Production of heavy quarks from W-gluon fusion

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We show that heavy-quark production via W-gluon fusion in high-energy pp and $\bar{p}p$ collisions is an important source of the heavier member of an $SU(2)_L$ doublet of quarks if the mass splitting within the doublet is large. W-gluon fusion exceeds the strong production of heavy quarks for mass splittings greater than 300—350 GeV at $\sqrt{s}=10$ TeV and 400—450 GeV at $\sqrt{s}=40$ TeV. An alternative way to regard W-gluon fusion is as the production of the heavier quark by fusing its light partner with a W boson. We use a distribution function for the light partner to show that this process gives results which agree qualitatively with W-gluon fusion. We also discuss the Drell-Yan production of an $SU(2)_L$ doublet of heavy quarks via a virtual W boson and corrections to this process from initial gluons. We find that at the Fermilab Tevatron energy $\sqrt{s}=2$ TeV, W-gluon fusion exceeds the Drell-Yan production of top quarks for masses above 100 GeV.

I. INTRODUCTION

The standard model[1] has taught us that the weak interaction is weak not because the coupling constant is small, but rather because the W-boson mass is large. Historically, the energies involved in our study of the weak interaction have been much smaller than the W-boson mass. Only with the advent of the production of W and Z bosons at the CERN $\bar{p}p$ collider[2] have we been able to study the weak interaction as a force of unsuppressed electromagnetic strength.

We hope that someday pp or $\bar{p}p$ colliders will be able to study the weak interaction at an energy considerably greater than the W-boson mass. If they do, we will be rewarded with a nice surprise: some weak processes are actually *enhanced* at very high energies. Cross sections for strong processes typically scale like $1/\hat{s}$, where $\hat{s}$ is the square of the center-of-mass energy of the parton-parton collision. Some weak processes, on the other hand, scale like $1/M_W{}^2$. Therefore, at sufficiently large energies, weak processes may actually be stronger than strong processes. Examples of enhanced weak processes in high-energy pp and $\bar{p}p$ collisions are Higgs-boson production from W-boson fusion[3] and heavy-lepton production from gluon fusion via an intermediate Z or Higgs boson.[4]

In this paper we will study the production of heavy quarks in high-energy pp collisions from weak processes. By a heavy quark we have in mind the top quark as well as any quarks belonging to further generations. We will show that the most important weak contribution to heavy-quark production is from W-gluon fusion, shown in Fig. 1. This is a striking example of an enhanced weak interaction since, unlike the production of Higgs bosons and heavy leptons mentioned above, heavy-quark production may proceed entirely via the strong interaction.

We also study a related weak process for heavy-quark production which proceeds by using a W boson to transform a heavy sea quark into its $SU(2)_L$ partner. This process is an alternative way to calculate W-gluon fusion, and we will show that it gives qualitatively similar results.

Heavy-quark production via gluon-quark fusion with a virtual W boson emitted from the quark is similar to W-gluon fusion in that it has the same initial and final states. We will show, however, that it is more proper to consider this process to be an order-α_s correction to the Drell-Yan process and to the quark distribution function. The Drell-Yan process is small since it involves the s-channel exchange of a W boson and therefore scales like $1/\hat{s}$.

Weak production of heavy quarks has the distinction that the final state contains both of the quarks in an $SU(2)_L$ doublet. This may offer certain advantages for heavy-quark detection. Thus, weak production of heavy quarks may be important even if it is smaller than the usual strong production.

The remainder of the paper is organized as follows. In Sec. II we calculate the cross section for heavy-quark production via W-gluon fusion and compare it with heavy-quark production via strong processes. In Sec. III we calculate heavy-quark production from the fusion of a W boson with a heavy sea quark. In Sec. IV we calculate the production of heavy quarks from the Drell-Yan process and the correction to this process from gluon-quark fusion. Section V is devoted to a discussion of our results at the Fermilab Tevatron energy $\sqrt{s}=2$ TeV. Finally, in Sec. VI we summarize our results for heavy-quark pro-

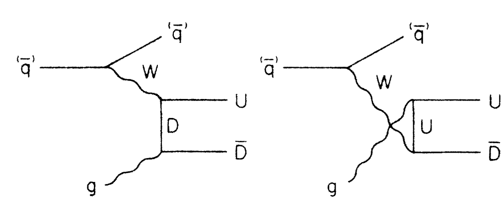

FIG. 1. Heavy-quark production via W-gluon fusion. We have denoted the $T_{3L}=+\frac{1}{2}$ quark by U and the $T_{3L}=-\frac{1}{2}$ quark by D. The W boson is positively charged; the same process with a negatively charged W boson results in a $\bar{U}D$ final state.

duction and we discuss some possibilities for heavy-quark detection.

