Classical cosmologies from ten-dimensional supergravity

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We study possible cosmological solutions to $N=1$, $D=10$ supergravity with the Yang-Mills field strength set to zero and show that the model accepts both power-law solutions and exponential solutions in the large-time limit. The stability of these solutions is investigated. It is found that a shrinking internal space is compatible with several field configurations. Using a stable power-law solution we analyze the conditions to obtain enough inflation in the physical space from the shrinking internal dimensions. We also show that for a flat topology a de Sitter phase is possible for late times. We used the consistency with the density perturbations to fix the inflationary parameter.

In the quest for the unification of the fundamental interactions, superstring theories[1] are presently considered as the best candidate. Indeed, while the alliance of local supersymmetry with higher-dimensional theories offered the possibility of obtaining the full $N=8$ supergravity action in four dimensions via dimensional reduction of $N=1$ supergravity in 11 dimensions,[2] the ultraviolet divergences that were already present in four-dimensional calculations of graviton and matter-loop corrections did not get appreciably milder when going to higher-dimensional supersymmetric theories. Thus, the proof that superstrings are anomaly-free at 1 loop for SO(32) or $E_8 \times E_8$,[3] plus their finiteness[4] to at least that order, has triggered a great deal of action in the complete formulation of the correct theory with acceptable phenomenological predictions.[5]

A related question of great importance is the cosmological implications of string theories. Beyond the Planck scale, the drastic changes that are brought up by the inclusion of the massive modes as intermediaries of gravitational interactions are surely going to have important consequences in our understanding of the initial singularity, as some recent work on the subject has shown,[6] although still in a superficial way. As we lower the energy, the massive modes are frozen out of equilibrium and the string is thought to collapse to a point, with the massless modes being related to massless fields described by a local field theory. The type of field theory obtained is related to the way the string theory is formulated, i.e., it being an open or closed string theory or by the number of supersymmetry generators (if any) involved.

As has been stressed in the literature,[7] the main concern one has when looking for cosmological solutions of higher-dimensional theories is to obtain an explanation for the enormous difference between the physical and the internal scale factors consistent with compactification. One may add that it is also very important to obtain a scenario which is consistent with the known limits on the time variation of the fundamental couplings. This implies that the internal space must have been constant or very nearly so since nucleosynthesis.[8]

In this paper, we propose to study possible classical cosmological solutions arising from the bosonic sector of $N=1$, $D=10$ supergravity theory[9] (or equivalently the Chapline-Manton action[10] with the Yang-Mills field strength set to zero) which describes the massless bosonic sector of the type-I superstrings and of the phenomenologically more promising heterotic string.[11]

We will show that this model admits power-law solutions for the scale factors and also de Sitter-type solutions at late times. The stability of these solutions will be analyzed as well as some of their physical implications.

We will start by writing the action as

$$S=-\int d^{10}z(-g^{10})^{-1/2}\Big(R-\tfrac{1}{2}g^{MN}\partial_M\phi\partial_N\phi -\tfrac{1}{6}e^{\phi}F_{MNP}F^{MNP}\Big)\ . \tag{1}$$

We will adopt the conventions of Weinberg's book with $G=c=\hbar=1$. By varying the action with respect to the dynamical fields we get the following field equations:

$$R_{MN}-\tfrac{1}{2}g_{MN}R=-T_{MN} \tag{2a}$$

with

$$T_{MN}=-\partial_M\phi\partial_N\phi+\tfrac{1}{2}(\partial\phi)^2g_{MN} -e^{\phi}\big(F_{MPQ}F_N^{PQ}-\tfrac{1}{6}F_{PQR}F^{PQR}g_{MN}\big)\ , \tag{2b}$$

$$\partial_M[(-g^{10})^{1/2}e^{\phi}F^{MNP}]=0\ , \tag{3}$$

$$\frac{1}{(-g^{10})^{1/2}}\partial_M[(g^{10})^{1/2}\partial^M\phi]=\tfrac{1}{6}e^{\phi}F_{PQR}F^{PQR}\ , \tag{4}$$

where capital latin indices run from 0 to 9, greek indices run from 0 to 3, i,j,k run from 1 to 3, and m,n,p from 4 to 9.

To solve this system of equations we must make an ansatz on the Kalb-Ramond and the scalar fields. The simplest choice comes from assuming that the dilaton field is a constant throughout space and time. If we do so then Eq. (4) compels us to take a zero value for the Kalb-Ramond three-index field. Clearly then the problem reduces to solving Einstein's equations in vacuum.

