Radiative angular distributions from $c\bar{c}$ states directly produced by $p\bar{p}$ annihilation

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(Received 23 May 1986)

We present formulas, based on the helicity formalism, for the angular distributions of the radiative decay products involving η_c', ψ, χ_J, and ${}^1P_1$ $c\bar{c}$ states formed in $p\bar{p}$ collisions. Normalized joint angular observables are expressed in terms of allowed multipole transitions and the ratio of $p\bar{p}$ production helicity amplitudes.

I. INTRODUCTION

Experiments in which charmonium states are directly formed by $p\bar{p}$ annihilation will provide a new perspective to heavy-quark physics. One of the most interesting possibilities of such experiments is to determine to high accuracy the electromagnetic radiation multipole structure for transitions between these states. We show these transitions in Fig. 1. Another by-product of these angular distributions will be the measurement of relative annihilation through the possible helicity states. Finally, even in those cases where only one multipole and helicity state is allowed by conservation laws, observation of characteristic angular distributions will serve to verify the correct J^{PC} assignment for a newly detected state such as the ${}^1P_1$.

An advantage of formation from $p\bar{p}$ is that we are no longer confined, as with e^+e^-, to one type of state: the ψ and its radial excitations. From $p\bar{p}$ all states with quark-antiquark quantum numbers can be excited. Once produced the radiative transitions between these states can be analyzed in terms of transition multipoles. These multipoles probe the electromagnetic structure of the heavy quark, and further test the quark model including wave functions and relativistic corrections.

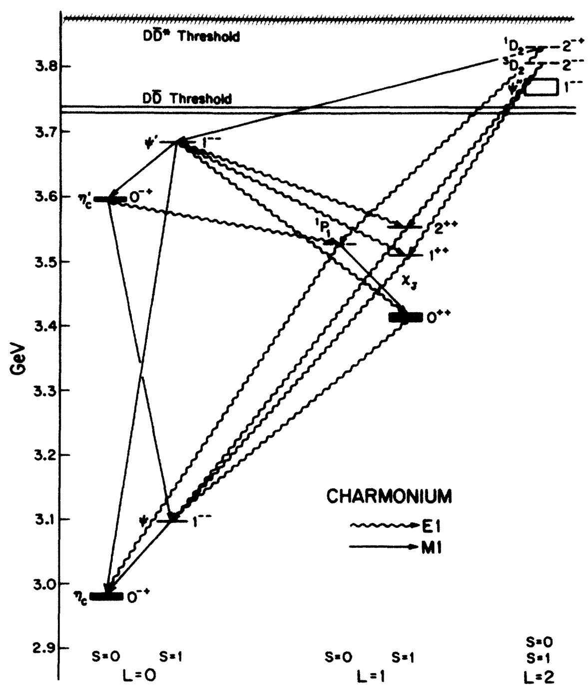

FIG. 1. Allowed radiative transitions for sharp charmonium states. The total widths of the states are roughly indicated by the thickness of the lines. States are labeled by ${}^{2S+1}L_J$ or J^{PC} notation where S, L, J, P, and C are spin, orbital and total angular momenta, parity, and charge parity, respectively.

In this paper we present the joint radiative angular distributions involving most states which will be observable in $p\bar{p}$ formation experiments. The relation between the decay helicity amplitudes and the transition multipoles is presented in each case.

Following the notation of Martin, Olsson, and Stirling[1,2] the joint angular distribution for the process $p\bar{p}\to(\chi_J,\eta_c')\to\gamma\psi\to\gamma e^+e^-$ is

$$W(\theta;\theta',\phi')=\sum_{\lambda}B_{|\lambda|}{}^2\sum_{\nu,\nu'=-J}^{J}\sum_{\mu=\pm1}d^J_{\lambda\nu}(\theta)d^J_{\lambda\nu'}A_{|\nu|}A_{|\nu'|}\times\rho^{\sigma'\sigma}(\theta',\phi')\,, \quad (1)$$

where the ψ helicity $\sigma\equiv\nu-\mu$ and $\sigma'\equiv\nu'-\mu$, and the density matrix for the ψ decay into an unpolarized e^+ and e^- is

$$\rho^{\sigma'\sigma}(\theta',\phi')=\sum_{\kappa=\pm1}D^1_{\sigma'\kappa}(\phi',\theta',-\phi')D^{1*}_{\sigma\kappa}(\phi',\theta',-\phi')\,. \quad (2)$$

