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Strangeness production in ultrarelativistic heavy-ion collisions. II. Evolution of flavor composition in scaling hydrodynamics

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Bjorken's hydrodynamic description of the space-time evolution of the central rapidity region in ultrarelativistic heavy-ion collisions is extended to incorporate the chemical processes which affect the strangeness abundance. Scaling hydrodynamic equations which contain the rate equation for strangeness production and annihilation both in the plasma phase and in the hadron gas phase are integrated numerically assuming an almost adiabatic first-order phase transition from plasma to hadron gas. It is found that if a plasma is initially formed the resultant K/π ratio will be enhanced by about a factor of 3 from that in pp collisions. However, this ratio is still smaller than that in an equilibrium hadron gas and hence cannot be considered a direct signal of plasma formation.

I. INTRODUCTION

Heavy-ion collisions at ultrarelativistic energies offer a unique opportunity to explore the large-scale properties of quantum chromodynamics, in particular, of its phase structure at high temperatures and densities.[1] It is expected that at sufficiently high beam energies the produced matter will initially take the form of a plasma of unconfined quarks and gluons. This plasma would immediately begin a rather complex evolution, culminating in its decay into ordinary hadrons. This poses a very difficult question: How can we unambiguously confirm the creation of the plasma in an experiment? Can we find any relics of plasma formation in the final decay products?

It has been proposed by several authors[2–5] that an enhancement in strange-particle production would be one signal of quark-gluon plasma formation. In pp collisions strange-particle production comprises only about a tenth of the total multiplicity.[6] In a quark-gluon plasma in equilibrium, on the other hand, the abundance of strange quarks and antiquarks is expected to be highly enhanced. This is mainly because the s-quark mass ($m_s \approx 150$ MeV) is comparable to the expected temperature of the plasma. In the baryon-rich plasma which may be formed from compressed nuclear matter in the nuclear fragmentation region (or in "stopping-regime" collisions at lower energies), $s\bar{s}$ pairs would be more abundant than $\bar{u}$ and $\bar{d}$ because of the Pauli blocking of light-quark pair creation. It has been argued[4] that the relaxation time to reach chemical equilibrium in the plasma is sufficiently short that chemical equilibrium will indeed be achieved in a heavy-ion collision and enhanced strangeness abundance will be reflected in the final particle composition.

In the baryon-free central rapidity region, however, essentially all particles are produced after the collision. In this case a more natural assumption is that the initial state of the plasma will *already* possess flavor composition close to that in thermodynamic equilibrium. Consider for example a dynamical model[7] in which the plasma constituents are produced by the quantum creation of $q\bar{q}$ pairs from the vacuum by the confining color-electric field. In such a model the suppression of strange-particle production in pp collisions is explained by the small tunneling probability of $s\bar{s}$ pairs, due to the large s-quark mass. Applying this model to nucleus-nucleus collisions[8,9] suggests that the mass suppression will be less important because of a stronger background color field, which will also increase both the total multiplicity and the energy density and hence will work in favor of plasma formation. In this model the plasma would be created close to thermodynamic equilibrium. It is indeed unlikely, in view of the flavor blindness of QCD, that the plasma will be born at a very high temperature ($T \gg m_s$) but with flavor composition considerably in violation of flavor-SU(3) symmetry.

In order to test such conjectures experimentally, we have to describe the dynamical evolution of the plasma into a hadronic final state. The key question is, how is the supposed symmetry in the initial flavor composition reflected in the observed particle spectrum? As has been noted by Glendenning and Rafelski,[5] the K/π ratio may be sensitive to details of the expansion. In particular, the number of pions reflects the entropy of the system. At late times, most of the entropy may reside in pions, and for this reason, even if the number of $s\bar{s}$ pairs is conserved in the expansion stage the K/π ratio will not necessarily become very large. On the other hand, if thermodynamic equilibrium is maintained throughout the expansion of the system, the flavor composition of the emergent hadrons will reflect only the freeze-out conditions. To make this issue more quantitative, there are two competing dynamical processes to be understood: how the system deviates from chemical equilibrium as it cools and hadronizes, and how the system reacts to return to equilibrium.

In a preceding paper[10] (henceforth I) we formulated a

QCD kinetic theory for the chemical reaction processes which affect the flavor composition of the quark-gluon plasma. The rates of production and annihilation of strange-quark pairs were computed in lowest-order QCD perturbation theory. The hydrodynamic equations coupled to the rate equation were then solved for a high-temperature homogeneous plasma and the relaxation time to reach chemical equilibrium in such a system was computed. The purpose of this paper is to apply this description to the dynamical situation which we expect to encounter in the central rapidity region of ultrarelativistic heavy-ion collisions. Our study is similar to the work recently reported by Kapusta and Mekjian,[11] but it differs in several important points, as we shall see.

Now let us briefly review the standard picture for the space-time evolution of very-high-energy ($E_{\mathrm{c.m.}} \geq 50$ GeV per nucleon) nucleus-nucleus collisions.[1,12–14] Prior to the collision two Lorentz-contracted nuclear pancakes approach from opposite directions along the light cone, $t = \pm z$ (see Fig. 1). After the collision at $t = z = 0$ two highly excited nuclear pancakes, containing most of the baryon number, recede along the same light-cone lines, leaving a highly excited volume in between. The excited region will soon decay (materialize) into individual excitations of fundamental quanta, supposedly unconfined quarks and gluons rather than hadrons, which form an expanding plasma. The collisions among these excited quanta may bring the system into local thermodynamic equilibrium and, if so, the hydrodynamic expansion follows. The prehydrodynamic regime of the matter evolution has been studied in a semiclassical transport theory,[15] extended to incorporate particle formation.[16]

The evolution of the central rapidity region appears almost invariant under longitudinal boosts as long as one is not too far away from the pp center-of-mass frame. In this paper we assume that hydrodynamic expansion of the plasma starts at a certain proper time $\tau = \sqrt{t^2 - z^2} = \tau_0$ with Lorentz-boost-invariant initial conditions. This symmetry in the initial conditions will be preserved by the hydrodynamical evolution of the system if it is not broken significantly by chaotic fluctuations[17] generated by the dynamical hadronization transition, and will eventually be reflected in the final particle distribution as a central rapidity plateau.

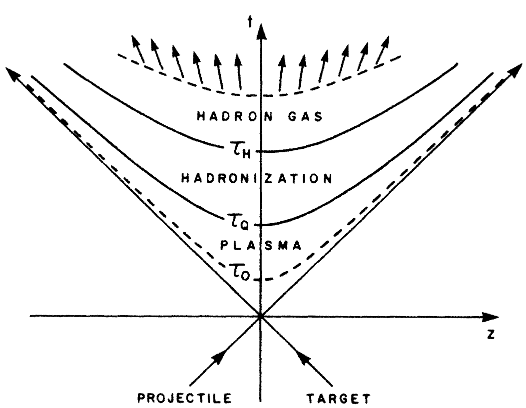

FIG. 1. A space-time picture of an ultrarelativistic nucleus-nucleus collision in the pp center-of-mass frame. The Lorentz-contracted target and projectile nuclei approach along light-cone trajectories and collide at the origin of the coordinate system. The hadronic matter (initially quark-gluon plasma) of the central region is formed between the two receding excited nuclei, and reaches thermal equilibrium on or before the hyperbola at $\tau = \sqrt{t^2 - z^2} = \tau_0$. The subsequent hydrodynamic expansion preserves the Lorentz-boost symmetry of the initial conditions.

In Bjorken's hydrodynamical description of the longitudinal expansion[12] and in its later extensions,[13,14] local thermodynamic equilibrium is assumed to be maintained throughout. This implies that there is only one independent thermodynamic variable (e.g., temperature or entropy) for baryon-free hadronic matter.[18] As we have seen in I, however, if the system is not in chemical equilibrium with respect to its flavor content there appears another thermodynamic variable, which can be taken to be either the s- (or $\bar{s}$-) quark density n_s or the s-quark chemical potential μ. (μ gauges the deviation of the system from chemical equilibrium; it should *not* be confused with the chemical potential coupled to baryon number, which is assumed to be zero in the baryon-free central region.) In the next section we extend Bjorken's description of the one-dimensional longitudinal hydrodynamic expansion to incorporate this new feature explicitly. The hydrodynamic equations which contain the rate equation for the s-quark current are introduced and examined in the scaling limit.

A major difficulty in studying the space-time evolution of the quark-gluon plasma arises from our uncertainty of the hadronization mechanism. Several attempts have been made to obtain a plausible scenario of plasma hadronization, including a macroscopic description within the context of fluid mechanics[19,17] and microscopic model calculations[20] based on phenomenological descriptions of color confinement. It is not our purpose here to study the consequences of all these different scenarios; such an extensive study should wait until full multidimensional hydrodynamic calculations become available. Instead we shall adopt a simple plausible model for plasma hadronization which consists of the following assumptions and approximations.

(1) It is not known at this time whether there is a phase transition separating the high-temperature quark-gluon plasma from the low-temperature hadronic regime. The various possibilities include a first-order transition, a second-order transition, and a continuous crossover without thermodynamic singularities.[21] Our first assumption is a sharp first-order transition, which allows us to approximate the thermodynamics on either side as weakly interacting, almost ideal gases.