II. W-GLUON FUSION

The production of heavy quarks in pp and $\bar{p}p$ collisions may proceed entirely via the strong interaction. The two strong processes which contribute to heavy-quark production are shown in Fig. 2. We have denoted the heavy quark by the generic label Q. Strong processes produce a heavy quark-antiquark pair since the strong interaction conserves flavor. Although there have been several other mechanisms proposed for the strong production of heavy quarks, a recent study concluded that the processes above give the dominant contribution at lowest order.[5] However, there are indications that the process $gg \to Q\bar{Q}g$ could give a large correction to the lower-order process $gg \to Q\bar{Q}$ (Ref. 6). We should therefore regard the fusion processes in Fig. 2 as giving a lower bound to the production of heavy quarks from the strong interaction.

As discussed in the Introduction, weak processes which scale like $1/M_W{}^2$ may be important at high energies. This scaling behavior may be achieved by exchanging a spacelike weak boson. We will consider the exchange of a W rather than a Z boson, for two reasons. First, W-boson exchange will result in the production of an $SU(2)_L$ pair of quarks, which is favorable for the production of the heavier member of the doublet if its partner is much lighter. Second, the vector coupling of the Z boson to quarks is much smaller than that of the W boson, and therefore Z-boson processes are usually weaker.

The production of heavy quarks from exchange of a spacelike W boson is shown in Fig. 1. We will refer to this process as W-gluon fusion. We have denoted the heavy $T_{3L} = +\frac{1}{2}$ quark by U and the heavy $T_{3L} = -\frac{1}{2}$ quark by D. The W boson is positively charged, and thus must be emitted from a positively charged quark or antiquark. One could also consider the emission of a negatively charged W boson from negatively charged quarks and antiquarks, which would result in a $\bar{U}D$ final state.

In Figs. 3 and 4 we show the total cross section for heavy-quark production in pp collisions via W-gluon fusion and via the strong interaction at center-of-mass energies of 10 and 40 TeV, respectively. We are concentrating on the production of the heavier member of an $SU(2)_L$ doublet, which we presume to be the U quark; our results are independent of this assumption, however. The cross sections are graphed as a function of the U-quark mass,

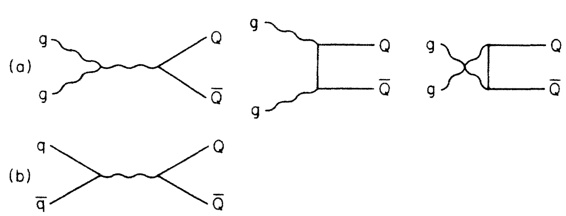

FIG. 2. Strong processes which contribute to heavy-quark production in pp and $\bar{p}p$ collisions: (a) gluon fusion, (b) quark-antiquark annihilation.

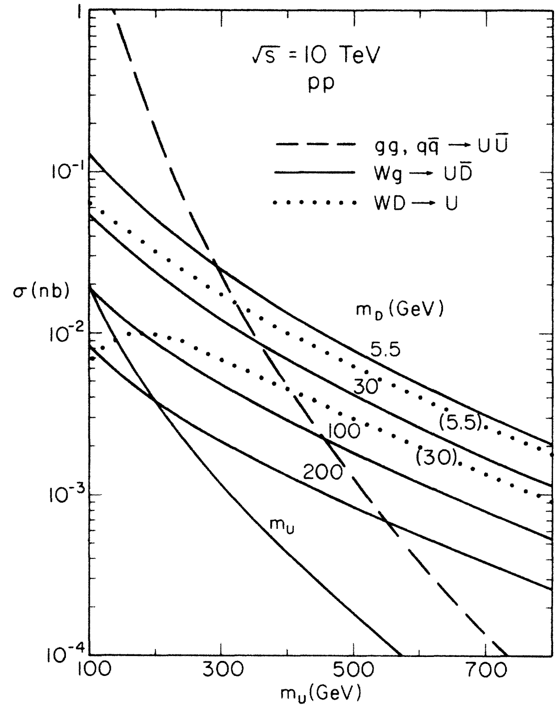

FIG. 3. Total cross sections at $\sqrt{s} = 10$ TeV for various processes which produce heavy quarks in pp collisions. U and D denote a heavy $SU(2)_L$ doublet of quarks. In the case of $m_D = 5.5$ GeV, U denotes the top quark; otherwise, U and D refer to fourth-generation quarks. The $Wg \to U\bar{D}$ and $WD \to U$ lines are labeled by the mass of the D quark.

