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Description of the gluon condensate

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We develop an effective Lagrangian for QCD in terms of order parameters of the gluon condensate. (A useful analogy to the Ginzburg-Landau theory of superconductivity may be made.) In this work the order parameters are identified with averages of the low-momentum components of the vector potential. The theory is formulated such that only a single order parameter, which describes the number of gluon pairs in the condensate (of all colors), is relevant. This order parameter becomes spatially dependent in the presence of quarks and one obtains a dynamical model for the formation of hadrons as nontopological soliton solutions of the effective Lagrangian of our model.

I. INTRODUCTION

If we are to introduce order parameters (or Higgs fields) to describe the condensates in the QCD vacuum, it is necessary to say something about length scales. For example, let us consider the corresponding situation in the Ginzburg-Landau theory of superconductivity.[1] There a (complex) order-parameter field $\Psi(x)$ is introduced so that $|\Psi(x)|^2$ is proportional to the density of condensed electron *pairs*, n_s^*. However, it is clear that $\Psi(x)$ cannot exhibit an arbitrary spatial variation. For example, if we are to construct a local theory, we cannot consider a variation of $\Psi(x)$ over a length scale small compared to the coherence length of the condensate, ξ_0. Therefore, we see that the introduction of condensate order parameters requires a specification of the length scale at which the theory is to be used.

If we turn to the theory of quantum chromodynamics (QCD) we see no length scale in the Lagrangian since the coupling constant is dimensionless; however, the theory develops its own length scale. For example, the string tension or the parameter Λ, which sets the scale for the running coupling constant, could be used to specify the length scale. Another scale might be the coherence length of the gluon condensate, $\xi_0^{\rm QCD}$. We do not have a precise value of $\xi_0^{\rm QCD}$; however, this quantity will play an important role in our discussion. (A value of $\frac{1}{5}$ or $\frac{1}{10}$ fm might be appropriate—see the Appendix.) We will be interested in discussing momenta that are either smaller or greater than $1/\xi_0^{\rm QCD}$. It is only the former momenta that will be relevant in characterizing the condensate structure.

As we will see, the introduction of condensate order parameters, which tell us how the system is correlated over a *finite* space-time volume, leads to a loss of local gauge invariance at the original length scale. Under certain assumptions we can restore gauge invariance to the model by specifying the gauge transformation properties of the order parameters at the new length scale appropriate to the effective Lagrangian of our model. This is quite analogous to the gauge invariance exhibited in the Ginzburg-Landau theory of superconductivity.

Our notation is such that $p^\mu = i\partial^\mu = i\partial/\partial x_\mu$, $x_\mu = (t, -\mathbf{x})$, and $x^\mu = (t, \mathbf{x})$. In the case of electromagnetism we achieve gauge invariance by replacing p^μ by $[p^\mu - qA^\mu_{\rm em}(\mathbf{x},t)]$, where q is the charge of the particle coupled to the electromagnetic field. Thus, in a relativistic version of the Ginzburg-Landau theory, the covariant derivative $\mathscr{D}^\mu$ is given by

$$i\mathscr{D}^\mu = [p^\mu - e^* A^\mu_{\rm em}(\mathbf{x},t)] , \tag{1.1}$$

where $e^* = 2e < 0$, e being the charge of an electron. Therefore the gauge invariance of that theory requires that the order parameter $\Psi(x)$ should transform as

$$\Psi(x) \to \Psi'(x) = e^{ie^*\gamma(x)}\Psi(x) , \tag{1.2}$$

if the vector potential undergoes a gauge transformation

$$A^\mu_{\rm em}(x) \to A^\mu_{\rm em}(x) - \partial^\mu\gamma(x) . \tag{1.3}$$

[This U(1) invariance implies that the phase of $\Psi(x)$ is arbitrary and not a physical observable.] We see that we must assume specific transformation properties of the order-parameter field if gauge invariance is to be preserved at the new length scale. The transformation property given in Eq. (1.2) is seen to be consistent with the BCS microscopic theory since the Ginzburg-Landau wave function is proportional to the gap parameter $\Delta(\mathbf{x})$ which is related to the electron field operators $\psi(\mathbf{x})$ and $\psi^\dagger(\mathbf{x})$ by

$$\Delta^*(\mathbf{x}) = -G\langle \psi_\downarrow^\dagger(\mathbf{x})\psi_\uparrow^\dagger(\mathbf{x})\rangle \tag{1.4}$$

or

$$\Delta(\mathbf{x}) = -G\langle \psi_\uparrow(\mathbf{x})\psi_\downarrow(\mathbf{x})\rangle . \tag{1.5}$$

Under gauge transformation, we have

$$\psi(x) \to \psi'(x) = e^{ie\gamma(x)}\psi(x) , \tag{1.6}$$

from which Eq. (1.2) follows.

When we consider QCD, we find that $\langle {\rm vac}|(\alpha_s/\pi)G^a_{\mu\nu}(0)G_a^{\mu\nu}(0)|{\rm vac}\rangle$ might be a useful *gauge-invariant* order parameter to describe the gluon condensate. It is much more convenient, however, to work with a "pairing" order parameter $\langle {\rm vac}|g^2A^a_\mu(0)A_a^\mu(0)|{\rm vac}\rangle$, which is clearly not gauge invariant. However, we can show that, *if the condensate has only low-momentum components,*

$$\langle \text{vac} \mid (\alpha_s/\pi) G^a_{\mu\nu}(0) G^{\mu\nu}_a(0) \mid \text{vac} \rangle$$

is proportional to the square of $\langle \text{vac} \mid g^2 A^a_\mu(0) A^\mu_a(0) \mid \text{vac} \rangle$. To the extent that this is true, $\langle \text{vac} \mid g^2 A^a_\mu(0) A^\mu_a(0) \mid \text{vac} \rangle$ is a useful, essentially gauge invariant, characterization of the condensate. We have seen in an earlier work[2] how a specification of a nonperturbative value for

$$\langle \text{vac} \mid (\alpha_s/\pi) G^a_{\mu\nu}(0) G^{\mu\nu}_a(0) \mid \text{vac} \rangle$$

allows us to specify the value of dynamical masses for quarks and gluons of low momentum. (A dynamical gluon mass has been extensively discussed by Cornwall and collaborators[3] and the number suggested for that quantity is consistent with the value obtained in our model.)

In Sec. II we begin with a discussion of the local gauge invariance of QCD. We then introduce order parameters which are used to describe the gluon condensate. In Sec. III we construct an effective Lagrangian in terms of these order parameters. In that section we specify matrix elements of the condensate field in the QCD vacuum and use these elements in the development of our effective Lagrangian. We then show that (essentially) the same Lagrangian may be obtained if the condensate field is replaced by a classical field. This field must be averaged in a specific manner to reproduce the effective Lagrangian obtained using the first method.

In Sec. IV we introduce still another method to obtain a similar effective Lagrangian. In that section we develop the analogy to the Ginzburg-Landau theory of superconductivity. We show that averaging over the gauge group in a specific fashion leads to the effective Lagrangian obtained in Sec. II.

In Sec. V we describe the modification of the theory required in the presence of quarks. Finally, Sec. VI contains some concluding remarks and a summary of the properties of the effective Lagrangians developed at various stages of this analysis.