(2) Our next assumption is that the passage from plasma to hadrons is smooth, i.e., never far from equilibrium. In the case of a first-order transition, this specifically excludes the possibility of supercooling and/or superheating which lead to rather complex scenarios.[17,22] Instead we assume that the transition proceeds quasiadiabatically, developing a "uniform mixture" of plasma and hadron gas via the Maxwell construction. This means that we calculate extensive thermodynamic variables in the transition region by averaging the volume fractions taken up by

plasma and by hadron gas. This assumption implies that there is little difference between assuming a first-order transition and assuming a steep continuous crossover.

(3) Our next hypothesis concerns the effect of the hadronization process on the chemical equilibrium and is, therefore, a key ingredient in the calculation. We assume detailed balance in the mixed phase with respect to those hadronization processes which conserve the number of s quarks, and we neglect $s\bar{s}$ pair creation in the hadronization. Since the chemical potentials of the light quarks and gluons are always kept at zero (because of the assumed fast rate of the reactions which change their numbers), this implies that the chemical potential of a given species of meson is equal to the sum of the two chemical potentials of its $q\bar{q}$ constituents. Hence the pion chemical potential vanishes and the kaon chemical potential becomes equal to the s-quark chemical potential. This relative equilibrium is consistent with the assumption of a quasiadiabatic hadronization transition. In the mixed phase, the chemical reaction rate for the creation or annihilation of $s\bar{s}$ pairs is calculated by taking the volume average of the rates in the plasma and in the hadron gas.

The numerical results are presented and discussed in Sec. III. We choose initial conditions for the hydrodynamic equations which meet the requirements of the uncertainty principle and which give a final total particle multiplicity roughly consistent with the currently available cosmic-ray data. We first display the time evolution of the system in the absence of chemical reactions, and show how the system deviates from chemical equilibrium by cooling and undergoing hadronization. Contrary to naive expectations, a shortage of s quarks arises as the system goes through the hadronization transition, characterized by a fall of the s-quark chemical potential into negative values. This is due to the small strangeness/entropy ratio of the equilibrium quark-gluon plasma in comparison to that of the equilibrium hadron gas, a quite amusing fact first noted by Redlich,[23] and more recently by Kapusta and Mekjian.[11] In other words, as the plasma hadronizes the large amount of entropy originally carried by thermal gluons and light quarks goes into extra pions; this results in a K/π ratio which, to our surprise, turns out to be less than that of an ideal gas of pions and kaons in equilibrium at the transition temperature.

This means that inclusion of chemical reactions will lead to overall production, rather than annihilation, of $s\bar{s}$ pairs during the mixed phase. In fact, the mixed phase is where all significant chemical production takes place. For one thing, the mixed phase is long-lived because the large entropy of the plasma has to be taken up by expansion as conversion to the low-entropy hadron gas takes place. For another, our estimate of the chemical reaction rate in the hadron gas implies that the K/π ratio will change very slowly after completion of the hadronization transition. We find an eventual value of the K/π ratio significantly larger than that observed in high-energy pp collisions. This result is not strongly dependent on the hypothesis of chemical equilibrium in the initial plasma.

We discuss the significance of our results in Sec. IV. The enhanced strangeness signal unfortunately does not prove that a plasma was formed, since a high-temperature hadron gas in equilibrium will yield a still larger K/π ratio. Nevertheless, any effects of equilibration of the strangeness abundance point toward a long lifetime for the high-temperature fluid in the central region, which is indirect evidence of plasma formation. Various effects which will dilute the K/π ratio, such as resonance production and entropy generation through viscosity, have yet to be studied.

II. SCALING HYDRODYNAMICS WITH THE RATE EQUATION

In this section we discuss the equations of relativistic hydrodynamics and the incorporation of the chemical processes which change the flavor composition of the expanding central region. We review the basic formalism obtained in I and Bjorken's one-dimensional scaling ansatz in the context of the early, plasma phase. We then discuss the chemical kinetics of the hadron gas which will be created later by the hadronization of the plasma. Finally we set forth our treatment regarding the evolution of the mixed phase which exists while the system undergoes a first-order confinement phase transition.

A. Evolution in the plasma phase

We discussed in I the evolution of the quark-gluon plasma when it is in local equilibrium except for the relatively slow chemical processes which change the number of s (and $\bar{s}$) quarks. Semiclassical kinetic theory implies that the system is governed by the hydrodynamic equations

$$\partial_\mu T^{\mu\nu}(x)=0 , \tag{2.1}$$

$$\partial_\nu n_s^\nu(x)=\partial_\nu n_{\bar{s}}^\nu(x)=R_{\text{gain}}-R_{\text{loss}} . \tag{2.2}$$

In the absence of dissipation, the energy-momentum tensor is written in terms of the proper energy density $\epsilon(x)$, the pressure $p(x)$, and the local flow velocity $u^\mu(x)$ as

$$T^{\mu\nu}=-g^{\mu\nu}p(x)+[\epsilon(x)+p(x)]u^\mu(x)u^\nu(x) , \tag{2.3}$$

and the s-quark current is given in terms of the proper s-quark density $n_s(x)$ as

$$n_s^\mu=n_s(x)u^\mu(x) . \tag{2.4}$$

The two terms on the right-hand side of (2.2) represent, respectively, the production and annihilation rates of $s\bar{s}$ pairs per unit volume. We have assumed that the net baryon-number density and the net strangeness density, $n_s(x)-n_{\bar{s}}(x)$, are always zero.

We calculated in I the reaction rates on the right-hand side of (2.2) as functions of the temperature $T=1/\beta$ and the strange-quark chemical potential $\mu_s=\mu_{\bar{s}}\equiv\mu$. Taking into account only the binary processes $g+g\rightleftarrows s+\bar{s}$ and $q+\bar{q}\rightleftarrows s+\bar{s}$ (where $q=u$ or d), the result is

$$R_{\text{gain}}-R_{\text{loss}}=(e^{-2\beta\mu}-1)I(T,\mu) , \tag{2.5}$$

where the reduced collision integral $I(T,\mu)$ is the sum of contributions from gluon and quark-antiquark processes:

$$I(T,\mu)=I_{\text{gluon}}+I_{\text{quark}} , \tag{2.6}$$

with

$$I_{\text{gluon}}=\frac{1}{2}\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,|\mathcal{M}_{gg\rightleftarrows s\bar{s}}|^2$$
$$\times f_g(p_1)f_g(p_2)f_s(p_3)f_{\bar{s}}(p_4)\exp[\beta(E_1+E_2)]\ , \tag{2.7a}$$

$$I_{\text{quark}}=\int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,|\mathcal{M}_{q\bar{q}\rightleftarrows s\bar{s}}|^2$$
$$\times f_q(p_1)f_{\bar{q}}(p_2)f_s(p_3)f_{\bar{s}}(p_4)\exp[\beta(E_1+E_2)]\ . \tag{2.7b}$$

Since I is positive definite, Eq. (2.5) shows explicitly that when $\mu>0$, indicating an excess of s quarks, we have $s\bar{s}$ annihilation, while when $\mu<0$, signifying a deficiency, we have production. Detailed balance of the two inverse processes is achieved when μ vanishes.

Contracting (2.1) with the fluid four-velocity gives

$$\begin{aligned}0=u_\nu\partial_\mu T^{\mu\nu}&=u_\nu\partial^\nu\epsilon+(\epsilon+p)\partial_\nu u^\nu\\&=T\partial_\nu(\sigma u^\nu)+2\mu\partial_\nu(n_s u^\nu)\ ,\end{aligned} \tag{2.8}$$

where in deriving the second line we have used the thermodynamic relations

$$\begin{aligned}\epsilon+p&=T\sigma+\mu_s n_s+\mu_{\bar{s}}n_{\bar{s}}=T\sigma+2\mu n_s\ ,\\ d\epsilon&=T\,d\sigma+2\mu\,dn_s\ .\end{aligned} \tag{2.9}$$

With the rate equations (2.2) and (2.5), we derive an equation for the entropy current:

$$\partial_\nu(\sigma u^\nu)=-2\beta\mu(e^{-2\beta\mu}-1)I(T,\mu)\ . \tag{2.10}$$

The right-hand side of this equation is always positive, in accordance with the second law of thermodynamics; when the system is in chemical equilibrium, namely, $\mu=0$, entropy is conserved.

Bjorken's scaling ansatz presumes invariance under Lorentz boosts in the beam ($\hat{z}$) direction, and demands that all local thermodynamic quantities be functions only of the proper time $\tau=\sqrt{t^2-z^2}$. The fluid four-velocity is taken to be

$$u^\nu(x)\equiv(\gamma,\gamma\mathbf{v})=(t/\tau,0,0,z/\tau)\ , \tag{2.11}$$

so that $v_z=z/t$ and the fluid rapidity is given by

$$y\equiv\operatorname{arctanh}(v_z)=\frac{1}{2}\ln\left|\frac{t+z}{t-z}\right|\ . \tag{2.12}$$

Other choices for the fluid rapidity, related to (2.12) by $y'=y+f(\tau)$ for any function f, are also allowed by Lorentz invariance. Our choice is motivated by the inside-outside cascade picture[24] for the underlying dynamics of particle formation.