The angles and helicities are indicated in Fig. 2; θ',ϕ' specify the $\psi\to e^+e^-$ decay in the ψ rest frame with the z axis aligned with the ψ direction in the χ_J or η_c' rest frame. The angles and helicities for

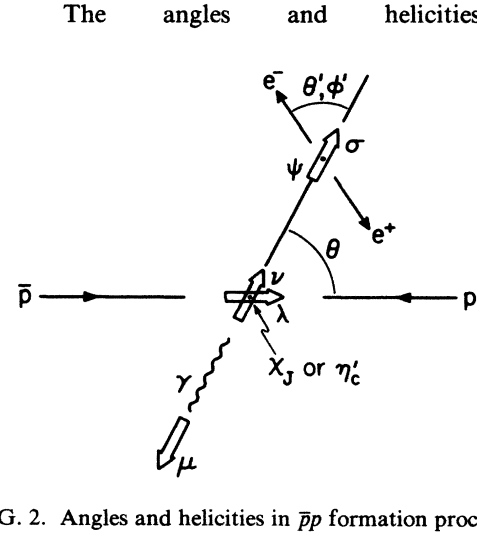

FIG. 2. Angles and helicities in $\bar{p}p$ formation processes.

$p\bar{p}\to(\psi,{}^1P_1)\to\gamma_1\eta_c\to\gamma_1\gamma\gamma$ are found from Fig. 2 by replacing χ with ψ or ${}^1P_1$, ψ with η_c, and e^+e^- with $\gamma\gamma$. The joint angular distributions for these processes are different but it is straightforward to find[4]

$$W(\theta;\theta',\phi')=\sum_{\lambda} B_{|\lambda|}{}^2 \sum_{\mu,\mu'=\pm 1} d^1_{\lambda\mu}(\theta)d^1_{\lambda\mu'}(\theta)A_0{}^2 \ . \quad (3)$$

For simplicity we define a normalized, joint angular distribution,

$$\hat{W}(\theta;\theta',\phi')=C\frac{W(\theta;\theta',\phi')}{B_0{}^2+2B_1{}^2} \quad (4)$$

so that the total rate is normalized to unity

$$\int d\Omega\, d\Omega' \hat{W}(\theta;\theta',\phi')=1 \ . \quad (5)$$

We also define a constant R, which measures the fractional contribution of the helicity-one initial production process:

$$R\equiv\frac{2B_1{}^2}{B_0{}^2+2B_1{}^2} \ . \quad (6)$$

The factor of 2 appears because helicities ± 1 contribute equally.

For each transition, we obtain the joint distribution $\hat{W}(\theta;\theta',\phi')$ (all are independent of ϕ) in terms of observables $\{K_i\}$ and elementary trigonometric functions. These $\{K_i\}$ can be expressed in terms of the helicity amplitudes A_i which in turn can be written in terms of the multipole transition amplitudes a_i. These amplitudes are normalized to one,

$$\sum_{i=0}^{J} A_i{}^2=1 \ , \quad (7a)$$

$$\sum_{i=1}^{J+1} a_i{}^2=1 \ , \quad (7b)$$

and by convention a_1 is taken to be positive. The general relation[3] between the two sets of amplitudes is

$$A_\nu=\sum_k a_k \left[\frac{2k+1}{2j'+1}\right]^{1/2} \langle k,1;1,\nu-1\,|\,j',\nu\rangle \quad (8)$$

and we shall list the particular relations for each transition. It is important to note that the helicity and multipole amplitudes for different transitions are independent, e.g., $A_0(\chi_2)\neq A_0(\chi_1)$, $a_2(\chi_2)\neq a_2(\chi_1)$, $R(\chi_1)\neq R({}^1P_1)$.

