Radiative corrections to the ratio of Z- and W-boson production

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We calculate the order-α_s^2 corrections to Z-boson production in $p\bar{p}$ collisions for which there are no corresponding corrections to W-boson production. These corrections affect the ratio of Z- and W-boson production, a quantity which is useful for counting the number of species of neutrinos. We find that these corrections are much smaller than the uncertainty in the ratio stemming from the quark and gluon distribution functions and the value of $\Lambda_{\rm QCD}$.

I. INTRODUCTION

One of the great puzzles confronting elementary-particle physics is the apparent replication of fundamental fermions in nature. The fermions observed thus far may be grouped into three generations of quarks and leptons. Each generation contains a neutral lepton, the neutrino, which is massless or very light. It is reasonable to expect that the neutrinos associated with any further generations will also be massless, or nearly so. If we can count the number of neutrino species in nature, it will reveal the number of generations, a quantity of the most profound significance.

Recently, several authors have suggested an ingenious method to count the number of neutrino species using the CERN $p\bar{p}$ collider.[1-3] The method is based on the fact that every species of neutrino contributes to the total Z width via $Z\to\nu\bar{\nu}$. An accurate measurement of the Z width therefore provides us with a reliable way to count the number of neutrino species.

In order to understand the method proposed to obtain the Z width, let us first consider another, more straightforward, method. The Z width may be deduced from the measurement of the cross section for inclusive Z production followed by leptonic decay $\sigma(p\bar{p}\to Z\to l^+l^-)$. The total width is then given by

$$\Gamma_Z=\frac{\sigma(p\bar{p}\to Z)}{\sigma(p\bar{p}\to Z\to l^+l^-)}\Gamma(Z\to l^+l^-)\ , \tag{1}$$

where $\Gamma(Z\to l^+l^-)$ is the partial width for leptonic Z decay and $\sigma(p\bar{p}\to Z)$ is the inclusive Z production cross section. Although the partial width may be calculated very accurately, the calculation of the total cross section is subject to large uncertainties. This prevents us from using this method to obtain an accurate measurement of the Z width, and hence the number of neutrino species.

The uncertainties in the total Z production cross section may be grouped into two categories: those associated with the calculation of the parton-model subprocess cross section for inclusive Z production, and those associated with the quark and gluon distribution functions.[2] The order-α_s corrections to the Z production cross section have been calculated and are known to be large, about 30% (Ref. 4). One thus expects the order-α_s^2 corrections, which have not been calculated, to be around 10%. The uncertainty in the distribution functions may be as large as 25%. Both of these uncertainties are compounded by our lack of knowledge of $\Lambda_{\rm QCD}$ (Refs. 2 and 4).

The new method[1-3] to obtain the Z width was designed to overcome these uncertainties. The idea is to supplement the above information with the observed cross section for W production followed by leptonic decay, $\sigma(p\bar{p}\to W\to l\bar{\nu})$. One then calculates the ratio of the Z and W widths via

$$\frac{\Gamma_Z}{\Gamma_W}=\frac{\sigma(p\bar{p}\to W\to l\bar{\nu})}{\sigma(p\bar{p}\to Z\to l^+l^-)}\frac{\sigma(p\bar{p}\to Z)}{\sigma(p\bar{p}\to W)}\frac{\Gamma(Z\to l^+l^-)}{\Gamma(W\to l\bar{\nu})}\ . \tag{2}$$

If we assume that any charged leptons belonging to further generations are too heavy to contribute to the W width via $W\to L\bar{\nu}$, or if we know the masses of the new leptons which are not too heavy to contribute, the equation above yields the Z width, and hence the number of neutrino species.

The advantage of this method is that only the *ratio* of the Z and W production cross sections is needed. While the calculation of each cross section is subject to the large uncertainties described above, it has been found that these uncertainties largely cancel in the ratio.[2-4] It has been concluded that the ratio of cross sections may be calculated to within $\pm 6\%$, the uncertainty stemming almost entirely from the distribution functions and the value of $\Lambda_{\rm QCD}$.

The intensitivity of the ratio of Z and W production cross sections to radiative corrections is due to the fact that most corrections are common to both processes. Hikasa[2] has pointed out, however, that there are contributions to Z production for which there are no corresponding contributions to W production. These contributions first arise at order α_s^2, and are represented by the interference of the tree diagrams with the one- and two-loop diagrams in Fig. 1. They all involve the effective coupling of the Z to two gluons via a quark loop. Clearly no such coupling exists for the W boson.