Using $u_\nu\partial^\nu\tau=1$ and $\partial_\nu u^\nu=1/\tau$, we readily find

$$\frac{d\sigma}{d\tau}+\frac{\sigma}{\tau}=-2\beta\mu(e^{-2\beta\mu}-1)I(T,\mu)\ , \tag{2.13a}$$

$$\frac{dn_s}{d\tau}+\frac{n_s}{\tau}=(e^{-2\beta\mu}-1)I(T,\mu)\ . \tag{2.13b}$$

Equations (2.13) are the basic equations for our analysis. Their physical meaning becomes clear if we rewrite them as

$$\frac{d(\sigma\tau)}{d\tau}=-2\tau\beta\mu(e^{-2\beta\mu}-1)I(T,\mu)\ , \tag{2.14a}$$

$$\frac{d(n_s\tau)}{d\tau}=\tau(e^{-2\beta\mu}-1)I(T,\mu)\ . \tag{2.14b}$$

These equations give the rates of change of the entropy and the s-quark number contained in an expanding volume $V=V_0\times(\tau/\tau_0)$.

We note here that $\sigma\tau$ and $n_s\tau$ are also proportional to the entropy and the s-quark number *per unit rapidity*. To see this we calculate the entropy and the number of s quarks contained in a cylindrical volume at a given time t, which are given, respectively, by

$$S(t)=\int dz\int d^2r\,\sigma(t,z)u^0=\int dy\,\pi R^2\sigma\tau\ , \tag{2.15a}$$

$$N_s(t)=\int dz\int d^2r\,n_s(t,z)u^0=\int dy\,\pi R^2 n_s\tau\ , \tag{2.15b}$$

where we have used the relation $(dy/dz)_t=t/\tau^2$ and fixed the transverse radius of the cylinder at the initial radius R of the colliding nuclei, neglecting the transverse expansion of the system. From this we find

$$\frac{dS}{dy}=\pi R^2\sigma\tau,\quad \frac{dN_s}{dy}=\pi R^2 n_s\tau\ . \tag{2.16}$$

In order to solve the coupled differential equations (2.13), we need information from the equation of state, which gives n_s and σ as functions of T and μ. We employ, both for the plasma and, below, for the hadron gas, an ideal gas equation of state. We note that use of the semiclassical kinetic equation usually leads to an ideal gas approximation to the thermodynamic quantities which appear in the hydrodynamic equations.[10] This is because kinetic theory treats the particles as free particles between collisions; interactions are taken into account only in collisions, where particles change momenta and internal quantum numbers. The ideal gas approximation for the equation of state is thus inherent in all Boltzmann-type kinetic theories,[25] and systematic improvement would require development of a more elaborate quantum transport theory. This is far beyond the scope of the present study.

The entropy density σ_a of a relativistic ideal gas is given by

$$\sigma_a = -\int \frac{d^3p}{(2\pi)^3}\{f_a(p)\ln f_a(p) \mp [1\pm f_a(p)]\ln[1\pm f_a(p)]\} , \tag{2.17}$$

where

$$f_a(p) = [\exp(\beta\sqrt{p^2+m_a{}^2} - \beta\mu_a) \pm 1]^{-1} \tag{2.18}$$

is the thermal distribution function of species a with mass m_a and chemical potential μ_a. In the above expressions the upper sign refers to bosons and the lower sign to fermions. For light quarks and gluons, the masses and chemical potentials of which vanish, the integral in (2.17) yields the simple formula

$$\sigma_q + \sigma_g = 4(\gamma_g + \tfrac{7}{8}\gamma_q)\frac{\pi^2}{90}T^3 , \tag{2.19}$$

where $\gamma_g = 2\times 8\times 1\times 1$ and $\gamma_q = 2\times 3\times 2\times 2$ are the products of spin, color, isospin, and particle-antiparticle degeneracy factors. The entropy σ_s carried by the massive s and $\bar{s}$ quarks has to be evaluated numerically. The total entropy density σ is hence given, in the pure plasma phase, by the sum of these contributions:

$$\sigma(T,\mu) = \sigma_q(T) + \sigma_g(T) + \sigma_s(T,\mu) . \tag{2.20}$$

We note that the μ dependence of the entropy density comes only from σ_s and is therefore weak.

Similarly, the s-quark density is given in the ideal gas approximation by

$$n_s(T,\mu) = 6\int \frac{d^3p}{(2\pi)^3} f_s(p) , \tag{2.21}$$

which again can only be evaluated numerically.

With these relations between (σ, n_s) and (T,μ), it is now straightforward to solve (2.13) for $T(\tau)$ and $\mu(\tau)$ with given initial conditions $T(\tau_0)$ and $\mu(\tau_0)$. As we shall see later the system cools very rapidly in the plasma phase, essentially according to

$$T(\tau) = T(0)(\tau_0/\tau)^{1/3} . \tag{2.22}$$

Hence this pure plasma era lasts only for a short period.

B. The chemical reactions in the hadron gas

Once the temperature of the system reaches the transition temperature $T_{\rm tr}$, the expansion of the system takes place isothermally, gradually converting dense plasma into dilute hadron gas. We first examine the chemical processes which are active in the hadron gas component.

We will treat the hadron gas as being composed entirely of pions and kaons. We assume that the two species are in local thermal equilibrium with each other and with the plasma component of the system. Isospin invariance and particle-antiparticle symmetry imply that $\mu_{K^+} = \mu_{K^-} = \mu_{K^0} = \mu_{\bar{K}^0} \equiv \mu_K$ and that $\mu_{\pi^\pm} = \mu_{\pi^0} \equiv \mu_\pi$.

To estimate the reaction rates in the hadron gas we assume that changes in the kaon number are due predominantly to the following binary processes:

$$\begin{aligned} &\pi^+ + \pi^- \rightleftarrows K^+ + K^- , \\ &\pi^+ + \pi^- \rightleftarrows K^0 + \bar{K}^0 , \\ &\pi^0 + \pi^0 \rightleftarrows K^+ + K^- , \\ &\pi^0 + \pi^0 \rightleftarrows K^0 + \bar{K}^0 , \\ &\pi^+ + \pi^0 \rightleftarrows K^+ + \bar{K}^0 , \\ &\pi^- + \pi^0 \rightleftarrows K^- + K^0 . \end{aligned} \tag{2.23}$$

The gain and loss rates for these reaction processes can be calculated as for the chemical processes in the plasma. The sum of the gain and loss terms for each process is written in terms of a reduced collision integral as in (2.5) with μ_K taking the place of μ on the right-hand side. (We set $\mu_\pi = 0$, as explained in Sec. II C below.) For example, for the first process in (2.23),

$$R_{\pi^+\pi^-\to K^+K^-} - R_{K^+K^-\to\pi^+\pi^-} = (e^{-2\beta\mu_K} - 1)I_{\pi^+\pi^-\rightleftarrows K^+K^-} , \tag{2.24}$$

where the reduced collision integral is defined by

$$I_{\pi^+\pi^-\rightleftarrows K^+K^-} \equiv \int\frac{d^3p_1}{(2\pi)^3 2E_1}\int\frac{d^3p_2}{(2\pi)^3 2E_2}\int\frac{d^3p_3}{(2\pi)^3 2E_3}\int\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,|\mathcal{M}_{\pi^+\pi^-\rightleftarrows K^-K^+}|^2 \times f_\pi(p_1)f_\pi(p_2)f_K(p_3)f_K(p_4)\exp[\beta(E_1+E_2)] . \tag{2.25}$$

The other processes in (2.23) are governed by similar expressions. Since the chemical potentials of all kaons involved in these processes are equal, the total reduced collision integral for all the hadronic processes may be expressed as

$$R_{\pi\pi\to K\bar{K}} - R_{K\bar{K}\to\pi\pi} = (e^{-2\beta\mu_K} - 1)I_H , \tag{2.26}$$

where $I_H(T,\mu_K)$ is the sum of collision integrals for all processes (2.23).

