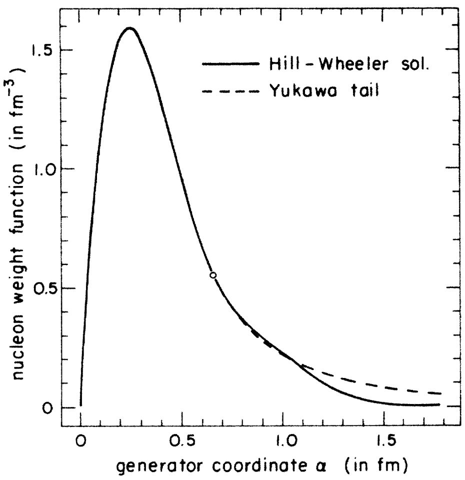

FIG. 4. Solution of the Hill-Wheeler equation $F_{NN}(\alpha)$ for the nucleon.

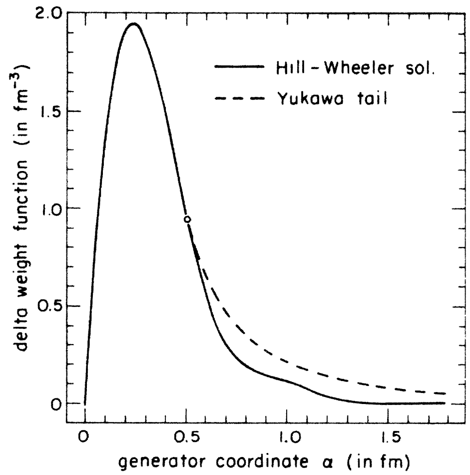

FIG. 5. Solution of the Hill-Wheeler equation $F_{N\Delta}(\alpha)$ for the nucleon.

where $\mathscr{L}(f)$ is a functional of f and λ is a regularization parameter, to be chosen carefully: large enough so that the solution is numerically stable and small enough so that it remains an approximate solution of the original equation (3.18). The new regularization term forces the solution to be smooth. For our particular problem, we found good results with the following choice of $\mathscr{L}(f)$ and λ:

$$\mathscr{L}(f)=||\nabla f||^2 , \tag{3.22}$$

$$\lambda=10^{-4}\ \mathrm{fm}^{-6} . \tag{3.23}$$

The method can be easily generalized for a three-dimensional problem. We replace (3.17) by the regularized version

$$\frac{4\pi}{3}\int_0^\infty \alpha^2 d\alpha[\mathscr{H}_1^j(\alpha,\beta)-E\eta_1(\alpha,\beta)]\left[\frac{4\pi}{3}\int_0^\infty \alpha'^2 d\alpha'[\mathscr{H}_1^j(\alpha,\alpha')-E\eta_1(\alpha,\alpha')]F_{ij}(\alpha')+C_i\langle f^\alpha | f\rangle S_{ij}(\alpha)\right]$$
$$-\lambda\left[\frac{1}{\beta}\frac{d^2}{d\beta^2}[\beta F_{ij}(\beta)]-\frac{2}{\beta^2}F_{ij}(\beta)\right]=0 . \tag{3.24}$$

Figures 4 and 5 show the smooth behavior of the functions $F_{NN}(\alpha)$ and $F_{N\Delta}(\alpha)$, respectively, obtained using this regularization technique. As expected, they start out linear in α, as must be the case for p waves, and die out exponentially without any further node, as is required for the ground state. Also shown in these figures is a Yukawa tail proportional to

$$Y(\alpha)=\left[1+\frac{1}{m_\pi\alpha}\right]\frac{e^{-m_\pi\alpha}}{m_\pi\alpha} , \tag{3.25}$$