Since these contributions arise at order α_s^2, the possibility exists that they are as large as 10%. An uncertainty of 10% in the ratio of Z and W production cross sections is disastrous, since each additional neutrino species contri-

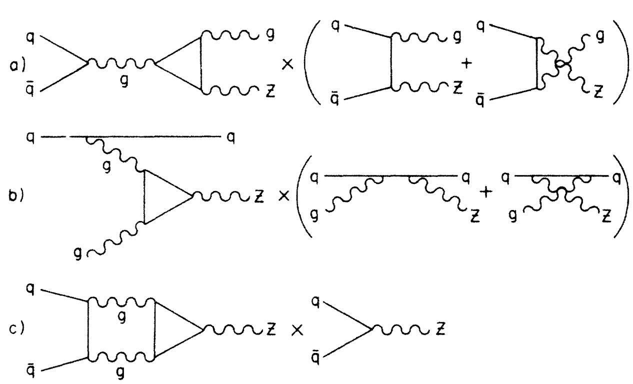

FIG. 1. Radiative corrections to Z production for which there are no corresponding graphs for W production. The corrections are of order $\alpha_s{}^2$ and are given by the interference of the tree graphs with the one- or two-loop graphs. Each process also includes a graph with the gluon lines crossed on the effective ggZ coupling.

butes only about 6% to the total Z width. Hence it is important that we calculate the above diagrams.

The remainder of the paper is organized as follows. In Sec. II we make some general comments on the calculations. In Sec. III we analyze the one-loop contributions and in Sec. IV the two-loop contribution. Section V is devoted to conclusions and a discussion. The calculations all involve the effective ggZ coupling, which we give in Appendix A. Appendix B contains two integrals which are used to evaluate the one-loop cross sections. Appendix C contains the analytic expression for the two-loop cross section.

II. PRELIMINARY REMARKS

The set of Feynman diagrams common to both Z and W production must form an independent, gauge-invariant set in order for the calculation of W production to be consistent. Hence the additional contributions to Z production must be consistent by themselves. This may be checked explicitly by showing that each set of graphs in Figs. 1(a)–1(c) is invariant under SU(3)-color gauge transformations.

The additional contributions to Z production all contain the effective coupling of the Z to two gluons via a quark loop, with one or both gluons off shell. Since the gluons have only vector couplings to fermions, and the effective coupling of the Z to two gluons is a color singlet, Furry's theorem tells us that only the axial-vector coupling of the Z contributes. The axial-vector couplings of the Z to the quarks in an $SU(2)_L$ doublet differ by a sign, so the contributions to the effective ggZ coupling from the quarks in a given generation would cancel if the quarks were degenerate in mass. One thus expects the effective coupling to be suppressed by factors like $(m_U{}^2-m_D{}^2)/M_Z{}^2$, where m_U and m_D are the masses of the quarks with $SU(2)_L$ quantum number $T_{3L}=+\frac{1}{2}$ and $T_{3L}=-\frac{1}{2}$, respectively.[2]

The argument above indicates that the contribution to the effective ggZ coupling from the first two generations of quarks in greatly suppressed. The contribution from the third generation could be large, depending on the mass of the top quark. Since we are trying to bound the number of generations, we should also consider the contributions from additional doublets of heavy quarks as well.

As is well known, the effective coupling of three gauge bosons via a fermion loop is ambiguous, due to a "surface term," which is related to the triangle anomaly.[5] This surface term is independent of the fermion mass and is therefore canceled by including the contributions from both of the quarks in an $SU(2)_L$ doublet. This is a consequence of the fact that the standard model is free of anomalies. This also guarantees the gauge independence of the effective ggZ coupling.

The amplitude for each process is the sum of contributions from each flavor of quark in the loop. Upon squaring the matrix element for each process, one finds that only the real part of the loop integral survives in the interference term. Since, in the interference term, each flavor of quark in the loop contributes incoherently, we may consider the contribution from each quark doublet separately.