II. ORDER PARAMETERS OF THE QCD VACUUM

In quantum chromodynamics the covariant derivative D_μ is given by

$$iD_\mu = \left[i\partial_\mu + g A^a_\mu(x) \frac{\lambda^a}{2} \right] . \tag{2.1}$$

It is useful to write the vector potential as

$$A^\mu(x) = \sum_a A^\mu_a \lambda^a/2 , \tag{2.2}$$

so that if the quark field $q(x)$ transforms as

$$q(x) \rightarrow q'(x) = U(x) q(x) , \tag{2.3}$$

with

$$U(x) = e^{i\omega^a(x)\lambda^a/2} , \tag{2.4}$$

we must have

$$A^\mu(x) \rightarrow A'^\mu(x) = U(x) A^\mu(x) U^{-1}(x) - \frac{i}{g} \partial_\mu U(x) U^{-1}(x) . \tag{2.5}$$

If $\omega^a(x) \rightarrow \delta\omega^a(x)$, an infinitesimal quantity, Eq. (2.5) becomes

$$A^a_\mu(x) \rightarrow A'^\mu_a(x) = A^\mu_a(x) - f^{abc} \delta\omega_b(x) A^\mu_c(x) + \frac{1}{g} \partial_\mu [\delta\omega_a(x)] , \tag{2.6}$$

where we have used

$$\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2} \right] = i f^{abc} \frac{\lambda^c}{2} . \tag{2.7}$$

Now, the spatial variation of $\omega_a(x)$ can, in principle, be extremely rapid, that is, the Fourier transform of $\omega_a(x)$ can have very-high-momentum components. Such rapid spatial variation is certainly a possible characteristic of a local gauge transformation however, in order to discuss the gluon condensate we should limit our consideration to distances greater than ξ_0^{QCD} or momenta smaller than $1/\xi_0^{\text{QCD}}$ as discussed in the previous section.

We now decompose the modes of the gluon field into those with momenta greater than and less than $1/\xi_0^{\text{QCD}}$. We will write

$$A^\mu_a(x) = \mathbb{A}^\mu_a(x) + \mathscr{A}^\mu_a(x) , \tag{2.8}$$

where $\mathbb{A}^\mu_a(x)$ contains only low momenta, as defined above. Indeed, let us introduce the Fourier transform of Eq. (2.8),

$$A^\mu_a(k) = \mathbb{A}^\mu_a(k) + \mathscr{A}^\mu_a(k) , \tag{2.9}$$

where $\mathbb{A}^\mu_a(k)$ is nonzero for $k < 1/\xi_0^{\text{QCD}}$ and $\mathscr{A}_a(k)$ is nonzero for $k > 1/\xi_0^{\text{QCD}}$. We will see that in this model $\mathbb{A}^\mu_a(k)$ and $\mathscr{A}^\mu_a(k)$ have quite different magnitudes.

For definiteness let us consider the condensate to be in the $k=0$ mode. We note that *local* gauge transformations will mix the $\mathbb{A}^\mu_a$ and $\mathscr{A}^\mu_a$ modes. Therefore in order to make a separation such as that in Eq. (2.9), we should work in a fixed gauge. In any gauge we will find that the relevant physical quantities will be proportional to the total number of gluons (of all colors) in the condensate and the order parameters of the theory will be real. For the model discussed here we can also see that the condensate is color neutral and thus there is no color current associated with the flow of the condensate. (This is in contrast with the theory of superconductivity where the condensate is charged and one therefore requires a complex order parameter.[1])

In order to keep the notation simple let us take ξ_0^{QCD} to be "large" and concentrate on the $k=0$ mode. Thus we will write $\mathbb{A}^a_\mu(0)$ rather that $\mathbb{A}^a_\mu(k)$ in the following. As a next step we will assume that the mode $\mathbb{A}^a_\mu(0)$ is *macroscopically occupied.* This assumption is characteristic of any analysis of a boson condensate. For example, in the nonrelativistic theory of a boson condensate one separates off the $\mathbf{k}=0$ mode in the expansion of the field:[4]

$$\theta(\mathbf{x})=e^{i\alpha}\theta_0+\sum_{k\neq 0} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} . \tag{2.10}$$

The phase may be taken equal to zero and θ_0 is then seen to be $\sqrt{n_0}$, where $n_0=N_0/(\mathrm{vol})$ is the density of particles in the condensate. Therefore if N_g is the number of condensed gluons in some volume, the magnitude of $\mathbb{A}_\mu^a$ averaged over that volume is of order $\sqrt{N_g}$ larger than the characteristic magnitude of the average of $\mathscr{A}_\mu^a(x)$. In QCD we have an octet of fields, $\mathbb{A}_\mu^a(0)$ and these are (Lorentz) vector fields. Therefore we can consider gluons condensed into each of eight color states and also consider various spatial directions. However, *we should describe the condensate in a manner such that gauge invariance and Lorentz invariance are preserved.*

Let us consider the field operator

$$G_a^{\mu\nu}(x)=\partial^\mu A_a^\nu(x)-\partial^\nu A^\mu(x)+gf^{abc}A_b^\mu(x)A_c^\nu(x) , \tag{2.11}$$

and write

$$G_a^{\mu\nu}(x)=\mathbb{G}_a^{\mu\nu}(x)+\mathscr{G}_a^{\mu\nu}(x) , \tag{2.12}$$

where

$$\mathbb{G}_a^{\mu\nu}(x)=gf^{abc}\,\mathbb{A}_b^\mu(x)\,\mathbb{A}_c^\nu(x) . \tag{2.13}$$

We now recall that from the QCD sum-rule analysis one obtains[5]

$$\left\langle \mathrm{vac}\left|\frac{\alpha_s}{\pi}G_{\mu\nu}^a(0)G_a^{\mu\nu}(0)\right|\mathrm{vac}\right\rangle=0.012\ \mathrm{GeV}^4 . \tag{2.14}$$

We identify this quantity, which is fundamentally nonperturbative, with[2]

$$\left\langle \mathrm{vac}\left|\frac{\alpha_s}{\pi}\mathbb{G}_{\mu\nu}^a(0)\,\mathbb{G}_a^{\mu\nu}(0)\right|\mathrm{vac}\right\rangle=\frac{1}{4\pi^2}f^{abc}f^{ade}\langle \mathrm{vac}\,|\,g^4\,\mathbb{A}_b^\mu(0)\,\mathbb{A}_c^\nu(0)\,\mathbb{A}_\mu^d(0)\,\mathbb{A}_\nu^e(0)\,|\,\mathrm{vac}\rangle . \tag{2.15}$$

As discussed in an earlier work,[2] we equate b and d and c and e to obtain the coherent part of the last expression:

$$\left\langle \mathrm{vac}\left|\frac{\alpha_s}{\pi}\mathbb{G}_{\mu\nu}^a(0)\,\mathbb{G}_a^{\mu\nu}(0)\right|\mathrm{vac}\right\rangle=\frac{3}{32\pi^2}[\langle \mathrm{vac}\,|\,g^2\,\mathbb{A}_b^\mu(0)\,\mathbb{A}_\mu^b(0)\,|\,\mathrm{vac}\rangle]^2 \tag{2.16}$$

or

$$\langle \mathrm{vac}\,|\,g^2\,\mathbb{A}_b^\mu(0)\,\mathbb{A}_\mu^b(0)\,|\,\mathrm{vac}\rangle=-1.12\ \mathrm{GeV}^2 . \tag{2.17}$$

We have chosen a negative sign in Eq. (2.17), since that sign leads to positive dynamical masses for quarks and gluons. Indeed, we have shown in a previous work[2] that the value of the condensate parameters given in Eq. (2.17) leads to a dynamical gluon mass of $m_G=649$ MeV and a dynamical quark mass of $m_q^G=\frac{2}{3}m_G=434$ MeV. (In Ref. 2 we also discussed the momentum dependence of the dynamical mass parameters. In order to describe that momentum dependence we must discuss the nature of the theory at length scales significantly smaller than those considered here.) As we will see, m_G can be put into correspondence with the inverse of the Landau penetration length (λ_L) in the theory of superconductivity. Similarly the dynamical quark mass m_q^G is analogous to the gap parameter Δ in the theory of superconductivity.

