Remarks on the dimension-four chiral Lagrangian

Hai-Yang Cheng
Physics Department, Indiana University, Bloomington, Indiana 47405
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The profound fact that the dimension-four operators in the effective chiral Lagrangian, including quartic-derivative terms, are uniquely determined by the integration of spurious (nontopological) chiral anomalies has rich applications to low-energy hadron physics. The form factor f_- and the Callan-Treiman relation in K_{l3} decays, the axial-vector K_{l4} form factors, and the structure-dependent form factor ξ in $\pi \to e\nu e^+e^-$ are derived from the effective Lagrangian. A discussion of the parameter γ in weak radiative π and K decays is also given.

Although thus far the equations of QCD have not been solved, they can be studied in two extreme cases: one is the short-distance regime where perturbative QCD is applicable, the other is the low-energy domain where the chiral structure of QCD together with PCAC (partial conservation of axial-vector current) enables us to elaborate on the infrared properties of QCD. It is known that the integration of the proper chiral (Bardeen) anomaly gives the Wess-Zumino (WZ) effective action.[1,2] Likewise, it has been shown[3–6] that the integration of spurious (nontopological) chiral anomalies arising from the quark loop yields a unique dimension-four low-energy chiral Lagrangian for QCD, which cannot be eliminated by adding some local counterterms. The nontopological chiral anomaly is uniquely determined and independent of the choice of the regularization scheme as long as it respects vector gauge invariance.[5] A quartic-derivative Lagrangian was obtained by Nepomechie[3] by evaluating the finite variation of the fermionic determinant before and after the chiral rotation. A complete dimension-four chiral Lagrangian including external gauge fields V_μ and A_μ was independently derived by Balog,[5] by Seo,[4] and by Andrianov.[6] Following the notation of Ref. 5, the chiral Lagrangian reads

$$\begin{aligned}
\mathscr{L} &= \mathscr{L}_0 + \mathscr{L}_2 , \\
\mathscr{L}_0 &= \frac{f^2}{8}\mathrm{Tr} D_\mu U^\dagger D^\mu U , \qquad (1)\\
\mathscr{L}_2 &= \frac{1}{64\pi^2}\mathrm{Tr}[(D_\mu U^\dagger D_\nu U)^2 \\
&\quad + 4(R^{\mu\nu} D_\mu U^\dagger D_\nu U + L^{\mu\nu} D_\mu U D_\nu U^\dagger) \\
&\quad + 2U^\dagger L^{\mu\nu} U R_{\mu\nu}] + \mathscr{L}_{\mathrm{WZ}} ,
\end{aligned}$$

where

$$\begin{aligned}
&D_\mu U = \partial_\mu U + L_\mu U - U R_\mu , \\
&L_\mu = V_\mu + A_\mu , \quad R_\mu = V_\mu - A_\mu , \\
&R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu + [R_\mu, R_\nu] , \qquad (2)\\
&L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu + [L_\mu, L_\nu] , \\
&U = \exp(i\sqrt{2}\phi^a\lambda^a/f), \quad f \approx 130 \text{ MeV} .
\end{aligned}$$

$\mathscr{L}_{\mathrm{WZ}}$ is the Wess-Zumino effective Lagrangian,[7,2] and $\phi = \phi^a \lambda^a$ describes the Goldstone-boson fields. To write down $\mathscr{L}_2$, use has been made of three colors for quarks and the lowest-order equation of motion:

$$U^\dagger D_\mu D_\mu U = -D_\mu U^\dagger D_\mu U . \qquad (3)$$

It should be noticed that the chiral Lagrangian $\mathscr{L}_2$ can be derived in many different ways.[8,9] From Eq. (1) it is evident that contributions from the quartic-derivative Lagrangian are suppressed by factors of p^2/Λ^2 at low energies, where

$$\Lambda = (64\pi^2 f^2/8)^{1/2} = 2\sqrt{2}\pi f \sim 1 \text{ GeV} \qquad (4)$$

is the scale of chiral-symmetry breaking.[10]

Many low-energy phenomena beyond the current algebra or the lowest-order effective Lagrangian can now be studied. Some applications of the chiral Lagrangian $\mathscr{L}_2$ have been discussed by Balog.[5] The purpose of this paper is to elaborate on the physics and to add some further applications. Before proceeding to the low-energy dynamics, we would like to get some insight into the chiral Lagrangian $\mathscr{L}_2$.