In the absence of any solid knowledge of the hadronic matrix element in (2.25), we shall take a more phenomenological approach to the reduced collision integral. First we approximate the pion and kaon distribution functions in (2.25) by classical Boltzmann distributions, viz.,

$$\begin{aligned} f_\pi(p) &= e^{-\beta(p^2+m_\pi{}^2)^{1/2}} , \\ f_K(p) &= e^{\beta\mu_K} e^{-\beta(p^2+m_K{}^2)^{1/2}} . \end{aligned} \tag{2.27}$$

Then (2.25) reduces to the classical expression (see I)

$$I_{\pi^+\pi^-\rightleftarrows K^+K^-}=e^{2\beta\mu_K}\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}f_\pi(p_1)f_\pi(p_2)\times\sigma_{\pi^+\pi^-\to K^+K^-}v_{12}\ . \tag{2.28}$$

Approximating σv_{12} by its average in the relevant energy regime, we get

$$I_{\pi^+\pi^-\rightleftarrows K^+K^-}\simeq e^{2\beta\mu_K}\left[\frac{n_\pi}{3}\right]^2\langle\sigma_{\pi^+\pi^-\to K^+K^-}v_{12}\rangle\ , \tag{2.29}$$

where n_π is the total pion density (i.e., summed over isospin). An alternative reduction of (2.25) gives

$$I_{\pi^+\pi^-\rightleftarrows K^+K^-}=\int\frac{d^3p_1}{(2\pi)^3}\int\frac{d^3p_2}{(2\pi)^3}f_K(p_1)f_K(p_2)\times\sigma_{K^+K^-\to\pi^+\pi^-}v_{12}$$
$$\simeq\left[\frac{n_K}{4}\right]^2\langle\sigma_{K^+K^-\to\pi^+\pi^-}v_{12}\rangle\ , \tag{2.30}$$

with $n_K\equiv n_{K^+}+n_{K^-}+n_{K^0}+n_{\bar K^0}$. The total reduced collision integral (2.28) can thus be approximated either by

$$I_H\simeq\tfrac{2}{3}e^{2\beta\mu_K}n_\pi^{\,2}\langle\sigma_{\pi^+\pi^-\to K^+K^-}v_{12}\rangle \tag{2.31a}$$

or by

$$I_H\simeq\tfrac{3}{8}n_K^{\,2}\langle\sigma_{K^+K^-\to\pi^+\pi^-}v_{12}\rangle\ , \tag{2.31b}$$

where we have assumed that the collision integrals for all processes (2.23) are roughly equal. In the following analysis, we will use the formula (2.31b), with the kaon density n_K computed from the ideal gas expression

$$n_K(T,\mu_K)=4\int\frac{d^3p}{(2\pi)^3}f_K(p)\ , \tag{2.32}$$

where f_K is the Bose-Einstein distribution. We will discuss estimates for the hadronic cross section in Sec. III.

C. Evolution in the mixed phase

We are now able to give a detailed prescription for describing the mixed phase. The basic idea is to use the Maxwell construction both for the equation of state and for the chemical reaction rates, averaging over the plasma component and the hadron gas component according to their respective volumes.

Let $f(\tau)$ be the fraction of the volume occupied by plasma at time τ. We define τ_Q to be the time when the temperature reaches the transition temperature $T_{\rm tr}$ and hadronization begins, and τ_H to be the time when the hadronization transition is completed. Thus by definition

$$f(\tau)=1\quad\text{if }\tau\le\tau_Q$$
$$=0\quad\text{if }\tau\ge\tau_H\ . \tag{2.33}$$

For $\tau_Q\le\tau\le\tau_H$, $f(\tau)$ should decrease monotonically from 1 to 0. We write the entropy density and the s-quark density in the mixed phase as

$$\sigma=f\sigma_Q+(1-f)\sigma_H\ , \tag{2.34}$$

$$n_s=fn_Q+(1-f)n_H\ . \tag{2.35}$$

Here σ_Q and n_Q are the entropy density and the s-quark density in the plasma component as given by (2.20) and (2.21) at temperature $T_{\rm tr}$,

$$\sigma_Q=\sigma_q(T_{\rm tr})+\sigma_g(T_{\rm tr})+\sigma_s(T_{\rm tr},\mu)\ , \tag{2.36}$$

$$n_Q=n_s(T_{\rm tr},\mu)\ , \tag{2.37}$$

and n_H and σ_H are the s-quark density and the entropy density of the hadron gas:

$$n_H=n_{K^-}(T_{\rm tr},\mu_K)+n_{\bar K^0}(T_{\rm tr},\mu_K)\ , \tag{2.38}$$

$$\sigma_H=\sigma_\pi(T_{\rm tr},\mu_\pi)+\sigma_K(T_{\rm tr},\mu_K)\ , \tag{2.39}$$

where σ_π and σ_K are the entropy densities carried by pions and kaons, respectively. As for the plasma phase, we will use the ideal gas relation for the entropy and kaon number density of the hadron gas.

In order to obtain a closed system of equations, the chemical potentials in the hadron gas must be related to those in the plasma. To do this we postulate a very fast hadronization process which converts the unconfined quarks and gluons into hadrons *without* creating or annihilating $s\bar s$ pairs. Examples of relevant reactions are

$$\pi^+\rightleftarrows u+\bar d\pm X \tag{2.40}$$

and

$$K^+\rightleftarrows u+\bar s\pm X\ , \tag{2.41}$$

where X refers to extra gluons, light $q\bar q$ pairs, or collective plasma excitations (e.g., color plasmons) involved in the reactions. We assume that the processes in which X contains extra $s\bar s$ pairs are negligible. If we demand that local detailed balance with respect to (2.40) and (2.41) be maintained in the mixed phase, then the pion and kaon chemical potentials must be related to the quark and gluon chemical potentials by relations such as

$$\mu_{\pi^+}=\mu_u+\mu_{\bar d}\pm\mu_X\ , \tag{2.42}$$

$$\mu_{K^+}=\mu_u+\mu_{\bar s}\pm\mu_X\ , \tag{2.43}$$

where μ_X is the sum of the chemical potentials of the extra gluons and $q\bar q$ pairs. Since the light-quark and gluon chemical potentials vanish, it immediately follows that $\mu_X=0$ and hence that

$$\mu_\pi=0 \tag{2.44}$$

(as promised) and

$$\mu_K=\mu_s=\mu\ . \tag{2.45}$$

Inserting these conditions into (2.38) and (2.39) we express all thermodynamic quantities in the mixed phase as functions of the plasma volume fraction f and the s-quark chemical potential μ.

The chemical reaction rate in the mixed phase is also given by the average of that in the plasma and that in the hadron gas. The hadronization processes themselves preserve the strange-quark number, as we assumed, and hence contribute nothing. We have written the chemical reaction rates both in the plasma and in the hadron gas in terms of reduced collision integrals [see (2.5) and (2.26)] using the same chemical potential μ [according to (2.45)]. Thus we can proceed with the calculation in the mixed phase by simply writing the collision integral I which appears on the right-hand side of (2.13) as

$$I = f I_Q + (1-f) I_H \quad \text{(mixed phase)} . \tag{2.46}$$

Here I_Q is the reduced collision integral in the plasma, given by (2.6), while I_H is its counterpart in the hadron gas, given by (2.31). We are now able to determine the evolution of the system in the mixed phase by solving (2.13).

III. NUMERICAL RESULTS AND DISCUSSION

In this section we present numerical solutions to the hydrodynamic equations (2.13). After a discussion of initial conditions, we show the evolution of the system in the absence of the flavor-changing chemical reactions in order to provide a background for their inclusion. We then proceed to include the chemical reactions in the plasma and in the hadron gas.

A. Initial conditions

The initial conditions consist of three parameters: the proper time τ_0 for the hyperbola $\tau=\tau_0$ where the initial conditions are imposed, and the temperature T_0, and the s-quark chemical potential μ_0 on this hyperbola. We first discuss constraints on these parameters stemming from the uncertainty principle and from the observed multiplicity of secondaries in cosmic-ray events.

One constraint on T_0 and τ_0 follows from the uncertainty principle:

$$T_0\tau_0 \gtrsim 1 . \tag{3.1}$$

This relation follows from the assumption that the thermal fluctuations cannot be smaller than the quantum fluctuations in the initial state; since this is only a rough statement, we do not choose to fix the right-hand side of (3.1) more precisely. Another constraint comes from the empirical knowledge of dN/dy in cosmic-ray events. The final total multiplicity is related to the total entropy at the time that the particle number is frozen out by

$$\frac{dN}{dy} \simeq \frac{1}{4}\left(\frac{dS}{dy}\right)_f . \tag{3.2}$$

As seen from (2.14a) and (2.15a), the entropy per unit rapidity $dS/dy \simeq \pi R^2 \sigma\tau$ is roughly a constant of motion in the scaling hydrodynamic expansion. This implies that τ_0 and T_0 have to satisfy[26]

$$\sigma(T_0,\mu_0)\tau_0 \simeq 4\frac{dN/dy}{\pi R^2} \tag{3.3}$$

in order to reproduce a given final particle multiplicity dN/dy. For $T_0 \gg m_s$, we assume the massless ideal gas relation $\sigma_0 \simeq a T_0{}^3$, with $a = 4(47.5)(\pi^2/90) \doteq 20.8$ for three massless flavors [see (2.19)]. Thus (3.3) becomes

$$T_0{}^3\tau_0 \simeq \frac{4}{a}\frac{dN/dy}{\pi R^2} . \tag{3.4}$$

High-energy cosmic-ray events yield the empirical relation[27]

$$\frac{dN/dy}{\pi R^2} \simeq \frac{3}{2}\frac{dN_{\rm ch}/dy}{\pi R^2} \simeq 2.5 A^{1/3}\ {\rm fm}^{-2} , \tag{3.5}$$

whence

$$T_0{}^3\tau_0 \simeq 0.5 A^{1/3}\ {\rm fm}^{-2} . \tag{3.6}$$

Combining (3.1) and (3.6) gives independent constraints

$$T_0 \lesssim 140 A^{1/6}\ {\rm MeV} , \tag{3.7}$$

$$\tau_0 \gtrsim 1.4 A^{-1/6}\ {\rm fm} . \tag{3.8}$$

It is interesting to note that an $A^{\pm 1/6}$ dependence of these initial conditions is predicted by a flux-tube model with a random walk ansatz for the color charging mechanism.[9] For central collisions of lead nuclei the inequalities (3.7) and (3.8) give the upper bound $T_0 = 340$ MeV for the initial plasma temperature and the lower bound $\tau_0 = 0.6$ fm for the time when the hydrodynamic expansion starts. In light of the uncertainty in (3.1), these bounds might be relaxed by a factor of 2.