The most information is obtained from determining the full distribution from experiment but this is not always possible. We may, however, gain some information from the integrated distributions

$$\hat{W}(\theta)\equiv\int d\phi\, d\Omega' \hat{W}(\theta;\theta',\phi') \ , \quad (9a)$$

$$\hat{W}(\theta')\equiv\int d\Omega\, d\phi' \hat{W}(\theta;\theta',\phi') \ , \quad (9b)$$

$$\hat{W}(\phi')\equiv\int d\Omega\, d(\cos\theta') \hat{W}(\theta;\theta',\phi') \ , \quad (9c)$$

which we calculate for each transition as well.

II. $\bar{p}p\to\chi_2\to\gamma\psi$

These results have been published previously[5] but for completeness and to correct a misprint we have

$$\frac{64\pi^2}{15}\hat{W}(\theta;\theta',\phi')=K_1+K_2\cos^2\theta+K_3\cos^4\theta+(K_4+K_5\cos^2\theta+K_6\cos^4\theta)\cos^2\theta' +(K_7+K_8\cos^2\theta+K_9\cos^4\theta)\sin^2\theta'\cos2\phi'+(K_{10}+K_{11}\cos^2\theta)\sin2\theta\sin2\theta'\cos\phi' \ , \quad (10)$$

where the eleven observables $\{K_i\}$ are given by

$$\begin{aligned}
8K_1&=2A_0{}^2+3A_2{}^2-R(2A_0{}^2-4A_1{}^2+A_2{}^2) \ ,\\
\tfrac{4}{3}K_2&=-2A_0{}^2+4A_1{}^2-A_2{}^2+R(4A_0{}^2-6A_1{}^2+A_2{}^2) \ ,\\
8K_3&=(6A_0{}^2-8A_1{}^2+A_2{}^2)(3-5R) \ ,\\
8K_4&=2A_0{}^2+3A_2{}^2-R(2A_0{}^2+4A_1{}^2+A_2{}^2) \ ,\\
\tfrac{4}{3}K_5&=-2A_0{}^2-4A_1{}^2-A_2{}^2+R(2A_0{}^2+6A_1{}^2+A_2{}^2) \ ,\\
8K_6&=(6A_0{}^2+8A_1{}^2+A_2{}^2)(3-5R) \ ,\\
4K_7&=\sqrt{6}(R-1)A_0A_2 \ ,\\
4K_8&=\sqrt{6}(4-6R)A_0A_2 \ ,\\
4K_9&=\sqrt{6}(5R-3)A_0A_2 \ ,\\
(4/\sqrt{3})K_{10}&=A_0A_1+\sqrt{3/2}A_1A_2 -R(2A_0A_1+\sqrt{3/2}A_1A_2) \ ,\\
4\sqrt{3}K_{11}&=(5R-3)(3A_0A_1+\sqrt{3/2}A_1A_2) \ .
\end{aligned} \quad (11)$$

The partially integrated angular distributions are, up to a normalization constant,

$$\begin{aligned}
&\hat{W}(\theta)=\hat{W}(\pi/2)(1+\alpha\cos^2\theta+\beta\cos^4\theta) \ ,\\
&\alpha=6\frac{-A_2{}^2+2A_1{}^2-2A_0{}^2+R(A_2{}^2-3A_1{}^2+4A_0{}^2)}{3A_2{}^2+2A_0{}^2+R(-A_2{}^2+2A_1{}^2-2A_0{}^2)} \ ,\\
&\beta=\frac{(3-5R)(A_2{}^2-4A_1{}^2+6A_0{}^2)}{3A_2{}^2+2A_0{}^2+R(-A_2{}^2+2A_1{}^2-2A_0{}^2)} \ ;
\end{aligned} \quad (12a)$$

$$\begin{aligned}
&\hat{W}(\theta')=\hat{W}(\pi/2)(1+\alpha'\cos^2\theta') \ ,\\
&\alpha'=\frac{1-3A_1{}^2}{1+A_1{}^2} \ ;
\end{aligned} \quad (12b)$$

$$\begin{aligned}
&\hat{W}(\phi')=\hat{W}(\pi/4)(1+\alpha''\cos2\phi') \ ,\\
&\alpha''=-A_2A_0/\sqrt{6} \ .
\end{aligned} \quad (12c)$$