Up to this point we have not been careful to distinguish between the operator $\mathbb{A}_a^\mu$ and a corresponding classical variable. We make this distinction in the next section where we present values for vacuum matrix elements of the operator $\mathbb{A}_a^\mu$ and various products of such operators. We then go on to show how one can replace $\mathbb{A}_a^\mu$ by a c number:

$$[\,\mathbb{A}_a^\mu]_{\mathrm{cl}}=\varphi_0\eta_a^\mu . \tag{2.18}$$

We will see that one has to average the Lorentz vector η_a^μ in a specific manner to reproduce the results of our original analysis which is based on specifying vacuum matrix elements involving the $\mathbb{A}_a^\mu$.

III. AN EFFECTIVE LAGRANGIAN

In this section we will show how the separation of the vector potential into a condensate field plus an additional field allows one to construct an effective Lagrangian. Since color symmetry is not broken we require special techniques for the description of the condensate fields, which will become apparent as we proceed.

We note that in the conventional approach to spontaneous symmetry breaking one writes a Lagrangian which has some continuous symmetry. This Lagrangian may contain a Higgs field or chiral fields such as $\sigma(x)$ and $\pi(x)$. One then has a continuous infinity of degenerate vacuum states. By choosing a specific vacuum state one breaks the symmetry of the ground state. Now we note that this procedure *cannot* be used without modification in the case of QCD since the gauge symmetry is not broken.

The procedure we adopt is as follows. We write the vector potential as

$$A_\mu^a(x)=\mathbb{A}_\mu^a+\mathscr{A}_\mu^a(x) , \tag{3.1}$$

where, for simplicity, we will take $\mathbb{A}_\mu^a$ to represent the zero-momentum mode. From symmetry considerations we must have

$$\langle \mathrm{vac}\,|\,\mathbb{A}_\mu^a\,|\,\mathrm{vac}\rangle=0 . \tag{3.2}$$

However, we can put, with $\mathbb{A}_a^0=0$, i.e., in the temporal gauge,

$$\langle \text{vac} \,|\, g^2 \mathrm{A}^i \mathrm{A}^j \,|\, \text{vac} \rangle = g^2 \delta_{ab} \delta_{ij} \varphi_0^2 / 24 \; . \tag{3.3}$$

Indeed, our assumptions concerning the character of the QCD vacuum can be translated into assumptions concerning matrix elements of the operator A_a^μ. We now assume that, in addition to Eqs. (3.2) and (3.3), we may write

$$\langle \text{vac} \,|\, \mathrm{A}_a^i \mathrm{A}_b^j \mathrm{A}_c^k \,|\, \text{vac} \rangle = 0 \tag{3.4}$$

and

$$\langle \text{vac} \,|\, \mathrm{A}_a^i \mathrm{A}_b^j \mathrm{A}_c^k \mathrm{A}_d^l \,|\, \text{vac} \rangle = \frac{\varphi_0^4}{(24)(26)} (\delta_{ij}\delta_{kl}\delta_{ab}\delta_{cd} + \delta_{ik}\delta_{jl}\delta_{ac}\delta_{bd} + \delta_{il}\delta_{jk}\delta_{ad}\delta_{bc}) \; . \tag{3.5}$$

From the last relation we have

$$\langle \text{vac} \,|\, \mathrm{A}_b^i \mathrm{A}_c^j \mathrm{A}_i^{b'} \mathrm{A}_j^{c'} \,|\, \text{vac} \rangle = \frac{\varphi_0^4}{(24)(26)} (3\delta_{bc}\delta_{b'c'} + 9\delta_{bb'}\delta_{cc'} + 3\delta_{bc'}\delta_{cb'}) \; , \tag{3.6}$$

from which we can obtain a result we will need later:

$$\tfrac{1}{4} \langle \text{vac} \,|\, f^{abc} \mathrm{A}_b^i \mathrm{A}_c^j \mathrm{A}_i^{b'} \mathrm{A}_j^{c'} f^{ab'c'} \,|\, \text{vac} \rangle = \frac{1}{4} \frac{\varphi_0^4}{(24)(26)} [9(f^{abc})^2 + 3 f^{abc} f^{acb}] \; , \tag{3.7}$$

$$= \frac{6}{4} \frac{\varphi_0^4}{(24)(26)} (f^{abc})^2 \; , \tag{3.8}$$

$$= \tfrac{3}{32} \varphi_0^4 [\tfrac{8}{13}] \; . \tag{3.9}$$

Note that the term in the brackets in Eq. (3.9) arises from considering the "exchange terms" of Eq. (3.6), that is, those terms with coefficient 3.

We digress at this point to remark that if A_μ^a were to be treated as a classical variable we could put

$$[\mathrm{A}_i^a]_{\text{cl}} = \varphi_0 \hat{\eta}_i^a \; , \tag{3.10}$$

where φ_0 is a constant. Now $\hat{\eta}^a$ cannot be a simple classical vector since the vacuum expectation value of A^a is zero, which implies $\hat{\eta}^a = 0$ for finite φ_0. We may also consider the correspondence

$$\langle \text{vac} \,|\, g^2 \mathrm{A}_a^i \mathrm{A}_a^i \,|\, \text{vac} \rangle \Longrightarrow g^2 \varphi_0^2 \hat{\eta}^a \cdot \hat{\eta}^a \; . \tag{3.11}$$

This then suggests that we should constrain the vector $\hat{\eta}^a$ such that

$$\hat{\eta}_a \cdot \hat{\eta}_a = 1 \; . \tag{3.12}$$

The characterization of the vacuum state given by Eqs. (3.2)–(3.9) may be translated into further specifications concerning the behavior of the vector $\hat{\eta}_a$. The various assumptions concerning the character of matrix elements of the operator A_a^i translate into the following prescription. One uses Eqs. (3.1) and (3.10) to reexpress the QCD Lagrangian in terms of $\mathscr{A}^a(x)$, φ_0, and $\hat{\eta}^a$. One then averages the result over all directions of the vector $\hat{\eta}_a$ with the constraint of Eq. (3.12).