A few words should be said about this case as it has been the object of intense study. This case is important because at early times the curvature terms might have dominated the dynamics, making the contribution from matter and fields negligible. For the case of anisotropic models this is certainly the situation, as the energy associated with the curvature anisotropy is going to dominate over any other contribution.[12]

The case where the underlying metric is that of a product of two anisotropic spaces (for $D=4$) was studied in Ref. 13, it was found that the field equations accepted a Kasner-type power-law solution where the internal dimension contracted while the physical space expanded isotropically like a radiation-dominated Friedmann model. The general form of the vacuum anisotropic solution for $D \geq 4$ has been studied in the context of the mixmaster models in Ref. 12. In the general case the topology of space is not that of a product space but instead that of a simply connected manifold. The solution presented in Ref. 13 is a particular case of these models.

In this paper we will only consider the case where the topology of space-time is that of a product of manifolds of the form $M^4 \times B^6$ with a metric described by the appropriate Robertson-Walker line element:[7]

$$ds^2 = -dt^2 + R_3(t)^2 \tilde{g}_{ij}(\mathbf{x}) dx^i dx^j + R_6(t)^2 \tilde{g}_{mn}(\mathbf{y}) dy^m dy^n \ . \tag{5}$$

The general solution for the case where both manifolds are flat is given by a Kasner-type power-law solution similar to that found in Ref. 13:

$$R_3 = R_{30} t^{-1/3}, \quad R_6 = R_{60} t^{1/3} \ , \tag{6a}$$

$$R_3 = R_{30} t^{5/9}, \quad R_6 = R_{60} t^{-1/9} \ . \tag{6b}$$

So the internal space becomes smaller while the physical space expands (as in a radiation-dominated model).

There is a trivial solution where both radii are static, so describing a ten-dimensional Minkowski space-time of the form $M^{10} = M^4 \times M^6$.

If we now allow the curvatures to be nonzero, then we can find the following power-law solutions:

$$R_3 = R_{30} t, \quad R_6 = R_{60} t \tag{7}$$

with $k_3 = -4 R_{30}^2, k_6 = -\frac{8}{5} R_{60}^2$. This is not a very interesting solution as it predicts an expansion for both the internal and the physical radii.

If, on the other hand, we now take $F_{MNP} = 0$, we do not require $\phi = \phi_0$. Then Eq. (3) is identically satisfied and the action describes Einstein gravity with a massless scalar field. Equation (4) now gives

$$\partial_M [(g^{10})^{1/2} \partial^M \phi] = 0 \tag{8}$$

if we now assume the dilaton is a homogeneous field we get

$$\dot{\phi} = \frac{1}{R_3{}^3 R_6{}^6} = \frac{1}{V} \tag{9a}$$

and the field equations now accept the solution

$$R_3 = R_{30} t^{-4/9}, \quad R_6 = R_{60} t^{7/8} \tag{9b}$$

and

$$R_3 = R_{30} t^{2/3}, \quad R_6 = R_{60} t^{-1/6} \tag{9c}$$

with

$$\phi = \phi_0 \ln(t), \quad k_3 = k_6 = 0 \ . \tag{9d}$$

The second solution is much more interesting than the first. We can understand the similarity of this solution with a dust solution by noting that the scalar field scales like the inverse of the volume, just like the energy density of matter (dust). There are no power-law solutions for nonflat manifolds.

The next step in complication arises if we assume both the ϕ and the F_{MNP} fields are only functions of time, and the F field takes values only in the physical three-space (by no means is this the only feasible ansatz, for some different ones see Ref. 14), i.e.,

$$F_{ijk} = \epsilon_{ijk} (g^3)^{1/2} F(t) \ , \tag{10a}$$

$$F_{mnp} = F_{\mu\nu p} = F_{\mu n p} = 0 \ . \tag{10b}$$

This ansatz is consistent with having a torsion-free internal manifold if we want to preserve SU(3) holonomy.

We note that the no-go theorem for 10 into 4 compactification[15] will not apply here since the four-dimensional space-time is not maximally symmetric and the dilaton plus the Kalb-Ramond field can, according to the ansatz, vary in time.

