Comment on "Time variation of fundamental constants, primordial nucleosynthesis, and the size of extra dimensions"

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It is pointed out that the constraints on the mean radius of the extra dimensions at the time of primoridal nucleosynthesis obtained by Kolb, Perry, and Walker get modified if primordial neutrinos are degenerate. Similarly, the constraints on the neutrino degeneracy parameters obtained from primordial nucleosynthesis also change if the volume of the extra dimensions at the time of nucleosynthesis is different than its present value.

In theories of the early Universe with extra dimensions the fundamental constants such as the fine-structure constant α, the gravitational constant G_N, the Fermi coupling constant G_F, etc., depend on the volume of the extra dimensions. The primordial nucleosynthesis yield therefore depends on the value of this volume at the time of primordial nucleosynthesis. From the observed range in the primordial abundance of $^4\mathrm{He}(Y_P)$, Kolb, Perry, and Walker[1] (KPW) have obtained the following limits on the ratio of the mean radius R of the extra dimensions at the time of nucleosynthesis to its present value R_0:

$$\begin{aligned} &0.99 \le R/R_0 \le 1.01 , \\ &0.995 \le R/R_0 \le 1.005 \end{aligned} \tag{1}$$

for a superstring theory with six extra dimensions (SS6) and for a Kaluza-Klein theory with two to seven extra dimensions (KK2 and KK7), respectively. In this Comment I want to show that the effect of extra dimensions can get canceled if the primordial neutrinos are degenerate and, as a result, a much larger range of values of R/R_0 is allowed depending on the values of the degeneracy parameter. I also show that the constraints on the neutrino degeneracy parameter obtained from nucleosynthesis considerations would change significantly if the Universe indeed had extra dimensions, the volume of which would be different at the time of nucleosynthesis from its present value.

In view of our limited knowledge about the extra dimensions and behavior of particle densities and statistics during compactification I have confined myself to simple analytic approximations for estimating the effect of extra dimensions as well as the neutrino degeneracy on the helium yield, which in addition to being illustrative is clearly adequate for the purpose of this Comment.

The effect of the change in the values of the fundamental constants during compactification of the extra dimensions on Y_P is essentially through[1] a change in freeze-out temperature and $Q(=M_n - M_p)$. It can be approximately evaluated as follows.

The freezing temperature T_f depends on G_F and G_N as

$$T_f \propto (G_F{}^{-2} G_N{}^{1/2})^{1/3} \tag{2}$$

giving

$$\frac{T_f}{T_{f_0}} = \left(\frac{G_F}{G_{F_0}}\right)^{-2/3} \left(\frac{G_N}{G_{N_0}}\right)^{1/6} . \tag{3}$$

Here T_{f_0}, G_{F_0}, and G_{N_0} are the values of T_f, G_F, and G_N for the present value (R_0) of the mean radius (R) of the extra dimensions. Following KPW we assume

$$\frac{Q}{Q_0} = \frac{\alpha}{\alpha_0} \tag{4}$$

giving

$$\begin{aligned} \frac{Q}{T_f} &= \frac{Q_0}{T_{f_0}} \frac{Q}{Q_0} \frac{T_{f_0}}{T_f} \\ &= \frac{Q_0}{T_{f_0}} \left[\frac{\alpha}{\alpha_0} \left(\frac{G_F}{G_{F_0}}\right)^{2/3} \left(\frac{G_N}{G_{N_0}}\right)^{-1/6} \right] . \end{aligned} \tag{5}$$

Using the R/R_0 dependence of α/α_0, G_F/G_{F_0}, and G_N/G_{N_0} as given in Table II of KPW we get

$$\frac{Q}{T_f} = \frac{Q_0}{T_{f_0}} (R/R_0)^B , \tag{6}$$

where $B = -9$ for SS6 and $B = (D-20)/6$ for KK theories with D extra dimensions.