This averaging procedure for an operator $O(\hat{\eta})$ may be written as

$$\langle O(\hat{\eta}) \rangle = \frac{\int \prod_{a'} d\hat{\eta}_{a'} \delta(\hat{\eta}_a \cdot \hat{\eta}_a - 1) O(\hat{\eta})}{\int \prod_{a'} d\hat{\eta}_{a'} \delta(\hat{\eta}_a \cdot \hat{\eta}_a - 1)} \; . \tag{3.13}$$

We now turn to the construction of an effective Lagrangian. First we will use Eqs. (3.1)–(3.9) to develop a Lagrangian. Then we will carry out an analysis based upon Eqs. (3.10)–(3.13). We begin by inserting Eq. (3.1) into Eq. (2.11) to find

$$G_{\mu\nu}^a = \partial_\mu \mathscr{A}_\nu^a - \partial_\nu \mathscr{A}_\mu^a + g f^{abc} (\mathscr{A}_\mu^b \mathscr{A}_\nu^c + \mathscr{A}_\mu^b \mathrm{A}_\nu^c + \mathrm{A}_\mu^b \mathscr{A}_\nu^c + \mathrm{A}_\mu^b \mathrm{A}_\nu^c) \; . \tag{3.14}$$

Thus the gluon part of the QCD Lagrangian is

$$\begin{aligned} \mathscr{L}_G = -\tfrac{1}{4} [& \mathscr{G}_{\mu\nu}^a \mathscr{G}_a^{\mu\nu} + 2g f^{abc} \mathscr{G}_a^{\mu\nu} (\mathscr{A}_\mu^b \mathrm{A}_\nu^c + \mathrm{A}_\mu^b \mathscr{A}_\nu^c + \mathrm{A}_\mu^b \mathrm{A}_\nu^c) \\ & + g^2 f^{abc} f^{ab'c'} (\mathscr{A}_\mu^b \mathrm{A}_\nu^c + \mathrm{A}_\mu^b \mathscr{A}_\nu^c + \mathrm{A}_\mu^b \mathrm{A}_\nu^c) \\ & \times (\mathscr{A}_\mu^{b'} \mathrm{A}_\nu^{c'} + \mathrm{A}_\mu^{b'} \mathscr{A}_\nu^{c'} + \mathrm{A}_\mu^{b'} \mathrm{A}_\nu^{c'})] \; , \end{aligned} \tag{3.15}$$

where

$$\mathscr{G}_{\mu\nu}^a(x) \equiv \partial_\mu \mathscr{A}_\nu^a(x) - \partial_\nu \mathscr{A}_\mu^a(x) + g f^{abc} \mathscr{A}_\mu^b(x) \mathscr{A}_\nu^c(x) \; . \tag{3.16}$$

As a next step we replace various products of the A_a^μ by their coherent values. (That is, we insert the "contractions" of these operators, to obtain an effective Lagrangian.) This procedure leads to the effective Lagrangian

$$\mathscr{L}_G(\mathscr{A}_\mu^a, \varphi_0) = -\tfrac{1}{4} [\mathscr{G}_{\mu\nu}^a(x) \mathscr{G}_a^{\mu\nu}(x)] + \frac{m_G^2}{2} \mathscr{A}_\mu^b(x) \mathscr{A}_b^\mu(x) - b\varphi_0^4 \; , \tag{3.17}$$

where

$$m_G^2 = [\tfrac{2}{3}] \tfrac{3}{8} g^2 \varphi_0^2 \; , \tag{3.18}$$

$$b = [\tfrac{8}{13}] \tfrac{3}{32} g^2 \; . \tag{3.19}$$

Again the factors in brackets arise from keeping the "exchange terms."

We may now show how the same result may be obtained using Eq. (3.10) plus a specific averaging procedure such as that defined in Eq. (3.13). This will strate to what extent we can treat φ_0 and η_μ^a as classical variables. To avoid repetition let us consider a more general model where we write

$$[A_a^\mu]_{\rm cl}=\varphi(x)\eta_a^\mu \;, \tag{3.20}$$

with $\varphi(x)=\varphi_0+\mathcal{X}(x)$. We now have

$$\begin{aligned} G_{\mu\nu}^a(x)=\mathcal{G}_{\mu\nu}^a(x)&+\partial_\mu\varphi(x)\eta_\nu^a-\partial_\nu\varphi(x)\eta_\mu^a \\ &+gf^{abc}\varphi^2(x)\eta_\mu^b\eta_\nu^c \\ &+gf^{abc}\varphi(x)(\eta_\mu^b\mathcal{A}_\nu^c+\mathcal{A}_\mu^b\eta_\nu^c) \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} \mathcal{L}_G(\varphi,\eta)=-\tfrac{1}{4}\{&\mathcal{G}_{\mu\nu}^a(x)\mathcal{G}_a^{\mu\nu}(x)-2[\partial_\mu\varphi(x)\partial^\mu\varphi(x)+\partial_\mu\varphi(x)\partial_\nu\varphi(x)\eta_a^\mu\eta_a^\nu]+g^2f^{abc}f^{ab'c'}\varphi^4(x)\eta_\mu^b\eta_\nu^c\eta_{b'}^\mu\eta_{c'}^\nu \\ &+2g^2f^{abc}f^{ab'c'}\varphi^2(x)[\eta_\mu^b\eta_{b'}^\mu\mathcal{A}_\nu^c(x)\mathcal{A}_{c'}^\nu(x)+\eta_\mu^b\eta_\nu^c\mathcal{A}_b^\mu(x)\mathcal{A}_c^\nu(x)]+\cdots\}\;. \end{aligned} \tag{3.22}$$

In Eq. (3.22) the ellipsis indicates various terms which will vanish upon averaging. We find, upon averaging,

$$\begin{aligned} \mathcal{L}_G(\varphi)=&-\tfrac{1}{4}\mathcal{G}_a^{\mu\nu}(x)\mathcal{G}_{\mu\nu}^a(x) \\ &+\frac{m_G{}^2}{2}\mathcal{A}_\mu^a(x)\mathcal{A}_a^\mu(x)\varphi^2(x)/\varphi_0{}^2 \\ &+\tfrac{1}{2}(\partial_0\varphi\partial_0\varphi-\tfrac{2}{3}\nabla\varphi\cdot\nabla\varphi)-b\varphi^4(x)\;. \end{aligned} \tag{3.23}$$

This result agrees with our previous result if $\varphi(x)=\varphi_0$.

The point of this development is to show that if we wish to use classical fields to represent the condensate, we have to perform a particular type of averaging of these fields to reproduce the results of the first method used.

In the next section we describe a model which exhibits a local gauge symmetry. We will show that if the Lagrangian of that model is averaged in a particular fashion, we can reproduce the effective Lagrangians obtained in this section.

IV. YANG-MILLS FIELDS COUPLED TO HIGGS FIELDS

We recall that in the Ginzburg-Landau theory of superconductivity one adds to the free energy density of the normal system, $F_n(\mathbf{x})$, the free energy density of the condensate[6]

$$\begin{aligned} F(\mathbf{x})=F_n(\mathbf{x})+\frac{1}{2m^*}&\left|\left[-i\nabla-\frac{e^*}{c}\mathbf{A}(\mathbf{x})\right]\Psi(\mathbf{x})\right|^2 \\ &+a\,|\Psi(\mathbf{x})|^2+b\,|\Psi(\mathbf{x})|^4+\frac{\mathbf{B}^2(\mathbf{x})}{8\pi}\;. \end{aligned} \tag{4.1}$$

We remark that the covariant version of the Ginzburg-Landau theory is an Abelian Higgs model:

$$\begin{aligned} \mathcal{L}_{\rm LG}(x)=&-\tfrac{1}{4}F_{\rm em}^{\mu\nu}(x)F_{\mu\nu}^{\rm em}(x)+\bar\psi(x)(i\not{D}^{\rm em}-m_e)\psi(x) \\ &+\tfrac{1}{2}[\mathcal{D}^\mu\Psi'(x)]^\dagger\mathcal{D}_\mu\Psi'(x) \\ &+a'\,|\Psi'(x)|^2+b'\,|\Psi'(x)|^4\;, \end{aligned} \tag{4.2}$$

where $\psi(x)$ is the electron field operator. Note that the mass $m^*=2m_e$ has been absorbed into the definition of the Higgs field $\Psi'(x)$, in Eq. (4.2). (Here $D_\mu^{\rm em}=[i\partial_\mu-eA_\mu^{\rm em}(x)]$, and $\mathcal{D}_\mu$ has been given in Eq. (1.1).)