Using (5) and the ansatz in (10) we can immediately integrate Eq. (3) to give

$$R_6{}^6 e^{\phi} F = f(t) \tag{11}$$

with $f(t)$ an arbitrary function of time. We require the Kalb-Ramond field defined in (10) to obey the Bianchi identities:

$$\partial_{[M} F_{NPQ]} = 0 \ . \tag{12}$$

This equation gives

$$(g^3)^{1/2} F = f_0 \tag{13}$$

with f_0 a constant. Clearly from (12) and (13) we get the form of $f(t)$:

$$f(t) = f_0 \frac{R_6{}^6}{R_3{}^3} e^{\phi} \ . \tag{14}$$

We are now in a position to write down the field equations:

$$3\frac{\ddot{R}_3}{R_3} + 6\frac{\ddot{R}_6}{R_6} = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2} e^{\phi} F^2 \ , \tag{15}$$

$$\frac{2k_3}{R_3{}^2} + \frac{d}{dt}\left[\frac{\dot{R}_3}{R_3}\right] + \left[3\frac{\dot{R}_3}{R_3} + 6\frac{\dot{R}_6}{R_6}\right]\frac{\dot{R}_3}{R_3} = -\tfrac{3}{2} e^{\phi} F^2 \ , \tag{16}$$

$$\frac{5k_6}{R_6{}^2} + \frac{d}{dt}\left[\frac{\dot{R}_6}{R_6}\right] + \left[3\frac{\dot{R}_3}{R_3} + 6\frac{\dot{R}_6}{R_6}\right]\frac{\dot{R}_6}{R_6} = \tfrac{1}{2} e^{\phi} F^2 \ , \tag{17}$$

and

$$\ddot{\phi} + \left[3\frac{\dot{R}_3}{R_3} + 6\frac{\dot{R}_6}{R_6}\right]\dot{\phi} = -2 e^{\phi} F^2 \ . \tag{18}$$

We shall see that this system of equations accepts several exact solutions. Then we will use one of these as an initial condition and integrate the system numerically. Based on this we can show that the model has an inflationary phase, by obtaining an exponential solution in the late time regime, when the Kalb-Ramond field is negligible. It is also interesting to note that the system does not accept a static solution unless we add a mass term or a self-interacting term to the dilaton field present in the action, which would break supersymmetry explicitly.

By direct substitution of a power-law ansatz into Eqs. (10)—(13) it is possible to show that two consistent solutions can be found:

$$R_3=R_{30}t,\quad R_6=R_{60}t^{-1},\quad \phi-\phi_0=4\ln(t)\ , \tag{19}$$

with $k_3=-4R_{30}^2$, $k_6=0$, and $\phi_0=\ln(8R_{30}^6/f_0^2)$, and

$$R_3=R_{30}t^{3/7},\quad R_6=R_{60}t^{-1/7},\quad \phi-\phi_0=\tfrac{4}{7}\ln(t)\ , \tag{20}$$

with $k_3=k_6=0$ and $\phi_0=\ln(8R_{30}^6/49f_0^2)$. R_{30} and R_{60} are arbitrary constants.

By perturbing the field equations around these two solutions we discover after some algebra that the first one is stable while the second is not. However, we should mention that the growth of the instabilities is like a power law: namely, very slow (for some discussion about this effect see Ref. 16). This instability could be associated with the fact that the model is flat in both spaces. It is a well-known fact that in four dimensions the flat Friedmann-Robertson-Walker solution is unstable under small perturbations. This is interpreted by saying that any extra energy put in or taken out from the Universe will close or open the model.

For the moment we will concentrate on the stable solution and its physical implications. First of all, it is a nice solution at least from a qualitative point of view since the three-physical space expands while the internal space contracts. However, if this expansion rate is not decreased at late times, the solution would be completely unacceptable since it would mess up nucleosynthesis. Nevertheless, we do expect the influence of other fields to become the leading terms in the energy-momentum tensor and slow down the expansion. Radiation at late times will undoubtedly change the expansion rate of both manifolds.