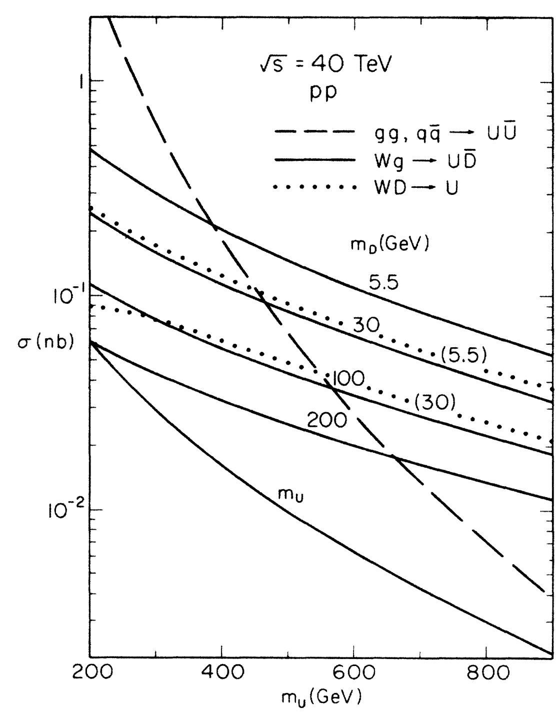

FIG. 4. Same as Fig. 3 except at $\sqrt{s} = 40$ TeV.

with the W-gluon fusion lines labeled by the mass of the D quark. The dotted lines, corresponding to $WD \to U$, will be discussed in the next section.

As the graphs show, the production of U-type quarks from W-gluon fusion actually surpasses that from the strong interaction for sufficiently heavy U quarks. The essential parameter is the mass difference $m_U - m_D$. We see that at $\sqrt{s} = 10$ TeV the W-gluon fusion process is larger if the mass difference is 300–350 GeV. At $\sqrt{s} = 40$ TeV somewhat larger mass differences are required, around 400–450 GeV. One should keep in mind that the W-gluon fusion cross sections will be almost doubled if we include the $\bar{U}D$ final state as well.

At first sight these results are somewhat surprising. Although W-gluon fusion is lower order in the strong interaction than the pure strong-interaction processes, it is second order in the weak interaction. W-gluon fusion also produces a three-body final state, and is therefore suppressed by a phase-space factor of about $(2\pi)^{-3} \times \pi/4$. We therefore expect W-gluon fusion to be suppressed relative to the strong processes by a factor[7] $\alpha^2/4\pi\alpha_s \sim 10^{-4}$.

There are several enhancements to W-gluon fusion, however, which overcome this suppression factor for sufficiently large mass differences $m_U - m_D$. Let us discuss each of these enhancements separately.

(i) As we have already mentioned, the W-gluon fusion cross section scales like $1/M_W{}^2$, while the strong processes scale like $1/\hat{s}$. The scaling of the W-gluon fusion process derives from the presence of a spacelike W-boson propagator: $(Q^2 + M_W{}^2)^{-1}$. The cross section is dominated by the region of phase space $Q^2 \ll M_W{}^2$, which corresponds to W bosons emitted in the direction of the incoming quark parton.

(ii) The luminosity of gluon-quark plus gluon-antiquark collisions is about the same as that of gluon-gluon collisions, and both are much greater than the luminosity of quark-antiquark collisions. This is demonstrated in Fig. 5, where we have graphed the quantity

$$\frac{1}{s}\frac{d\mathscr{L}}{d\tau} = \frac{1}{s}\int_\tau^1 \frac{dx}{x} \sum_{i,j} F_i(x) F_j(\tau/x)$$

as a function of the center-of-mass energy of the parton-parton collision. $F_i(x)$ is the distribution function of the ith parton; we have used set 2 ($\Lambda = 290$ MeV) of the distribution functions of Eichten, Hinchliffe, Lane, and Quigg[8] (EHLQ). The quark-antiquark luminosity is the sum over all flavor-singlet combinations $(u\bar{u} + d\bar{d} + \cdots)$ while the gluon-quark plus gluon-antiquark luminosity is the sum over all positively charged quarks and antiquarks. Because of its small luminosity, the quark-antiquark fusion process makes a negligible contribution to the strong production of quarks in comparison with gluon fusion. The W-gluon fusion process, which is initiated by gluon-quark and gluon-antiquark collisions, is not enhanced relative to the gluon fusion process, but it is important to note that it is not suppressed, as one might have guessed from the small quark-antiquark luminosity.

(iii) The strong-interaction gluon-fusion process is color suppressed. The suppression factor is $\frac{1}{3}$, which means that only one in three of all possible gluon combinations carries the correct quantum numbers to produce a quark-antiquark pair. The W-gluon fusion process is color unsuppressed, so we will consider this to be an enhancement to this process.