In the Ginzburg-Landau theory the current consists of a normal part, $\mathbf{j}_n(\mathbf{x})$, and a supercurrent, $\mathbf{j}_s(\mathbf{x})$,

$$\begin{aligned} \mathbf{j}_s(\mathbf{x})=&-\frac{(e^*)^2}{m^*}\,|\Psi(\mathbf{x})|^2\mathbf{A}_{\rm ext}(\mathbf{x}) \\ &+\frac{e^*}{2m^*i}[\Psi^*(\mathbf{x})\nabla\Psi(\mathbf{x})-\Psi(\mathbf{x})\nabla\Psi^*(\mathbf{x})]\;. \end{aligned} \tag{4.3}$$

The first term of Eq. (4.3) makes the photon field "massive" in the superconductor leading to the Meissner effect. The penetration length is seen to be

$$\lambda_L=\left[\frac{m_e}{4\pi n_s e^2}\right]^{1/2}, \tag{4.4}$$

where n_s is the density of superconducting electrons. This follows from Eq. (4.3) if we note that $e^*=2e<0$ and $m^*=2m_e$, and if we normalized $|\Psi|^2$ to be proportional to the density of superconducting electron pairs, n_s^*. (Note that $n_s^*=\frac{1}{2}n_s$ and that the factor of 4π arises from use of curl$B=4\pi\mathbf{j}_s$.) As we will see, in our analysis we will obtain a current analogous to the first term of Eq. (4.3) (Ref. 2),

$$J_{sc,a}^\mu(x)=m_G{}^2\left[\frac{\phi_0+\mathcal{X}(x)}{\phi_0}\right]^2A_a^\mu(x)\;. \tag{4.5}$$

Thus, there is an analogy between the dynamical gluon mass and $\lambda_L{}^{-1}$ which may be drawn. (We should remark that the coherence length ξ_0 and the penetration length λ_L can be quite different in the theory of superconductivity.[1])

We note that the *local* relations between the supercurrent and the vector potential in Eq. (4.3), and the corresponding relation in Eq. (4.5), are only valid if $\mathbf{A}(x)$ varies slowly over the coherence length. This may be seen in the Pippard generalization of the Landau theory, where[1]

$$\mathbf{j}_s(\mathbf{x})=-\frac{3}{4\pi}\frac{(e^*)^2}{m^*}\frac{n_s^*}{\xi_0}\int d\mathbf{y}\frac{\mathbf{r}[\mathbf{r}\cdot\mathbf{A}(\mathbf{y})]e^{-r/\xi_0}}{r^4}\;. \tag{4.6}$$

Here $\mathbf{r}=\mathbf{x}-\mathbf{y}$. For example, if we put $\mathbf{A}(\mathbf{y})=\mathbf{A}(\mathbf{x})$ and take $\mathbf{A}(\mathbf{x})$ outside the integral, we have

$$\mathbf{j}_s(\mathbf{x}) = -\frac{(e^*)^2}{m^*} n_s^* \mathbf{A}(\mathbf{x}) , \tag{4.7}$$

which is the Landau ansatz for the relation between the supercurrent and the vector potential. [The supercurrent of Eq. (4.7) corresponds to the first term of Eq. (4.3).]

We do not expect that the coherence length in QCD, $\xi_0^{\rm QCD}$, can be much less than $\frac{1}{10}$ fm since Bjorken scaling seems to begin in deep-inelastic scattering at momenta of the order of 2–4 GeV. On the other hand, if an order-parameter approach to hadron structure is to be viable, $\xi_0^{\rm QCD}$ should be significantly smaller than the diameter of a hadron, a number which is about 1 fm.

In analogy with the Ginzburg-Landau theory of superconductivity we add a Lagrangian describing the condensate to the QCD Lagrangian. (As in the case of the Ginzburg-Landau theory one must avoid double counting when using such a Lagrangian.) We write

$$\mathscr{L}(x) = -\tfrac{1}{4} G_a^{\mu\nu}(x) G_{\mu\nu}^a(x) - \tfrac{1}{2}[\mathbb{D}_\mu \Phi_\nu^a(x)]^\dagger [\mathbb{D}^\mu \Phi_a^\nu(x)] + a\Phi_\nu^{a\dagger}(x)\Phi_a^\nu(x) - b[\Phi_\nu^{a\dagger}(x)\Phi_a^\nu(x)]^2 , \tag{4.8}$$

where a and b are greater than zero, and

$$\mathbb{D}_\mu = \partial_\mu - igA_\mu^a(x) T^a . \tag{4.9}$$

Symmetry under gauge transformation may be obtained by *assuming* that the order parameter fields $\Phi_\nu^a(x)$ transform as

$$\Phi_\nu(x) \rightarrow \Phi_\nu'(x) = e^{i\omega_a(x)T^a} \Phi_\nu(x) . \tag{4.10}$$

In this model, $\omega_a(x)$ can have arbitrary spatial variation; however, considering that this model is only meaningful at a certain length scale, we should restrict the spatial variation of $\omega_a(x)$ so that only momenta $k < 1/\xi_0^{\rm QCD}$ appear in the Fourier transform. That is to say, $\omega_a(x)$ can only vary here over distances larger than the coherence length. We may call such restricted gauge transformations *quasilocal.*

The notion of a quasilocal gauge transformation seems generally useful. For example, it is natural to consider QCD to be an effective theory at some scale. Indeed, QCD is not a complete theory as the current quark masses are expected to arise from symmetry breaking at some large mass scale. Therefore the quarks and gluons of QCD may be composite fields of a more fundamental theory, and there is a characteristic length scale for which the QCD Lagrangian is a useful effective Lagrangian. It follows that the local gauge invariance of QCD is *quasilocal* in the sense defined above. Since we have introduced still another length scale, the QCD coherence length $\xi_0^{\rm QCD}$, we lose the "local" gauge invariance of QCD, but still have a quasilocal gauge invariance at a new length scale. Therefore our model is defined by the Lagrangian of Eq. (4.8). The order parameters are assumed to have the quasilocal gauge-transformation property of Eq. (4.10).

At the new length scale associated with the QCD coherence length, we can see that the gluons and quarks acquire dynamical masses. (This feature was discussed at some length in a previous work.[2]) Now we can obtain the Lagrangian of our model by choosing the $\Phi_a^\nu(x)$ of Eq. (4.8) to be

$$\Phi^\nu(x) = U(\omega_a)[\Phi^\nu(x)]_0 , \tag{4.11}$$

where $[\Phi_a^\nu(x)]_0$ "points" in an arbitrary direction in ordinary and color space. [The norm of this vector is specified below—see Eqs. (4.18) and (4.19).]

