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Finite-temperature instability for compactification

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We consider finite-temperature effects upon theories with extra dimensions compactified via vacuum stress-energy (Casimir) effects. For sufficiently high temperature, a static configuration for the internal space is impossible. At somewhat lower temperatures, there is an instability due to thermal fluctuations of the radius of the compact dimensions. For both cases, the Universe can evolve to a de Sitter-type expansion of all dimensions. Stability to late times constrains the initial entropy of the Universe.

I. INTRODUCTION

The current interest in supersymmetry and string theory has renewed discussion concerning theories with extra dimensions. An example is the superstring: Upon quantization it is found that Lorentz invariance is preserved (or, assuming Lorentz invariance, negative-norm states are absent) for ten space-time dimensions. If we are to take theories with more than four dimensions seriously, then a mechanism is needed for dynamically compactifying the extra dimensions so that they presently form a D-dimensional compact manifold with size of the order of the Planck length. The most natural setting for discussing the implications of compactification is the early Universe.[1]

In cosmology, one method used to stabilize the size of this compact manifold against small amplitude perturbations is to balance a positive, "bare" $(4+D)$-dimensional cosmological constant Λ against the vacuum stress energy of quantum and classical fields. By requiring that the effective four-dimensional cosmological constant vanishes in the ground-state configuration, one finds that the "radius" b_0 is a function of the bare cosmological constant. Compactification stabilized due to the vacuum stress energy of quantum fluctuations[2] is in analogy to the Casimir effect familiar in field theory, while compactification due to classical stress energy can, for example, arise due to the existence of nontrivial monopole configurations for the gauge and matter fields in the theory.[3] Another approach to stabilization is to include curvature-squared terms in the gravitational sector of the theory.[4] Such terms appear in the low-energy limit of superstrings.[5]

For these methods, the ground-state manifold, taken to be a product space of the form $R^1 \times Q^3 \times S^D$, is semiclassically unstable.[6] The classically stable ground state is, in fact, metastable (a "false vacuum") with nonzero probability for decay via quantum tunneling through a potential barrier. As a result of this barrier penetration to large values of the radius of the compact manifold, the effective four-dimensional cosmological constant is no longer zero and induces a de Sitter-type expansion in all $3+D$ spatial dimensions.

For compactification stabilized by the Casimir effect, Frieman and Kolb obtain an approximate form for the tunneling action: $S_4 \approx 165 m_{\rm Pl}{}^2/\Lambda$. The decay rate per unit 4-volume has the semiclassical form $\Gamma/V_4 \approx m_{\rm Pl}{}^4 \exp(-S_4)$. The probability that a given point will no longer be in the false, compactified state becomes large after a time $\tau \approx m_{\rm Pl}{}^{-1} \exp(41 m_{\rm Pl}{}^2/\Lambda)$. To avoid conflict with observation, τ should be longer than the present age of the Universe, which is possible if $\Lambda \leq 0.3 m_{\rm Pl}{}^2$. From the standpoint of naturalness, this constraint on Λ poses no difficulty, not requiring fine-tuning or implying an anomalous value for the size of the compact manifold ($\Lambda = 0.3 m_{\rm Pl}{}^2$ implies for $b_0 = 11 l_{\rm Pl}$), and so one might conclude that instabilities in the above models, though not a desirable feature, are phenomena one could learn to live with, at least for a few Hubble ages.

At nonzero temperature, there is an additional contribution to vacuum decay processes from finite-temperature effects,[7,8] and in general there exists a temperature T_0 above which the probability for thermally fluctuating over the potential barrier is greater than quantum tunneling through it. In this paper we consider the possibility of classically rolling over the potential barrier due to thermal fluctuations; we find that there exists a critical temperature $T_{\rm crit}$ for theories which balance the vacuum energy against a bare cosmological constant, above which there exists no stable point for compactification—the metastable ground state disappears; the Universe evolves directly into a $(4+D)$-dimensional space-time with exponential de Sitter-type expansion in all spatial dimensions. In addition, for $T_{\rm crit} > T$, the fraction of metastable vacuum $P(T)$ extant at temperature T, given that compactification occurs at $T_{\rm compact} \leq T_{\rm crit}$, is small if $-\ln P(T) < 1$. Except for compactification in a very small range of temperatures below $T_{\rm crit}$, stability of the compactified state against thermal decays does not impose any serious constraints on the initial entropy 3-volume $s_3{}^{\rm init}$ (i.e., Q^3) at compactification.