The effective ggZ coupling involves the four index antisymmetric pseudotensor $\epsilon^{\mu\nu\rho\sigma}$. Since there are only three independent four-momenta in the processes shown in Figs. 1(a) and 1(b), and only two in the process shown in Fig. 1(c), at least one of the indices must be contracted with another such tensor. This is generated by the axial-vector coupling of the Z to the quark in the tree diagram. Since this coupling depends only on the $SU(2)_L$ quantum number of the quark, we see that quark partons with $T_{3L}=+\frac{1}{2}$ and those with $T_{3L}=-\frac{1}{2}$ give opposite sign contribution to the interference term. This has the interesting consequence that the up and down sea-quark contributions exactly cancel each other. This is not terribly important at CERN energies, however, since the dominant contribution comes from valence quarks and antiquarks.

III. ONE-LOOP CONTRIBUTIONS

The one-loop contributions to the ratio of Z and W production are shown in Figs. 1(a) and 1(b). We calculate the interference between the tree and one-loop diagrams, which is of order $\alpha_s{}^2$.

The effective ggZ coupling vanishes if all three particles are on the mass shell. This is a consequence of Yang's theorem,[6] which forbids the construction of a state of total angular momentum one from two massless spin-one states. The effective ggZ coupling is therefore proportional to the square of the four-momentum of the internal gluon line in the diagram. This cancels the pole in the gluon propagator. The result is that the cross sections scale like $1/M_Z{}^2$ rather than $1/\hat{s}$ or $1/\hat{t}$ as one would naively expect. Note, in particular, that this eliminates the potential mass (or collinear) divergence in the process shown in Fig. 1(b) associated with emitting a massless gluon in the t channel from a massless quark.

One might also worry about the mass divergences associated with the t- and u-channel tree diagrams in Fig. 1(a) and the u-channel tree diagram in Fig. 1(b). The former diagrams also have a potential infrared divergence from the emission of a soft gluon. However, the consistency of perturbation theory and the parton model demands that these divergences do not appear in the calculation. Mass and infrared divergences are common in parton-model calculations, and we know how to handle them: The mass divergences are absorbed into the distribution functions, and the infrared divergences are canceled by similar divergences arising in loop diagrams.[4,7] The set of diagrams of order α_s^2 which are common to both Z and W production must be rendered finite by this prescription in order for the calculation of W production to be consistent. The Z production calculation will therefore be consistent only if the additional contributions are divergence free. It is indeed satisfying that these divergences do not appear in the actual calculations.

The spin- and color-averaged contributions to the Z production subprocess cross section from the interference of the tree and one-loop diagrams is

(a) $q\bar{q}\to Zg$,

$$\sigma=-\eta_e\frac{\alpha_s^2}{18}\frac{\alpha}{\sin^2\theta_W\cos^2\theta_W}\left[1-\left[\frac{M_Z^2}{\hat{s}}\right]^2\right]\sum_{\text{flavors}}\eta_i I; \tag{3}$$

(b) $gq\to Zq$ (or $g\bar{q}\to Z\bar{q}$),

$$\sigma=-\eta_e\frac{\alpha_s^2}{192}\frac{\alpha}{\sin^2\theta_W\cos^2\theta_W}\left[1-\frac{M_Z^2}{\hat{s}}\right]\times\int_{-1}^{1}dz\left[1-3\frac{M_Z^2}{\hat{s}}-z\left[1-\frac{M_Z^2}{\hat{s}}\right]\right]\times\sum_{\text{flavors}}\eta_i I\;; \tag{4}$$

where $\hat{s}$ is the square of the subprocess center-of-mass energy and z is the center-of-mass scattering angle. The symbol η_e equals ±1, corresponding to the sign of the $SU(2)_L$ quantum number of the incoming quark or antiquark parton. The summation is over all quark flavors, with $\eta_i=\pm1$ corresponding to the $SU(2)_L$ quantum number of the quark in the loop. A quark of mass m gives a contribution to this sum of

$$I=\frac{1}{(q^2-M_Z^2)^2}\left\{M_Z^2\left[I_1\left[\frac{M_Z^2}{m^2}\right]-I_1\left[\frac{q^2}{m^2}\right]\right]-2m^2\left[I_2\left[\frac{M_Z^2}{m^2}\right]-I_2\left[\frac{q^2}{m^2}\right]\right]\right\}, \tag{5}$$

where q is the momentum of the virtual gluon:

$$q^2=\begin{cases}\hat{s}\ \text{in (a)}\,,\\ -\frac{1}{2}\hat{s}\left[1-\dfrac{M_Z^2}{\hat{s}}\right](1-z)\ \text{in (b)}\,.\end{cases} \tag{6}$$

The integrals I_1 and I_2 are given in Appendix B.