We average $\mathscr{L}(x)$ (which now depends on the angles ω_a) over the gauge group to obtain $\mathscr{L}_M(x)$:

$$\mathscr{L}_M(x) = \frac{\int d[\omega_a] \mathscr{L}(x,\omega_a)}{\int d[\omega_a]} . \tag{4.12}$$

In order to perform this average one needs to use the result

$$\int d[\omega^a] U_{ij}^\dagger(\omega^a) U_{kl}(\omega^a) = \delta_{jk}\delta_{il}/8 \tag{4.13}$$

for the adjoint representations of SU(3). Here the ω^a are a set of parameters which specify an element of the group and 8 is the dimension of the adjoint representation. We then have

$$\frac{-g^2}{2} \int d[\omega^a] \Phi_\nu^a(x) U_{ab}^\dagger(\omega^a) T_{bc}^{i*} T_{cd}^j U_{de}(\omega^a) \Phi_e^\nu(x) A_\mu^i(x)$$

$$= \frac{-g^2}{2} \Phi_\nu^a(x) \frac{\delta_{ae}\delta_{bd}}{8} f^{bic} f^{cjd} \Phi_e^\nu(x) A_\mu^i(x) A_j^\mu(x) \tag{4.14}$$

$$= \frac{g^2}{2}(\tfrac{3}{8}) \Phi_\nu^a(x) \Phi_a^\nu(x) A_i^\mu(x) A_\mu^i(x) \tag{4.15}$$

$$= \tfrac{1}{2}(\tfrac{3}{8}) g^2 \varphi^2(x) A_a^\mu(x) A_\mu^a(x) \tag{4.16}$$

$$= \frac{m_G{}^2}{2} \frac{\varphi^2(x)}{\varphi_0{}^2} A_a^\mu(x) A_\mu^a(x) . \tag{4.17}$$

In passing from Eq. (4.16) to Eq. (4.17) we have put

$$\Phi_\nu^a(x)^\dagger \Phi_a^\nu(x) = -g^2\varphi^2(x) \tag{4.18}$$

$$= -g^2[\varphi_0 + \chi(x)]^2 . \tag{4.19}$$

We find

$$\bar{\mathscr{L}}_M(x) = -\tfrac{1}{4} G_{\mu\nu}^a(x) G_a^{\mu\nu}(x) + \tfrac{1}{2}\partial_\mu \chi(x) \partial^\mu \chi(x) - V(\varphi_0 + \chi(x)) + \frac{m_G{}^2}{2} \left[\frac{\varphi_0 + \chi(x)}{\varphi_0} \right]^2 A_a^\mu(x) A_\mu^a(x) , \tag{4.20}$$

where

$$V(\varphi_0 + \chi(x)) = -a[\varphi_0 + \chi(x)]^2 + b[\varphi_0 + \chi(x)]^4 , \tag{4.21}$$

with $b = a/2\varphi_0{}^2$ $(a > 0)$.

From our previous work we see that we can avoid double counting by replacing $\bar{\mathscr{L}}_M(x)$ of Eq. (4.20) by

$$\mathscr{L}'_M(x) = -\tfrac{1}{4} \mathscr{G}_{\mu\nu}^a(x) \mathscr{G}_a^{\mu\nu}(x) + \tfrac{1}{2}\partial_\mu \chi(x) \partial^\mu \chi(x) - V(\varphi_0 + \chi(x)) + \frac{m_G{}^2}{2} \left[\frac{\varphi_0 + \chi(x)}{\varphi_0} \right]^2 \mathscr{A}_a^\mu(x) \mathscr{A}_\mu^a(x) . \tag{4.22}$$

We note that this is essentially the same Lagrangian as that derived in Sec. III except for the term in the potential proportional to a. However, we see that this method does not produce the corrections due to "exchange terms" which were isolated by brackets in Sec. III. It is clear that *at the length scale at which $\mathscr{L}'_M(x)$ is meaningful*, the gluons have a mass m_G. We found that

$$m_G{}^2 = -\tfrac{3}{8}g^2[\Phi_a^\nu]_0^\dagger[\Phi_\nu^a]_0 \tag{4.23}$$

$$= \frac{3\pi^2}{2}\left[\left\langle \text{vac}\left|\frac{\alpha_s}{\pi}\,\mathbb{G}\,{}_a^{\mu\nu}(0)\,\mathbb{G}\,{}_{\mu\nu}^a(0)\right|\text{vac}\right\rangle\right]^{1/2} \tag{4.24}$$

$$= -\tfrac{3}{8}\langle \text{vac}\,|\,g^2\,\mathbb{A}\,{}_\nu^a(0)\,\mathbb{A}\,{}_a^\nu(0)\,|\,\text{vac}\rangle \tag{4.25}$$

$$= (649\ \text{MeV})^2\ . \tag{4.26}$$

The exchange correction is given in Eq. (3.21). Including that correction, one has $m_G = 529$ MeV. A more general version of Eq. (4.23) is

$$m_G{}^2(x) = -\tfrac{3}{8}g^2[\Phi_a^\nu(x)]^\dagger[\Phi_\nu^a(x)] \tag{4.27}$$

$$= \tfrac{3}{8}g^2[\phi_0 + \chi(x)]^2\ . \tag{4.28}$$

As we will see $[\phi_0+\chi(x)]$ can be quite small "inside" a hadron, and therefore $m_G \simeq 0$ there.

It is interesting to note that the dynamical mass m_G can be considered to be a function of momentum (as well as temperature and the quark chemical potential). The dependence on p^2 has been discussed recently by Larsson[7] for $t=-p^2$ large, that is, in the deep-Euclidean region. That discussion of the gluon propagator refers to a high-momentum gluon moving in the field generated by the quark and gluon condensates.

V. QUARK DEGREES OF FREEDOM

The addition of quarks to the model leads to the introduction of the effective Lagrangian

$$\begin{aligned}\mathscr{L}'_M(x) = &-\tfrac{1}{4}\mathscr{G}_{\mu\nu}^a(x)\mathscr{G}_a^{\mu\nu}(x)\\ &+\tfrac{1}{2}\partial^\mu\chi(x)\partial_\mu\chi(x) - V(\phi_0+\chi(x))\\ &+\frac{m_G{}^2}{2}\left[\frac{\phi_0+\chi(x)}{\phi_0}\right]^2\mathscr{A}_a^\mu(x)\mathscr{A}_\mu^a(x)\\ &+\bar{q}(x)(i\not{D}-m^{\text{cur}})q(x)\ ,\end{aligned} \tag{5.1}$$

where m^{cur} is a *current* quark mass matrix in flavor space. We also saw in our previous analysis[2] that, if one studies the quark field equation which is second order in the time variable, one generates a dynamical quark mass, $m_q^G = \frac{2}{3}m_G$. Indeed, if one includes in Eq. (5.1) a mass $m_q^{q\bar{q}}$, which arises from the breaking of chiral symmetry,[2] the quark field equation becomes (with $m = m_q^{q\bar{q}} + m^{\text{cur}}$), $m = m_q^{q\bar{q}} + m^{\text{cur}}$),

$$(i\not{D} - m)q(x) = 0\ , \tag{5.2}$$

or

$$[i\gamma^\mu\partial_\mu + g\gamma^\mu A_\mu^a(x)\lambda^a/2 - m]q(x) = 0\ . \tag{5.3}$$

This equation may be written as

$$\left[\boldsymbol{\alpha}\cdot\mathbf{p}_{\text{op}} + g\boldsymbol{\alpha}\cdot\mathbf{A}(x)\frac{\lambda^a}{2} + \gamma^0 m\right]q(x) = -\frac{1}{i}\frac{\partial}{\partial t}q(x) \tag{5.4}$$

in the temporal gauge. We can take the time derivative of this equation and retain only those terms which have a coherent vacuum value. That procedure leads to

$$(\mathbf{p}_{\text{op}}{}^2 + \tfrac{1}{6}g^2\,\mathbb{A}\,{}_a^i\,\mathbb{A}\,{}_a^i + m^2)q(x) = -\frac{\partial^2}{\partial t^2}q(x)\ . \tag{5.5}$$