where m_π is the physical pion mass ($m_\pi=140$ MeV). The tail $Y(\alpha)$ is attached to the functions $F_{NN}(\alpha)$ and $F_{N\Delta}(\alpha)$ at a point of least sensitivity (where the functions and their derivatives match). The functions $F_{NN}(\alpha)$ and $F_{N\Delta}(\alpha)$ should be proportional to $Y(\alpha)$ as α becomes large. However, we see in the graphs that $F_{NN}(\alpha)$ and $F_{N\Delta}(\alpha)$ fall off faster than $Y(\alpha)$. This discrepancy is due to the rearrangement energy: the rate of falloff is determined by the energy required to remove one pion completely from the nucleon. In reality, when one pion is removed the system gets rearranged and the nucleon becomes dressed with another pion. Such effects are missing in this calculation and therefore the removal energy is *not* the physical pion mass as it should be.

IV. HADRONIC PROPERTIES

The model we have studied contains five parameters: the four original soliton bag parameters a, b, c, and g plus the strong coupling constant α_s. We were able to determine three of those parameters by fitting three known properties of nucleons and Δ's, namely, the nucleon and Δ masses and the proton root-mean-square radius. We choose the two remaining parameters to be c and the dimensionless combination

$$f\equiv b^2/ac . \tag{4.1}$$

Table I gives a summary of all the hadronic properties we have calculated for the case $f=3.1$ and $c=4000$, and the

TABLE I. Summary of hadronic properties in the case $a=51.6$, $b=-799.9$, $c=4000$, and $g=14.8$, and comparison between the MFA and the pionic dressing calculations.

Property	MFA	With pion	Experiment
λ (nucleon)	1	1.14	
λ (Δ)	1	1.11	
λ (pion)	1	1.18	
α_s (N,Δ)		2.72	
α_s^* (pion)		3.17	
Nucleon mass	1736 MeV	939 MeV	939 MeV
N-Δ splitting	271 MeV	293 MeV	293 MeV
Pion mass	1330 MeV		140 MeV
with α_s		252 MeV	140 MeV
with α_s^*		140 MeV	140 MeV
$f_{NN\pi}$	0	0.14	0.28
$f_{N\Delta\pi}$	0	0.27	0.59
$P_{N\pi}$	0	0.28	
$P_{\Delta\pi}$	0	0.17	
$\langle r^2\rangle^{1/2}$ (proton)	0.69 fm	0.83 fm	0.83 fm
$\langle r^2\rangle$ (neutron)	0	-0.08 fm^2	-0.12 fm^2
g_A	0.97	0.86	1.26
D	0.45 fm^{-1}	-0.09 fm^{-1}	small

comparison with the mean-field calculation of Goldflam and Wilets[4] for the same set of parameters. Where experimental values are available, they are also given.

The results presented there go one step beyond the simple generator-coordinate method, which is a perturbative-like treatment building upon the mean-field solutions. But instead of a full self-consistent treatment, some of the effects of the pion cloud on the quark and soliton fields have been incorporated by considering a simple rescaling. For a given set of soliton bag parameters we can define new quark and soliton fields, scaled according to their dimensions. Let us denote them by $\tilde{\Psi}$ and $\tilde{f}(f\equiv\sigma-\sigma_v)$:

$$\tilde{\Psi}(\lambda\mathbf{r})=\lambda^{-3/2}\Psi(\mathbf{r})\ , \qquad (4.2)$$

$$\tilde{f}(\lambda\mathbf{r})=\lambda^{-1}f(\mathbf{r})\ . \qquad (4.3)$$

Starting with these scaled functions, we can treat the dressed nucleon problem by solving the Hill-Wheeler equations and determining the new ground-state eigenenergy $E(\lambda)$. The scale λ is then determined by requiring E to be stationary

$$\frac{\partial E(\lambda)}{\partial\lambda}=0 \quad \text{for } \lambda=\lambda_0\ . \qquad (4.4)$$

This will tell us in a very crude way (using only one degree of freedom) how much the pion cloud shrinks or dilates the bag. The results of Table I show a slight increase in bag size coming from pionic effects.