To calculate the size of the contribution of these interference terms to Z production, we integrate the subprocess cross sections over the distribution functions of Eichten, Hinchliffe, Lane, and Quigg[8] (EHLQ), set 2 ($\Lambda=290$ MeV). We also calculate the lowest-order contribution $p\bar{p}\to Z$. In Fig. 2 we have graphed the ratio of the interference term to the lowest-order contribution as a function of the mass of the $T_{3L}=+\frac{1}{2}$ quark in the loop at energies of 540 GeV and 2 TeV. We have fixed the $SU(2)_L$ partner of this quark to have a mass of 5 GeV (we have in mind the bottom and top quarks); however, the contribution of any doublet of quarks may be read off this graph in the following manner. The contribution of the $T_{3L}=-\frac{1}{2}$ quark is found by treating it as the heavier member of a doublet with a 5-GeV partner. This contribution is then subtracted from that of the $T_{3L}=+\frac{1}{2}$ quark with a 5-GeV partner.

Note that the cross section approaches a constant as the mass of the $T_{3L}=+\frac{1}{2}$ quark approaches infinity. This is because the effective ggZ coupling scales like $1/m^2$ for large m, and hence the contribution of the $T_{3L}=+\frac{1}{2}$ quark decouples in this limit.

The graph in Fig. 2 shows that the contribution from any quark doublet is at most 0.025% for $q\bar{q}\to Zg$ and at most 0.005% for $gq\to Zq$ at the CERN energy $\sqrt{s}=540$ GeV. The results at $\sqrt{s}=630$ GeV are only slightly different. At the Fermilab Tevatron energy $\sqrt{s}=2$ TeV the $q\bar{q}\to Zq$ curve has changed only qualitatively, while the

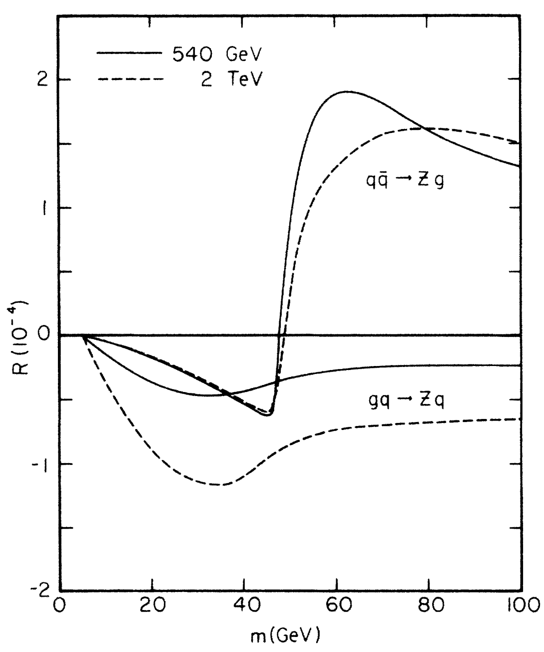

FIG. 2. Ratio of the one-loop Z production radiative corrections shown in Figs. 1(a) and 1(b) to the lowest-order Z production cross section as a function of the mass of the $T_{3L}=+\frac{1}{2}$ quark in the loop. The $SU(2)_L$ partner of the quark has a mass of 5 GeV. The contribution from any doublet of quarks may be found by subtracting the contribution of the $T_{3L}=-\frac{1}{2}$ quark from that of the $T_{3L}=+\frac{1}{2}$ quark, as explained in the text. The curves are labeled by the corresponding process in Figs. 1(a) and 1(b). The solid (dashed) curve corresponds to $p\bar{p}$ collisions at a center-of-mass energy of 540 GeV (2 TeV); the results at 630 GeV are very close to those at 540 GeV.

$gq \to Zq$ curve has more than doubled. This is due to the large density of gluons which reside at small values of the parton fractional momentum. In any case, these contributions are very small, much less than the $\pm 6\%$ uncertainty associated with the distribution functions. We therefore conclude that these one-loop, order-α_s^2 corrections to the ratio of Z and W production are negligible.