Thus we can define

$$m_q{}^2 = (m_q^{q\bar{q}} + m^{\text{cur}})^2 + (m_q^G)^2\ , \tag{5.6}$$

with

$$(m_q^G)^2 = \tfrac{1}{6}g^2\,\mathbb{A}\,{}_a^i\,\mathbb{A}\,{}_a^i = \tfrac{1}{6}g^2\varphi_0{}^2\ . \tag{5.7}$$

For simplicity let us consider the case of $m^{\text{cur}}=0$. Now in the presence of a hadron we can replace $\mathbb{A}^i\,\mathbb{A}^i$ by $[\varphi_0+\chi(x)]^2$. We have also argued[2] that $m_q^{q\bar{q}}$ may be taken to be proportional to $[\varphi_0+\chi(x)]$. Therefore we define $m_q(x) = m_q[\varphi_0+\chi(x)]/\varphi_0$ and write

$$\left[\mathbf{p}_{\text{op}}{}^2 + m_q{}^2\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right]^2\right]q(x) = -\frac{\partial^2}{\partial t^2}q(x)\ . \tag{5.8}$$

We may rewrite this equation as

$$\left[-\frac{\partial^2}{\partial t^2} + \nabla^2 - m_q{}^2\right]q(x) = \frac{2m_q{}^2}{\varphi_0}q(x)\chi(x) + \frac{m_q{}^2}{\varphi_0{}^2}q(x)\chi^2(x)\ . \tag{5.9}$$

Using techniques developed extensively in earlier works,[8–10] we may study the structure of mesons by taking matrix elements of this field equation between states of the meson, $|\,\mathbf{P})$ and (on-shell) quark states $|\,\mathbf{k},s)$. We find

$$\begin{aligned}[(P-k)^2 - m_q{}^2](\mathbf{k},s\,|\,q(0)\,|\,\mathbf{P}) = &\frac{2m_q{}^2}{\varphi_0}\int\frac{d\mathbf{P}'}{2\omega(\mathbf{P}')}(\mathbf{k},s\,|\,q(x)\,|\,\mathbf{P}')(\mathbf{P}'\,|\,\chi(0)\,|\,\mathbf{P})\\ &+\frac{m_q{}^2}{\varphi_0{}^2}\int\frac{d\mathbf{P}'}{2\omega(\mathbf{P}')}(\mathbf{k},s\,|\,q(0)\,|\,\mathbf{P}')(\mathbf{P}'\,|\,\chi^2(0)\,|\,\mathbf{P})\ .\end{aligned} \tag{5.10}$$

The analysis of equations such as Eq. (5.10) has been described in great detail elsewhere.[8-10] We had also suggested that rather than study the second-order quark field equation, it is convenient to add a mass term $[m_q+g_\chi\chi(x)]\bar{q}(x)q(x)$ directly to $\mathscr{L}'_M(x)$ of Eq. (4.23). Therefore, having exhibited the effect of the most coherent part of the gluon field, we can take as a starting point for the study of hadron structure the Lagrangian

$$\mathscr{L}_{\rm eff}(x)=\tfrac{1}{2}\partial^\mu\chi(x)\partial_\mu\chi(x)-V(\phi_0+\chi(x))+\bar{q}(x)[i\gamma^\mu\partial_\mu-m_q-g_\chi\chi(x)]q(x)\ , \tag{5.11}$$

with $q_\chi=m_q/\phi_0$. The field equations are

$$\partial^\mu\partial_\mu\chi(x)+\frac{\delta V}{\delta\chi(x)}=-g_\chi\bar{q}(x)q(x) \tag{5.12}$$

and

$$(i\gamma^\mu\partial_\mu-m_q)q(x)=g_\chi q(x)\chi(x)\ . \tag{5.13}$$

We have also shown that a Lagrangian such as that of Eq. (5.11) may be used to give a good account of the structure of the nucleon[8,9] and a large number of meson states:[10] ρ, ω, and several states of the charmonium and Υ systems. The effects of "single-gluon exchange" have also been considered; however, a gluon mass term was not taken into consideration in our earlier studies.[10]

We note that we have broken chiral symmetry in $\mathscr{L}_{\rm eff}(x)$ of Eq. (5.11). We can avoid using such an effective Lagrangian if we introduce order parameters associated with chiral-symmetry breaking (pion and σ fields). We can write

$$\begin{aligned}\mathscr{L}_{q,\rm ch}(x)=&-\tfrac{1}{4}\mathscr{G}^a_{\mu\nu}(x)\mathscr{G}^{\mu\nu}_a(x)+\frac{m_G{}^2}{2}\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right]^2\mathscr{A}^\mu_a(x)\mathscr{A}^a_\mu(x)+\tfrac{1}{2}\partial^\mu\chi(x)\partial_\mu\chi(x)-V(\varphi_0+\chi(x))\\&+\bar{q}(x)[i\not{D}-m^{\rm cur}-g_\pi(\sigma(x)+i\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}\gamma_5)]q(x)+\tfrac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x)\\&+\tfrac{1}{2}\partial_\mu\boldsymbol{\pi}(x)\cdot\partial^\mu\boldsymbol{\pi}(x)-\lambda[\sigma^2(x)+\boldsymbol{\pi}^2(x)-f_\pi{}^2(x)]^2\ .\end{aligned} \tag{5.14}$$

This effective Lagrangian exhibits chiral symmetry. We remark that the minimum of the potential term for the chiral fields is not unique, but is usually given by

$$\sigma^2(x)+\boldsymbol{\pi}^2(x)=f_\pi{}^2\ . \tag{5.15}$$

The left-hand side of Eq. (5.15) is a chiral invariant. Our modification of this equation is based upon the assumption that the chiral condensate and the gluon condensate are closely coupled. We write

$$\sigma^2(x)+\boldsymbol{\pi}^2(x)=f_\pi{}^2\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right]^2 \tag{5.16}$$

and define

$$f_\pi(x)=f_\pi\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right], \tag{5.17}$$

where in vacuum $f_\pi(x)=f_\pi=93$ MeV. This equation defines the quantity $f_\pi{}^2(x)$ which appears in Eq. (5.14).

Now, as usual, we can assume that while our Lagrangian exhibits chiral symmetry, this symmetry is broken in the ground state. Therefore one chooses a particular vacuum state in which $\sigma(x)$ has a finite value and for which the expectation value of $\boldsymbol{\pi}(x)$ is zero. The pion is then seen to be the Goldstone boson of this model. (We recall that in discussing the gluon condensate we did not choose a specific vacuum state since the gauge symmetry is unbroken.)