Our discussion assumes a product space manifold for the ground state of the form $R^1\times Q^3\times S^D$, where Q^3 is R^3, S^3, or a 3-hyperboloid as $k=0,1,-1$. The metric on this manifold is $g_{MN}=\text{diag}(-1,a^2(t)\tilde{g}_{mn},b^2(t)\tilde{g}_{\mu\nu})$, where $a(t)$ and $b(t)$ are the scale factors for Q^3 and S^D, and $\tilde{g}_{mn}$, $\tilde{g}_{\mu\nu}$ are metrics on the maximally symmetric unit 3-space and D-sphere. The indices M,N run over all values, the indices $m,n=1,2,3$ and $\mu,\nu=5,6,\ldots,4+D$. The Einstein equations are

$$R_{MN}=\tfrac{1}{2}Rg_{MN}-\frac{\Lambda}{2}g_{MN}=-8\pi\bar{G}T_{MN} \tag{1}$$

with T_{MN} the stress-energy tensor for classical and quantum fields, including thermal terms, and $\bar{G}$ is the gravitational "constant" in $4+D$ dimensions (the four-dimensional Newton constant is $m_{\text{Pl}}{}^{-2}=\bar{G}/\Omega_D^0$, where Ω_D^0 is the static volume of the internal D-sphere). Consistent with the symmetry of the product-space metric, the stress-energy tensor has nonzero components $T_{MN}=\text{diag}(\rho,p_3\tilde{g}_{mn},p_D\tilde{g}_{\mu\nu})$, with ρ, p_3, and p_D functions of $b(t)$ and temperature. From Eq. (1), the equations of motion for the scale factors are

$$3\frac{\ddot{a}}{a}+D\frac{\ddot{b}}{b}=\frac{1}{D+2}\{\Lambda-8\pi\bar{G}[(D+1)\rho+3p_3+Dp_D]\}\ , \tag{2}$$

$$\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}+D\frac{\dot{a}\dot{b}}{ab}+\frac{2k}{a^2}=\frac{1}{D+2}\{\Lambda-8\pi\bar{G}[-\rho+Dp_D+(1-D)p_3]\}\ , \tag{3}$$

$$\frac{\ddot{b}}{b}+(D-1)\frac{\dot{b}^2}{b^2}+3\frac{\dot{b}\dot{a}}{ba}+\frac{D-1}{b^2}=\frac{1}{D+2}[\Lambda-8\pi\bar{G}(-\rho+3p_3-2p_D)]\ . \tag{4}$$

The paper is organized as follows. In Sec. II we discuss the finite-temperature equations of motion for the specific example of a model which compactifies due to quantum effects. In Sec. III we outline the calculation of thermal decays for this model. In Sec. IV we discuss our results and briefly consider extensions to models constructed from the low-energy limits of string theories.

II. CASIMIR COMPACTIFICATION AT FINITE TEMPERATURE

Manifolds with nontrivial topology, such as D-spheres, can have their curvature associated with stress-energy tensors derived from 1-loop quantum fluctuations in matter fields defined on them. We will assume that these matter fields are noninteracting, spinless, and in thermal equilibrium at a temperature T. The free energy of such a system to one-loop order is

$$\beta F=\tfrac{1}{2}\ln\det(-\Box+\mu^2)\ ,$$

where $\beta=1/T$, $\Box$ is the Laplacian on the Euclideanized product-space manifold $S^1\times Q^3\times S^D$, and μ is a mass parameter. Since the system is at finite temperature, the time direction is compactified to a circle of radius $r=\beta/2\pi$. By assuming that $Q^3=S^3$ (this is chosen for simplicity; our results are equally valid for a $k=0$ space), the $(4+D)$-dimensional space is compact and $\Box$ has discrete eigenvalues on each n sphere of the manifold. The free energy becomes[9]

$$\beta F=\tfrac{1}{2}\sum_{r=-\infty}^{\infty}\sum_{m,n=0}^{\infty}D_{mn}\ln[(2\pi T)^2r^2+m(m+2)a^{-2}+n(n+D-1)b^{-2}+\mu^2]\ , \tag{5}$$

where

$$D_{mn}=\frac{(m+1)^2(2n+D-1)(n+D-2)!}{(D-1)!n!}\ .$$

Equation (5) is formally infinite and requires regularization. The finite part is[9]

$$\beta F=\frac{d}{ds}\left[\frac{1}{2\Gamma(-s)}\int_0^\infty dt\,t^{-s-1}\times\exp(-t\mu^2)\sigma_1(4\pi^2t^2\beta)\sigma_3(ta^{-2})\sigma_D(tb^{-2})\right]_{s=0} \tag{6}$$

with

$$\sigma_i(X)=\sum_{n=0}^{\infty}\left[\frac{(2n+i-1)(n+i-2)!}{(i-1)!n!}\right]\times\exp[-n(n+i-1)X]\ .$$