IV. TWO-LOOP CONTRIBUTION

The two-loop contribution to the ratio of Z and W production is shown in Fig. 1(c). We calculate the interference between the tree and two-loop diagrams, which is of order α_s^2. As we shall see, this contribution is larger than the one-loop terms discussed in the preceding section.

It is easy to see that this two-loop graph [Fig. 1(c)] must be finite, although it is naively logarithmically divergent. The set of graphs common to both Z and W production must be rendered finite by renormalization in order for the calculation of W production to be consistent. Hence there are no counterterms available to absorb a divergence in this additional contribution to Z production. A similar conclusion may be reached if one considers the incoming quark-antiquark pair to be an $SU(2)_L$ singlet with no direct coupling to the Z. This two-loop graph [Fig. 1(c)] then induces an effective $\bar{q}qZ$ coupling which must be finite, since there are no counterterms available.

We have performed this two-loop calculation by first integrating over the quark loop momentum to form an effective ggZ vertex, then combining the denominators via the usual Feynman technique in order to integrate the remaining loop. As we discussed earlier, Yang's theorem[6] guarantees that all the terms in the effective ggZ vertex are proportional to the square of the momentum of one of the gluons. We use this to cancel the corresponding gluon propagator, thereby decreasing the number of denominators and hence the number of Feynman parameters. We then integrate these two parameters, leaving just the two parameter integrals associated with the effective ggZ vertex.

In performing the second loop integral, it is imperative that one include the contribution to the effective ggZ vertex from both of the quarks in an $SU(2)_L$ doublet. The contribution from just one quark causes some of the terms in the loop integral to diverge logarithmically, which is the naive degree of divergence of the graph. This divergence is canceled by the contribution from the quark's $SU(2)_L$ partner. The two-loop contribution is therefore finite as a consequence of the fact that the standard model is anomaly free.

The spin- and color-averaged contribution to the Z production subprocess cross section from the interference of the tree and two-loop diagrams is

$$\sigma = \eta_e \frac{\alpha_s^2}{3} \frac{\alpha}{\sin^2\theta_W \cos^2\theta_W} \delta(\hat{s} - M_Z^2) \sum_{\text{flavors}} \eta_i J \; , \qquad (7)$$

where $\hat{s}$ is the square of the subprocess center-of-mass energy. As in the one-loop case, $\eta_e = \pm 1$ corresponding to the $SU(2)_L$ quantum number of the incoming partons, and $\eta_i = \pm 1$ corresponding to that of the quark in the loop. Each flavor contributes an amount J to the sum, with J given in Appendix C. In the expression for J we have dropped all mass-independent terms since they cancel in the sum.

The ratio of the two-loop contribution and the lowest-order contribution to $p\bar{p} \to Z$ is shown in Fig. 3 as a function of the mass of the $T_{3L} = +\frac{1}{2}$ quark at energies of 540 GeV and 2 TeV. Again, we have used the distribution functions of EHLQ (Ref. 8) set 2 ($\Lambda = 290$ MeV), to obtain this result. As before, we have fixed the mass of the $T_{3L} = -\frac{1}{2}$ quark at 5 GeV. The contribution of any doublet of quarks may be read off this graph (Fig. 3) by subtracting the contribution of the $T_{3L} = -\frac{1}{2}$ quark from that of the $T_{3L} = +\frac{1}{2}$ quark.

As the graph in Fig. 3 shows, the two-loop contribution to Z production is larger than the one-loop contributions. It also has a qualitatively different behavior as the mass of the $T_{3L} = +\frac{1}{2}$ quark increases. Instead of decoupling, the heavy-quark contribution grows logarithmically. One may show that in the limit $m^2/M_Z^2 \to \infty$ the complicated expression given for J in Appendix C reduces to

