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Flux-tube model of quark deconfinement at high temperature

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The SU(2) flux-tube model proposed by Patel is used to calculate the effective string tension $\sigma(T)$ as a function of the temperature T. The main results are (i) $\sigma(T)/\sigma(0)\simeq 1-\text{const}\times T$ for T near 0, (ii) $\sigma(T)/\sigma(0)$ vanishes like $(T_c-T)^{1/2}$ as T approaches a critical temperature T_c, and (iii) the mean length $\langle l\rangle$ of the string spanned between a static quark-antiquark pair with distance r behaves like $\text{const}\times r(T_c-T)^{-1/2}$ as $T\to T_c$ from below, implying a scaling law that $\langle l\rangle/r$ is independent of r for large r and T near T_c.

It is widely believed that a phase transition from hadronic matter to quark matter will take place when the temperature or the baryon-number density becomes sufficiently high. As for the existence and the nature of the transition at high temperature, most efforts have been devoted to the calculation of the critical temperature and to the investigation of the order of the transition.[1-3] Other important tasks are to draw a precise phase diagram and to calculate the behavior of the relevant quantities such as the order parameters near the critical temperature and to determine the relevant critical exponents. In fact, the temperature dependence of the energy density and of the expectation value of the thermal Wilson loop have been investigated extensively by using the Monte Carlo simulation method for lattice gauge theories.[2] In this paper, we calculate the effective string tension $\sigma(T)$ as a function of the temperature T by using the SU(2) flux-tube model proposed by Patel.[4] Our classical result will then be compared with recent results from a Monte Carlo lattice SU(2) gauge theory[5] and a quantum string model with the $1/d$ expansion,[6] where d is the dimension of the space-time. Our result is also compared with the prediction from the correspondence between gauge theories and spin models.[7]

Consider a static quark(q)-antiquark($\bar{q}$) pair in a color-SU(2)-singlet combination with distance r. A color flux tube with a finite thickness is formed between q and $\bar{q}$. The configurations of the flux tubes are then given by random walks from q to $\bar{q}$ on a three-dimensional cubic lattice with lattice spacing a. Patel gave an estimate that $a\simeq m_g/[4\sigma(0)]$, where m_g is the mass of the lowest-lying 0^+ glueball and $\sigma(0)$ is the usual string tension at zero temperature. The value of $\sigma(0)$ is related to the Regge slope α' by the relation $\sigma(0)=(2\pi\alpha')^{-1}$. Then a choice $\alpha'=0.9$ GeV^{-2} and $m_g=1$ GeV gives $a=0.28$ fm. The partition function for the system is

$$Z=\int_r^\infty dl\,\Omega(l,r)e^{-\sigma l/T}\ , \tag{1}$$

where $\sigma=\sigma(0)$ and $\Omega(l,r)dl$ is the number of the random walks of length between l and $l+dl$. In the case of unconditional random walks that start from an origin, the probability that the end point of a random walk with length na lies in a small-volume element $d\mathbf{v}$ around $\mathbf{r}$ is

$$P(\mathbf{r})d\mathbf{v}=\left[\frac{3}{2\pi na^2}\right]^{3/2}\exp\left[-\frac{3r^2}{2na^2}\right]d\mathbf{v}\ , \tag{2}$$

for $n\gg\max[r/a,1]$. On the other hand, the number of random walks with n steps is equal to 6^n if backtracking is allowed. Therefore, $\Omega(l,r)$ is approximately given by $\text{const}\times 6^nP(\mathbf{r})\,|_{n=l/a}$, i.e.,

$$\Omega(l,r)=\text{const}\times 6^{l/a}(la)^{-3/2}\exp\left[-\frac{3r^2}{2la}\right]\ . \tag{3}$$

From (1) and (3), one obtains

$$Z=C\int_r^\infty dl\,l^{-3/2}\exp\left[-\left[\frac{1}{T}-\frac{1}{T_c}\right]\sigma l-\frac{3r^2}{2la}\right]\ , \tag{4}$$

where C is an irrelevant normalization constant and

$$T_c\equiv\sigma a/\ln 6\ . \tag{5}$$

A change of the integration variable from l to $x=r/l$ yields

$$Z=Cr^{-1/2}\int_0^1 dx\,x^{-1/2}\exp\left\{-\left[\left[\frac{1}{T}-\frac{1}{T_c}\right]\frac{\sigma}{x}+\frac{3x}{2a}\right]r\right\}\ . \tag{6}$$

The expression (6) is useful for numerical integration.

It is evident from (4) that the partition function becomes singular as the temperature increases toward T_c, indicating a phase transition at $T=T_c$. This observation is in agreement with the argument of Patel.[4] However, he has mainly examined the partition function for pure SU(2) gauge theory and has observed that the effective string tension defined as $\sigma_{\text{eff}}=\sigma-T\ln 5/a$ approaches zero continuously and linearly as $T\to T_c=\sigma a/\ln 5$. [His formula contains a factor $\ln(2d-3)=\ln 5$ instead of $\ln 6$ because backtracking is forbidden in his case. This point is unimportant.] Our formula (4) or (6) with a more rigorous definition of the effective string tension gives a nonlinear result as will be shown in the following.