In general, the free energy has a complex form; however, it simplifies in the limits of high and low temperatures. In these cases, with $a\gg b$ (the "flat-space" limit), the σ_i take simple forms allowing the evaluation of the integral in Eq. (6). In the flat-space limit $\sigma_3\approx\frac{1}{4}\sqrt{\pi}a^3t^{-3/2}$ and for low temperatures, $1/2\pi b>T>1/2\pi a$, $\sigma_1\approx(4\pi tT^2)^{-1/2}$. The free energy for a single scalar field reduces (in odd dimensions) to[9]

$$F\approx\Omega_3\left[c_Nb^{-4}-\frac{\pi^2}{90}T^4\right]\ , \tag{7}$$

where Ω_3 is the volume of physical 3-space. Note that the volume of an n-sphere of radius R is $\Omega_n=V_nR^n$, $V_n=(2\pi)^{(n+1)/2}/\Gamma(n+1/2)$. The first term in Eq. (7) is the one-loop zero-temperature quantum (Casimir) correction whose coefficient c_N has been computed for various models by Candelas and Weinberg.[10] When T is greater than all scales in the theory (we will always assume $\mu<T$), the system reduces to radiation in $4+D$ dimensions. For a single scalar field F then becomes

$$F\approx-\frac{\zeta(D+4)}{\pi^{(D+2)/2}}\Gamma\left[\frac{D+4}{2}\right]\Omega_D\Omega_3T^{D+4}\ . \tag{8}$$

Generalizing to a set of spinless, noninteracting fields in thermal equilibrium, the free energy can be approximated by a function of the form[11]

$$F=\frac{\Omega_3}{b^4}[c_1-c_2(2\pi bT)^4-c_3(2\pi bT)^{D+4}]\ , \tag{9}$$

where the coefficient c_1 is the c_N of Candelas and Weinberg, while c_2 and c_3 are thermal terms.[12] This result assumes $\dot{a}=\dot{b}=0$, $a\gg b$. Equation (9) has the correct form in the high- ($T>1/2\pi b$) and low- ($T<1/2\pi b$) temperature limits and is expected to be qualitatively correct. Since we will be mainly concerned in the following sections with the high-temperature limit, we expect the approximation of the free energy by Eq. (9) to be adequate. Using Eq. (9), the total entropy S in a comoving $(3+D)$-volume can be written

$$S=-\left.\frac{\partial F}{\partial T}\right|_{a,b}=\frac{\Omega_3}{b^4}[c_2(2\pi bT)^3+(D+4)c_3(2\pi bT)^{D+3}]\ . \tag{10}$$

The functions ρ, p_3, p_D can be computed using standard thermodynamic relations ($U=F+TS$):

$$\rho=\frac{U}{\Omega_3\Omega_D}=\frac{1}{\Omega_D b^4}[c_1+3c_2(2\pi T)^4+(D+3)c_3(2\pi T)^{D+4}]\ , \tag{11a}$$

$$p_3=-\left.\frac{a}{3\Omega_3\Omega_D}\frac{\partial U}{\partial a}\right|_{b,S}=\frac{1}{\Omega_D b^4}[-c_1+c_2(2\pi T)^4+c_3(2\pi T)^{D+4}]\ , \tag{11b}$$

$$p_D=-\left.\frac{b}{D\Omega_3\Omega_D}\frac{\partial U}{\partial b}\right|_{a,S}=\frac{1}{\Omega_D b^4}\left[\frac{4}{D}c_1+c_3(2\pi T)^{D+4}\right]. \tag{11c}$$

At $T=0$ a static solution for Eqs. (2)–(4) requires the balancing of the $4+D$ cosmological constant against the one-loop fluctuations:

$$b_0^{D+2}=\frac{8\pi\bar{G}c_1(D+4)}{V_nD(D-1)}\ , \tag{12a}$$

$$\Lambda=\frac{D(D-1)(D+2)}{b_0^2(D+4)}\ . \tag{12b}$$

Here, b_0 is the static radius of the D-sphere. The effective four-dimensional cosmological constant has the form

$$\Lambda_{\rm eff}(b)=\frac{D(D-1)}{(D+4)}\left[\frac{1}{b_0^2}-\frac{b_0{}^{D+2}}{b^{D+4}}\right]\ , \tag{13}$$

