# Note on baryon masses in the Skyrme model

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Adopting a view that the Skyrmion and quarks may play complementary roles in the hadrons, a two-body spin-spin interaction term is added to the hadron masses. The obtained fit is in agreement with experiment.

There are indications that the Skyrmion,[1] a classical soliton solution of an effective field theory of mesons, may provide a description of the nucleon. The Skyrmion model has been extended to the SU(3) sector[2–4] also. Static properties such as masses, magnetic moments, $G_A/G_V$, the baryon decays, etc., have been shown to be reproduced in the model. The agreement is, however, to within about 30%, suggesting that the Skyrmion may not provide a complete picture of baryons. There may be other features in the baryon structure. One can think of a number of improvements such as a hard-pion correction,[5] quark contributions at short distances,[6] a subdominant mass term,[7] hybrid bag models,[8] etc.

In this note, we consider the baryon masses in the Skyrmion picture where the short-distance effects are modified due to the inclusion of an explicit $\sigma_i \cdot \sigma_j$ term which may arise as the perturbative part of a single-gluon-exchange effect. We shall first discuss the results of the constituent-quark model and the Skyrmion model, and then give the results in our picture.

In the quark model the splitting between baryon and meson spin multiplets results from one-gluon exchange:[9]

$$\delta H = -\frac{g^2}{N_c} \sum_{i,j} b_{ij} (\lambda_i \cdot \lambda_j)(\sigma_i \cdot \sigma_j) , \quad (1)$$

where $\lambda_i$ ($\sigma_i$) are the color (spin) matrices of the $i$th quark, $g^2$ is normalized to remain fixed as $N_c \to \infty$, and $b_{ij}$ are responsible for any flavor breaking. With this term and the constituent-quark mass term $\sum_i m_{q_i}$, the baryon masses are found to obey the following mass sum rules:

$$\tfrac{1}{3}(\Omega-\Delta) = (\Xi^* - \Sigma^*) , \quad (2)$$
(147 MeV) (149 MeV)

$$(\Sigma-\Lambda) = \tfrac{2}{3}(\Delta - N) y , \quad (3)$$
(78 MeV) (68 MeV)

$$(\Sigma^* - \Delta) = \tfrac{1}{2}(3\Lambda - \Sigma - 2N) , \quad (4)$$
(152 MeV) (137 MeV)

$$(\Xi^* - \Sigma^*) = (\Xi - \Sigma) , \quad (5)$$
(149 MeV) (125 MeV)

$$(2N + 2\Xi - 3\Lambda - \Sigma) = \tfrac{1}{3}(\Delta - N) y^2 , \quad (6)$$
(−24 MeV) (11 MeV)

where

$$y = \left[ 1 + \frac{\Delta + N}{6(\Lambda - N)} \right]^{-1} = 1 - \frac{m_u}{m_s} = \tfrac{1}{3} . \quad (7)$$

Though the first two relations (2) and (3) are well satisfied, the relations (4)–(6) involving mass differences between the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons do not agree that well. The general feeling is that in the nonrelativistic quark model, we do not have a satisfactory description of the long-range forces.[9]

In the Skyrme picture, although quarks do not appear explicitly, the effective Lagrangian does possess[6] some of the physics associated with quarks, for instance, the short-range repulsive $\Delta - N$ splitting.[3,4] The SU(3) breaking in the model is introduced through the meson masses,[3] i.e.,

$$\Delta H^{(8)} = \int \frac{m_K{}^2 - m_\pi{}^2}{8\sqrt{3}} F_\pi{}^2 \, \mathrm{Tr}[\lambda_8 (U + U^\dagger)] d^3x . \quad (8)$$