For this process, a_1, a_2, and a_3 correspond to $E1$, $M2$, and $E3$ transitions, respectively, and

$$a_1=\sqrt{1/10}A_0+\sqrt{3/10}A_1+\sqrt{3/5}A_2\ ,$$
$$a_2=\sqrt{1/2}A_0+\sqrt{1/6}A_1-\sqrt{1/3}A_2\ , \qquad (13)$$
$$a_3=\sqrt{2/5}A_0-\sqrt{8/15}A_1+\sqrt{1/15}A_2\ .$$

The inverse transformation is

$$A_0=\sqrt{1/10}a_1+\sqrt{1/2}a_2+\sqrt{2/5}a_3\ ,$$
$$A_1=\sqrt{3/10}a_1+\sqrt{1/6}a_2-\sqrt{8/15}a_3\ , \qquad (14)$$
$$A_2=\sqrt{3/5}a_1-\sqrt{1/3}a_2+\sqrt{1/15}a_3\ .$$

III. $\bar{p}p \to \chi_1 \to \gamma\psi$

Formation of the χ_1 state takes place in either $\lambda=0$ or ± 1 $\bar{p}p$ helicity states and the decay multipoles are $E1$ or $M2$. The joint angular distribution is given by

$$\frac{64\pi^2}{9}\hat{W}(\theta;\theta',\phi')=K_1+K_2\cos^2\theta+(K_3+K_4\cos^2\theta)\cos^2\theta' + K_5\sin 2\theta\sin 2\theta'\cos\phi'\ . \qquad (15)$$

The five observable coefficients in the angular distributions are

$$K_1=A_1{}^2+\tfrac{1}{2}R(A_0{}^2-A_1{}^2)\ ,$$
$$K_2=(1-\tfrac{3}{2}R)(A_0{}^2-A_1{}^2)\ ,$$
$$K_3=-A_1{}^2+\tfrac{1}{2}R\ , \qquad (16)$$
$$K_4=1-\tfrac{3}{2}R\ ,$$
$$4K_5=A_1A_0(3R-2)\ ;$$

$$\hat{W}(\theta)=\hat{W}(\pi/2)(1+\alpha\cos^2\theta)\ ,$$
$$\alpha=\frac{(2A_0{}^2-A_1{}^2)(2-3R)}{2A_1{}^2+R(2A_0{}^2-A_1{}^2)}\ ; \qquad (17a)$$

$$\hat{W}(\theta')=\hat{W}(\pi/2)(1+\alpha'\cos^2\theta')\ ,$$
$$\alpha'=\frac{1+A_1{}^2}{1-3A_1{}^2}\ ; \qquad (17b)$$

$$\hat{W}(\phi')=\frac{1}{2\pi}\ . \qquad (17c)$$

For this process, a_1 and a_2 correspond to $E1$ and $M2$ transitions, respectively, and

$$a_1=\sqrt{1/2}(A_0+A_1)\ ,$$
$$a_2=\sqrt{1/2}(A_0-A_1)\ . \qquad (18)$$

The inverse transformation is

$$A_0=\sqrt{1/2}(a_1+a_2)\ ,$$
$$A_1=\sqrt{1/2}(a_1-a_2)\ . \qquad (19)$$

IV. $\bar{p}p \to (\chi_0,\eta_c') \to \gamma\psi$

The χ_0 decays only by $E1$ transitions and the η_c' decays only by $M1$ transitions for these processes so we have

$$a_1=A_0=1\ . \qquad (20)$$

The absorption helicity state B_1 does not appear in these processes because the χ_0 and η_c' only have helicity zero which implies $R=0$. Because of this, these states can be created only if $\bar{p}p$ can couple to helicity-zero states. The normalized joint angular distribution is

$$\frac{64\pi^2}{3}\hat{W}(\theta;\theta',\phi')=K_0(1+\cos^2\theta')\ ,$$
$$K_0=1-R=1\ , \qquad (21)$$