so that $\Lambda_{\rm eff}(b=b_0)=0$. For finite temperature, choosing $k=0$ in the external space, the evolution equations for $a(t)$ and $b(t)$, Eqs. (3) and (4), can now be written

$$\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}+D\frac{\dot{a}\dot{b}}{ab}=\Lambda_{\rm eff}(b)+\frac{D(D-1)}{(D+4)}\frac{b_0{}^{D+2}}{b^{D+4}}\left[\frac{c_2}{c_1}(2\pi bT)^4+\frac{c_3}{c_1}(2\pi bT)^{D+4}\right]\ , \tag{14a}$$

$$\frac{\ddot{b}}{b}+(D-1)\frac{\dot{b}^2}{b^2}+3\frac{\dot{b}\dot{a}}{ba}=-\frac{D-1}{b^2}+\frac{D(D-1)}{(D+4)}\left[\left[b_0^{-2}+\frac{4}{D}\frac{b_0{}^{D+2}}{b^{D+4}}\right]+\frac{b_0{}^{D+2}}{b^{D+4}}\frac{c_3}{c_1}(2\pi bT)^{D+4}\right]\ . \tag{14b}$$

For $T<T_{\rm crit}$ (see below) the static solution is stable against small perturbations $\delta b(t)\rightarrow b(t)+b_0$. In the limit $b(t)\rightarrow\infty$, $a(t)\rightarrow\infty$, Eqs. (14a) and (14b) have the de Sitter-type solutions $a,b\approx\exp(\pm Ht)$ with $H^2=\Lambda/(D+2)(D+3)$. From Eq. (13), as $b(t)\rightarrow\infty$, $\Lambda_{\rm eff}(b)\rightarrow D(D-1)b_0{}^{-2}/(D+4)$, i.e., $\Lambda_{\rm eff}(b)\rightarrow\Lambda$. There are two regions of interest: $T>\max[(1/2\pi a),(1/2\pi b)]$ and $(1/2\pi b)>T>(1/2\pi a)$. In the high-temperature region, a power-law solution of the form $a(t)\approx\alpha t^{2/(D+4)}$, $b(t)\approx\beta t^{2/(D+4)}$ obtains for $t\rightarrow 0$. The dependence of temperature on the scale factors can be found from Eq. (10):

$$T=K\times(a^3b^D)^{-1/(D+3)}\ , \tag{15}$$

with

$$K=\left[S\frac{1}{(D+4)}\frac{1}{c_3}(2\pi)^{D+2}\left[\frac{a^3(t)}{\Omega_3}\right]\right]^{1/(D+3)}\ .$$

When Q^3 does not have positive spatial curvature, Ω_3 is the volume within a casual horizon size. If the internal dimensions have stabilized then in the low-temperature region ($T<1/2\pi b$), the Universe is effectively four dimensional since the temperature is now less than the energy scale for exciting the compactified dimensions ("freeze-out" of the extra dimensions). Conservation of entropy then implies the familiar relation $T\propto a^{-1}(t)$.

III. CALCULATION OF THERMAL DECAY

How does the stability of the compactification point depend upon temperature? Specifically, how is stability affected by compactification in the high- or low-temperature regions? To answer this note that the equation of motion for the scale factor $b(t)$ can be written in the form of an equation of motion similar to that for a scalar field minimally coupled to gravity in four dimensions.[13] As dictated by the requirement that there can be a canonical kinetic term for the scalar field, define $\phi(b)\equiv m_{\rm Pl}(b/b_0)^{D/2}[(D-1)/2\pi D]^{1/2}$. With $\phi_0=\phi(b_0)=m_{\rm Pl}[(D-1)/2\pi D]^{1/2}$, we can define $\Phi\equiv\phi/\phi_0$. In terms of this new scalar field Φ, Eq. (14b) becomes

$$\ddot{\Phi}+3\frac{\dot{a}}{a}\dot{\Phi}+\frac{\dot{\Phi}^2}{\Phi}=-\frac{dV}{d\Phi} \tag{16}$$

which, aside from the term $\dot{\Phi}^2/\Phi$, is the equation of motion for a minimally coupled scalar field. The form of the potential can be read off:

$$V(\Phi,T)=\frac{(D-1)\Lambda m_{\rm Pl}^2}{8\pi(D+2)}\left[\frac{(D+4)}{(D-2)}(\Phi^{(2/D)(D-2)}-1)+\Phi^{-8/D}-\left[1+\frac{c_3}{c_1}(2\pi b_0T)^{D+4}\right]\Phi^2+\frac{c_3}{c_1}(2\pi b_0T)^{D+4}\right]. \tag{17}$$