Sandwiching this SU(3)-breaking Hamiltonian $\Delta H^{(8)}$ between baryon wave functions, one obtains

$$\langle B_{\alpha\beta} | \Delta H^{(8)} | B_{\alpha\beta} \rangle = -\Delta m \int d\mu(g) D^{8*}_{\alpha\beta}(g) \times D^{8*}_{\gamma\gamma}(g) D^{8}_{\alpha\beta}(g) , \quad (9)$$

where $\alpha\beta$ represent SU(3) flavor and spin quantum numbers and

$$\Delta m = \frac{2\pi}{3} (m_K{}^2 - m_\pi{}^2) F_\pi{}^2 \int_0^\alpha (1 - \cos F) r^2 dr . \quad (10)$$

The baryon masses depend on the three parameters $m_8$, $m_{10}$, and $\Delta m$,[3,4] which yield the following mass relations:

$$2(\Xi - \Sigma) = (\Sigma - \Lambda) = (\Lambda - N) , \quad (11)$$
(250 MeV) (78 MeV) (176 MeV)

$$(\Omega - \Xi^*) = (\Xi^* - \Sigma^*) = (\Sigma^* - \Delta) , \quad (12)$$
(139 MeV) (149 MeV) (152 MeV)

$$(\Xi^* - \Sigma^*) = \tfrac{5}{4}(\Xi - \Sigma) . \quad (13)$$
(149 MeV) (156 MeV)

One retains the Gell-Mann–Okubo and equal-spacing rules, but the other mass relations do not agree well with the experimental values. This is because the predicted $F/D$ ratio is incorrect and the short-distance spin-spin interaction is suppressed[4] in the $N_c \to \infty$ limit. Since the real world ($N_c=3$) is obviously far from the large-$N_c$ limit, this may necessitate additional terms in the Hamiltonian.

One may take the view that the low-energy large-$N_c$ limit correctly represents the long-distance structure of the baryons and the short-distance effects signal the quark degrees of freedom.[10] So the quark and the Skyrmion may play complementary roles in the baryon. The quarks keep the Skyrmion from collapsing, while the Skyrmion keeps quarks confined.[10] Following this view, we introduce a two-body spin-spin interaction Hamiltonian:

$$4B_{ij}(\sigma_i \cdot \sigma_j) , \tag{14}$$

where $B_{ij}$ include all the relevant factors.

Combining this contribution with the Skyrmion masses, we obtain the relations

$$\underset{(147\text{ MeV})}{\tfrac{1}{3}(\Omega-\Delta)}=\underset{(149\text{ MeV})}{(\Xi^*-\Sigma^*)}=\underset{(147\text{ MeV})}{(\Xi-\Sigma)+\tfrac{1}{8}(\Lambda-N)} , \tag{15}$$

$$\underset{(153\text{ MeV})}{(\Sigma^*-\Delta)}=\underset{(159\text{ MeV})}{(13\Lambda-9N-4\Sigma)} , \tag{16}$$

and the values of the parameters are

$$\begin{aligned} M_8-B_{uu}&=1204\text{ MeV} , \\ M_{10}+B_{uu}&=1342\text{ MeV} , \\ \Delta m&=883\text{ MeV} , \\ B_{us}-B_{uu}&=74\text{ MeV} , \\ B_{ss}-B_{uu}&=109\text{ MeV} . \end{aligned} \tag{17}$$

Since some short-distance effects are already present in the Skyrmion, i.e., $m_8 \neq m_{10}$, we may consider the additional term (14) to be responsible for flavor breaking in the spin-spin interaction, i.e.,

$$B_{ij}=\left[\frac{m_u{}^2}{m_i m_j}-1\right]b . \tag{18}$$

Then $m_u/m_s=0.5$ follows from (17) and

$$m_8=1204\text{ MeV},\quad m_{10}=1342\text{ MeV} ,$$
$$\Delta m=883\text{ MeV},\quad b=-148\text{ MeV} .$$

Notice that the unsatisfactory results (4) and (5) of the De Rújula-Georgi-Glashow (DGG) model do not follow and the violations (11) of the Skyrmion model are also removed. That is to say that by combining the physics of the quark and Skyrme models, a good fit is obtained.

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