Using the SU(3) trace identity

$$\mathrm{Tr}(ABAB) = -2\,\mathrm{Tr}(A^2B^2) + \tfrac{1}{2}\mathrm{Tr}A^2\mathrm{Tr}B^2 + [\mathrm{Tr}(AB)]^2 , \qquad (5)$$

one may write, for the 3-flavor case,

$$\begin{aligned}
\mathscr{L}_2 = {} & g_1[\mathrm{Tr}(D^\mu U^\dagger D_\mu U)]^2 + g_2[\mathrm{Tr}(D^\mu U^\dagger D_\nu U)]^2 \\
& + g_3 \mathrm{Tr}(D^\mu U^\dagger D_\mu U)^2 \\
& + g_9 \mathrm{Tr}(R^{\mu\nu} D_\mu U^\dagger D_\nu U + L^{\mu\nu} D_\mu U D_\nu U^\dagger) \\
& - g_{10} \mathrm{Tr}(U^\dagger L^{\mu\nu} U R_{\mu\nu}) , \qquad (6)
\end{aligned}$$

where

$$8g_1 = 4g_2 = -2g_3 = g_9 = -2g_{10} = \frac{1}{16\pi^2} . \qquad (7)$$

It was observed by Gasser and Leutwyler[11] that in the chiral limit and in the absence of external gauge fields, the most general expression for the quartic-derivative

Langrangian contains three independent terms. For the 2-flavor case, we have the trace identity

$$\mathrm{Tr}(A^2B^2)=\tfrac{1}{2}\mathrm{Tr}A^2\mathrm{Tr}B^2 \tag{8}$$

in addition to Eq. (5). This explains why in the SU(2) case there exist only two independent quartic-derivative terms:

$$\mathscr{L}_2^{\mathrm{SU}(2)}=-\frac{1}{128\pi^2}[\mathrm{Tr}(\partial^\mu U^\dagger\partial_\mu U)]^2+\frac{1}{64\pi^2}[\mathrm{Tr}(\partial^\mu U^\dagger\partial_\nu U)]^2\ . \tag{9}$$

Denoting $l_\mu=U^\dagger\partial_\mu U$, we may write

$$\mathscr{L}_2^{\mathrm{SU}(2)}=\frac{1}{32e^2}\mathrm{Tr}[l_\mu,l_\nu]^2+\frac{\gamma}{8e^2}[\mathrm{Tr}(\partial^\mu U^\dagger\partial_\mu U)]^2\ , \tag{10}$$

with

$$e=2\pi,\ \ \gamma=\tfrac{1}{4}\ .$$

The first term of (10) is the celebrated Skyrme term.[12] At the SU(2) level the Skyrme coupling constant e was estimated to be 5.45 by fitting the mass of the soliton baryon to N and Δ (Ref. 13), and the parameter γ was extracted to be 0.16 ± 0.04 from the measured D-wave $\pi\pi$ scattering data.[14]

Explicit chiral-symmetry breaking adds a term $f^2\mathrm{Tr}(M^\dagger U+MU^\dagger)/8$ to $\mathscr{L}_0$, where (isospin symmetry is assumed)

$$\begin{gathered}M_{ij}=0\ \ (i\neq j)\ ,\\ m_\pi{}^2=M_{11}=M_{22},\ \ m_K{}^2=(M_{11}+M_{33})/2\ ,\end{gathered} \tag{11}$$

and the following terms to $\mathscr{L}_2$ (Ref. 11):

$$\begin{aligned}\mathscr{L}_2=\text{Eq. (6)}&+g_4\mathrm{Tr}(D^\mu U^\dagger D_\mu U)\mathrm{Tr}(M^\dagger U+MU^\dagger)+g_5\mathrm{Tr}[D^\mu U^\dagger D_\mu U(M^\dagger U+MU^\dagger)]+g_6[\mathrm{Tr}(M^\dagger U+MU^\dagger)]^2\\&+g_7[\mathrm{Tr}(M^\dagger U-MU^\dagger)]^2+g_8\mathrm{Tr}(M^\dagger UM^\dagger U+MU^\dagger MU^\dagger)\ .\end{aligned} \tag{12}$$