We have chosen the constant of integration such that $V(\Phi=1,T)=0$; this is a local minimum of the $T=0$ potential and in this limit Eq. (17) corresponds to Eq. (10) of Ref. 6. The temperature dependence of the potential is illustrated in Fig. 1. For $\Phi>\Phi_M$, where $\Phi_M>\Phi_0$ is the local maximum of $V(\Phi,T)$, the potential is unbounded from below. For sufficiently low temperature ($T<T_{\rm crit}$), the potential has a local minimum Φ_0, while for sufficiently high temperature ($T>T_{\rm crit}$) the potential is monotonically decreasing for increasing Φ. The potential barrier separating the stable compactification point from the unstable (unbounded) region decreases as T increases. Note that at $T\neq 0$, $\Phi=1$ is not the minimum of $V(\Phi,T)$ (here and in the rest of the paper, $m_{\rm Pl}=1$):

$$V'(\Phi,T)\,|_{\Phi=1}=-2\frac{(D-1)\Lambda}{8\pi(D+2)}\frac{c_3}{c_1}(2\pi b_0T)^{D+4}\phi_0^{-1}\,.$$

Therefore, the true local minimum, Φ_0 is greater than one. Similarly, $\Phi_M(T\neq 0)$ is less than $\Phi_M(T=0)$. However, we have found numerically that for $T\leq T_{\rm crit}$, the relative difference between the points Φ and $\Phi=1$ is less than 1, so that the choice $\Phi_0\approx 1$ is reasonable and allows a rough estimate of $T_{\rm crit}$ by requiring that at that temperature, $\Phi_0\approx 1$ be an inflection point:

$$T_{\rm crit}\approx\frac{1}{2\pi b_0}\left\{\frac{c_1}{c_3}\left[\frac{4}{D}\left[\frac{4}{D}+1\right]\right]\right\}^{1/(D+4)}.$$

This relation overestimates $T_{\rm crit}$ (found numerically) by $\approx 20\%$ (since $\Phi_0>1$). The range of temperatures in which $V(\Phi,T)$ has no (meta)stable compactification point is $T>T_{\rm crit}$, while the region of high temperature is defined by $T>T_H=1/2\pi b$. In terms of the scalar field Φ

$$T_H=(2\pi b_0\Phi^{2/D})^{-1}\,,$$

and, in particular, when $\Phi_0\approx\Phi=1$, $T_H=1/2\pi b_0$. Since $T_{\rm crit}$ is proportional to T_H, as $\Lambda\to 0$, $T_{\rm crit}$ and $T_H\to 0$. Decreasing Λ shifts the region $T_{\rm crit}\geq T\geq T_H$ downward. At zero temperature, the requirement that $V(\Phi,T)$ be stable against semiclassical decay implies that $\Lambda\leq 0.3$. This gives the values $T_H\leq 1.49\times 10^{-2}$ and $T_{\rm crit}\leq 2.44\times 10^{-2}$ ($\leq 1.975\times 10^{-2}$ numerically).

As noted by Frieman and Kolb, if the gravitational degrees of freedom $a(t)$ are treated as a classical background, we must look for barrier penetration solutions (bubbles) which are "thick walled" due to the unbounded nature of the potential.[14] The bubble interior is approximately de Sitter, while the exterior is asymptotically flat. In the present case we must consider false vacuum decay at finite temperature.[15] For finite-temperature field theory, a formal equivalence can be established with Euclidean field theory, the Euclidean time being periodic in β. Rather than requiring solutions with O(4) symmetry, O(3)-symmetric solutions periodic in β must be found. At high temperatures, the time integration in the four-dimensional action, S_4, is trivial: $S_4=\beta S_3$ with S_3, the three-dimensional action. For a minimally coupled scalar field,

$$S_3=\int d^3x\left[\tfrac{1}{2}(\nabla\Phi)^2+V(\Phi,T)\right]. \tag{18}$$

The Euclidean equation of motion satisfied by Φ becomes, at finite temperature,

$$\frac{d^2\Phi}{dr^2}+\frac{2}{4}\frac{d\Phi}{dr}=\frac{dV(\Phi,T)}{d\Phi}\,. \tag{19}$$