We argued in the Introduction that if the initial temperature of the plasma is sufficiently high compared to the s-quark mass in the plasma ($m_s \simeq 150$ MeV), then the initial flavor composition should be close to the equilibrium composition. Therefore we set (to begin with) $\mu_0 = 0$ which corresponds to complete chemical equilibrium in the initial state. We will vary μ_0 later to see how the final results depend on this hypothesis.

B. Evolution in the absence of chemical reactions

We first present results obtained by setting $I = 0$ in (2.13) in order to show how the system deviates from chemical equilibrium when the chemical reaction processes which change the s-quark number in the system are absent. In this case both the total entropy and the number of s quarks are conserved: σ and n_s decrease as $1/\tau$ and hence dS/dy and dN_s/dy are constant. The time evolution of the temperature T, the plasma fraction f, and the s-quark chemical potential μ are plotted in Fig. 2 as functions of τ/τ_0. The hydrodynamic equations with $I = 0$ are invariant under the scale transformation $\tau \to \lambda\tau$, which implies that the time scale for the expansion is set simply by the initial time τ_0. We set $T_0 = 340$ MeV and $\mu_0 = 0$ at $\tau = \tau_0$ and choose several different values for the transition temperature $T_{\rm tr}$. The s-quark mass is fixed at $m_s = 150$ MeV.

In the pure plasma phase ($\tau_0 \le \tau \le \tau_Q$), the temperature of the system decreases very rapidly, reaching $T_{\rm tr}$ at τ_Q. The simple formula

$$T(\tau) = T(0)(\tau_0/\tau)^{1/3} \tag{3.9}$$

obtained for the massless ideal gas in equilibrium ($\sigma = aT^3$) turns out to fit very well in spite of the inclusion of the massive s quarks with their nonzero chemical potential. Hence τ_Q is approximately determined by $\tau_Q = \tau_0 (T_0/T_{tr})^3$.

In the mixed phase, the temperature of the system stays constant while the plasma fraction in the system decreases from 1 to 0. Upon completion of the phase transition at $\tau = \tau_H$, the system starts to cool again in the pure hadron gas phase. Since $\sigma \propto \tau^{-1}$ at all times, we have

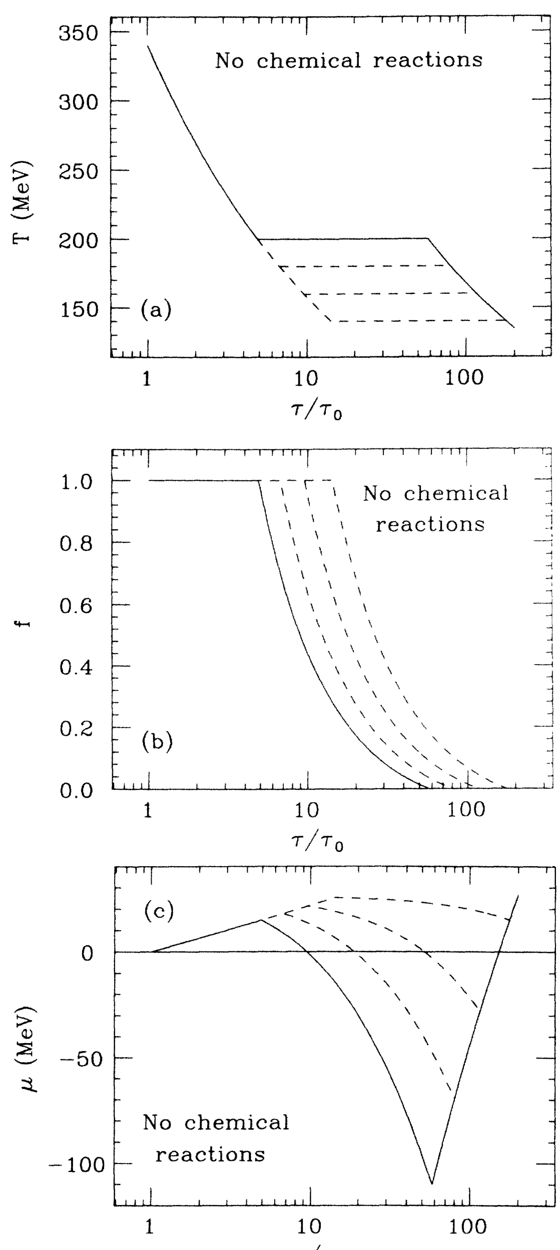

FIG. 2. Time evolution of (a) the temperature T, (b) the volume fraction f of plasma, and (c) the s-quark chemical potential μ, in the absence of flavor-changing chemical reactions. The initial conditions are $T_0 = 340$ MeV and $\mu_0 = 0$ at $\tau = \tau_0$. The solid curve is for the transition temperature $T_{tr} = 200$ MeV, while the dashed curves are for $T_{tr} = 180$ MeV, 160 MeV, and 140 MeV. [The sequence of curves is top to bottom in Fig. 2(a), and left to right in Figs. 2(b) and 2(c).]

$$\frac{\tau_H}{\tau_Q} = \frac{\sigma(\tau = \tau_Q)}{\sigma(\tau = \tau_H)} . \tag{3.10}$$

We can see from the figures that $\tau_H/\tau_Q \simeq 10$, reflecting the large entropy of the plasma compared to the hadron gas at the transition temperature.

The time evolution of μ shows how the system deviates from complete chemical equilibrium because of cooling and hadronization. Initially, as the plasma cools, μ increases. This increase turns into a rapid decrease as the system enters the mixed phase at $\tau = \tau_Q$ and μ eventually becomes negative when $T_{tr} > 150$ MeV. The chemical potential reaches its minimum when the phase transition is completed at $\tau = \tau_H$ and from this time on it again increases monotonically.

To understand this behavior, recall that positive μ means an excess of s quarks in the system compared to the number in the chemical equilibrium, and negative μ means a shortage. The initial excess of s quarks arises because of the falling temperature: The equilibrium s-quark density $n_s^{eq}(T)$ decreases exponentially, because of the finite s-quark mass, while the entropy density of the system, dominated by the massless particles, decreases only by the power law $\sigma \propto T^3 \propto 1/\tau$. Since n_s/σ is conserved in the absence of chemical reactions, this leads to the excess of s quarks in the expanding plasma. This behavior is repeated later, in the evolution of the pure hadron gas.

The decrease in μ in the mixed phase, culminating in a shortage of s quarks, is rather puzzling at first sight. Indeed, the shortage peaks at $\tau = \tau_H$ when the system is entirely a gas of pions and kaons. This looks peculiar since one would think that the plasma is strangeness rich compared to the hadron gas because of the small s-quark mass in the plasma.

Why is this so? The crucial quantity in this calculation is not n_s itself but the ratio n_s/σ of the s-quark density to the *total* entropy density. In the absence of chemical reactions, this ratio is conserved during the expansion and the phase transition. One finds that in the equilibrium plasma this ratio is much *smaller* than in the equilibrium gas of pions and kaons at the same temperature for $T_{tr} > 150$ MeV (see Table I; this important fact was noted by Redlich,[23] and Kapusta and Mekjian[11]). There are two reasons for this. One is the quark degeneracy factor: In the plasma, there are twice as many light-quark states as s-quark states, while in the hadron gas there are four kaon states to the three pion states. The more important reason is that the plasma carries a lot of entropy in the form of thermal gluon excitations, while the color degrees of freedom are frozen in the hadron gas phase. (To see the importance of the two effects we show in Table I the n_s/σ ratio in the plasma *excluding* the gluon entropy.) The exponential Boltzmann factor due to the K-π mass difference does *not* dominate the n_s/σ ratio for $T \geq m_s$.

Since the entropy is conserved during the hadronization transition, the entropy lost through the confinement of the gluons and quarks must be regained by the production of large numbers of mesons. One of our assumptions concerning the phase transition is that the number of K's produced is determined entirely by the number of strange quarks present in the plasma. Thus the extra mesons can

TABLE I. Equilibrium n_s/σ ratio in plasma and hadron gas. n_s in the hadron gas is defined by the sum of the number densities of K^- and $\bar{K}^0$.

T (MeV)	200	180	160	140
Plasma	0.0241	0.0236	0.0230	0.0221
(without gluons)	(0.0366)	(0.0359)	(0.0350)	(0.0338)
Hadron gas	0.0378	0.0337	0.0290	0.0236

only be pions. This causes a significant dilution of the K/π ratio, which comes out *smaller* than the equilibrium hadron gas value: The conservation of entropy via pion production brings about a shortage of kaons in the hadron gas.