In principle, the coupling constants $g_4,\ldots,g_8$ are derivable from QCD. It has been shown that Eq. (12) is the most general expression for the dimension-four effective Lagrangian. The coupling constants $g_1,\ldots,g_{10}$ have been determined in Ref. 11 from available experimental data. However, a direct comparison of them with the theoretical values (7) is dangerous because one-loop corrections are taken into account in Ref. 11 in extracting the coupling constants, as we are going to discuss. Although an exponential form for the unitary matrix U is chosen in Eq. (2), all nonlinear realizations are equivalent for the physical amplitudes. Up to fourth order in ϕ, the general expression for U is[15]

$$U=1+2i\phi/f-2\phi^2/f^2-ia_3\phi^3/f^3+2(a_3-1)\phi^4/f^4\ . \tag{13}$$

Realization independence is ensured by the Chisholm theorem,[16] which states that S-matrix elements (i.e., on-mass-shell amplitudes) are independent of the value of a_3.

In spite of the fact that the effective Lagrangian (12) is nonrenormalizable, the presence of the meson loop, which is necessary to preserve unitarity in the low-energy limit, is not an obstacle. Since at the one-loop level the necessary counterterms are of the same structure as that of $\mathscr{L}_2$ (Refs. 11 and 17) the divergence of the one-loop graphs can be eliminated by defining renormalized coupling constants

$$g_i(\mu)=g_i^{(0)}-b_i\ln(\Lambda/\mu)^2\ \ (i=1,\ldots,10)\ , \tag{14}$$

where $g_i^{(0)}$ are the bare coupling constants. The resulting amplitude is thus finite and calculable. Since the physical results must be independent of the arbitrary renormalization scale μ, the renormalization-group method leads to[17]

$$g_i(\mu)=b_i\ln(\mu/\mu_0)^2\ , \tag{15}$$

The coefficients b_i have been calculated in the paper of Gasser and Leutwyler.[11] The unknown scale μ_0 can be determined if the relation between the renormalized coupling constants $g_i(\mu)$ and the regularization-independent "physical" constants g_i [i.e., Eq. (7)] is known.[18] In Ref. 11 the coupling constants $g_i(\mu)$ [which we call $g_i(\mu)_{\mathrm{expt}}$] rather than g_i are obtained from the analyses of experimental low-energy information. Unfortunately, a direct comparison of $g_i(\mu)_{\mathrm{expt}}$ with $g_i(\mu)_{\mathrm{theory}}$ is not possible at present because we have not been able to derive μ_0, and hence $g_i(\mu)_{\mathrm{theory}}$. However, it is found empirically that $g_i(\mu)_{\mathrm{expt}}$ are close to the theoretical values of g_i [Eq. (7)] when $\mu=m_\eta$ (except for g_{10}) (Ref. 11). Unitarity corrections are normally expected to be small compared to the contribution from the dimension-four Lagrangian due to the large-N_c argument:[19] The meson loop is suppressed by at least a factor of $1/N_c$ (N_c being the number of the colors) relative to the quark loop in the large-N_c limit. Indeed, for all physical quantities we shall consider later, contributions from the effective dimension-four Lagrangian dominate over the loop corrections. Nevertheless, it is known that the one-loop correction can be of the nonanalytic form $m_\pi{}^2\ln m_\pi{}^2$ or even $\ln m_\pi{}^2$, which is not negligible owing to the small mass of the pion.[20] A complete and satisfactory agreement with experiment generally requires the inclusion of meson loop contributions.

Some applications of the effective Lagrangian $\mathscr{L}_2$ are already discussed in Ref. 5. In the following we elaborate on the details of the physics and add some further examples.

