Magnetic moments of light, charmed, and *b*-flavored baryons in a relativistic logarithmic potential

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A simple independent-quark model based on the Dirac equation with logarithmic confining potential of the form $V(r)=(1+\gamma^0)[a\ln(r/b)]$ with $a,b>0$ is used to calculate the magnetic moments of light, charmed, and b-flavored baryons. Not only do the results obtained for light baryons agree reasonably well with experiment, but also the overall predictions for the charmed and b-flavored baryons compare very well with other model predictions.

I. INTRODUCTION

A great deal of work has been done on the calculation of magnetic moments of old light baryons in the nucleon octet[1] and also of the recent heavier baryons in the charmed and b-flavored sectors,[2–5] yet the study of baryon magnetic moments is not a closed chapter. The various constituent-quark models are able to reproduce well the general pattern of magnetic moments of light baryons in the nucleon octet, but there are significant quantitative failures. However, the phenomenology of constituent-quark dynamics in hadrons has been reasonably successful through the "bag-model" approach,[6] which implies that the observed properties of hadrons are not too inconsistent with the picture of constituent quarks moving relatively independently within the hadron. In these models the confinement of the relativistic quarks is achieved by the fixed finite boundary in terms of the bag radius or equivalently by the vacuum pressure. Keeping the essential features of this successful bag model the same, one can make some alternative scheme, which can provide a simple, yet unified approach to the understanding of constituent-quark dynamics, particularly in the context of the magnetic-moment study of the light, charmed, and b-flavored baryons. With the idea of independent constituent quarks in the hadrons and the mechanism of confinement of these quarks to the hadronic dimensions as the two basic ingredients of the bag models leading to their success, one can make a simpler alternative approach based on the independent-quark Dirac equation with some average quark interaction potential of suitable Lorentz structure. Such schemes with equally mixed scalar and vector parts of the potential in harmonic,[7] linear,[7] and non-Coulombic power-law[8] form have been followed by many authors in the recent past. The confinement of individual constituent quarks in hadrons in such schemes has been achieved through some average potential with suitable Lorentz structure without any finite boundary restrictions of the bag models. Here the confining potential replaces the effects of external pressure on the bag. Implications of such a scheme in the context of quark confinement and relativistic consistency have been studied by Magyari[9] in reference to heavy-meson spectra. It has been found there that the logarithmic potential with a Lorentz structure in the form of an equal admixture of scalar and vector parts not only can guarantee relativistic quark confinement but also can generate charmonium and Υ bound-state masses in reasonable agreement with experiments. The logarithmic potential has been investigated in the past by Quigg and Rosner with remarkable success in the nonrelativistic potential-model studies of heavy mesons.[10] The success of this purely phenomenological logarithmic potential in the above-mentioned works makes it tempting to use this potential for the study of magnetic moments of baryons. Therefore in the present work, we intend to understand the magnetic moments of light, charmed, and b-flavored baryons in the framework of the independent-quark model based on the Dirac equation with the confining logarithmic potential.

II. THEORETICAL FRAMEWORK

Here we assume that the constituent quarks of baryons move independently in an average potential taken in the form

$$V_q(r)=(1+\gamma^0)V(r)=(1+\gamma^0)[a\ln(r/b)]\ , \tag{2.1}$$

where $a,b>0$ and r is the radial distance from the baryon center of mass. It is further assumed that the independent quark of rest mass m_q obeys the Dirac equation, so that the four-component quark wave function $\psi_q(\mathbf{r})$ satisfies the equation (with $\hbar=c=1$)

$$[\gamma^0E_q-\boldsymbol{\gamma}\cdot\mathbf{p}-m_q-V_q(r)]\psi_q(\mathbf{r})=0\ . \tag{2.2}$$