In the absence of neutrino degeneracy, the neutron to proton ratio at the freeze-out and the helium abundance is given by[2]

$$n/p = \exp(-Q/T_f) , \tag{7}$$

$$\begin{aligned} Y_P &= \frac{2}{1+(n/p)^{-1}} \\ &= \frac{2}{1+\exp[(Q_0/T_{f_0})(R/R_0)^B]} . \end{aligned} \tag{8}$$

For $R/R_0 = 1$, requiring $0.23 \le Y_P \le 0.25$, Eq. (8) gives

$$1.946 \le \frac{Q_0}{T_{f_0}} \le 2.041 . \tag{9}$$

For $R/R_0 \ne 1$, Eq. (8) and the above constraints on Y_P and Q_0/T_{f_0} give

$$\begin{aligned} &0.995 \le R/R_0 \le 1.005 , \\ &0.98 \le R/R_0 \le 1.02 \end{aligned} \tag{10}$$

for SS6 and KK (2–7), respectively, in rough agreement with results[1] of the detailed calculations [Eq. (1)] mentioned above.

The effect of neutrino degeneracy on nucleosynthesis has been studied by several workers.[3] Limits have been obtained on the degeneracy parameter ($\xi_\nu = \phi_\nu / KT_\nu$, ϕ_ν being the chemical potential and T_ν the temperature of the neutrinos) by comparing the theoretical predictions of Y_P with its observed values. A narrow range is allowed when only one type of neutrino is degenerate, the range being narrowest[3] for degenerate electron neutrinos, $-0.05 \le \xi_{\nu(e)} \le 0.1$ for $0.23 \le Y_P \le 0.25$. When two or more neutrino species are degenerate (not equally) the limits, particularly on $\xi_{\nu(\mu)}$ and $\xi_{\nu(\tau)}$, become very weak. In what follows I will therefore assume only the electron neutrinos to be degenerate, with a few comments at the end regarding degenerate ν_μ and ν_τ.

Following Beaudet and Goret[4] we have

$$(n/p)_{\xi_{\nu(e)} \neq 0} = (n/p)_{\xi_{\nu(e)} = 0} \exp(A\,\xi_{\nu(e)}) \ , \tag{11}$$

where $A = 1$ for $\xi_{\nu(e)} < 0$ and 1.2 for $\xi_{\nu(e)} > 0$. Thus,

$$Y_P = \frac{2}{1 + \exp(Q/T_f)\exp(A\,\xi_{\nu(e)})} \ . \tag{12}$$

For $R/R_0 = 1$, requiring $0.23 \le Y_P \le 0.25$ and using (9) this gives

$$-0.09 \le \xi_{\nu(e)} \le 0.095 \ , \tag{13}$$

in rough agreement with results of more detailed calculations mentioned above.

For $R/R_0 \neq 1$ we therefore have

$$Y_P = \frac{2}{1 + \exp[(Q_0/T_{f_0})(R/R_0)^B]\exp(A\,\xi_{\nu(e)})} \ . \tag{14}$$

It is clear from this equation that for a given value of Y_P a large range of values is permissible for R/R_0 depending on the value of $\xi_{\nu(e)}$. Clearly neither of the constraints obtained above on R/R_0 or $\xi_{\nu(e)}$ [Eqs. (10) and (13)] are valid when the effects of extra dimensions as well as neutrino degeneracy are considered together. In Fig. 1, I have plotted $\xi_{\nu(e)}$ vs R/R_0 for $Y_P = 0.24$ for SS6, KK2, and KK7 theories.

It can be seen from the figure as well as Eq. (14) that an upper limit of 1.66 (1.7) obtains for $\xi_{\nu(e)}$ in the limit of large R/R_0 for $Y_P = 0.24$ (0.23). No limit can be obtained for negative values of $\xi_{\nu(e)}$. The only constraint that then remains valid for $\xi_{\nu(e)} < 0$ is that obtained from energy density considerations:[2] namely, $|\xi_{\nu(e)}| \le 45$. This can be used along with Eq. (14) to obtain lower limits on values of R/R_0: namely, $R/R_0 \ge 0.7$, 0.35, and 0.23 for SS6, KK2, and KK7, respectively.