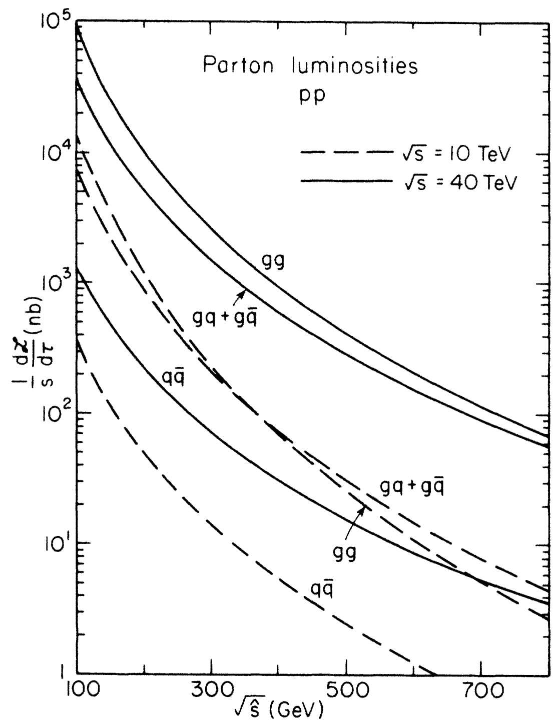

FIG. 5. Parton luminosities for pp collisions at $\sqrt{s} = 10$ TeV (dashed lines) and $\sqrt{s} = 40$ TeV (solid lines) as a function of the center-of-mass energy of the parton-parton collision. The luminosities are shown for gluon-gluon (gg), gluon-quark plus gluon-antiquark ($gq + g\bar{q}$), and quark-antiquark ($q\bar{q}$) collisions.

(iv) W-gluon fusion produces only one heavy particle in the final state (assuming $m_U - m_D$ is large), while the strong-interaction processes produce two. The result is that the typical value of x at which the distribution functions are evaluated in the W-gluon fusion process is half that of the strong processes. Since the distribution functions go roughly like $1/x$ at small x, and since there are two distribution functions, this gives an enhancement of a factor of 4.

(v) The W-gluon fusion process has a logarithmic enhancement of $\ln(\hat{s}/m_D{}^2)$ associated with the collinear singularity which would be present for a massless D quark. Since $\hat{s} \geq (m_U + m_D)^2$, this factor is typically $\ln(m_U{}^2/m_D{}^2)$. If this factor is too large, perturbation theory breaks down and one must use a D quark distribution function. This will be discussed in Sec. III.

We calculated the W-gluon fusion cross section from the diagrams in Fig. 1 by integrating over the $U\bar{D}$ phase space analytically and performing the remaining quark (antiquark) phase-space integral as well as the integrals over the parton-distribution functions numerically. The cross section differs for quarks and antiquarks in the initial state. We assumed that the Kobayashi-Maskawa matrix element associated with the UD vertex is unity. We used set 2 ($\Lambda = 290$ MeV) of the distribution functions of

EHLQ (Ref. 8) and evaluated them at $Q^2=M_W{}^2$ for the quarks and antiquarks and at $Q^2=(m_U+m_D)^2$ for the gluons. The gluon-fusion and quark-antiquark annihilation cross sections may be found in Refs. 8 and 9; we used the distribution functions above, evaluated at $Q^2=4m_U{}^2$.

In the W-gluon cross section for $m_D=5.5$ GeV, we mean U to be the top quark and D to be the bottom quark. Otherwise, we mean U and D to belong to a fourth generation. If there is appreciable mixing among the third and fourth generations, the case $m_D=5.5$ GeV could refer to the production of a $U\bar{b}$ pair; however, one must then multiply the cross section by the square of the corresponding Kobayashi-Maskawa matrix element. Similarly, one may also consider the production of a $t\bar{D}$ pair.

Strictly speaking, our calculation is reliable only for $m_U \lesssim 700$ GeV, which is the unitarity bound on the mass of a quark with a relatively light $SU(2)_L$ partner.[10] Quarks with masses exceeding this bound are strongly coupled and do not permit a perturbative analysis in the weak sector. Nevertheless, we expect large cross sections for quarks whose masses exceed the unitarity bound.

We should also note that there is a phenomenological bound on the mass splitting in an $SU(2)_L$ doublet of quarks in the context of the minimal standard model.[11] This bound, which arises from radiative corrections to the W- and Z-boson masses from heavy fermion loops, is[8]

$$|m_U{}^2-m_D{}^2|^{1/2} \lesssim 350 \text{ GeV} .$$

It is important to keep in mind that this bound may be relaxed or eliminated altogether in extensions of the standard model.

One may perform an approximate calculation of the W-gluon fusion process using the effective-W approximation.[12,13] This has been done[12] for the case $m_D=m_U$, which corresponds to the solid line labeled m_U in Figs. 3 and 4. The effective W approximation appears to underestimate the cross section; however, we hesitate to compare the calculations because the effective W calculation included only the t-channel diagram, corresponding to the first diagram in Fig. 1. A different set of distribution functions was used as well [set 1 ($\Lambda=200$ MeV) instead of set 2 ($\Lambda=290$ MeV) of EHLQ (Ref. 8)].