$$J = \frac{1}{4} \ln \frac{m^2}{M_Z^2} + \text{const} \; . \qquad (8)$$

The curve in Fig. 3 at $\sqrt{s} = 540$ GeV is then described in this limit by

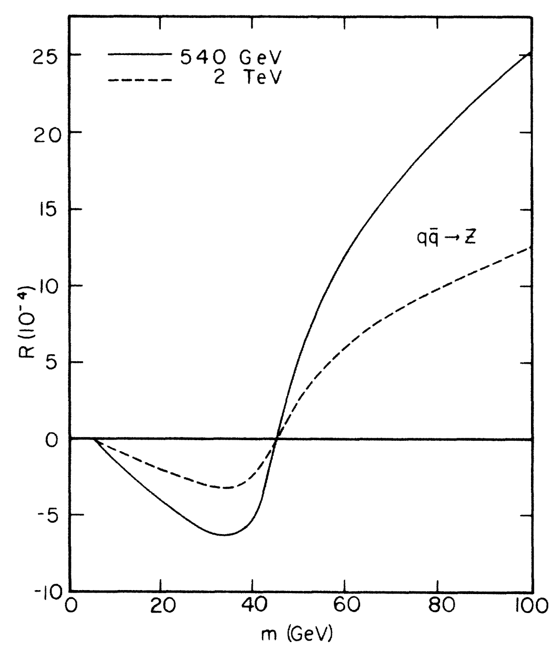

FIG. 3. Ratio of the two-loop Z production radiative correction shown in Fig. 1(c) to the lowest-order Z production cross section as a function of the mass of the $T_{3L} = +\frac{1}{2}$ quark in the loop. The $SU(2)_L$ partner of the quark has a mass of 5 GeV. The contribution from any doublet of quarks may be found by subtracting the contribution of the $T_{3L} = -\frac{1}{2}$ quark from that of the $T_{3L} = +\frac{1}{2}$ quark, as explained in the text. The solid (dashed) curve corresponds to $p\bar{p}$ collisions at a center-of-mass energy of 540 GeV (2 TeV); the results at 630 GeV are very close to those at 540 GeV.

$$R=\left[25.5+9.92\ln\frac{m^2}{M_Z{}^2}\right]\times 10^{-4}\ , \tag{9}$$

where the coefficient of the logarithm is calculated from the asymptotic expression for J. This logarithmic behavior sets in at about $m=100$ GeV, i.e., near the Z mass.

The logarithmic growth may be traced back to the logarithmic divergence associated with the second loop integral.[9] Recall that this divergence is canceled when we include the contributions from both of the quarks in an $SU(2)_L$ doublet. At low loop momenta the contribution from the heavier of the two $SU(2)_L$ quarks is suppressed, and the integral begins to diverge. When the loop momentum reaches the mass of the heavy quark its contribution is no longer suppressed and it effectively cuts off the logarithmic divergence, leading to a factor $\ln m^2/M_Z{}^2$. This logarithmic behavior is therefore related to the cancellation of the triangle anomaly in the standard model. We should also mention that this logarithmic growth does not violate our notion of the decoupling of heavy particles from low-energy phenomena since the decoupling theorem does not hold if anomaly cancellation occurs between heavy and light particles.[10]

The two-loop contribution at $\sqrt{s}=630$ GeV is only slightly smaller than at $\sqrt{s}=540$ GeV. The values at $\sqrt{s}=2$ TeV are almost exactly $\frac{1}{2}$ as big. This reflects the cancellation of the up and down sea-quark contributions, which are more important at higher energies.

Although larger than the one-loop contributions, this two-loop correction is still much smaller than the $\pm 6\%$ uncertainty in the ratio of Z and W production cross sections associated with the distribution functions, even for very-heavy-quark masses. For example, the contribution of a quark doublet with masses of 5 and 1000 GeV is only 0.73% at $\sqrt{s}=540$ GeV. Furthermore, we really cannot trust our calculation beyond a quark mass of about 700 GeV, the unitarity bound on the mass of a quark with a light $SU(2)_L$ partner.[11] Quarks whose masses exceed this bound are strongly coupled and do not permit a perturbative analysis.

We therefore conclude that the two-loop, order-$\alpha_s{}^2$ correction to the ratio of Z- and W-boson production is negligibly small at CERN energies and even smaller at the Fermilab Tevatron energy $\sqrt{s}=2$ TeV.

V. CONCLUSION AND DISCUSSION

We have seen that the order-$\alpha_s{}^2$ correction to the ratio of Z- and W-boson production is quite small, less than 1% at current $p\bar{p}$ energies. This gives us more faith in the calculated value of this ratio and hence in the reliability of the method for counting neutrino species[1–3] which uses this ratio.