In case $\nu(\mu)$ or $\nu(\tau)$ are degenerate we have[4]

$$(n/p)_{\xi_{\nu(\mu)},\xi_{\nu(\tau)} \gg 1} = \exp(-4.2\,|\xi_{\nu(\mu)},\xi_{\nu(\tau)}|^{-2/3}) \ . \tag{15}$$

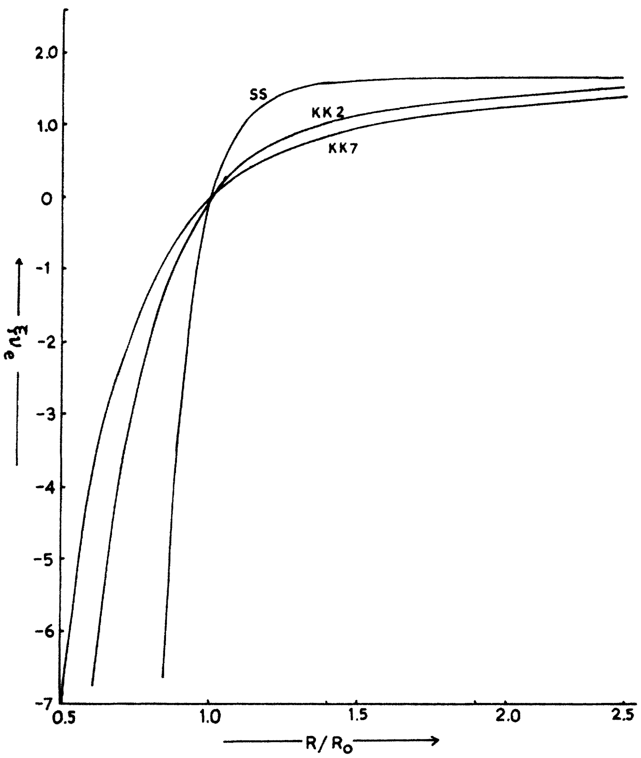

FIG. 1. A plot of the degeneracy parameter ($\xi_{\nu(e)}$) for primordial electron neutrino as a function of R/R_0 for $Y_P = 0.24$ for the superstring theory with six extra dimensions and for Kaluza-Klein theories with two and seven extra dimensions.

In presence of extra dimensions this will be modified to

$$(n/p)_{\xi_{\nu(\mu)},\xi_{\nu(\tau)} \gg 1} = \exp[-4.2\,|\xi_{\nu(\mu)},\xi_{\nu(\tau)}|^{-2/3}(R/R_0)^B] \ . \tag{16}$$

No limits can thus be obtained either on R/R_0 or on $\xi_{\nu(\mu),\nu(\tau)}$ from constraints on Y_P. Using the constraint $|\xi_{\nu(\mu),\nu(\tau)}| \le 45$ as obtained from energy density considerations lower limits $R/R_0 \ge 0.82$, 0.55, and 0.44 are obtained for SS6, KK2, and KK7 theories, respectively.

In conclusion, we have shown that the only constraints on the electron degeneracy parameter that can be obtained from nucleosynthesis calculations in theories with extra dimensions is $\xi_{\nu(e)} \le 1.7$. The only constraint that can be obtained in R/R_0 at the time of nucleosynthesis is $R/R_0 \ge 0.7$, 0.35, and 0.23 for SS6, KK2, and KK7, respectively.

[1] E. W. Kolb, M. J. Perry, and T. P. Walker, Phys. Rev. D **33**, 869 (1986).

[2] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 15.

[3] See the references given in A. M. Boesgard and G. Steigman, Annu. Rev. Astron. Astrophys. **23**, 319 (1985).

[4] G. Beaudet and P. Goret, Astron. Astrophys. **49**, 415 (1976).