III. $WD \to U$

Inspection of the diagrams in Fig. 1 for W-gluon fusion leads us to a different interpretation of the process. In the first diagram we see a gluon split into a $D\bar{D}$ pair with the D quark subsequently converted to a U quark by fusing with a W boson. Thus we may interpret this process as U-quark production from a W boson fusing with a D quark which is in the proton sea, with the sea generated by gluons splitting into $D\bar{D}$ pairs. This process is shown in Fig. 6.

Distribution functions for heavy quarks may be generated by a modified Altarelli-Parisi equation. Here we will use the heavy-quark distribution functions given in set 2 ($\Lambda=290$ MeV) of EHLQ (Ref. 8) for quarks of mass 5.5 and 30 GeV.

The cross section for the process in Fig. 6 is

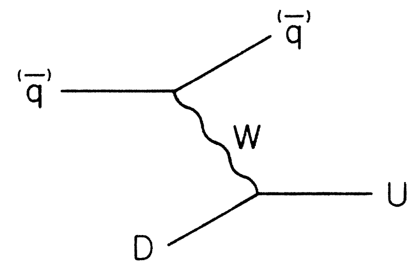

FIG. 6. Heavy-quark production by fusing a virtual W boson with a heavy D quark in the proton sea to form a heavy U quark. The W boson is positively charged; the same process with a negatively charged W boson converts a $\bar{D}$ into a $\bar{U}$.

$$\frac{d\hat{\sigma}}{d\hat{t}}=\frac{\pi}{4}\frac{\alpha^2}{\sin^4\theta_W}\frac{\hat{s}-m_U{}^2}{(\hat{s}-m_D{}^2)(\hat{t}-M_W{}^2)^2} \tag{3.1}$$

for a quark-quark or antiquark-antiquark collision, and

$$\frac{d\hat{\sigma}}{d\hat{t}}=\frac{\pi}{4}\frac{\alpha^2}{\sin^4\theta_W}\frac{\hat{u}-m_D{}^2}{(\hat{s}-m_D{}^2)^2}\frac{\hat{u}-m_U{}^2}{(\hat{t}-M_W{}^2)^2} \tag{3.2}$$

for a quark-antiquark collision, where

$$\hat{t}=-\tfrac{1}{2}\hat{s}\left[1-\frac{m_D{}^2}{\hat{s}}\right]\left[1-\frac{m_U{}^2}{\hat{s}}\right](1-z)$$

and $\hat{s}+\hat{t}+\hat{u}=m_U{}^2+m_D{}^2$. As usual, $\hat{s}$ is the square of the energy and z is the cosine of the scattering angle in the parton-parton center-of-mass frame.

The dotted lines in Figs. 3 and 4 show the result of our calculation of $WD\to U$ for pp collisions at $\sqrt{s}=10$ and 40 TeV and for D masses of 5.5 and 30 GeV. We have evaluated the light-quark-parton distribution function at $Q^2=M_W{}^2$ and the heavy-sea-quark distribution function at $Q^2=m_U{}^2$. The suppression of the cross section at small values of m_U, which is particularly evident for $m_D=30$ GeV, is due to the vanishing of the sea-quark distribution function at $Q^2=4m_D{}^2$.

Our results show that the $WD\to U$ process using the D quark distribution function somewhat underestimates the W-gluon fusion production of heavy quarks. This is perhaps related to the prescription used to include mass effects in the Altarelli-Parisi evolution of the heavy-quark sea. Nevertheless, the qualitative agreement between the two approaches is a satisfying consistency check on the calculation.

A similar comparison between two related processes was recently performed by Barger, Jacobs, Woodside, and Hagiwara.[14] They calculated scalar-quark ($\tilde{q}$) production via $gq\to\tilde{q}\tilde{g}$ and via $\tilde{g}q\to\tilde{q}$, the latter process employing a gluino ($\tilde{g}$) distribution function. Their result was the opposite of ours; they found that using a gluino distribution function *overestimated* the production of scalar quarks. However, they did not use the same prescription for generating the sea distribution function as was used in Ref. 8.

There is one regime where the W-gluon fusion calculation is suspect and the $WD\to U$ process gives a better estimate of the cross section. This is when the ratio m_U/m_D

is very large. As we pointed out in remark (v) of Sec. II, the W-gluon fusion process has an enhancement factor of $\ln(m_U{}^2/m_D{}^2)$, and if this factor becomes too large we cannot trust our perturbative calculation. It is difficult to say how large is too large; a reasonable guess is that $\alpha_s/\pi \times \ln(m_U{}^2/m_D{}^2)$ must be somewhat less than unity.[15] This criterion is satisfied by the masses we used in our calculations. If the mass ratio m_U/m_D is too large, one would have to use a D-quark distribution function and the process $WD \to U$ to calculate U-quark production. In that case the W-gluon fusion diagram would be an order-α_s correction to the $WD \to U$ process and the D-quark distribution function, with the large logarithm being absorbed in the latter.[16]

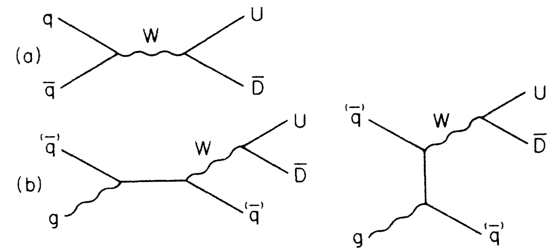

FIG. 7. Drell-Yan production of heavy quarks from a W boson: (a) lowest-order process, and (b) corrections to the lowest-order process from gluon-quark fusion.