Solutions to Eq. (19) with boundary conditions $\Phi\to 0$ as $r\to\infty$ are finite-temperature bounce solutions which extremize (minimize) S_3. The extremized action gives the (thermal) decay probability per unit 4-volume:

$$\frac{\Gamma}{V_4}=\beta^{-4}\exp[-\beta S_3(\Phi,T)]\,,$$

where (again for high temperatures) we have set the prefactor equal to β^{-4}, since the relevant energy scale of the calculation (at the moment of bubble formation) is

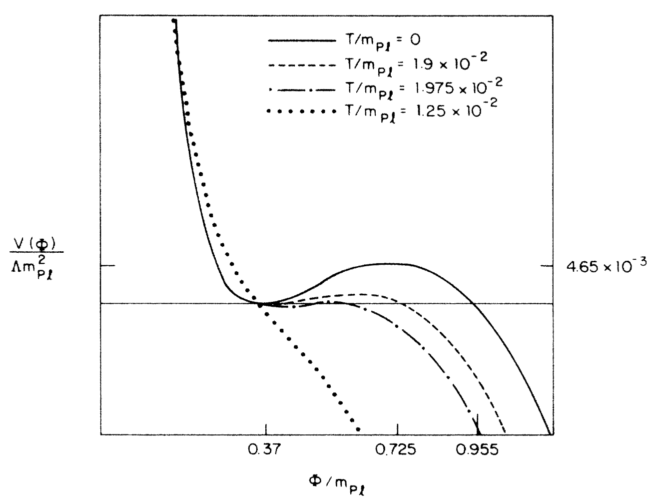

FIG. 1. The temperature dependence of the potential given in Eq. (12).

$\beta^{-1}=T$. Thermal decays, characterized by bounce solutions to Eq. (19), dominate quantum decays (and so dominate the total decay probability) when $\beta S_3 \le S_4$, equality between the actions holding when $T=T_{eq}$. Using the value $S_4=165/\Lambda$ from Ref. 6, $T>T_{eq}$ ($T<T_{eq}$) implies $\beta S_3<165/\Lambda$ ($\beta S_3>165/\Lambda$). Because of the exponential nature of Γ/V_4 (and the strong power dependence of the temperature term) thermal decays are dominant in the region $T>T_{eq}$ while for $T<T_{eq}$, semiclassical decays are dominant.

Because of the complexity of the equations, the bounce solutions and actions were evaluated numerically. A representative sample of our results are presented in Table I. To place the metastable minimum near the origin, the potential is evaluated with respect to $\overline{\Phi}=\Phi-1$. Initially, $\overline{\Phi}=0$, so that $\overline{\Phi}$ is nonsingular at the origin (the initial kinetic energy of Φ can be damped by particle production). For our model, the internal space is a 7-sphere[16] and, consistent with semiclassical (low-temperature) constraints, $\Lambda\le 0.3$. The massless matter fields which give rise to the Casimir and thermal terms are assumed to be minimally coupled so that $c_3/c_1=3.81\times10^{-3}$ which is kept fixed since both coefficients scale with the number of matter fields. From Eqs. (12a) and (12b), Λ is a function of the number of matter fields (for $\Phi\approx 1$):

$$\Lambda=\frac{[D(D-1)]^2(D+2)}{8\pi c_1(D+4)^2}\ .$$

With $c_1=8.16\times10^{-4}$ (a single field), $\Lambda\approx 6.4\times10^3$. In preparing Table I, we have adjusted the number of matter fields such that $\Lambda\approx 0.3$, 0.1, 0.01, 0.001. Finally, note that $T_{eq}>T_H$ so that we need only consider the high-temperature region. For each value of Λ, there exists a temperature T_{crit} at which βS_3 vanishes. If $\Phi=\Phi_0$ for $T>T_{crit}$, stabilization is impossible. Though T decreases as Φ increases, this only assures that $\Phi>\Phi_M$ when $T<T_{crit}$.

Compactification at high temperature brings with it a finite probability for decay. Given the temperature dependence of the problem, we take the following approach: The fraction of false vacuum remaining by the time t is[17]

$$P(t)=\exp\left[-\int_{t_0}^{t}dt_1\frac{\Gamma}{V_4}R^3(t_1)V(t_1,t)\right] \tag{20}$$

TABLE I. Decay actions at finite temperature.