When the general formalism described in Sec. III is applied to the case where no baryon is present, i.e., for a free pion, one obtains a simple center-of-mass projection, including OGE effects. The value of λ for that case is also greater than 1 (dilation). In order to get the physical pion mass we were forced to use a slightly greater value of the strong coupling constant than for the baryon case. The resulting α_s^* is an effective coupling constant that simulates higher-order gluon exchanges. However, as shown in Table I, it is not much greater than α_s. If one were to use the same value of α_s as for the baryon case, one would find a pion that is 80% too heavy.

The comparison of hadron masses in the MFA and pionic dressing calculations shows a substantial reduction in mass from the MFA. This is especially true for the pion, where the reduction is about a factor of 10. When compared to the mean N-Δ mass, the N-Δ splitting is increased when pions are included (almost doubled). This is in qualitative agreement with other pionic dressing calculations such as the cloudy bag model (CBM),[15] where it is shown that a large fraction of the N-Δ mass difference is due to the pion cloud.

The nucleon-nucleon-pion and N-Δ-pion coupling constants are quantities that were not accessible in the MFA, but that we can now compute. To do so, we first construct a state $|B_j,\underline{\pi};T_i,t_i,\mathbf{k}\rangle$ consisting of a baryon $|B_j\rangle$ dressed with a pion of good momentum $\mathbf{k}$: $|\underline{\pi}_{\mathbf{k}}\rangle$,

$$|B_j,\underline{\pi};T_i,t_i,\mathbf{k}\rangle=N_k\int d^3\alpha\, e^{i\mathbf{k}\cdot\boldsymbol{\alpha}}|B_j,\underline{\pi};T_i,t_i,\boldsymbol{\alpha}\rangle\ , \qquad (4.5)$$

where N_k is a normalization constant required to satisfy

$$\langle B_j,\underline{\pi};T_i,t_i,\mathbf{k}\,|\,B_j,\underline{\pi};T_i,t_i,\mathbf{k}\rangle=\Omega\ . \qquad (4.6)$$

In general N_k can depend on k, and Ω is the volume in which we normalize the plane wave (box normalization). The general $BB\pi$ vertex can be computed using (3.8):

$$v_r^{ij}=4\pi N_k k^m\frac{\langle S^i 1 s_i m\,|\,S_j s_j\rangle}{(2S_j+1)^{1/2}} \times\int_0^\infty \alpha^2 d\alpha\, j_1(k\alpha)\langle f^\alpha\,|\,f\rangle S_{ij}(\alpha)\ . \qquad (4.7)$$

Comparing this result to the Chew-Low theory[21] and taking the zero-momentum limit, one can identify the coupling constants $f_{NN\pi}$ and $f_{N\Delta\pi}$:

$$f_{NN\pi}=\frac{N_0}{9}\sqrt{2\pi}m_\pi{}^{3/2}\int_0^\infty \alpha^2 d\alpha\,\alpha\langle f^\alpha\,|\,f\rangle S_{NN}(\alpha)\ , \qquad (4.8)$$

TABLE II. Quark and pionic contributions to magnetic moments (in nucleon magnetons) in the case $a=51.6$, $b=-799.9$, $c=4000$, and $g=14.8$.

	"Quark effects"	Pion (not modified)	Pion (modified)	Total (not modified)	Total (modified)	Experiment
${}^d\langle p\,|\,\mu\,|\,p\,\rangle^d$	1.91	0.10	0.95	2.01	2.86	2.76
${}^d\langle n\,|\,\mu\,|\,n\,\rangle^d$	−1.37	−0.10	−0.95	−1.47	−2.32	−1.91
${}^d\langle p\,|\,\mu\,|\,\Delta^+\rangle^d$	1.58	0.05	0.48	1.63	2.06	3.25

$$f_{N\Delta\pi}=\frac{N_0}{6}\sqrt{2\pi}m_\pi{}^{3/2}\int_0^\infty \alpha^2 d\alpha\,\alpha\langle f^\alpha\,|\,f\,\rangle S_{N\Delta}(\alpha)\ , \tag{4.9}$$

where N_0 is the $k\to 0$ limit of N_k. It is given by

$$N_0=\frac{\sqrt{\Omega}}{4\pi}\left[\int_0^\infty \alpha^2 d\alpha\int_0^\infty \alpha'^2 d\alpha'\eta_0(\alpha,\alpha')\right]^{-1/2}. \tag{4.10}$$