If we accept this fact we can now turn this rapid expansion to our advantage and use it to produce inflation. Entropy production in the noncompact dimensions is obtained by decreasing the mean volume due to the contraction of the compactified dimensions.[17]

For the solution we are interested the mean spatial volume decreases like

$$V\sim(R_3{}^3R_6{}^6)^{1/9}\sim t^{-1/3}\ . \tag{21}$$

We would like to develop a rough calculation to show that even a power-law solution may produce sufficient inflation. We shall assume that the total entropy is adiabatically conserved and has the required value (how this total entropy was generated may depend on the quantum era prior to this classical approximation; in any case this is a standard assumption for these models[18]),

$$S\sim10^{88}\sim R_{3i}{}^3R_{6i}{}^6T_i{}^9\simeq R_{3f}{}^3R_{6f}{}^6T_f{}^9 \tag{22}$$

where the i and f labels refer to two times, before and after inflation. Following Abbott *et al.*,[18] we take the final value of R_{6f}, to be of the order of the Planck length, thus assuming some sort of quantum mechanism to stop the collapse of the internal radius to a singularity. Also, in order to avoid massive excitations of the internal modes, we take $T_f\leq(1/R_{6f})\leq1(L_{\rm Pl}\sim1)$. According to the authors in Ref. 18, there are two ways of obtaining enough inflation; the first is to have a large number of internal dimensions (around 40) and the second is to have a very large initial size for the internal manifold (for an alternative approach see Ref. 19). In our case, the number of dimensions is fixed thus forcing us to adopt the second point of view. Although assuming a highly asymmetric set of initial conditions for the physical and internal radii may sound rather unnatural, our ignorance of the proper initial conditions allows us to do so, if the internal dimensions decouple after a not so long period (again, safely before nucleosynthesis). Also, the fact that we obtain a t^{-1} behavior for the internal space, makes it more natural to have a "cigarlike" configuration for the initial singularity.

Thus, taking $R_{3i}\sim1$ we obtain

$$R_{3f}\sim10^{88/3}\ . \tag{23}$$

For a compactification temperature close to the Planck energy, $T_i\leq1$, we see that the ratios between the initial and final values for the internal radius and time are

$$\frac{R_{6i}}{R_{6f}}\sim\frac{t_f}{t_i}\sim10^{44/3}\ . \tag{24}$$

Thus, to obtain sufficient inflation, the internal radius has initially to be 15 orders of magnitude bigger than the physical radius, with the situation being reversed after the above period of time. If $t_i\sim t_{\rm Pl}$, we get that the inflationary epoch ends at the grand-unified-theory scale, an interesting result.

We can now look for exponential solutions of the field equations. So, with an exponential ansatz for the evolution of the scale factors we find a consistent solution of the form

$$R_3=R_{30}e^{1/3\alpha t};\quad R_6=R_{60}e^{-1/6\alpha t};\quad \phi=\pm\alpha(t+t_0) \tag{25}$$

with t_0 an arbitrary integration constant and α a free parameter in the solution. These can be fixed once we impose some boundary condition. What is worth pointing out is the fact that if R_3 expands then R_6 contracts and vice versa. However, for this solution to be consistent we require $e^{\phi}F^2$ to be very small, thus effectively neglecting the right-hand side of Eqs. (15)—(18). Recalling that for de Sitter-type solutions we can set the curvature terms in Einstein's equations to zero, we could then use the power-law solution previously obtained as an initial condition to the numerical integration of the equations, to check if indeed the decay of the $e^{\phi}F^2$ term is going to trigger inflation. In Figs. 1 and 2 we show that this is indeed the case. We can also show that if we introduce a new variable $\psi=\phi-6\ln R_3$, we can rewrite Eq. (18) as

$$\ddot{\psi}+\left[3\frac{\dot{R}_3}{R_3}+6\frac{\dot{R}_6}{R_6}\right]\dot{\psi}=7e^{\psi}\ . \qquad (26)$$

Now we can obtain the solution of Eq. (26) in the limit where the friction term is negligible:

$$\psi=-2\ln[(\tfrac{7}{4})^{1/2}(t-t_0)]\ . \qquad (27)$$

As can be seen from Eq. (27) ψ will become more negative as it evolves in time. This fact, when translated into the original variables imply that the $\ln R_3$ term dominates over the ϕ term, showing that the term $e^{\phi}F^2$ on the right-hand side of the field equations becomes negligible, confirming the above result.

The above calculations seem to indicate that the inflationary behavior for the physical space-time is a result of the interplay between the dilaton and the shrinking internal radius. We should mention at this point that the model does not terminate the inflationary phase in a spontaneous way due to the fact that the potential for ψ has no stable minimum.