This result, though obtained in the absence of the chemical reactions, has important implications: Whatever the rate of the chemical reactions, *their inclusion should result in overall production, rather than annihilation, of $s\bar{s}$ pairs in the course of the expansion.*

C. Inclusion of chemical reactions

Now we switch on the chemical reactions which create and annihilate $s\bar{s}$ pairs and kaon pairs, and see how the above results change. In the plasma, we evaluate the collision integrals (2.7) via QCD perturbation theory, as described in I. There is ambiguity in the choice of the running coupling $\alpha_s(Q^2)$, since the momentum transfer Q^2 depends on which diagram one uses for its definition.[28] We take Q^2 to be the average s-channel Q^2 at the temperature T. This may be evaluated roughly as

$$\langle s\rangle \simeq (2\langle\sqrt{m_s{}^2+q^2}\rangle)^2 \simeq 4(m_s+\tfrac{3}{2}T)^2\ , \tag{3.11}$$

where $\langle\sqrt{m_s{}^2+q^2}\rangle$ is the average s-quark energy and in evaluating it we have used a nonrelativistic approximation. Hence we run the coupling constant according to

$$\alpha_s(T)=\frac{2\pi}{9\ln[(2m_s+3T)/\Lambda]}\ . \tag{3.12}$$

For $\Lambda=150$ MeV this choice gives $\alpha_s=0.40$ at the transition temperature $T_{\rm tr}=200$ MeV, while at $T=300$ MeV it gives $\alpha_s=0.34$. We could just fix α_s at one of these values, since its variation is a higher-order effect, beyond the precision of our tree-level calculation; nevertheless, we will let it run with T. The running of the strange-quark mass m_s is likewise an $O(\alpha_s)$ correction, and we neglect it.

The reaction rates in the hadron gas are determined by the average empirical cross section $\sigma_{K^+K^-\to\pi^+\pi^-}$ [see Eq. (2.31)]. A rough estimate of this cross section may be obtained from the simple additive quark model as follows. The measured $p\bar{p}$ total cross section, $\sigma_{p\bar{p}}(s>50\ \text{GeV}^2)=40$ mb, gives for the elementary $q\bar{q}$ scattering cross section

$$\sigma_{q\bar{q}}=(\tfrac{1}{3})^2\sigma_{p\bar{p}}\simeq 4\ \text{mb}\ \ (\text{light quarks})\ . \tag{3.13}$$

From the Kp cross section, $\sigma_{Kp}=20$ mb, we deduce the cross section for scattering of strange antiquarks:

$$\sigma_{\bar{s}q}=\tfrac{1}{3}\sigma_{Kp}-\sigma_{qq}\simeq 2\ \text{mb}\ . \tag{3.14}$$

Assuming factorization

$$\sigma_{\bar{s}s}=\frac{\sigma_{\bar{s}q}}{\sigma_{qq}}\sigma_{\bar{s}q} \tag{3.15}$$

gives a *total* $s\bar{s}$ cross section of 1.5 mb. To obtain the cross section for

$$K^+(u\bar{s})+K^-(s\bar{u})\to\pi^+(u\bar{d})+\pi^-(d\bar{u})$$

we note that the s and $\bar{s}$ must annihilate into d and $\bar{d}$. Thus the total $s\bar{s}$ cross section of 1.5 mb gives an *upper bound* for this process. Given the rough nature of quark additivity, we will take this value as the most probable one, and vary it to see its implication for our results.[29]

The relaxation times to achieve chemical equilibrium in the plasma and in the hadron gas may be computed from the formula

$$\tau_{\rm rel}=\left[\frac{n_s}{2I}\right]_{\mu=0} \tag{3.16}$$

(see I) and the result is shown in Fig. 3. The relaxation

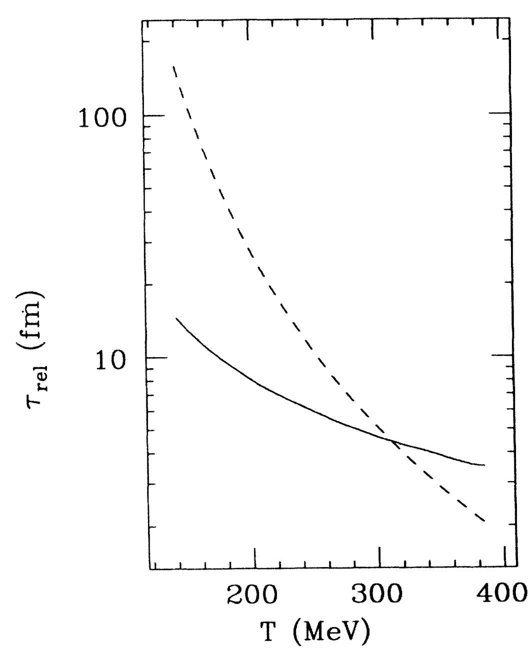

FIG. 3. Relaxation time to chemical equilibrium [defined by (3.15)] in the plasma (solid curve) and in the hadron gas (dashed curve). The hadron gas relaxation time is computed using the formula (2.31b) with $\sigma_{KK\to\pi\pi}=1.5$ mb.

time in the hadron gas is 3–10 times as long as that in the plasma in the relevant temperature range. The flavor-changing chemical reaction processes are more active in the plasma than in the hadron gas.

We first note that the collision term breaks the invariance of the hydrodynamic equations under the scale transformation $\tau \rightarrow \lambda\tau$ and therefore the solution, even though plotted against τ/τ_0, now depends on the initial time τ_0 as well as on the initial temperature. Here we present the results only for $\tau_0 = 0.6$ fm which corresponds to the lower bound for central Pb-Pb collisions. For smaller τ_0 the system will deviate further from equilibrium and there will be less variation in n_s/σ. This is because the scaling expansion rate is still controlled by τ_0:

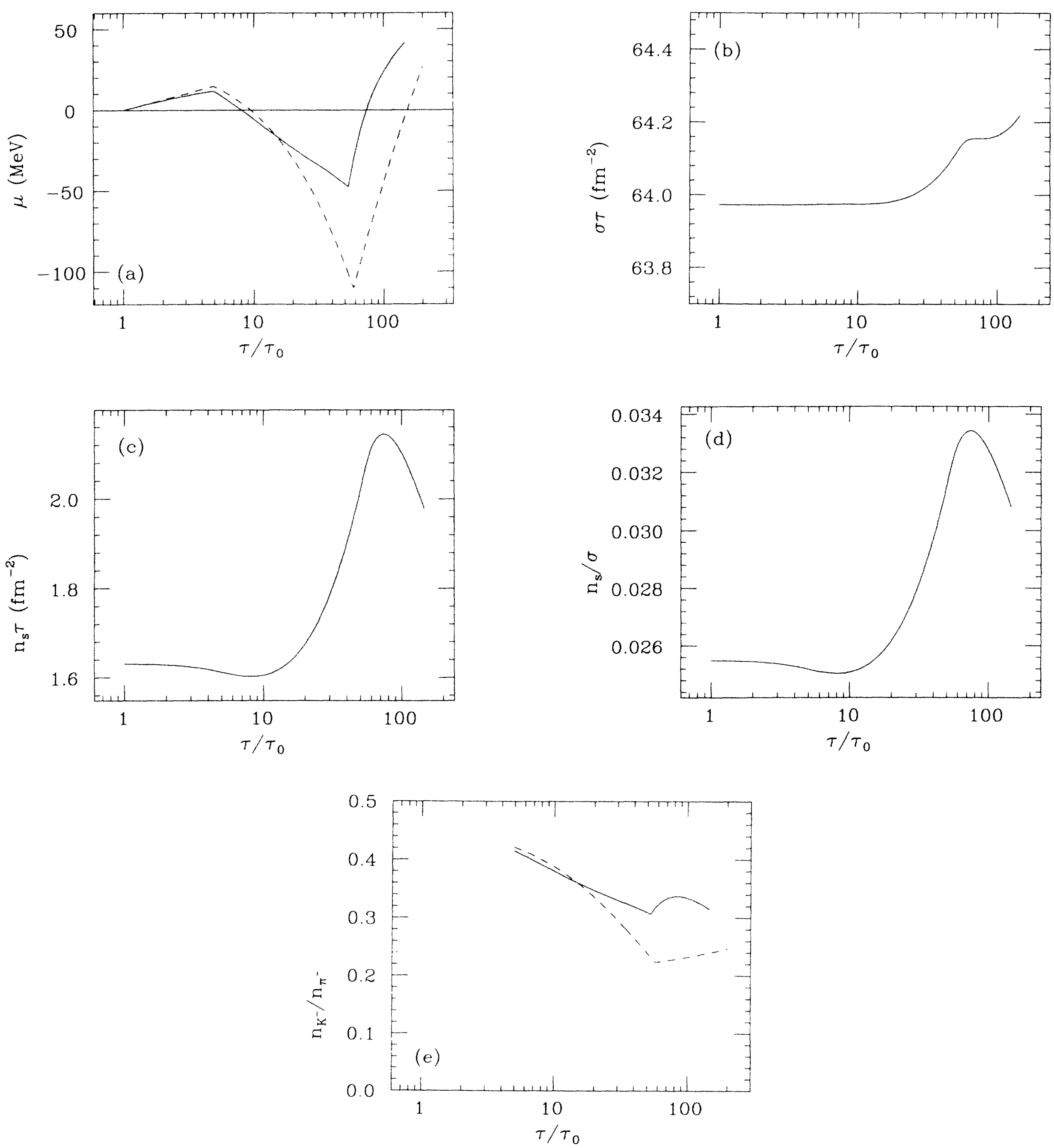

FIG. 4. Time evolution in the presence of chemical reactions in both the plasma and the hadron gas phases. Shown are (a) the chemical potential μ, (b) the scaled entropy density $\sigma\tau \propto dS/dy$, (c) the scaled strange-quark density $n_s\tau \propto dN_s/dy$, (d) the strangeness-to-entropy ratio n_s/σ, and (e) the K^-/π^- ratio in the hadron gas component. The initial conditions are $T_0 = 340$ MeV and $\mu_0 = 0$ at $\tau_0 = 0.6$ fm. The dashed curve in (a) shows evolution of the chemical potential for the case without chemical reactions, and is identical to the solid curve in Fig. 2(c). The dashed curve in (e) is also for the case without chemical reactions.