These results do not take into account the distortion from a plane wave of the pion field as it comes close to the nucleon or the Δ. A treatment allowing for such distortion would require solving the scattering problem discussed above. It has not been investigated here. These also do not take into account multimeson effects, which might be important.

$P_{N\pi}$ and $P_{\Delta\pi}$ are the probabilities of having a nucleon turn into nucleon + pion and Δ + pion, respectively. They are given by

$$P_{j\pi}=\frac{1}{4}\left[\frac{4\pi}{3}\right]^2\int_0^\infty \alpha^2 d\alpha\int_0^\infty \alpha'^2 d\alpha' F_{Nj}(\alpha)\times\eta_1(\alpha,\alpha')F_{Nj}(\alpha') \tag{4.11}$$

if the eigenstate $[C_N,F_{Nj}(\alpha)]$ is normalized so that

$$1=|\,C_N\,|^2+\sum_j P_{j\pi}\ . \tag{4.12}$$

The total pionic dressing probability is about 45% in our model, which is slightly higher than the cloudy bag model result (where it depends on the bag radius but is around 33%).

The neutron charge radius results entirely from the pionic dressing here (it vanishes in the mean-field approximation). As can be seen in Table I, the values we obtain are in reasonable agreement with the experiment value of $\langle r^2\rangle_n=-0.12$ fm^2, considering that other mechanisms could be invoked to explain the negative charge radius (such as the up-down quark mass difference).

The proton and neutron magnetic moments as well as the $M1$ proton-Δ^+ transition are shown next in Table II. They are all smaller in absolute value than the experimental quantities. Their expression in this model is

$${}^d\langle p\,|\,\mu\,|\,p\,\rangle^d=\mu_0\left[\,|\,C_N\,|^2+\tfrac{1}{27}P_{N\pi}+\tfrac{20}{27}P_{\Delta\pi}+\frac{16\sqrt{2}}{27}P_{N\Delta\pi}\right]+\tfrac{4}{9}\mathcal{M}_N+\tfrac{1}{9}\mathcal{M}_\Delta\ , \tag{4.13}$$

$${}^d\langle n\,|\,\mu\,|\,n\,\rangle^d=\mu_0\left[-\tfrac{2}{3}\,|\,C_N\,|^2-\tfrac{4}{27}P_{N\pi}-\tfrac{5}{27}P_{\Delta\pi}-\frac{16\sqrt{2}}{27}P_{N\Delta\pi}\right]-\tfrac{4}{9}\mathcal{M}_N-\tfrac{1}{9}\mathcal{M}_\Delta\ , \tag{4.14}$$

$${}^d\langle p\,|\,\mu\,|\,\Delta^+\rangle^d=\mu_0\left[\frac{2\sqrt{2}}{3}\,|\,C_N\,|^2+\tfrac{10}{27}P_{N\pi}+\tfrac{5}{27}P_{\Delta\pi}+\frac{11\sqrt{2}}{27}P_{N\Delta\pi}\right]+\tfrac{1}{9}\mathcal{M}_N+\tfrac{5}{18}\mathcal{M}_\Delta\ , \tag{4.15}$$

where μ_0 is the proton magnetic moment in the MFA and $P_{N\Delta\pi}$ is defined in analogy with $P_{f\pi}$ (4.11) by

$$P_{N\Delta\pi}=\frac{1}{4}\left[\frac{4\pi}{3}\right]^2\int_0^\infty \alpha^2 d\alpha\int_0^\infty \alpha'^2 d\alpha' F_{NN}(\alpha)\times\eta_1(\alpha,\alpha')F_{N\Delta}(\alpha')\ . \tag{4.16}$$