Following the usual approach of the bag models, if we now assume that all three constituent quarks of the baryons are in their ground state with $J^P=\frac{1}{2}^+$ and $J_z=\frac{1}{2}$, then a solution to the independent-quark wave function $\psi_q(\mathbf{r})$ can be written in the two-component form as[8]

$$\psi_q(\mathbf{r})=\frac{N_q}{\sqrt{4\pi}}\begin{pmatrix} ig_q(r)/r \\ \sigma\cdot\hat{\mathbf{r}}f_q(r)/r\end{pmatrix}\chi\uparrow\ . \tag{2.3}$$

Here N_q stands for the overall normalization of $\psi_q(\mathbf{r})$, and it can be easily obtained as

$$N_q^2=[1+(E_q-m_q+2a\ln b-2a\langle \ln r\rangle_q)/\lambda_q]^{-1}\ , \tag{2.4}$$

where $\lambda_q=(E_q+m_q)$ and the angular brackets $\langle \ln r\rangle_q$ mean the expectation value with respect to the normalized radial-angular part of the upper component of $\psi_q(\mathbf{r})$. Finally the reduced radial parts $f_q(r)$ and $g_q(r)$ can be found to satisfy the equations

$$f_q(r)=\frac{1}{\lambda_q}\left[\frac{d}{dr}-\frac{1}{r}\right]g_q(r)\ , \tag{2.5}$$

$$\frac{d^2g_q(r)}{dr^2}+\lambda_q[(E_q-m_q+2a\ln b)-2a\ln r]g_q(r)=0\ . \tag{2.6}$$

Now choosing a suitable length scale,

$$r_{0,q}=(2a\lambda_q)^{-1/2}\ , \tag{2.7}$$

Eq. (2.6) can be expressed in terms of a dimensionless variable $\rho=(r/r_0)$ as

$$\frac{d^2g_q(\rho)}{d\rho^2}+(\epsilon_q-\ln\rho)g_q(\rho)=0\ , \tag{2.8}$$

where

$$\epsilon_q=\left[\left[\frac{E_q-m_q}{a}\right]+\ln(2ab^2\lambda_q)\right]\Big/2\ . \tag{2.9}$$

Equation (2.8) provides the basic eigenvalue equation whose solution by any standard numerical method or the WKB approximation method would give ϵ_q and the normalized function $g_q(r)$.

We find the WKB solution as

$$\epsilon_q=\ln(3\sqrt{\pi}/2) \tag{2.10}$$

and

$$g_q(\rho)=\frac{A_q}{(\epsilon_q-\ln\rho)^{1/4}}\cos\phi_q(\rho)\ , \tag{2.11}$$

where the phase factor is

$$\phi_0(\rho)=\int_0^{\rho_e}d\rho'(\epsilon_q-\ln\rho')^{-1/2}-\pi/4 \tag{2.12}$$

and the normalization constant A_q is given by

$$A_q^2(8a\lambda_q/\pi)^{1/2}\exp(-\epsilon_q)\ . \tag{2.13}$$

Once ϵ_q is known, Eq. (2.9) can give the individual-quark binding E_q, which shall now depend on the parameters a, m_q, and b through the relation

$$E_q=m_q-a\ln c+ax_q\ , \tag{2.14}$$

where $c=2a^2b^2$ and x_q is the solution of the root equation obtained through substitutions from (2.9) in the form

$$x_q+\ln(x_q+K)=2\epsilon_q\ , \tag{2.15}$$

where

$$K=\frac{2m_q}{a}-\ln C\ . \tag{2.16}$$

Thus the simple model under discussion provides a complete description of the relativistic bound states of the confined constituent quarks in the baryons with the quark wave function $\psi_q(\mathbf{r})$ given as in (2.3) and the corresponding binding energy E_q given by (2.14).