IV. GLUON-QUARK FUSION

The W-gluon fusion process which we discussed in Sec. II is not the only weak process which produces the desired $U\bar{D}$ final state. In fact, heavy-quark production via the Drell-Yan process,[17] shown in Fig. 7(a), is lower order in the strong interaction than W-gluon fusion. It is well known that this process is important if the W boson is real; i.e., the total mass of the $U\bar{D}$ pair is less than the W mass. Here we are interested in a much heavier $U\bar{D}$ system, however, in which case the Drell-Yan contribution is small because it scales like $1/\hat{s}$ at high energies and it is initiated by a quark-antiquark annihilation, which has a small luminosity.

This leaves us to deal with the diagrams in Fig. 7(b), which are initiated by gq and $g\bar{q}$ collisions. These diagrams have the same initial and final states as the W-gluon fusion diagrams in Fig. 1, so one might think that they must be included for the calculation to be gauge invariant. As we shall see, this is not the case; in fact, these diagrams are more properly included as corrections to the Drell-Yan process and the quark distribution functions.

Since the $U\bar{D}$ pair in the W-gluon fusion process comes from a gluon, it is in a color-octet state. The $U\bar{D}$ pair in the process shown in Fig. 7(b), which we shall refer to as gluon-quark fusion, forms a color singlet. Hence there is no interference between the two processes, and they must be separately gauge invariant, as one may check explicitly.

Although it shares the same initial and final states as W-gluon fusion, gluon-quark fusion is kinematically rather different. It does not possess the enhancements (i) and (v) which we listed in Sec. II for W-gluon fusion. Therefore we do not expect it to be nearly as important as W-gluon fusion.

This still leaves us with the question of how to deal with the gluon-quark process. In particular, the second diagram in Fig. 7(b) possesses a u-channel singularity which must be handled carefully. Politzer has shown how to absorb this singularity into the quark and antiquark distribution functions by calculating initial-gluon corrections to deep-inelastic scattering.[18] This technique has been used by several authors to calculate order-α_s corrections to lepton pair[19,20] and W,Z boson[21] production. Here we follow the calculation of Ref. 19, Sec. II.

The gluon-quark cross section for a $U\bar{D}$ pair of invariant mass Q^2 is

$$\frac{d\hat{\sigma}}{dQ^2}=\frac{\alpha_s\alpha^2}{96\sin^2\theta_W}\frac{1}{(Q^2-M_W{}^2)^2}\frac{1}{\hat{s}}\lambda^{1/2}(Q^2,m_U{}^2,m_D{}^2)\left[1-\frac{1}{2}\frac{m_U{}^2+m_D{}^2}{Q^2}-\frac{1}{2}\frac{(m_U{}^2-m_D{}^2)^2}{Q^4}\right]$$
$$\times\left[2[(1-\hat{\tau})^2+\hat{\tau}^2]\ln\left[\frac{Q^2}{m^2}\frac{(1-\hat{\tau})^2}{\hat{\tau}}\right]+(1+7\hat{\tau})(1-\hat{\tau})\right], \quad (4.1)$$

where $\hat{\tau}=Q^2/\hat{s}$, m is the quark-parton mass, and $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$. The quark-parton mass regulates the u-channel singularity; all terms which vanish for $m^2=0$ have been dropped.

The cross section for the Drell-Yan process in Fig. 7(a) is[8]

$$\frac{d\hat{\sigma}}{dQ^2}=\frac{\pi}{12}\frac{\alpha^2}{\sin^4\theta_W}\frac{1}{(Q^2-M_W{}^2)^2}\lambda^{1/2}(Q^2,m_U{}^2,m_D{}^2)$$
$$\times\left[1-\frac{1}{2}\frac{m_U{}^2+m_D{}^2}{Q^2}-\frac{1}{2}\frac{(m_U{}^2-m_D{}^2)^2}{Q^4}\right]\delta(Q^2-\hat{s}) . \quad (4.2)$$

The correction to the quark and antiquark distribution functions from gluons splitting into quark-antiquark pairs in deep-inelastic scattering is

$$q(x,Q^2)=q_0(x)+\frac{\alpha_s}{4\pi}\int_x^1\frac{dy}{y}g_0(y)$$
$$\times\left[[(1-z)^2+z^2]\ln\left[\frac{Q^2}{m^2}\frac{1-z}{z}\right]-1+8z-8z^2\right] \quad (4.3)$$

$(z=x/y)$, which we have borrowed form Ref. 19, Eq. (2.16). Note the singularity at $m^2=0$.