It must be said that quantum effects coming from one-loop corrections on the matter fields, or the Casimir forces, will probably play an important role at these energies, and were neglected for simplicity. In fact, depending on the ansatz for the Kalb-Ramond field, it can be shown that these effects stabilize the potential for the internal radius (taken to be an effective scalar field), thus generating an effective cosmological constant that will drive an inflationary period which ends when the internal radius becomes constant.[20]

For our model to have a successful inflation we require the density perturbations to be within the acceptable value[21] when they leave the de Sitter-type phase and cross

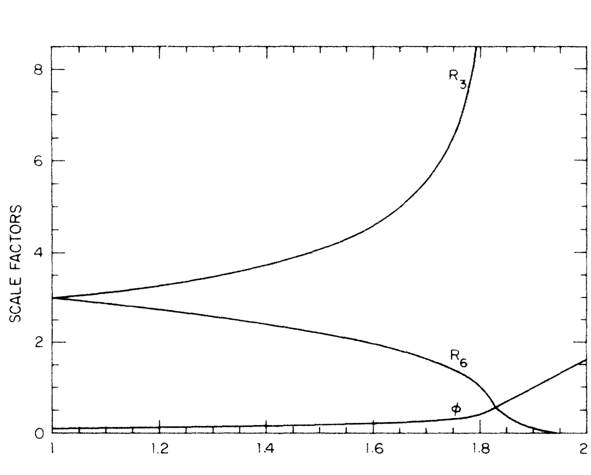

FIG. 1. The time evolution for the two scale factors and the dilaton field is shown. Time is in units of the Planck time.

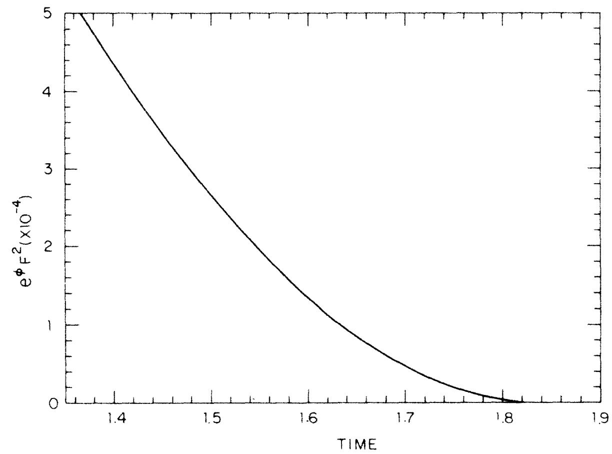

FIG. 2. We show the time behavior of the term $e^{\phi}F^2$ for the flat ($k_3=k_6=0$) geometry case. In the limit when this term is negligible, the exponential solution takes over.

the horizon (we will denote the horizon time by an H subscript):

$$\left[\frac{\delta\rho_3}{\rho_3}\right]_H\sim\left[\frac{{H_3}^2}{\dot{\phi}}\right]_H\sim\frac{\sqrt{2}}{3}\alpha\leq 10^{-4}\ . \qquad (28)$$

This condition fixes the value of α.

We shall now summarize our results. Taking the bosonic part of the superstring action in the low-energy limit we looked for cosmological solutions, in particular power-law and exponential solutions. Before studying the full action with the Kalb-Ramond and dilaton fields as functions of time, we analyzed briefly the simpler cases where either the dilaton field is a constant (which implies that $F_{MNP}=0$) or the Kalb-Ramond field is taken to be zero (which implies that $\dot{\phi}\sim V^{-1}$). For the full action we found that the power-law solution corresponds to a model with an open three-space (negative curvature) and a flat internal space while the exponential solution is consistent with having a flat three-dimensional space and a flat internal space. We also discussed the possibility of having a power-law-type inflation provided that we believe the total entropy is adiabatically conserved and that suitable initial conditions can be imposed on this model, in order to obtain enough entropy production in the physical space-time.

The flat model also contains a power-law behavior as a particular solution. However, this proved to be unstable. This was later used as a valid initial condition for numerical integration. From the information gathered in this way we discovered that the behavior for late times was exponential in time. This was confirmed in an analytical way by finding explicitly this solution. The solution contained one free parameter associated in a natural way with a cosmological constant. The value for this parameter was then restricted by demanding the density contrast to be consistent with observations.

Finally, it is interesting to note that the action considered here seems to be unique in giving a shrinking internal space for a variety of field configurations. To the best of our knowledge, this does not seem to be the case in other supergravity models at least in power-law form (see, for example, Refs. 7 and 14). This nice qualitative cosmological behavior coming from superstring models is very encouraging.

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