With the assumption that SU(3) is broken in the quark rest masses as $m_u=m_d\neq m_s$, we can now present some consequences of the model in terms of derived expressions of some of the measurable quantities of the S-wave baryons in the nucleon octet which are obtained simply by appropriately adding the contributions of each individual quark. With the ground-state quark wave function $\psi_q(\mathbf{r})$ given by (2.3) the expressions for (i) the axial-vector coupling constant $g_A(n)$ for neutron β decay, (ii) the mean-square charge radius $\langle r^2\rangle_p$ of the proton, and (iii) the confined quark magnetic moment μ_q can be obtained in the usual manner[7] as

$$g_A(n)=5(4N_u^2-1)/9\ , \tag{2.17}$$

$$\langle r^2\rangle_p=\sum_q e_q\langle r^2\rangle_q=\langle r^2\rangle_u\ , \tag{2.18}$$

and

$$\mu_q=(2M_pe_qN_q^2/\lambda_q)\mu_N\ , \tag{2.19}$$

where M is the proton mass, e_q is the electric charge of the quark in the unit of proton charge, and μ_N is the nuclear magneton. Here $\langle r^2\rangle_u$ is the individual contribution of u quark. The angular brackets $\langle \ln r\rangle_q$ in (2.4) and $\langle r^2\rangle_u$ in (2.18) can be easily obtained through the WKB approximation method as

$$\langle \ln r\rangle_q=\ln r_{0,q}+\epsilon_q-\tfrac{1}{2} \tag{2.20}$$

and

$$\langle r^2\rangle_u=r_{0,u}^2\exp(2\epsilon_q)/\sqrt{3}\ . \tag{2.21}$$

Now with the help of Eq. (2.20), the expression (2.4) for N_q^2 simplifies to

$$N_q^2=\lambda_q/(a+\lambda_q)\ . \tag{2.22}$$

If we make the usual assumption that the baryon moments arise solely from the constituent-quark moments, then, following Johnson and Shah-Jahan[3] and also the earlier work of Franklin,[11] the expressions for magnetic moments of light, charmed, and b-flavored baryons can be obtained in terms of the magnetic moments of the corresponding constitutent quarks in the following manner:

$$\mu_B=\sum_q\langle B\uparrow|\mu_q\sigma_z^q|B\uparrow\rangle\ , \tag{2.23}$$

where $|B\uparrow\rangle$ represents the state vectors of the baryons. In the case of octet nucleons $|B\uparrow\rangle$ represents the regular SU(6) state vectors. For the charmed or b-flavored baryons, the corresponding state vectors are the straightforward extensions as given by Singh.[2] The relations for the magnetic moments of baryons in terms of the constitutent-quark moments as given by Eq. (2.23) are well known[2,3,5,11] and can be used to compute the magnetic moments of baryons with the present model.

III. RESULTS AND CONCLUSION

The outcome of the present model depends very much on our choice of the potential parameters a and b and the quark mass parameters $m_u(=m_d)$, m_s, m_c, and m_b. Although these parameters are *a priori* unconstrainted, we have to make a suitable choice by reasonable assumptions.

We make the usual assumption that the average potential taken in this model for the confined independent quarks inside the hadrons is flavor independent. Therefore we use the same set of parameters and quark masses for the calculation of magnetic moments of light, charmed, and b-flavored baryons. The quark masses m_q for u, d, s, c, and b quarks are obtained by making appropriate reference to some hadronic ground-state masses, which in this independent-quark model approach would be given by the sum total of the constituent-quark energies in the form

$$M(\text{hadron})=\sum_q E_q \; . \tag{3.1}$$

Therefore with the values of the parameters a and b suitably fixed, the value of $m_u=m_d$ has to be chosen properly so that when confined within the nucleon by an average potential (2.1), the up and down quark would have the binding energy $E_u=E_d=\frac{1}{3}M_p$. Similarly for fixing m_s and m_c, we take masses of Λ and Λ_c, respectively, as inputs so as to obtain $E_s=(M_\lambda-2E_u)$ and $E_c=(M\lambda_c-2E_u)$. But in the absence of any knowledge of the b-flavored baryons, we simply refer to the $\Upsilon(b\bar{b})$ mass in fixing m_b so as to obtain $E_b=M_r/2$.