Λ	T	βS_3
	1.5×10^{-2}	3.4×10^3
	1.7×10^{-2}	2.5×10^3
$0.3m_{Pl}^2$	1.9×10^{-2}	1.1×10^3
($T_H\approx1.5\times10^{-2}$)	1.95×10^{-2}	5.5×10^2
($S_4=5.5\times10^2$)	1.96×10^{-2}	4.2×10^2
	1.97×10^{-2}	1.3×10^2
	1.975×10^{-2}	0
	8.6×10^{-3}	1.0×10^4
	9.0×10^{-3}	9.4×10^3
$0.1m_{Pl}^2$	9.4×10^{-3}	8.6×10^3
($T_H\approx8.6\times10^{-3}$)	9.8×10^{-3}	7.5×10^3
($S_4=1.65\times10^3$)	1.125×10^{-3}	1.65×10^3
	1.135×10^{-3}	6.5×10^2
	1.137×10^{-3}	0
	2.7×10^{-3}	1.0×10^5
	3.0×10^{-3}	8.4×10^4
$0.01m_{Pl}^2$	3.2×10^{-3}	6.7×10^4
($T_H\approx2.7\times10^{-3}$)	3.4×10^{-3}	4.3×10^4
($S_4=1.65\times10^4$)	3.56×10^{-3}	1.65×10^4
	3.60×10^{-3}	4.9×10^2
	3.601×10^{-3}	0
	8.6×10^{-4}	1.0×10^6
$0.01m_{Pl}^2$	9.0×10^{-4}	9.4×10^5
($T_H\approx2.7\times10^{-3}$)	1.0×10^{-3}	7.0×10^5
($S_4=1.65\times10^4$)	1.124×10^{-3}	1.65×10^5
	1.135×10^{-3}	5.60×10^4
	1.136×10^{-3}	0

with

$$V(t_1,t)=\frac{4\pi}{3}\left[\int_{t_1}^{t} dt_2 R^{-1}(t_2)\right]^3$$

the coordinate volume of a bubble at time t, formed at t_1. Since Eq. (20) can be rewritten as a function of temperature, if we assume freeze-out of the extra dimensions occurs instantaneously at $T=T_H$, the integral can be evaluated as a sum of separate integrations over regions (1) $T>T_H$ and (2) $T\leq T_H$. However, the contribution to the integral from region 2 can be dropped due to the exponential suppression of the decay rate. The fraction of false vacuum remaining at $T=0$ is (remembering that for the ground state $b\approx b_0$)

$$\ln P(T=0)=BI(T=0)\,, \tag{21}$$

where

$$I(T=0)\approx -\int_{T_H}^{T_{\text{compact}}} dT_1 T_1^{-(D+4)(D+9)/6} \times (T_1^{-(D+2)(D+3)/6} - T_H^{-(D+2)(D+3)/6})^3 \frac{\Gamma}{V_4}\,,$$

and

$$B=-\frac{4\pi}{3}\left[\frac{6}{(D+3)}\right]^4 \frac{C^{4(D+4)(D+3)/6}}{(D+2)^3(D+4)} b_0^{-4D(D+4)/6}\,.$$

The constant C is given by $C=K\alpha^{-3/(D+3)}$. If we take s_3^{init} to be the initial entropy per 3-volume (volume within a causal horizon size if $k\neq 1$ for Q^3) at compactification, then $s_3^{\text{init}}=(D+4)c_3C^{D+3}/(2\pi)^{D+2}$. From Eq. (20), decay at the temperature T of the ground state compactified at T_{compact} (the temperature for which $\Phi=\Phi_0$) is implied by the condition $-\ln P(T)>1$. To avoid this decay, we must require that

$$C<\left[\frac{3}{4\pi}(D+2)^3(D+4) \times \left[\frac{(D+3)}{6}\right]^4 I(T)^{-1}\right]^{6/[4(D+4)(D+3)]} b_0^{D/(D+3)}\,.$$

For a given Λ, we consider the compactification temperature in the range $T_{\text{crit}}\geq T_{\text{compact}}>T_H$. The case $T_{\text{compact}}=T_{\text{crit}}$ imposes the strongest constraints on C, i.e., for $\Lambda=0.001$, $C\leq 10^{-4}$ while $s_3^{\text{init}}\leq 10^{-45}$. In general, $\Lambda\to 0$ implies that the upper limit of C take on smaller values. When $T_{\text{compact}}<T_{\text{crit}}$ the constraint on C can be approximated by $C<\eta\exp[(6/440)\beta S_3(T_{\text{compact}})]b_0^{7/10}$ where $\eta\approx 10^{-2}$. As one expects, the constraint on C becomes rapidly less severe as $T_{\text{compact}}\to T_H$; when $\Lambda=0.3$, $C<1$ for $T_{\text{compact}}>1.96\times 10^{-2}$. The results for $T_{\text{compact}}=T_{\text{crit}}$ are summarized in Table II.