The hadronic cross section for a $U\bar{D}$ pair of invariant mass Q^2 in the parton model is calculated by folding the Drell-Yan process (4.2) with the quark and antiquark distribution functions $q_0(x)$ and $\bar{q}_0(x)$, using (4.3) to replace these with $q(x,Q^2)$ and $\bar{q}(x,Q^2)$ and the correction term from gluons, and then adding all of this to the gluon-quark cross section (4.1) folded with the quark [antiquark] and gluon distribution functions $q_0(x)$ [$q_0(x)$] and $g_0(x)$. The mass singularities cancel, leaving

$$\frac{d\sigma}{dQ^2}=\frac{d\sigma^{(\mathrm{DY})}}{dQ^2}+\frac{d\sigma^{(i)}}{dQ^2}\,, \tag{4.4}$$

where $d\sigma^{(\mathrm{DY})}/dQ^2$ is the Drell-Yan cross section (4.2) folded with the distribution functions $q(x,Q^2)$ and $\bar{q}(x,Q^2)$, and

$$\frac{d\sigma^{(i)}}{dQ^2}=\sum_q \frac{\alpha_s\alpha^2}{48\sin^4\theta_W}\frac{1}{(Q^2-M_W{}^2)^2}\frac{1}{Q^2}\lambda^{1/2}(Q^2,m_U{}^2,m_D{}^2)\left[1-\frac{1}{2}\frac{m_U{}^2+m_D{}^2}{Q^2}-\frac{1}{2}\frac{(m_U{}^2-m_D{}^2)^2}{Q^4}\right]$$
$$\times\int_0^1 dx_1\int_{\tau/x_1}^1 dx_2\{g(x_1,Q^2)[q(x_2,Q^2)+\bar{q}(x_2,Q^2)]+(x_1\longleftrightarrow x_2)\}$$
$$\times\hat{\tau}\{[(1-\hat{\tau})^2+\hat{\tau}^2]\ln(1-\hat{\tau})+\tfrac{3}{2}-5\hat{\tau}+\tfrac{9}{2}\hat{\tau}^2\}\,. \tag{4.5}$$

This is the order-α_s correction to the Drell-Yan process from initial gluons.

We evaluated the Drell-Yan process $\sigma^{(\mathrm{DY})}$ and the correction from initial gluons $\sigma^{(i)}$ by using set 2 ($\Lambda=290$ MeV) of the distribution function of EHLQ (Ref. 8). The cross section $\sigma^{(\mathrm{DY})}$ is negligibly small for heavy $U\bar{D}$ pairs, as anticipated. The correction from initial gluons, $\sigma^{(i)}$, is smaller still; it is typically about 10% as large as $\sigma^{(\mathrm{DY})}$, and negative. Thus our explicit calculation bears out our intuition that gluon-quark fusion is a correction to the Drell-Yan production of $U\bar{D}$ pairs and to the quark distribution functions, and is distinct from W-gluon fusion.

V. *W*-GLUON FUSION AT THE FERMILAB TEVATRON

Although it is somewhat outside the main theme of this paper, our analysis would not be complete without some discussion of W-gluon fusion at present collider energies. We will focus on the Tevatron energy $\sqrt{s}=2$ TeV since this is the highest-energy hadron collider that will be available for some time.

W-gluon fusion is not nearly as important at $\sqrt{s}=2$ TeV as it is at the higher energies $\sqrt{s}=10$ and 40 TeV. The subprocess energies involved in 2-TeV collisions are typically not much greater than the W-boson mass, so one does not gain from the scaling behavior of the subprocess cross section, which we discussed in the Introduction. Furthermore, the competing processes for heavy-quark production are more important at this lower energy. This is especially true at a $\bar{p}p$ collider, such as the Tevatron, since the quark-antiquark luminosity is no longer negligible.

The largest competing processes are once again strong interactions, as shown in Fig. 2. However, we cannot ignore quark-antiquark annihilation as we did previously; in fact, it is as large as gluon fusion at $m_U=120$ GeV, and larger for heavier quarks. As a result, the strong production of heavy quarks does not die off at large quark masses as rapidly as it does at $\sqrt{s}=10$ and 40 TeV, where gluon fusion dominates.

The Drell-Yan mechanism, Fig. 7(a), is also more important at the lower Tevatron energy because of the increased quark-antiquark luminosity and the lower subprocess energies involved. The latter is especially important if the intermediate W boson is real, producing a large resonant cross section.