$$\hat{W}(\theta)=\tfrac{1}{2}\ , \qquad (22a)$$

$$\hat{W}(\theta')=\tfrac{3}{8}(1+\cos^2\theta')\ , \qquad (22b)$$

$$\hat{W}(\phi')=\frac{1}{2\pi}\ . \qquad (22c)$$

V. $\bar{p}p \to (\psi,{}^1P_1) \to \gamma\eta_c$

These decays proceed by only one multipole, $M1$ for $\psi\to\gamma\eta_c$ and $E1$ for ${}^1P_1\to\gamma\eta_c$, so we have

$$a_1=A_0=1\ . \qquad (23)$$

For these states the angular distributions are

$$\frac{32\pi^2}{3}\hat{W}(\theta;\theta',\phi')=(K_1+K_2\cos^2\theta)\ ,$$
$$K_1=1-\tfrac{1}{2}R\ , \qquad (24)$$
$$K_2=\tfrac{3}{2}R-1\ ;$$

$$\hat{W}(\theta)=\hat{W}(\pi/2)(1+\alpha\cos^2\theta)\ ,$$
$$\alpha=\frac{3R-2}{2-R}\ ; \qquad (25a)$$

$$\hat{W}(\theta')=\tfrac{1}{2}\ ; \qquad (25b)$$

$$\hat{W}(\phi')=\frac{1}{2\pi}\ . \qquad (25c)$$

For $\psi\to\gamma\eta_c$, $\psi'\to\gamma\eta_c$, and $\psi'\to\gamma\eta_c'$ transitions, the angular distribution $\hat{W}(\theta)$ depends on R varying from $1+\cos^2\theta$ for $R=1$ to $\sin^2\theta$ for $R=0$.

Because of C-parity conservation the B_1 helicity state does not enter into ${}^1P_1$ production. Since the ${}^1P_1$ state is formed by pure $R=0$, the decay-angular distribution is uniquely $\sin^2\theta$.

VI. $\bar{p}p \to \psi \to e^+e^-$ AND $e^+e^- \to \psi \to \bar{p}p$

In the e^+e^--initiated formation process the ψ is produced with helicity ± 1 but the final $\bar{p}p$ state may have helicity 0 and ± 1 so the angular distribution is given by

$$\hat{W}_{\bar{p}p}(\theta)=\hat{W}_{\bar{p}p}(\pi/2)(1+\alpha\cos^2\theta)\ , \tag{26}$$

where

$$\alpha=\frac{3R-2}{2-R} \tag{27}$$

and R is defined to correspond to the $\bar{p}p$ initiated definition of Eq. (6) with B_i replaced by A_i. This angular distribution has been measured to good accuracy in a recent DM2 (Ref. 6) experiment which obtained

$$\alpha_{\rm expt}=0.60\pm0.08\pm0.03 \tag{28}$$

which by Eq. (27) implies that

$$R=\frac{2(\alpha+1)}{3+\alpha}=0.89\pm0.03\ . \tag{29}$$

By the principle of detailed balance, the angular distribution of the time-reversed reaction $p\bar{p}\to\psi\to e^+e^-$ should be the same as the forward reaction. In fact, the equations for these processes are identical and the R for the forward reaction is equal, by time reversal, to the R for the reverse reaction. This R should also be equal to the R appearing in Eq. (25a), obtained from a measurement of the process $p\bar{p}\to\psi\to\eta_c\gamma$. The angular distribution for $p\bar{p}\to\psi\to e^+e^-$ has been measured by R704 (Ref. 7) and is consistent with $\alpha=0.6$ but the statistics are too low to make any definitive statement.

VII. CONCLUSIONS

We have examined the kinematics of processes of the form $\bar{p}p\to(\bar{c}c)_1\to(\bar{c}c)_2\gamma$. The angular distribution of the decay products of the final $c\bar{c}$ state can be parametrized by a small number of real quantities. Conversely, measurement of these joint angular distributions allows a good determination of a variety of well-defined quantities which are susceptible to theoretical interpretation.

The heavy-quark model makes predictions for the various transition multipoles and exclusive QCD calculations should be able to account for the helicity absorption ratios. Finally, the observed angular distributions, even in a case where conservation laws mandate a unique result, is of interest as a test of state quantum numbers. This would be particularly important in verifying the existence of the $^1P_1$ state.

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