A. K_{l3}

Experimentally, the form factors in K_{l3} decays,

$$\langle\pi^-(q)\,|\,V_\mu\,|\,K^0(k)\rangle=f_+(t)(k+q)_\mu+f_-(t)(k-q)_\mu\ , \tag{16}$$

are often analyzed by assuming a linear dependence on $t=(k-q)^2$,

$$f_\pm(t)=f_\pm(0)\left[1+\lambda_\pm\frac{t}{m_\pi{}^2}\right]\ . \tag{17}$$

Slope parameters $\lambda_{\pm}(0)$ and the form factor $f_-(0)$, which are not predictable by current algebra, can now be calculated using the chiral Lagrangian, Eq. (1). The result is[5]

$$\langle \pi^-(q) | V_\mu | K^0(k) \rangle = \left[1 + \frac{t}{4\pi^2 f^2} \right] (k+q)_\mu - \frac{1}{4\pi^2 f^2}(k^2-q^2)(k-q)_\mu . \quad (18)$$

Hence

$$\lambda_+ = \frac{1}{4\pi^2}\frac{m_\pi^2}{f^2}, \quad \lambda_- = 0 . \quad (19)$$

The slope parameter λ_+ originates from the g_9 term of $\mathscr{L}_2$. When external mesons are on the mass shell, there is an additional contribution arising from the g_5 term in (12),

$$8\frac{g_5}{f}[(m_K^2+m_\pi^2)(k+q)_\mu + (m_K^2-m_\pi^2)(k-q)_\mu] . \quad (20)$$

Since the g_5 term also contributes to the kinetic term of a meson, it becomes necessary to normalize the meson field, $\phi \to \phi/Z$. For instance, for charged pions

$$Z_{\pi^\pm} = (1+16 g_5 m_\pi^2/f^2)^{-1/2} . \quad (21)$$

As a result, we have

$$f_+(0) \cong 1 , \qquad f_-(0) = -\frac{m_K^2-m_\pi^2}{4\pi^2 f^2} + 8g_5\frac{m_K^2-m_\pi^2}{f^2} . \quad (22)$$

In the SU(3) limit, $f_+(0)=1$ and $f_-(0)=0$, as it should be.

From $\langle 0 | A_\mu | P \rangle = i f_P p_\mu$ and the normalization of a meson field, it turns out that $f_K/f_\pi - 1$ is proportional to the coupling constant g_5 (Ref. 21):

$$\frac{f_K}{f_\pi} - 1 = 8g_5\frac{m_K^2-m_\pi^2}{f^2} . \quad (23)$$

Therefore,

$$\xi(0) \equiv \frac{f_-(0)}{f_+(0)} = -\frac{m_K^2-m_\pi^2}{4\pi^2 f^2} + \frac{f_K}{f_\pi} - 1 . \quad (24)$$

At $t=m_K^2$, we obtain the relation

$$f_+(m_K^2) + f_-(m_K^2) = f_K/f_\pi + O(m_\pi^2) , \quad (25)$$

which is known as the Callan-Treiman relation[22] for K_{l3} decays. Without the quartic-derivative Lagrangian, the Callan-Treiman relation would be valid at $t=0$ rather than $t=m_K^2$ (Ref. 23). In this case $f_-(0)$, and hence $\xi(0)$, would be positive, which is in contradiction with experiment. The slope parameter λ_0 of the scalar form factor,

$$f_0(t) = f_+(t) + \frac{t}{m_K^2-m_\pi^2} f_-(t) \equiv f_+(0)\left[1 + \lambda_0 \frac{t}{m_\pi^2} \right] ,$$

is given by

$$\lambda_0 = \frac{m_\pi^2}{m_K^2-m_\pi^2}\left[\frac{f_K}{f_\pi} - 1 \right] . \quad (26)$$

With the experimental value[24]

$$f_K/f_\pi = 1.22 \pm 0.01 , \quad (27)$$

we obtain

$$\xi(0) = -0.13, \quad \lambda_0 = 0.017 , \quad (28)$$

which are consistent with the measurements from $K^0_{\mu 3}$ decays,[25]

$$\xi(0) = -0.11 \pm 0.09, \quad \lambda_0 = 0.025 \pm 0.006 .$$

Unfortunately, current experimental results for $\xi(0)$ and λ_0 as determined from $K^+_{\mu 3}$ (Ref. 25) are not consistent with those determined from $K^0_{\mu 3}$:

$$\xi(0) = -0.35 \pm 0.15, \quad \lambda_0 = 0.004 \pm 0.007 .$$

This needs to be clarified by future precise experiments. Two remarks are in order.

(a) The quark model tends to predict a large f_-. For example, a calculation of $f_-(0)$ based on the Isgur model gives[26]

$$f_-^{K^0\pi^-} = \sqrt{2} f_-^{K^+\pi^0} = -0.30 .$$

(b) At the tree level the slope parameter λ_+ is uniquely determined by the spurious chiral anomaly. The excellent agreement between theory and experiment (measured from K_{e3} decays) for λ_+ leaves almost no room for the loop corrections.