It is easily confirmed numerically that for $0\le T<T_c$ the free energy $F=-T\ln Z$ increases linearly as a function of r for large r ensuring confinement of quarks below

T_c. Since there is an appreciable nonlinear correction at small r, it is appropriate to identify the slope at asymptotically large r with the effective string tension $\sigma(T)$. One thus obtains

$$\sigma(T) = -T \lim_{r\to\infty} \frac{\partial \ln Z}{\partial r} . \tag{7}$$

The numerical result for $\sigma(T)$ determined at $r/a = 10^4$ is shown in Fig. 1, where $\hat{T} \equiv T/(\sigma a)$. Our result confirms Patel's conclusion that a second-order phase transition takes place at $T = T_c$.

It is easy to derive approximate analytic expressions for $\sigma(T)$ for $T \simeq 0$ and $T \simeq T_c$ from (4) or (6). The result is

$$\sigma(T)/\sigma(0) \simeq 1 - (\ln 6 - \tfrac{3}{2})\hat{T} \text{ for } T/T_c \ll 1 , \tag{8}$$

and

$$\sigma(T)/\sigma(0) \simeq \sqrt{6}(\hat{T}_c - \hat{T})^{\nu} \text{ for } 1 - T/T_c \ll 1 , \tag{9}$$

where

$$\nu = \tfrac{1}{2} , \tag{10}$$

and $\hat{T}_c \equiv T_c/(\sigma a)$. Both (8) and (9) are shown in Fig. 1. Formula (8) is actually valid up to $T/T_c \simeq 0.61$, while (9) is valid down to $T/T_c \simeq 0.97$ within 1% error.

The above result may be compared with some other results obtained by different methods. McLerran and Svetitsky made a Monte Carlo analysis of an SU(2) gauge theory and estimated the value of $\sigma(T)$ at one point near T_c.[5] Their result is $\sigma(T)/\sigma(0) \simeq 0.4$ for $T/T_c = 0.98$. Our corresponding result is $\sigma(0.98T_c)/\sigma(0) = 0.257$. Pisarski and Alvarez calculated $\sigma(T)$ near $T = 0$ on the basis of a quantum string theory.[6] Their result is

$$\sigma(T)/\sigma(0) \simeq 1 - \frac{\pi(d-2)T^2}{6\sigma} \text{ for } T \ll \sqrt{\sigma} . \tag{11}$$

The quadratic dependence on T is different from our linear dependence in (8). However, our result (8) should be taken with some caution. The reason is that the dominant contribution to the partition fuction for $T/T_c \ll 1$ comes from the configurations with $0 \le l/r - 1 \ll 1$ for which the Gaussian distribution like (2) is not necessarily valid. Further careful investigation is necessary to establish the low-temperature behavior of $\sigma(T)$.

Making the $1/d$ expansion, Pisarski *et al.* have further calculated the behavior of $\sigma(T)$ near T_c.[6] The result for the $d \to \infty$ limit is

$$\sigma(T)/\sigma(0) = [1 - (T/T_c)^2]^{1/2} . \tag{12}$$

Now the result for the critical exponent ν agrees with ours though $d = 4$ in our case. In the case of Pisarski and Alvarez, the result $\nu = \frac{1}{2}$ will suffer, in general, a finite correction for a finite d. In our case, the only crucial point that leads to the result $\nu = \frac{1}{2}$ is the fact that the step number n appears in the form const$\times r^2/n$ on the exponent of the Gaussian factor in (2). This is a direct consequence from the well-known $\sqrt{n}$ law of random walks in a space with an arbitrary dimension. Hence, our result (10) is independent of d as long as $d \ge 2$.

On the other hand, our result (10) does not agree with the prediction given by Svetitsky and Yaffe.[7] They argue that, for nonzero temperature, there is correspondence between pure gauge theories that have a center symmetry and spin models invariant under the same symmetry. For example, the critical behavior of the SU(2) gauge theory in 3+1 dimensions is predicted to be the same as that of the three-dimensional Ising model. One then expects that $\nu = 0.63$ in disagreement with Eq. (10). This discrepancy may be due to neglect of quantum fluctuations in our classical picture. In fact, the mean-field theory which also neglects fluctuations gives $\nu = \frac{1}{2}$ for the three-dimensional Ising model.