First of all we adopt the WKB method of solving Eq. (2.8) and find that with

$$(a,b)=(124.768\text{ MeV},5.225\times 10^{-3}\text{ MeV}^{-1}) \tag{3.2}$$

and

$$m_u=m_d=234.35\text{ MeV}\; , \tag{3.3}$$

the energy eigenvalue condition (2.14) yields

$$E_u=E_d=312.76\text{ MeV}\; , \tag{3.4}$$

which results in the static nucleon properties

$$\mu_p=2.795\mu_N,\;\; g_A(n)=1.254,\;\; M_p=938.28\text{ MeV} \tag{3.5}$$

in close agreement with experiments. Then the parameters given by (3.2) and (3.3) along with the inputs.

$$(E_s,E_c,E_b)=(490.07,1647.38,4730)\text{ MeV} \tag{3.6}$$

yield from (2.14) the quark masses

$$(m_s,m_c,m_b)=(483.58,1798.59,5010.86)\text{ MeV}\; . \tag{3.7}$$

The above choice of potential parameters, quark masses, and the quark-binding energies give the constituent-quark magnetic moments from (2.19) as

$$\begin{aligned}&\mu_u=-2\mu_d=1.862\mu_N,\;\; \mu_s=-0.569\mu_N\; ,\\ &\mu_c=0.350\mu_N,\;\; \mu_b=-0.063\mu_N\; ,\end{aligned} \tag{3.8}$$

which can finally be used to compute the baryon mo-

TABLE I. (i) WKB, (ii) numerical results for magnetic moments of the nucleon octet calculated by the present model as compared with the results of the cloudy-bag model (CBM) (Ref. 12) and the experimental data (Ref. 13) (all numbers in nuclear magnetons).

Baryons	Present calculation (i)	Present calculation (ii)	CBM calculation	Experimental results
p	2.793	2.886	2.60	2.793
n	-1.862	-1.924	-2.01	-1.913
Λ	-0.569	-0.580	-0.58	-0.614 ± 0.005
Σ^+	2.672	2.758	2.34	2.33 ± 0.13
Σ^0	0.810	0.834		0.46 ± 0.28
Σ^-	-1.051	-1.089	-1.08	-0.89 ± 0.14
Ξ^0	-1.380	-1.414	-1.27	-1.25 ± 0.14
Ξ^-	-0.449	-0.452	-0.51	-0.69 ± 0.04
(Λ,Σ)	-1.613	-1.666		$-1.82^{+0.18}_{-0.25}$

TABLE II. (i) WKB, (ii) numerical results for magnetic moments of charmed baryons obtained in the present model as compared to calculations in the De Rujula-Georgi-Glashow (DGG) model and the bag model (all numbers in nuclear magnetons).

Baryon symbol (Ref. 3)	Quark content	Present calculation (i)	Present calculation (ii)	DGG model (Ref. 5)	Bag model (Ref. 4)
Σ_c^{++}	cuu	2.366	2.448	2.36	1.955
Σ_c^{+}	cud	0.504	0.524	0.43	0.363
Σ_c^{0}	cdd	-1.358	-1.400	-1.43	-1.23
Ξ_c^{+}	cus	0.745	0.779	0.73	0.475
Ξ_c^{0}	cds	-1.117	-1.145	-1.16	-1.09
Ω_c^{0}	ssc	-0.876	-0.890	-0.89	-0.98
Ξ_{cc}^{++}	ccu	-0.154	-0.172	-0.12	-0.167
Ξ_{cc}^{+}	ccd	0.778	0.790	0.82	0.865
Ω_{cc}^{+}	ccs	0.657	0.663	0.69	0.838
Λ_c^{+}	$c(ud)_a$	0.350	0.352	0.37	0.503
$\Xi_c^{\prime +}$	$c(us)_a$	0.350	0.352	0.37	0.503
$\Xi_c^{\prime 0}$	$c(ds)_a$	0.350	0.352	0.37	0.503

TABLE III. (i) WKB, (ii) numerical results for the magnetic moment of *b*-flavored baryons obtained in the present model as compared to other calculations (all numbers in nuclear magneton).