B. K_{l4}

There are four form factors needed to describe the K_{l4} decay:

$$\langle \pi^+(p_+)\pi^-(p_-) | V_\mu - A_\mu | K^+(k) \rangle = -\frac{h}{m_K^3}\epsilon_{\mu\nu\alpha\beta} k^\nu (p_+ + p_-)^\alpha (p_+ - p_-)^\beta + \frac{i}{m_K}[f(p_+ + p_-)_\mu + g(p_+ - p_-)_\mu + r(k - p_+ - p_-)_\mu] . \quad (29)$$

The vector form factor h, arising from the proper anomaly, can be calculated from the Wess-Zumino effective Langrangian:[27]

$$h = m_K^3/(2\pi^2 f_\pi^3) . \quad (30)$$

The lowest-order chiral-Lagrangian or current-algebra predictions for the axial-vector form factors f and g, as we shall see, are

$$f = g = m_K/f_\pi . \quad (31)$$

Numerically,

$$h = 2.77, \quad f = g = 3.80 ,$$

to be compared with the experimental values measured from K_{e4} decays,[28]

$$h=2.68\pm0.68,\quad f=5.59\pm0.14,\quad g=4.77\pm0.27\ . \tag{32}$$

It has been proposed that the discrepancy between theory and experiment for the axial-vector form factors could be improved by the so-called pole-enhanced chiral-Lagrangian model.[29] We argue, however, that this discrepancy is actually a test of the dimension-four chiral Lagrangian. Before proceeding to compute the spurious anomaly's contribution, we note that in order to describe the form factor r, it is necessary to include the tree graph of $K\to K\pi\pi$ followed by the weak $K\to e\nu$ transition. However, in K_{e4} decays one may neglect terms proportional to $q_\mu\equiv(k-p_+-p_-)_\mu$ due to the small electron mass. The relevant Lagrangian for K_{e4} is

$$\mathscr{L}=\frac{i}{2\pi^2 f_\pi^{\,3}}\mathrm{Tr}A_\mu[2\pi^2 f_\pi^{\,2}(\{\phi^2,\partial_\mu\phi\}-\{\phi,\partial_\mu\phi^2\}+\tfrac{2}{3}\partial_\mu\phi^3)-2\partial_\nu\phi\partial_\mu\phi\partial_\nu\phi$$
$$+\{\partial_\nu\phi,\partial_\mu\partial_\nu\phi^2\}-\{\partial_\nu\phi^2,\partial_\mu\partial_\nu\phi\}+\{\Box\phi,\partial_\mu\phi^2\}-\{\Box\phi^2,\partial_\mu\phi\}]\ . \tag{33}$$

From this we obtain

$$\langle\pi^+\pi^-|-A_\mu|K^+\rangle=\frac{2i}{3f_\pi}(2p_+-p_-+k)_\mu+\frac{i}{\pi^3 f_\pi^{\,3}}(k\cdot p_-+q^2/2)(p_+)_\mu\ . \tag{34}$$

To simplify the calculation we have dropped all terms proportional to q_μ. Comparing (34) with (29), it follows that

$$f=g=\frac{m_K}{f_\pi}\left[1+\frac{1}{2\pi^2 f_\pi^{\,2}}\left[k\cdot p_-+\frac{q^2}{2}\right]\right]. \tag{35}$$

Since the K_{e4} experiment is performed near the threshold,

$$k\cdot p_-\approx m_K m_\pi,\quad q^2\approx m_K^{\,2}+4m_\pi^{\,2}-4m_K m_\pi\ ,$$

hence

$$f=g\simeq\frac{m_K}{f_\pi}\left[1+\frac{1}{4\pi^2 f_\pi^{\,2}}(m_K^{\,2}+4m_\pi^{\,2}-2m_K m_\pi)\right]$$
$$=4.86\ . \tag{36}$$

Now the agreement with experiment for g is excellent, and the small remaining discrepancy for the form factor f might be attributed to the loop corrections. At any rate, the quartic-derivative Lagrangian (i.e., g_9 term of $\mathscr{L}_2$) accounts for the major corrections to soft-pion theorems.