TABLE II. Constraints on $C\,(T_{\text{compact}}=T_{\text{crit}})$.

Λ	T_{compact}	$C\langle$
0.3	1.975×10^{-2}	1.3×10^{-2}
0.1	1.137×10^{-2}	5.2×10^{-3}
0.01	3.601×10^{-3}	7.8×10^{-4}
0.001	1.136×10^{-3}	1.2×10^{-4}

IV. CONCLUSION

We end here with some comments concerning our approximations and results. Though curved-space corrections may serve to enhance the decay rate,[18] we have omitted them because their contribution does not significantly change our results. For $V(\Phi,T=0)$ the flat-space approximation for the solutions is acceptable since[19] $M>H$ where M is the mass parameter in the potential and H is the de Sitter–Hubble constant. In the present case this result is expected to hold since for Φ large, the finite-temperature mass parameter $M(T)\approx M$. The Casimir contribution to the free energy in Eq. (9), was computed for the static limit $\dot{a}=\dot{b}=0$. The time dependence of the scale factors will introduce corrections to both the potential and kinetic terms in the action.[20] As in Ref. 6 for $T=0$, we conclude that such corrections will not alter the existence of the $T\neq 0$ instabilities.

Our analysis leads us to the conclusion that there exists a range of temperatures for higher-dimensional cosmologies in which compactification via Casimir effects is unstable due to thermal fluctuations. For $T>T_{\text{crit}}$, there is no way to avoid this instability: the Universe evolves directly to a state of de Sitter-type expansion in all dimensions. For compactification in the region $T_{\text{crit}}\geq T_{\text{compact}}>T_H$, stability against thermal decays does not strongly constrain the parameter C except when $T_{\text{compact}}\approx T_{\text{crit}}$ (when $\Lambda=0.3$, $C<1$ for $T_{\text{compact}}>1.96\times 10^{-2}$). The low values for T_{crit} (i.e., $T_{\text{crit}}<1.975\times 10^{-2}$ for $\Lambda=0.3$) seem the most serious objections to hot initial conditions for such theories. Out of economy one might expect $T_{\text{compact}}\approx 1$ (compactification at the Planck scale) since it is the only scale available. However, this is not a strong objection; serious difficulties arise when these results are considered in light of theories which are more physically significant.

As in the case of the semiclassical instabilities found in Ref. 6, we believe our results have bearing on superstring theories.[21] Though the mass scales for the string tension m_{str}, compactification m_{compact}, and the Planck scale m_{Pl} are independent, very general arguments[22] based on the validity of a semiclassical approximation for the string and the strong coupling of the nonlinear σ model on the world sheet imply that $m_{\text{str}}\approx m_{\text{compact}}\approx m_{\text{Pl}}$. Our work indicates that compactification may not occur if there are hot initial conditions ($\Lambda\leq 0.3m_{\text{Pl}}^2$ implies $T_{\text{crit}}\leq 10^{-2}m_{\text{Pl}}$). Still, an obvious implication of the mass scale result for compactification in string theories is that the massive string modes can no longer be ignored and consistency would require study of the superstring in nontrivial background fields.[23] To first order in the string tension, the equations of motion for the background fields are the same as those obtained from the modified Chapline-Manton action.[24] The bosonic sector of the action contains the field strengths G_{MN} and H_{MNO}. Using the ansatz of Freund and Rubin,[25] the equations of motion for the scale factors contain, in addition to quantum and thermal contributions, terms of the form A/b^{2D} where A is a constant. This model will have thermal instabilities similar to what we have discussed if a cosmological constant is present. Note that we do not need to include a cosmological constant to stabilize compactification since this can be achieved by balancing monopole and Casimir terms.[26] It is possible to avoid decay of the ground state in this case when the effects of fermionic condensates are included.

Curvature squared terms appear at second order in the string tension in the equations of motion for the background fields, corresponding to the lowest-order massive modes. Introducing such corrections will not, in themselves, alter our results in the presence of a cosmological constant and possible vacuum contributions. For higher-dimensional curvature squared theories of the type considered by Shafi and Wetterich,[27] the effective four-dimensional action has a potential consisting of two terms: a scalar part, which has the same form found in theories with vacuum compactification, and a curvature-dependent part. For a particular choice of coefficients, the second term pulls the de Sitter region out to infinity. However, the thermal term enters in the potential with the same power of Φ as the curvature part and dominates at high temperature. For the specific case of the dimensionally continued Euler characteristic[28] (for which the de Sitter region is not at infinity) these considerations imply an instability, which is expected as well for the corrections obtained by Callan *et al.*

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