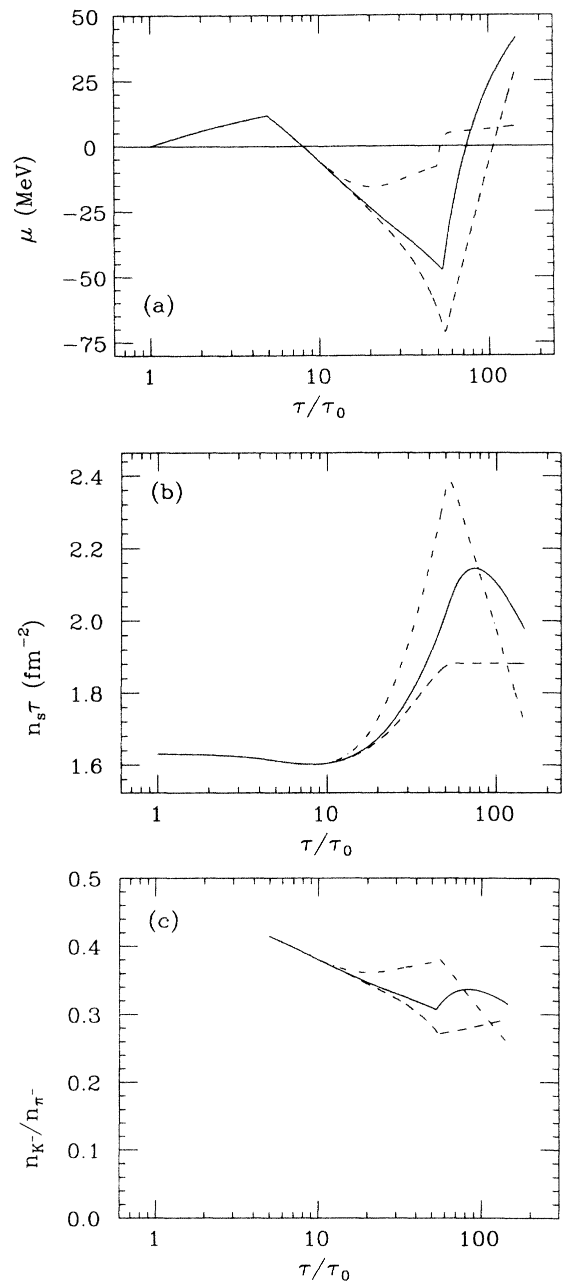

FIG. 5. Time evolution of (a) the chemical potential μ, (b) $n_s\tau \propto dN_s/dy$, and (c) n_{K^-}/n_{π^-} for the same conditions as Fig. 4, except that $\sigma_{KK\to\pi\pi}$ is varied: 0 (dashed), 1.5 mb (solid; same curves as in Fig. 4), 15 mb (dash-dotted).

the earlier the expansion starts, the faster the expansion, and hence the chemical processes have less time in which to act. We note that the hydrodynamic equations are invariant under the simultaneous transformations $\tau_0 \to \lambda\tau_0$ and $I \to \lambda^{-1} I$, so that halving τ_0 at fixed I has the same effect as halving the chemical reaction rate at fixed τ_0.

We present in in Fig. 4 the results of integrating the full hydrodynamic equations (2.13). As we have noted, the essential role of the chemical reactions is to bring the system toward chemical equilibrium. In the present case the system starts from chemical equilibrium and deviates from equilibrium because of the expansion; hence the chemical reactions compete with the expansion. This effect is clearly seen in Fig. 4(a) where we show the time evolution of the chemical potential: μ is pushed toward zero by the chemical reactions.

Figures 4(b) and 4(c) show the time evolution of $\sigma\tau \propto dS/dy$ and $n_s\tau \propto dN_s/dy$, respectively. When the right-hand sides of (2.14) are nonvanishing, neither quantity is conserved. The entropy production due to the chemical reactions is rather small, while dN_s/dy increases by about 20% by the end of the hadronization transition and by another 10% thereafter (the extremely slow fall off after the peak is presumably cut off by the transverse expansion of the system as we shall discuss later). In Fig. 4(d) we plot the time evolution of n_s/σ. It is seen that during the phase transition this ratio rises toward the equilibrium ratio of the hadron gas and reaches $n_s/\sigma \simeq 0.03$, which is between the plasma and hadron gas equilibrium values.

The time evolution of the K^-/π^- ratio in the *hadron gas component* of the system is plotted in Fig. 4(e). It is seen that this ratio starts from 0.4 and diminishes as the hadronization proceeds. To understand this behavior we first recall that in the present analysis the hadronized portion is always assumed to be in chemical equilibrium with the plasma, i.e., possessing the same chemical potential μ. Since μ is positive when the hadronization starts, the K^-/π^- ratio in the hadron gas is larger than the equilibrium ratio. This ratio eventually comes down, reflecting the overall shortage of strange particles characterized by the negative value of the chemical potential.

We compare in Fig. 5 the results obtained by turning off the chemical reactions *in the hadron gas*. It is clear that the hadronic chemical reactions, in spite of their small rates, are just as important as the plasma chemical reactions in increasing dN_s/dy. This is because (1) μ deviates from zero significantly only late in the mixed phase and (2) the time scale for expansion, $[d(\ln V)/d\tau]^{-1}$, grows as τ and is hence ten times as large at τ_H as at τ_Q. Thus the increase in dN_s/dy occurs mostly at the end of the mixed phase when the system consists mostly of hadron gas.

In view of the uncertainty in the hadronic reaction rates we also present in Fig. 5 the results of multiplying our assumed hadronic cross section by 10 (this large hadronic cross section is close to that used by Kapusta and Mekjian[11]). A very large hadronic reaction rate keeps the system very near chemical equilibrium [see Fig. 5(a)]. The K/π ratio [Fig. 5(c)] rises toward the equilibrium hadron gas value at τ_H, and then drops as the temperature drops. The smaller the hadronic reaction rate, the more indicative is the K/π ratio of plasma formation: The curve with zero KK cross section has the smallest ratio, reflecting the smaller value of n_s/σ in the plasma.

So far we have assumed complete chemical equilibrium in the initial state by setting $\mu_0=0$. The results of varying μ_0 are shown in Fig. 6. It is seen that the chemical potential ends up following the same curve whatever its initial value. Likewise $\sigma\tau$ and the K/π ratio end up within a

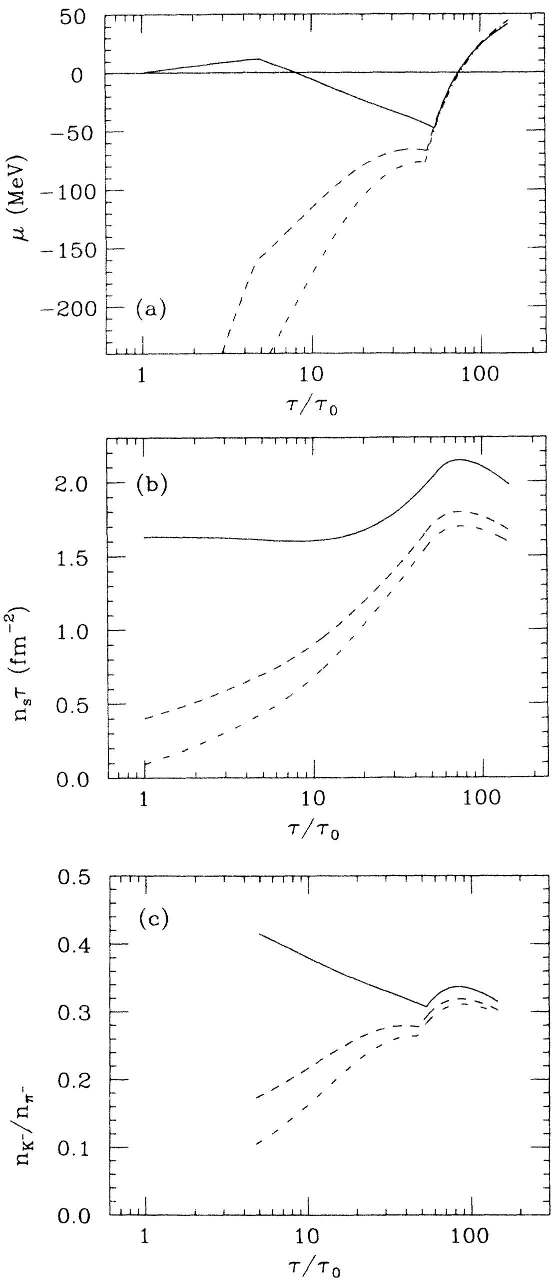

FIG. 6. As in Fig. 5, but with $\sigma_{KK\to\pi\pi}$ fixed at 1.5 mb and varying initial conditions: $\mu_0=0$ (solid; same curves as in Fig. 4), $\mu_0=-500$ MeV (dashed), $\mu_0=-1000$ MeV (dash-dotted).

narrow range of values. The same behavior is seen in the time evolution of n_s/σ. This is not very surprising because while the relaxation time in the plasma is comparable to the time scale of the longitudinal expansion, the lifetime of the mixed phase is perhaps ten times as long.