C. $\pi,K\to l\nu\gamma,\,l\nu e^+e^-$

The amplitude describing the weak radiative decay and the related process consists of two parts—one is the inner bremsstrahlung amplitude, the other is the structure-dependent amplitude. The structure-dependent amplitude for $\pi\to l\nu e^+e^-$ is of the form

$$A_{\mathrm{SD}}=\left[\frac{eG_F\cos\theta_C}{\sqrt{2}m_\pi}\right]l^\mu j^\nu[f_V\epsilon_{\mu\nu\alpha\beta}p^\alpha k^\beta+if_A(g_{\mu\nu}p\cdot k-k_\mu p_\nu)+i\tilde f_A g_{\mu\nu}k^2]\ , \tag{37}$$

where p,k are the momenta of pion, photon, respectively, and l_μ and j_μ are, respectively, the leptonic and electromagnetic currents. For the radiative decay the structure-dependent amplitude is of the same form, except that j_μ is replaced by the photon polarization vector ϵ_μ, and $k^2=0$. As in the K_{l4} decay, the vector form factor f_V arises from the topological chiral anomaly, i.e., the Wess-Zumino term; the axial-vector form factors f_A and $\tilde f_A$ originate from the spurious anomaly. Both the g_9 and g_{10} terms of $\mathscr{L}_2$ contribute to f_A, but $\tilde f_A$ receives a contribution only from the g_{10} term. In addition to the results for f_V and f_A in pion decays

$$f_V=\frac{1}{4\pi^2}\frac{m_\pi}{f_\pi},\quad f_A=8(g_9+g_{10})\frac{m_\pi}{f_\pi} \tag{38}$$

given in Ref. 5, we also obtain

$$\tilde f_A=-8g_{10}m_\pi/f_\pi\ . \tag{39}$$

From Eq. (7) it follows that

$$\gamma\equiv\frac{f_A}{f_V}=1,\quad \xi\equiv\frac{\tilde f_A}{f_V}=1\ . \tag{40}$$

It is of interest to note that there exists only one single-loop graph contributing to the structure-dependent amplitude as depicted in Fig. 1. A simple calculation shows

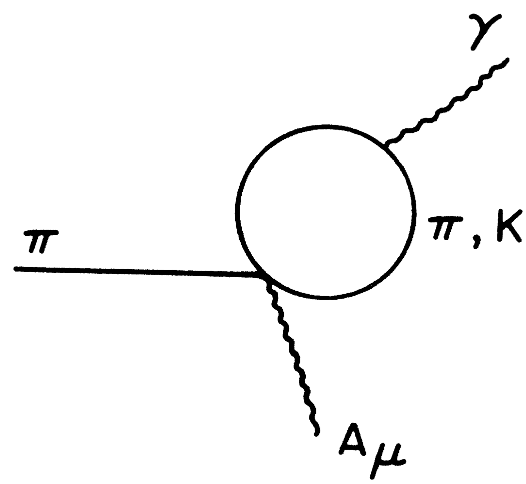

FIG. 1. The one-loop diagram which can contribute to the axial-vector form factors in Eq. (37).

that the (divergent) one-loop integral is proportional to k^2, and hence it contributes only to the form factor $\tilde{f}_A$; *the prediction* $\gamma=1$ *is thus free of meson-loop corrections.*