Baryon symbol	Quark content	Present calculation (i)	Present calculation (ii)	DGG model (Ref. 5)	Bag model (Ref. 4)
Σ_b^+	*uub*	2.504	2.586	2.5	2.318
Σ_b^0	*ubd*	0.642	0.662	0.61	0.587
Σ_b^-	*ddb*	−1.220	−1.261	−1.28	−1.117
Ω_b^-	*ssb*	−0.738	−0.752	−0.55	−0.838
Ξ_{cb}^+	*ucb*	1.496	1.538	1.5	2.04
Ξ_{cb}^0	*dcb*	−0.366	−0.385	−0.38	−0.39
Ω_{ccb}^+	*ccb*	0.488	0.491	0.51	0.894
Ξ_{bb}^0	*ubb*	−0.705	−0.726	−0.7	−0.614
Ξ_{bb}^-	*dbb*	0.226	0.236	0.23	0.14
Ω_{bb}^-	*sbb*	0.105	0.108	0.105	0.084
Ω_{cbb}^0	*cbb*	−0.201	−0.202	−0.21	0.31
Ξ_b^0	*sub*	0.883	0.917	0.87	0.73
Ξ_b^-	*sdb*	−0.979	−1.006	−1.05	−0.977
Ω_{cb}^0	*scb*	−0.125	−0.130	−0.11	−0.223

ments, in the light, charmed, and *b*-flavored sectors.

In order to get an indication as to the error one expects in the calculation of magnetic moments from the use of the WKB approximation, we repeat the above calculations by following the exact numerical methods. We find that for the same potential parameters and the quark-binding energies [Eqs. (3.2), (3.4), and (3.6)] taken as inputs, the quark masses turn out to be

$$(m_u=m_d,m_s)=(212.74,464.53)\ \text{MeV}\ ,$$
$$(m_c,m_b)=(m_c,m_b)=(1781.38,4994.06)\ \text{MeV}\ , \tag{3.9}$$

which are slightly different from the ones obtained in the WKB method. The magnetic moments of the constituent quarks as found by this method are

$$\mu_u=-2\mu_d=1.924\mu_N,\quad \mu_s=-0.580\mu_N\ ,$$
$$\mu_c=0.352\mu_N,\quad \mu_b=-0.064\mu_N\ . \tag{3.10}$$

We find these values, as well as the values for the magnetic moments of the baryons, are not drastically different from the corresponding WKB values.

The WKB and numerical results obtained for the magnetic moments of light baryons are presented in Table I, along with those of the cloudy-bag model[12] (CBM) and the experimental data for comparison. We find that our results agree well with experiment. In the absence of any experimental data for the magnetic moments of charmed and *b*-flavored baryons, we present our WKB and numerical results in Tables II and III in comparison with the predictions of some other models.[4,5] The symbols used in these tables for charmed and *b*-flavored baryons are according to Ref. 5. We observe that our predictions for charmed and *b*-flavored baryons are not drastically different from other model predictions. The root-mean-square charge radius of the proton is found to be $\langle r^2\rangle_p^{1/2}=1.072$ fm, as compared to its experimental value of 0.88 ± 0.03 fm. The neutron charge radius is obviously zero here in contradiction with the experimental value $\langle r^2\rangle_n^{1/2}=-0.12$ fm. This is of course the case with most of the models of this kind, including the bag model.

In the present work we thus find that a simple and unified approach for the study of magnetic moments of light, charmed, and *b*-flavored baryons is possible through an independent-quark model based on the Dirac equation with a logarithmic potential (2.1). In view of the simplicity of the model the results obtained are quite encouraging. It may take a long time for the experimental values of magnetic moments of baryons in charmed and *b*-flavored sectors to come to light; however, the predicted values of the same in this model may be used in some calculations, which may be verified experimentally.

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