IV. SUMMARY AND CONCLUSIONS

In this paper we have studied the time evolution of the flavor composition of the dense hadronic matter which would be produced in the central rapidity region of ultrarelativistic heavy-ion collisions. We assumed that the hadronic matter is formed initially as a dense plasma of unconfined quarks and gluons *near chemical equilibrium* and evolves according to the scaling hydrodynamics through three distinct phases: namely, the pure plasma phase, the mixed phase, and the pure hadron gas phase.

We have extended Bjorken's hydrodynamic description of the space-time evolution of the central rapidity region so as to incorporate the kinetics of the relatively slow chemical processes which change the s-quark abundance in the system. Our hydrodynamic equations thus contain the rate equation for the strangeness current which relates the change in the strange-particle density to the chemical reaction rates. It was shown that in the Lorentz-invariant one-dimensional scaling expansion, these hydrodynamic equations are reduced to the entropy equation and the rate equation which couple through the collision integrals of the chemical reaction processes. In solving the hydrodynamic equations, we used the results of a perturbative QCD calculation of the chemical reaction rates in the quark-gluon plasma, whereas the chemical reaction rates in the hadron gas were estimated phenomenologically using the observed hadron cross sections. One crucial assumption we made in converting the plasma to hadron gas is that the hadronization is a very rapid process and conserves the abundance of $s\bar{s}$ pairs.

We found that the hadronization transition causes a shortage of strange particles in the system and as a result strange particles are *produced* by the chemical reactions in the course of the expansion. This somewhat surprising result was shown to be the consequence of the fact that the strangeness/entropy ratio is larger in the equilibrium hadron gas than in the plasma. It is interesting to note that the small strangeness/entropy ratio and the high chemical activity of the plasma have the same origin, namely, the high abundance of thermal gluon excitations, one of the characteristic properties of the quark-gluon plasma.

To make predictions for the final K/π ratio from our analysis, an understanding of the late stages of the expansion is necessary. In particular, the system must fall out of equilibrium as densities and reaction rates drop. In the present calculation we have kept the pion chemical potential always at zero. This means that the pion density is adjusted to its equilibrium value at any stage of the hydrodynamic expansion. Although this may be a good approximation in the mixed phase where the pion gas is in strong thermal contact with the plasma droplets, it may not be valid in the pure hadron gas phase since the processes which change pion number are very slow. (Similarly, the photon gas in a blackbody is in equilibrium only because of interaction with the walls.) In fact the fastest process which changes the pion number is the $\pi\pi\to\pi\pi\pi\pi$ reaction whose cross section is as small as that of the $\pi\pi\rightleftarrows KK$ processes which we have shown to be *out* of detailed balance. This implies that the pion will acquire a nonzero chemical potential soon after the completion of the phase transition and that a proper treatment of the chemical evolution of the hadron gas requires an additional rate equation associated with the slowly changing pion number.

On the other hand, once the phase transition is over the system would fall apart very rapidly via the transverse rarefaction wave which moves inward at the speed of sound, $c_s \simeq 1/\sqrt{3}$. The time scale for this transverse rarefaction, R/c_s, is in fact much shorter than the relaxation time in the hadron gas (see Fig. 3). In contrast, our one-dimensional scaling hydrodynamics should be a good approximation *before* $\tau = \tau_H$ since the transverse expansion is slowed during the mixed phase: Sound waves cannot propagate in the system and the shocklike discontinuity can travel only very slowly.[30] Hence both the pion number and the kaon number will most likely freeze-out shortly after completion of the phase transition.[31]

For this reason we conclude that the final multiplicities of pions and kaons should reflect their values at $\tau = \tau_H$. We found that this K^-/π^- ratio is about three times as large as that seen in pp collisions. At first glance, this result appears very encouraging for experimentalists who are seeking signatures of plasma formation in high energy heavy-ion collisions. However, this K/π ratio is *smaller* than that which would emerge from an equilibrium hadron gas. Thus we are forced to conclude, along with Kapusta and Mekjian,[11] that enhanced strangeness is not a direct signal of plasma formation. The best that can be said[11] is that the enhancement is a sign that the hadronic fluid was sufficiently long-lived to allow the flavor composition to approach equilibrium. This is *indirectly* a signal of plasma formation, since the large entropy of the initial state dramatically increases the time needed for expansion in order to dissipate that entropy as freely streaming particles.

It thus becomes crucial to understand the effect of including resonances in the hadron gas. While it would appear that most resonances would be highly suppressed by their large masses, it must be remembered that the density of states increases rapidly with mass, and hence that the resonances contribute significantly to the entropy. Indeed, it was this observation that led Hagedorn to conclude long ago that we should expect novel physics at a temperature of only 140 MeV. It is quite possible that even entropy (as inferred from the expansion time) does not distinguish between a hadron gas and a quark-gluon plasma.

In any case, the K/π ratio we predict could be easily diluted by effects which we have neglected. Any further source of entropy, such as viscosity and other transport effects, would produce more pions in the same way as the gluon entropy we found to be so important. Hadronic resonance states formed during the expansion may decay late and also enhance the number of pions, since they are favored by phase space. Certainly, these effects as well as the transverse expansion and the different hadronization mechanisms deserve more extensive study. The framework we have presented, based on kinetic theory, can be used without major modification to deal with these interesting problems, as well as with the related problem of charm production.[32]

We make a final remark of a rather speculative nature. The Maxwell construction we used for the dynamics of the first-order phase transition is undoubtedly an oversimplified scenario. If bubble formation and phase separation occur on a large scale, the K/π ratio will fluctuate about its average value. Those bubbles of hadron gas which form earliest are created at positive chemical potential, and hence with a large local K/π ratio.[33] The regions formed later, on the other hand, have undergone supercooling and thus generated extra entropy density and extra final multiplicity. The fluctuations in the K/π ratio should thus be observed *out of phase* with the fluctuations of total multiplicity in rapidity space. Of course, these fluctuations could be smeared out by diffusion.

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APPENDIX

Our analysis differs from the earlier work of Kapusta and Mekjian[11] (KM) in several respects.

(1) KM solved the entropy equation (2.13a) in the adiabatic limit ($I=0$), neglecting the nonequilibrium chemical processes. The resulting thermodynamic profile was used to integrate the rate equation. Our solution, on the other hand, contains the effects of the chemical processes on the evolution of the temperature and the entropy. As is clear from Fig. 4(b), this effect is not significant, essentially because the entropy is dominated by the light quarks and gluons.

(2) KM used the result of Rafelsky and Müller[4] for the chemical reaction rates in the plasma. We recalculated the rates (see I) taking into account the effects of Pauli-blocking and correcting numerical errors in Ref. 4. (See also Ref. 34.) We used our new result in the present analysis. Because of a mistaken factor of 2 in Ref. 4 and a different choice of the strange-quark mass by KM, our estimate of the relaxation time in the plasma (with a running QCD coupling constant) is about three times as large as that used in Ref. 11.

(3) Our estimate of the reaction rates in the hadron gas is much smaller than that of KM. Specifically, the relaxation time in the hadron gas which is shown in Fig. 3 is about an order of magnitude larger than that used in Ref. 11. This difference stems from the uncertainty in deducing the kaon annihilation cross section from observed hadronic cross sections. We note here again that in our calculation the relaxation time in the hadron gas phase is always longer than the relaxation time in the plasma.

(4) KM considered two different scenarios for the course of the hadronization transition. In our work we examined only one scheme which is essentially the same as the "Maxwell scenario" in Ref. 11. We emphasize that the Maxwell scenario gives results that should not change much if it turns out that there is no phase transition at all, as long as ideal gas approximations are reasonably valid in both the plasma and the hadron regimes.

(5) Probably the most important difference between the two analyses is in the choice of initial conditions. KM set the initial strange-quark density to zero and studied the approach to chemical equilibrium in the subsequent ex-

pansion. We postulated, on the other hand, that the plasma is formed in the beginning with flavor composition near chemical equilibrium. We attempted to learn how this initial equipartitioning of the deposited energy among the flavor degrees of freedom is reflected in the final observable particle composition. It appears that the results (see Fig. 6) do not depend strongly on the initial conditions.

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