Vanishing of $\sigma(T)$ near T_c corresponds to a divergence of the mean length $\langle l \rangle$ of the flux tubes defined as

$$\langle l \rangle \equiv Z^{-1} \int_r^{\infty} dl\, l\, \Omega(l,r) e^{-\sigma l/T} = \frac{T^2}{\sigma} \frac{\partial \ln Z}{\partial T} . \tag{13}$$

It is interesting to see how it diverges at T_c. Both the ex-

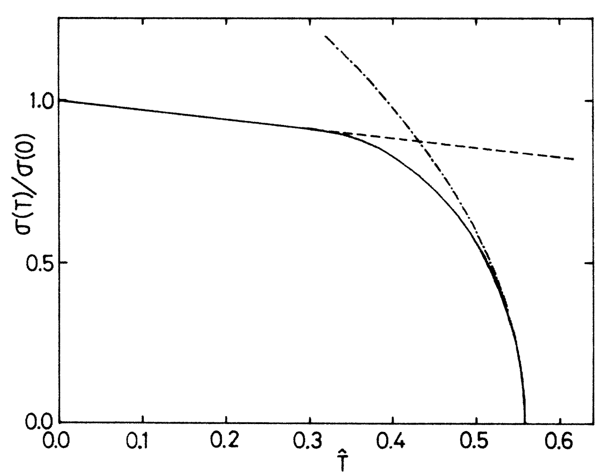

FIG. 1. Solid curve: the effective string tension normalized at $\hat{T} = 0$. Dashed curve: the low-temperature behavior given by (8). Dash-dotted curve: the behavior near the critical temperature given by (9) and (10).

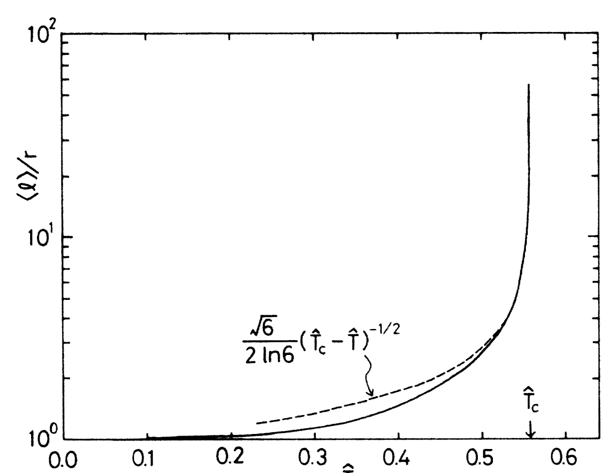

FIG. 2. The mean length of the flux tubes divided by the q-$\bar{q}$ distance. The dashed line shows the power-law behavior (14) near the critical temperature.

act numerical result for $r/a \gg 1$ and the approximate analytic formula

$$\langle l \rangle / r \simeq \frac{\sqrt{6}}{2\ln 6}(\hat{T}_c - \hat{T})^{-1/2} \text{ for } 1 - T/T_c \ll 1 \text{ and } r/a \gg 1 \tag{14}$$

are shown in Fig. 2. As (14) demonstrates explicitly, there is a remarkable scaling property that $\langle l \rangle / r$ becomes independent of r when $r/a \gg 1$ and T is sufficiently near T_c.

The increase of $\langle l \rangle$ implies increase of the radius of the flux-tube configuration. An interesting case here is the radius $R(T)$ of the quarkless configuration, i.e., the glueball. It is a configuration with $l \gtrsim l_{\min}$ and $r = 0$. It corresponds to a closed random walk with the total step number $n = l/a$. Here, $l_{\min} \equiv \lambda a$ is the minimum value of l for the glueball configuration and hence $\lambda = 2$ or 4 depending on whether or not backtracking is allowed. The expectation value of the distance between a starting point of the closed random walk and the farthest point is $\sqrt{n}a/2$. The radius is then etimated to be

$$R(T) \simeq (\langle l \rangle_{r=0})^{1/2} a^{1/2}/2 \ . \tag{15}$$

The lower bound for the l integration in (4) and (13) becomes λa instead of r while r in the integrand is set to zero when one calculates $\langle l \rangle_{r=0}$. Putting the result for $\langle l \rangle_{r=0}$ into (15), one obtains

$$R(T)/a \simeq (\pi\lambda)^{1/4} 2^{-3/2} (\ln 6)^{-1/2} (\hat{T}_c - \hat{T})^{-1/4} \tag{16}$$

for $\lambda(\hat{T}^{-1} - \hat{T}_c^{-1}) \ll 1$. It is remarkable that the divergent behavior of $R(T)$ at $\hat{T} = \hat{T}_c$ is just the same as that of the radius of a hadron bag at finite temperature for which the deconfinement transition is characterized by the divergence of the bag radius.[8]

To conclude, we have calculated the effective string tension $\sigma(T)$ as a function of the temperature using the SU(2) flux-tube model proposed by Patel. The result confirms the existence of the second-order deconfinement phase transition at some critical temperature and the critical exponent for $\sigma(T)$ was found to be equal to $\frac{1}{2}$, independent of the space-time dimension. Furthermore, it was found that $\sigma(T)$ decreases linearly for low T and the mean length of the flux tube diverges like $(\hat{T}_c - \hat{T})^{-1/2}$ as T approaches T_c from below.

It will be interesting to carry out a Monte Carlo calculation of $\sigma(T)$ on the basis of the SU(2) lattice gauge theory and to compare the result with our simple classical result. In fact, some result is already available for the SU(3) case, where it is suggested that $\sigma(T)$ is nonvanishing when T approaches T_c from below.[3] It is also interesting, if possible, to calculate $\sigma(T)$ by using a quantum string theory but without recourse to the $1/d$ expansion.

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