If one looks at the pion contributions to these magnetic moments [the last two terms of (4.13)–(4.15)], one finds that they are surprisingly small as compared to the CBM, for example. This is shown in Table II ("not modified" columns), where by quark effects we mean the terms proportional to μ_0. This is due to the fact that the pionic contributions are inversely proportional to the pion energy, and that in this calculation the mean-field pion we start with is very heavy (1330 MeV in this case). As explained above, momentum projection and one-gluon exchange reduce that value to the physical pion mass (140 MeV), but the pion *wave function* still corresponds to a mass of 1330 MeV. One can correct for that by doing a simple energy rescaling. The results are shown in the "modified" columns of Table II and turn out to be in much closer agreement with experiment for the diagonal matrix elements, but still too small for the $M1$ proton-Δ^+ transition.

The nucleon axial charge (g_A) is the least well-described physical property of this model. It is always lower than the experimental value $g_A=1.26$. Higher-order mesonic corrections (like two pions or ρ meson) and center-of-mass corrections are needed to bring it up to that value.

The last quantity we looked at is the divergence of the axial-vector current of a nucleon. It would vanish identically in a chiral-invariant theory, and therefore constitutes a nice test of PCAC. Figure 6 shows the radial dependence of that quantity in both the mean-field approximation, and in this dressed nucleon calculation for the case $c=4000$ and $f=3.1$. As one can see, the surface peak of the MFA, which is characteristic of PCAC-violating models has been strongly suppressed, even though the resulting divergence is far from being very small for all values of r. To get a better idea of the cancellation, we computed the radial integral of these functions. Writing

$$^{d}\langle N\,|\,\nabla\cdot\underline{\mathbf{A}}(\mathbf{r})\,|\,N\rangle^{d}=D(r)\underline{e}_{3}(\hat{\mathbf{r}}\cdot\hat{\mathbf{z}}) \tag{4.17}$$

we define the global quantity (which has the units of an energy)

$$D\equiv\frac{4\pi}{3}\int_0^\infty r^2\,dr\,D(r)\ . \tag{4.18}$$

Note that the volume integral of the divergence of $\underline{\mathbf{A}}(\mathbf{r})$ vanishes because of its angular p-wave dependence. Table I shows the values of D in both the MFA and this calculation. The corresponding values are $D=1.25$ and 0.16 fm^{-1} in the MIT and cloudy bag models, respectively (with a bag radius of 0.9 fm). This shows that our model gives indeed a very strong cancellation of $\nabla\cdot\underline{\mathbf{A}}$.

All the results presented so far have been obtained with the static ($\omega=0$) scalar and tensor propagators given by Eqs. (2.6) and (2.8) while the correct treatment would require considering a complete set of gluon modes, each characterized by a different frequency ω. An approximate solution is the one mode approximation: one truncates the sum over all gluon modes to one term, chosen to be the best possible mode by requiring that it extremizes the total energy. So in this approach ω is treated as a variational parameter. We have done an exploratory calculation for one given set of soliton bag parameters (the same as for Table I), that shows that in fact $\omega=0$ extremizes (minimizes in this case) the total energy (see Fig. 7). Moreover, the results are quite insensitive to ω in the vicinity of $\omega=0$.

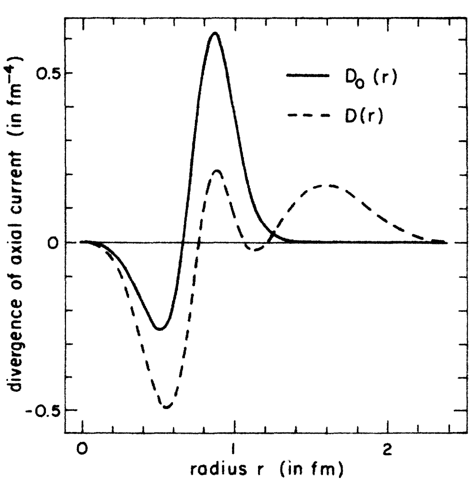

FIG. 6. Radial dependence of the divergence of the nucleon axial-vector current for the case $c=4000$ and $f=3.1$. $D_0(r)$ is the mean-field result and $D(r)$ the dressed calculation.