We note that in the presence of a hadron, which we can take to be a meson, we can write

$$(\mathbf{P}'\,|\,\sigma(0)\,|\,\mathbf{P})=f_\pi\delta(\mathbf{P}'-\mathbf{P})[2\omega(\mathbf{P}')]+\frac{f_\pi}{\varphi_0}(\mathbf{P}'\,|\,\chi(0)\,|\,\mathbf{P})\ . \tag{5.18}$$

The second-order quark field equation is now, with $m^{\rm cur}=0$,

$$[\mathbf{p}_{\rm op}{}^2+\tfrac{1}{6}g^2\,\mathbf{A}^i_a(x)\,\mathbf{A}^i_a(x)+g_\pi{}^2f_\pi{}^2(x)]q(x)=-\frac{\partial^2}{\partial t^2}q(x) \tag{5.19}$$

or

$$\left[\mathbf{p}_{\rm op}{}^2+[(m_q^G)^2+g_\pi{}^2f_\pi{}^2]\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right]^2\right]q(x)=-\frac{\partial^2}{\partial t^2}q(x)\ . \tag{5.20}$$

Contact with the previous analysis is made by identifying

$$m_q^{q\bar{q}}=g_\pi f_\pi\ . \tag{5.21}$$

We note that in the case $m^{\rm cur}\neq0$, Eq. (5.20) is to be replaced by

$$\left\{\mathbf{p}_{\rm op}{}^2+(m_q^G)^2\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right]^2+\left[m_q^{q\bar{q}}\left[\frac{\varphi_0+\chi(x)}{\varphi_0}\right]+m^{\rm cur}\right]^2\right\}q(x)=-\frac{\partial^2}{\partial t^2}q(x)\ . \tag{5.22}$$

We see that Eq. (5.20) is equivalent to Eq. (5.8). For $m^{\rm cur}\neq 0$ we can develop an equation similar in structure to Eq. (5.10) by starting from Eq. (5.22).

In Ref. 8 we also presented some conjectures concerning the dependence of the order parameter φ_0 on the temperature and quark chemical potential. That discussion led to the assignment of a temperature and mass dependence to the parameter m_q. Correspondingly we could discuss the density and temperature dependence of hadron size since the scale of hadron size is set by $m_q{}^{-1}$ in our theory of hadron structure, covariant soliton dynamics.[9] Our predictions of increased nucleon size in nuclei and the consequences for the interpretation of a broad range of experimental data have been reviewed elsewhere.[2,11]

VI. DISCUSSION

At this point it is useful to survey the various Lagrangians introduced in this work. The first effective Lagrangian is given by $\mathcal{L}_G$ of Eq. (3.17) and does not contain quark degrees of freedom. There we see a gluon mass term and another term which arises from the quartic coupling of the gluon fields. We remark that the mass is expressed in terms of a gauge-invariant quantity in Eq. (3.18) [see also Eqs. (4.23)–(4.26)].

The second Lagrangian is similar to the first and is given by Eq. (3.23). The method of derivation was different in this case and we also generalized our model to include a spatial variation of the order parameter $\varphi(x)$. This spatial variation is associated with the presence of quarks as may be seen from the developments of Sec. V.

The third Lagrangian is given in Eq. (4.8) and is written in analogy to the Ginzburg-Landau theory of superconductivity. We then demonstrate how the previous results may be obtained by averaging this Lagrangian over the gauge group. This analysis ultimately yields the Lagrangian of Eq. (4.22)

A new feature introduced in Sec. IV is the addition of a term quadratic in $\varphi(x)$. This term cannot be obtained by our elementary techniques since the original Lagrangian has no dimensional parameter, while the coefficient of the $\varphi^2(x)$ term has the dimension of (mass)^2. Such a term can be obtained in a dynamical calculation after a mass scale is introduced to regulate the theory. (This appearance of a mass scale when one goes beyond the "tree" approximation is usually called "dimensional transmutation."[12])

In Sec. V we extend our effective Lagrangian to include quark degrees of freedom [see Eq. (5.1)]. In Eq. (5.14) we further generalize the model to include order parameters associated with chiral-symmetry breaking. The Lagrangian $\mathcal{L}_{q,\rm ch}(x)$ of Eq. (5.14) is chirally symmetric if we put $m_q^{\rm cur}=0$. That Lagrangian represents a central result of this analysis. However, we have also discussed an extremely simple model which is defined by the Lagrangian $\mathcal{L}_{\rm eff}(x)$ of Eq. (5.11). This Lagrangian is not chirally symmetric, but represents a useful approximation to the more complete theory. If one is particularly concerned with understanding the role of chiral-symmetry breaking in a model of the type developed here, one should use $\mathcal{L}_{q,\rm ch}(x)$ of Eq. (5.14) as a starting point for further analysis.

In summary, we may note that we have used techniques of many-body theory to provide a description of the gluon condensate and have also indicated how quarks can be included in the model. The order parameter $\varphi_0+\chi(x)$ plays a central role in this model, and all masses, other than the current quark masses, are proportional to this quantity. The presence of quarks excites the field $\chi(x)$ and reduces the quantity $\varphi_0+\chi(x)$ inside a hadron. This mechanism provides a model for the formation of a hadron described as a nontopological soliton.[8–10]

We have stressed that our model represents QCD at low-momentum transfer. When quarks and gluons have large momentum, $-p^2>1-2$ GeV^2, the gluon and chiral condensates modify the propagators only to a minor degree.[7] Therefore, the high-momentum properties of the theory are largely unmodified by the presence of the condensates.

We have seen that the condensates provide dynamical masses for the quarks and gluons. While the mass generated by the quark condensate breaks chiral symmetry, the mass generated by the gluon condensate does not. Therefore the dynamical quark mass has two components. The simultaneous consideration of both components leads to a value for the dynamical quark mass for up and down quarks which is quite close to the phenomenological value we have determined in previous studies ($m_q\sim 500$–600 MeV) (Refs. 8–10). It can also be shown that the parameters which specify $V(\varphi_0+\chi(x))$ may also be obtained using the techniques introduced in this work.[2]

APPENDIX

We have stressed that the coherence (or correlation) length is an important quantity for the study of QCD. One may argue that this length may be used to characterize the domains where perturbative and nonperturbative techniques are useful.

We note that the correlation length of the vacuum condensate has been measured in a Monte Carlo simulation of SU(2) gauge theory by Campostrini, DiGiacomo, and Mussardo.[13] They consider

$$G_2(x)=\left\langle 0\left|\frac{\alpha_s}{\pi}:G_{\mu\nu}(x)\exp\left[iT^a\int_0^1 dt\, A_\mu^a(xt)x^\mu\right]\right.\right.$$
$$\left.\left.\times G^{\mu\nu}(0):\right|0\right\rangle, \tag{A1}$$

where T^a is a matrix in the adjoint representation of SU(2). Further, a lattice operator $O(r)$ is constructed which is proportional to $G_2(ra)$ in the limit that $a\to 0$. A perturbative contribution is also subtracted from the vacuum expectation value. The function

$$\varphi(ra)=\frac{\langle :O(r):\rangle}{\langle :O(0):\rangle}, \tag{A2}$$

defined in terms of suitable lattice averages, is calculated with the result that

$$\varphi(x)\sim e^{-x/\lambda}. \tag{A3}$$

It is found that

$$\sigma\lambda^2 = 0.064 \pm 0.003 \quad (A4)$$

in terms of the string tension σ. The scale is fixed by assuming $\sigma = (2\pi)^{-1}$ GeV, which gives $\lambda = 0.12$ fm. A value for the correlation length of this characteristic size has been suggested in our work on the basis of physical arguments.

The correlation length in a field theory is an inverse mass. Therefore we may consider either the glueball mass or the gluon mass as defining a correlation length. The glueball mass is calculated to be about 1 GeV in Monte Carlo studies. The gluon mass for the SU(2) theory is discussed above ($1/\lambda \simeq 1644$ MeV). The gluon mass for the SU(3) gauge theory is calculated to be about 750 MeV with a significant theoretical uncertainty.[14] That value is close to the value of 649 MeV we had calculated in an earlier work.[2] We have also calculated the Higgs-boson mass for our effective Lagrangian and obtained a value of 918 MeV (Ref. 2). If we identify that quantity with the glueball mass, we find we are in general agreement with results of lattice gauge studies.

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