Recently a method for counting neutrino species related to the one described in the text has been proposed which makes use of the observed cross section for the monojet events.[12] This method relies on a calculation of the ratio of Z+jet and W+jet cross sections. Since the processes shown in Figs. 1(a) and 1(b) contribute to the Z+jet cross section, our calculation also bears on this ratio. Again, we conclude that these order-$\alpha_s{}^2$ corrections to the ratio are negligible.

Other order-$\alpha_s{}^2$ corrections which apply to weak-gauge-boson production exist in the literature. The corrections to virtual-photon production at nonzero transverse momentum have been calculated for the case of incident quarks and antiquarks.[13] These corrections do not include processes corresponding to those in Fig. 1 because the photon has only vector coupling. Also, corrections to the double-logarithmic approximation have been evaluated for nonzero transverse momentum.[14] We do not expect any of these corrections to significantly effect the *ratio* of Z- and W-boson production.

We might also ask how large the corrections to the ratio of Z- and W-boson production are at much higher energies. At current $p\bar{p}$ energies the dominant parton interactions involve valence quarks. At multi-TeV energies the emphasis shifts to the sea quarks. Since the corrections we are calculating depend on the sign of the $SU(2)_L$ quantum number of the incident quark, the up and down sea-quark contributions largely cancel. We therefore expect these corrections to be even less important at higher eneriges. We have confirmed this using the distribution functions of EHLQ (Ref. 8) set 2 ($\Lambda=290$ MeV), and Duke and Owens,[15] set 1 ($\Lambda=200$ MeV) and set 2 ($\Lambda=400$ MeV). We would like to note, however, that although these two sets of distribution functions give similar results at CERN and Fermilab Tevatron energies, the results at multi-TeV energies are very different. This is because the cancellations which occur between $T_{3L}=+\frac{1}{2}$ and $-\frac{1}{2}$ quark partons are sensitive to the slight differences in the quark distributions. Nevertheless, the different distribution functions agree qualitatively that the corrections are even less important at higher energies.

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APPENDIX A

We present here the result for the effective ggZ coupling.[16] The graph in Fig. 4 corresponds to the momentum integral

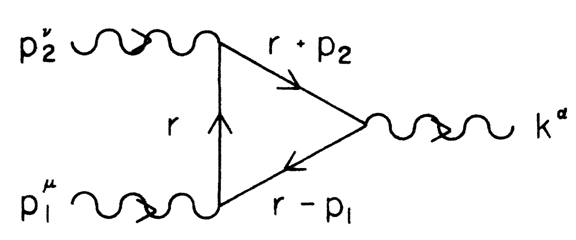

FIG. 4. Feynman diagram which, along with a graph with the gluon lines crossed, constitutes the effective ggZ coupling.

$$\Gamma^{\alpha\mu\nu}(p_1,p_2)=\int \frac{d^4r}{(2\pi)^4}\mathrm{Tr}\gamma_5\gamma^\alpha\frac{1}{\not r+\not p_2-m}\gamma^\nu\frac{1}{\not r-m}\gamma^\mu\frac{1}{\not r-\not p_1-m} . \tag{A1}$$

Evaluating the trace and carrying out the momentum integral yields

$$\begin{aligned}\Gamma^{\alpha\mu\nu}(p_1,p_2)=\frac{1}{2\pi^2}\int_0^1 dx\int_0^{1-x} dy[p_1{}^2x(1-x)+p_2{}^2y(1-y)+2p_1\cdot p_2xy-m^2]^{-1}\\ \times[k^\alpha\epsilon^{\rho\sigma\mu\nu}p_{1\rho}p_{2\sigma}xy+(p_1{}^\mu\epsilon^{\alpha\nu\rho\sigma}x-p_2{}^\nu\epsilon^{\alpha\mu\rho\sigma}y)p_{1\rho}p_{2\sigma}(1-x-y)\\ -\epsilon^{\alpha\mu\nu\rho}(p_{1\rho}p_2{}^2y-p_{2\rho}p_1{}^2x)(1-x-y)] .\end{aligned} \tag{A2}$$