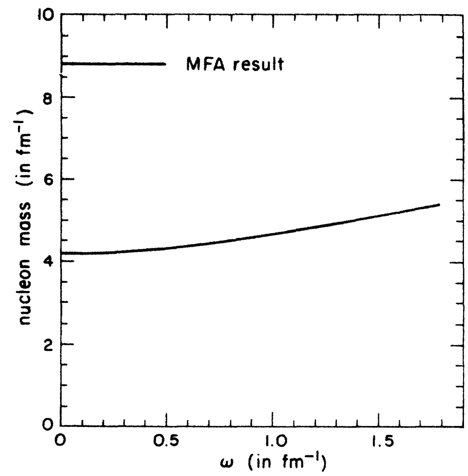

FIG. 7. The nucleon mass as a function of the gluon mode frequency ω for the soliton bag parameters $a=51.6$, $b=-799.9$, $c=4000$, $g=14.8$, and $\alpha_s=2.72$. The MFA result for the same set of parameters is given for comparison.

The results shown in Fig. 7 were obtained by using the frequency-dependent tensor propagator (2.7). To simplify the calculation, only the ω dependence of the dominant direct diagram represented in Fig. 2 was considered. Note that as ω becomes very large, all gluon effects should disappear and we should recover the MFA results.

V. CONCLUSIONS

We have presented a new model of the N and Δ allowing for the presence of one pion in the ground state of these baryons. Its distinctive features are that this pion is treated as a quark-antiquark pair (created by a one-gluon-exchange interaction), and that therefore no additional degree of freedom is needed. We obtain the experimental pion mass for reasonable values of the strong coupling constant (both center-of-mass corrections and one-gluon-exchange reduce the pion mass). The model is able to reproduce many experimental static properties of the N and Δ quite well.

We should stress that the ability to treat the bag as a dynamical variable in the soliton bag model was crucial to develop this calculation: we could not have done this with the MIT bag model. Indeed, we needed to have an unambiguous mathematical model to describe the dynamics of two colliding bags.

This calculation could be improved in many ways and some of the approximations we have used relaxed. For example, we could use a confined gluon propagator, calculated in a σ-dependent color-dielectric model instead of the free propagator we have used. We could also consider an adiabatic approximation, where the quarks would be assumed to adjust instantly to new soliton field configurations. We have used, on the other hand, a sudden approximation where the quarks do not react at all to changing soliton fields. Neither of these approximations is *a priori* better than the other (the real world lies somewhere in between), but their comparison would be interesting. An even better treatment of the quark fields would consist of considering a large basis of linearly independent quark states (for example, the eigenstates of a single bag) and diagonalizing in that basis. Moreover, to obtain a better agreement with experiment, a complete treatment of the center-of-mass problem is clearly needed. We believe that these improvements might necessitate a readjustment of the parameters of the model but probably will not affect the generally good agreement with experimental data.

The one-gluon-exchange Hamiltonian H' is the only interaction creating the $q\bar{q}$ pion in our treatment. The quark-σ coupling term $\mathscr{L}_{q\sigma}$ (1.4) also contributes to diagrams of the kind described in Figs. 2 and 3 where a soliton quanta instead of a gluon is now being exchanged. Such effects have not been considered here since they involve higher-order perturbation effects (two quark-quark-soliton vertices are required), or alternatively another, intermediate state in the ansatz (2.10).

ACKNOWLEDGMENTS

We wish to thank Dr. M. C. Birse for his active participation in the discussions concerning many aspects of this work. This work was supported in part by the U.S. Department of Energy. The work reported here forms part of a doctoral thesis submitted by Jean-Louis Dethier to the University of Washington.

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