Figure 8 shows the results of our numerical calculations. The calculational details are as discussed in Sec. II for $\sqrt{s}=10$ and 40 TeV. The dot-dashed lines, corresponding to the Drell-Yan process,[22] are labeled by the D-quark mass, with arrows pointing to the corresponding W-gluon fusion curves. One should keep in mind that

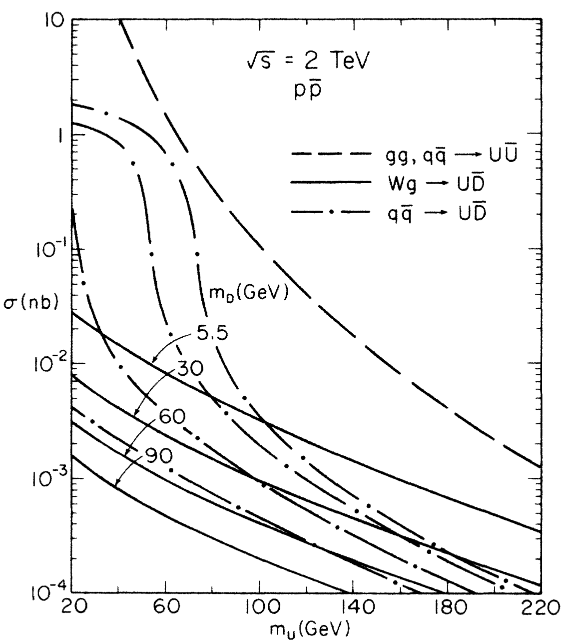

FIG. 8. Total cross sections at $\sqrt{s}=2$ TeV for various processes which produce heavy quarks in $\bar{p}p$ collisions. U and D denote a heavy $SU(2)_L$ doublet of quarks. In the case $m_D=5.5$ GeV, U denotes the top quark; otherwise, U and D refer to fourth-generation quarks. The $Wg\to U\bar{D}$ and $q\bar{q}\to U\bar{D}$ lines are labeled by the mass of the D quark.

both of these sets of curves are doubled if we include both $U\bar{D}$ and $\bar{U}D$ final states.

Figure 8 demonstrates our remarks regarding W-gluon fusion at the relatively low energy of the Tevatron. It does not surpass the strong processes until very large U-quark masses, which are inaccessible due to the limited luminosity ($\mathscr{L}=10^{30}$ cm^{-2}s^{-1}) of the machine. For D-quark masses above 30 GeV, corresponding to a fourth generation, W-gluon fusion is at best a correction to the Drell-Yan process. The one place where W-gluon fusion could be nontrivial is for top-quark production, corresponding to $m_D=5.5$ GeV. W-gluon fusion surpasses the Drell-Yan process at a top-quark mass of about 100 GeV. Even in this case it is questionable whether the cross section is large enough to be useful.

VI. CONCLUSIONS

We have shown that heavy-quark production via W-gluon fusion is an important source of U quarks ($T_{3L}=+\frac{1}{2}$) at high-energy pp and $\bar{p}p$ colliders if the mass splitting between the U and the lighter D quark ($T_{3L}=-\frac{1}{2}$) is large. In particular, W-gluon fusion exceeds the strong production of U quarks for mass splittings greater than 300–350 GeV at $\sqrt{s}=10$ TeV and 400–450 GeV at $\sqrt{s}=40$ TeV, depending on the D-quark mass. The W-gluon fusion cross section is even larger, by about a factor of 2, if we include $\bar{U}$ production as well.

Although W-gluon fusion does not exceed the strong production of heavy quarks at the Tevatron ($\sqrt{s}=2$ TeV), it is comparable to the Drell-Yan process if the sum of the quark masses exceeds the W-boson mass. The production of top quarks via W-gluon fusion exceeds production via the Drell-Yan mechanism for top quarks heavier than 100 GeV.

Even if W-gluon fusion does not exceed the strong production of heavy U quarks, it may be useful since it results in a different final state. Strong-interaction processes yield a $U\bar{U}$ final state, while W-gluon fusion yields a $U\bar{D}$ pair. However, the $\bar{D}$ quark is usually produced at small angles because of the t-channel pole present in the first diagram in Fig. 1. Hence W-gluon fusion yields a single U quark, which may be easier to reconstruct than a $U\bar{U}$ pair. The U quarks will decay via $U\to DW$, with the W virtual if $m_U-m_D<M_W$ and real if $m_U-m_D\geq M_W$. There may also be an appreciable amount of $U\to bW$ decays if the U and D quarks are nearly degenerate in mass.[23]

Note added in proof. A recent paper by J. C. Collins and W.-K. Tung (Report No. FERMI-PUB-86/39-T) suggests that the heavy-sea-quark distribution functions of EHLQ (Ref. 8) are small by roughly a factor of 2. This could account for the discrepancy between our calculation of heavy-quark production from W-gluon fusion and from W-boson fusion with a heavy sea quark, as shown by the solid and dotted curves in Figs. 3 and 4.

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