Note that we have not included the coupling constants nor the factor of (-1) from the closed fermion loop. However, we have taken into account a factor of i from Wick rotating the dr_0 integral and a factor of i from the trace. Our conventions are such that

$$\mathrm{Tr}\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma=4i\epsilon^{\mu\nu\rho\sigma} . \tag{A3}$$

In deriving this result we have dropped a term which is independent of the mass of the quark in the loop. This term cancels when we include the contributions from both of the quarks in an $SU(2)_L$ doublet to the effective coupling. By dropping this term our expression satisfies the Ward identities $p_{1\mu}\Gamma^{\alpha\mu\nu}=0$ and $p_{2\nu}\Gamma^{\alpha\mu\nu}=0$. Our expression is manifestly Bose symmetric, so the diagram with the gluon lines crossed yields the same result. Our expression agrees with Adler's and reduces to the result of Bell and Jackiw in the limit of on-shell gluons.[5]

The first term does not contribute to real Z production since it is proportional to the Z momentum, and $\epsilon\cdot k=0$ where ϵ is the Z polarization vector. The next two terms, proportional to $p_1{}^\mu$ and $p_2{}^\nu$, do not contribute if the gluons are real, for the same reason. Furthermore, they also do not contribute if the gluon momentum is contracted with a conserved current, since $p_{1\mu}J^\mu=0$. This is the case in the graphs of Fig. 1. Therefore, the first three terms do not contribute to the calculation of these graphs. The last set of terms are proportional to $p_1{}^2$ or $p_2{}^2$ in accordance with Yang's theorem,[6] as discussed in the text.

APPENDIX B

Below we list the integrals[17] associated with the effective ggZ vertex in the one-loop interference terms in Figs. 1(a) and 1(b):

$$\begin{aligned}I_1(a)&=\mathrm{Re}\int_0^1 dx\ln[1-ax(1-x)]\\ &=2\left[\frac{a-4}{a}\right]^{1/2}\mathrm{arcsinh}\left[-\frac{a}{4}\right]^{1/2}-2,\quad a<0\\ &=2\left[\frac{4-a}{a}\right]^{1/2}\arcsin\left[\frac{a}{4}\right]^{1/2}-2,\quad 0<a<4\\ &=2\left[\frac{a-4}{a}\right]^{1/2}\mathrm{arccosh}\left[\frac{a}{4}\right]^{1/2}-2,\quad a>4 ;\end{aligned} \tag{B1}$$

$$\begin{aligned}I_2(a)&=\mathrm{Re}\int_0^1 dx\frac{1}{x}\ln[1-ax(1-x)]\\ &=2\left[\mathrm{arcsinh}\left[-\frac{a}{4}\right]^{1/2}\right]^2,\quad a<0\\ &=-2\left[\arcsin\left[\frac{a}{4}\right]^{1/2}\right]^2,\quad 0<a<4\\ &=2\left[\mathrm{arccosh}\left[\frac{a}{4}\right]^{1/2}\right]^2-\frac{\pi^2}{2},\quad a>4 .\end{aligned} \tag{B2}$$

APPENDIX C

We present here the expression for the function J which appears in the two-loop interference term.[18] It is a two-dimensional integral, the integrand of which we have split into two terms. The first term f results from the logarithmically divergent part of the second loop integral, as discussed in the text. We write the integrand in terms of the three functions

$$\begin{aligned}a&=-xy+\frac{m^2}{M_Z{}^2} ,\\ b&=-xy+\frac{m^2}{M_Z{}^2}(x+y) ,\\ c&=-x(1-x)+\frac{m^2}{M_Z{}^2} .\end{aligned} \tag{C1}$$

The expression for J is then

$$J=\int_0^1 dx\int_0^{1-x} dy(f+g) , \tag{C2}$$

where

$$\begin{aligned}f=&\frac{1}{x^2y(1-x-y)^2(x+y)}\\ &\times[x(x+y)a^2\ln a(x+y)-b^2\ln b\\ &+y(x+y)c^2\ln c(x+y)] ,\end{aligned} \tag{C3a}$$

$$\begin{aligned}g=&\frac{-2}{3x^2y(1-x-y)^2}(a+c)\\ &\times[xa\ln a(x+y)-b\ln b+yc\ln c(x+y)] .\end{aligned} \tag{C3b}$$

The logarithms which appear in f and g are to be evaluated at the absolute value of their argument.

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