As one can see from the literature, both theoretical predictions and experimental measurements of the form factor f_A are very diverse and confusing (for an extensive review, see Ref. 30). This is one of the reasons that this problem receives constant attention from time to time. Let us first focus on the parameter γ and denote the fermion- and meson-loop contributions by γ^F and γ^M, respectively, so that $\gamma=\gamma^F+\gamma^M$. The value of γ^F depends on the type of fermion involved. For the color quark loop the prediction of γ^F in the relativistic quark model (or the σ-quark model) should be the same as that of Eq. (40) since the dimension-four chiral Lagrangian is equivalent to the nonlinear σ-quark model with the quark fields being integrated out. Realistic calculations in the quark model indeed give $\gamma^F=1$ (Refs. 31–33). (An earlier calculation in the relativistic quark model[34] leads to $\gamma=-1$. The sign ambiguity is clarified in Refs. 31 and 32.) If the fermion in the loop is a nucleon (as in the σ-nucleon model), $\gamma^F=\frac{1}{3}$ (Ref. 35) due to the lack of color. To calculate the meson-loop contribution to γ, it becomes necessary to consider the model in which the PCAC relation is respected and the chiral symmetry is realized. If the realization of the spontaneously broken chiral symmetry is nonlinear, then $\gamma^M=0$ because of the absence of the meson-loop correction, as we discussed in passing. In the linear σ model in which the chiral symmetry is realized linearly, $\gamma^M=-\frac{1}{3}$ in the soft-pion limit (or, equivalently, in the $m_\sigma\to\infty$ limit).[36] It should be stressed that the linear σ model with $m_\sigma\to\infty$ is not equivalent to the nonlinear σ model at the one-loop level. Consequently, we have $\gamma=1$ $(\frac{2}{3})$ in the nonlinear (linear) σ-quark model, and $\gamma=\frac{1}{3}$ (0) in the nonlinear (linear) σ-nucleon model.

We will not pursue the σ-nucleon model further, since the use of the nucleon loop would lead to a charge radius of a pion, $\langle r^2\rangle_\pi=(2\pi^2 f_\pi^{\,2})^{-1}$ (Ref. 37), which disagrees with the experimental measurement[38] by at least a factor of 3, the number of colors.[39] Now we have reduced the problem to that of deciding which model, the renormalizable linear σ-quark model or the nonrenormalizable nonlinear σ-quark model, gives the correct description. As emphasized by Gasser and Leutwyler,[11] despite the fact that to lowest order the low-energy behavior of the two models is the same, renormalizability of the linear σ model does not guarantee that it is still the correct model when goes beyond the leading order. (By contrast, nonrenormalizability does not prevent the nonlinear σ model having a finite and calculable S matrix.) Some realistic linear σ models in the one-loop approximation have been worked out[11,40] and lead to results which are not borne out by experiment. We thus conclude[41] that the axial-vector form factor f_A should be equal to f_V, as predicted by the chiral Lagrangian or by the nonlinear σ-quark model. It would be striking if the prediction $\gamma=1$ were not confirmed by future experiments.

For the weak radiative decay of a pion, two values of γ, 0.44 ± 0.12 and -2.36 ± 0.12, were obtained by Stetz *et al.*[43] Two experiments at SIN and TRIUMF are under way. For kaons, there are two measurements:

$$|f_V+f_A|=0.153\pm0.11$$

[Heintze *et al.* (1979) (Ref. 44)],

$$|f_V+f_A|<0.23,\quad -2.5<(f_V-f_A)<0.3$$

[Akiba *et al.* (1985) (Ref. 45)],

to be compared with the theoretical value

$$f_V=f_A=m_K/(4\pi^2 f_\pi)=0.092\ .$$

The ξ parameter has not been measured. Various model predictions of ξ are discussed in details by Lee.[31]

In all the above three examples, we did not calculate one-loop corrections (except for the γ parameter in the weak radiative decay of π and K which receives no contribution from the meson loop) for the reason that we have not been able to derive the constant μ_0 in Eq. (15), so we could not have predictions for the coupling constants $g_i(\mu)$. Nevertheless, the agreement of theoretical values derived from the tree Lagrangian with experiment implies that corrections to the soft-pion theorem are dominated by the dimension-four chiral Lagrangian, as expected from the large-N_c argument.[46] Of course, this does not mean the meson-loop contribution is always negligible at all. As we noted in passing, a complete agreement with the experimental measurements for physical quantities, such as the form factor f in K_{l4} decays, the charge radius of a pion, ..., etc., is possible only if the loop correction is taken into account.

Note added in proof. The prediction $\gamma=\frac{1}{4}$ [Eq. (10)] was also noticed by I. J. R. Aitchison, C. M. Fraser, and P. J. Miron [Phys. Rev. D 33, 1994 (1986)]. This parameter was discussed recently by B. A. Li and M. L. Yan [*ibid.* 33, 1492 (1986)]; M. Mashaal, T. N. Pham, and T. N. Truog [Phys. Rev. Lett. **56**, 436 (1986)]; A. A. Andrianov, V. A. Andrianov, and V. Yu. Novozhilov [*ibid.* **56**, 1882 (1986)].

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