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Nucleon axial-vector form factor in perturbative QCD

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We calculate the axial-vector form factor $g_A(Q^2)$ using leading-order perturbative QCD and a variety of distribution amplitudes (wave functions). We learn that g_A falls asymptotically like $1/Q^4$ and its normalization is determined if we use the same distribution amplitudes that account for the nucleon magnetic form factors. The data on g_A extrapolated to higher Q^2 are in accord with the normalization determined in this way.

I. INTRODUCTION

The number of applications of perturbative QCD to exclusive processes is expanding. The common feature in these applications is the factorizability of the amplitude into two parts.[1,2] One is a hard-scattering amplitude T_H where all the interactions involve propagators far off shell and which can be computed in perturbation theory. The other piece involves low-momentum-transfer interactions which bind the quarks into hadrons. This part is described by a wave function ψ for the quarks, and it is the "distribution amplitude" ϕ which is the wave function with transverse-momentum integrated, that enters the exclusive amplitude. Neither ψ nor ϕ can be calculated *ab initio.* (We should however mention work using QCD sum rules leading to moments of distribution amplitudes for the nucleon[3] and pion[2] and progress being made using lattice gauge theory where some moments of distribution amplitudes are available for the pion.[4]) However, once the distribution amplitude for a hadron is obtained, perhaps using one or another exclusive process, it can be used in any process. The distribution amplitude is the universal link among different exclusive processes, while the hard-scattering amplitude T_H is calculated perturbatively process by process.

The processes for which T_H has been calculated include decays of heavy mesons into lighter-meson pairs[5] and baryon pairs,[6] two-photon production of meson pairs[7] and baryon pairs,[8] and electromagnetic form factors of mesons[9] and baryons.[10] One would like to have still more processes to intercompare and/or use to determine distribution amplitudes. We shall here contribute by calculating a quantity that can be measured via a weak interaction, namely, the axial-vector form factor $g_A(Q^2)$.

Some special interest is given to the nucleon form factors by a special cancellation that occurs for the proton magnetic form factor. If the nucleon distribution amplitude takes its ultimate asymptotic form, then the proton form factor becomes zero to leading order in α_s and $(\text{mass})^2/Q^2$. The observed proton form factor must then be due to higher-order corrections in perturbation theory, to higher-twist corrections, or (as we may hope) to the leading-order calculation but with a distribution which is not yet close to its asymptotic form.[3,11]

Leading-order perturbative QCD (PQCD) can work[12] for the nucleons if one uses a broader distribution function than the asymptotic one and the observed ratio G_{Mp}/G_{Mn} seems to require an asymmetric spatial wave function. One objective here is to see if these distribution amplitudes are compatible with experimental data on $g_A(Q^2)$ and to predict its course at still higher Q^2.

In Sec. II we obtain T_H, calculate g_A both for a ϕ expanded in the first six Appel polynomials and for a popular symmetric power-law wave function, and give numerical results for a few relevant distribution amplitudes. Section III compares the calculated results to the experimental data, and we conclude in Sec. IV.

II. CALCULATIONS OF $g_A(Q^2)$

The form factor g_A is defined by a matrix element of the axial-vector current $A^\mu(x)$. Taking the component $A^+ \equiv A^0 + A^3$, we have[13]

$$\langle n_\uparrow | A^+(0) | p_\uparrow \rangle = 2p^+ g_A(Q^2) ,$$

where p^μ is the momentum of the proton and the arrows indicate positive helicity.

At the constituent level, Fig. 1(a), g_A may be calculated as a convolution

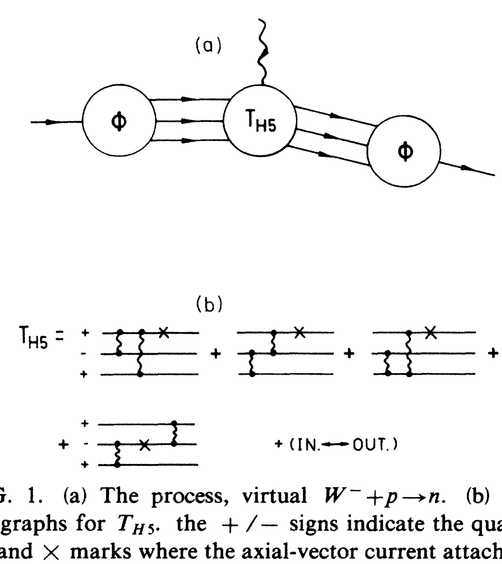

FIG. 1. (a) The process, virtual $W^- + p \to n$. (b) Lowest-order graphs for T_{H5}. the $+/-$ signs indicate the quark helicities and $\times$ marks where the axial-vector current attaches.

$$g_A(Q^2)=\int [dx][dy]\phi(y,Q^2)T_{H5}(x,y,Q)\phi(x,Q^2) .$$

Our notation involves light-cone longitudinal-momentum fractions x_i, transverse momenta k_{iT}, measures

$$[dx]=dx_1dx_2dx_3\delta(1-x_1-x_2-x_3) ,$$

$$[dk_T]=\prod_{i=1}^{3}\frac{d^2k_{iT}}{16\pi^3}16\pi^3\delta^2\left[\sum_{j=1}^{3}k_{iT}\right] ,$$

and

$$\phi(x,Q^2)=\int^{Q^2}[dk_T]\psi(x,k_T) ,$$

where ϕ is the distribution amplitude and ψ the wave function for the three-quark sector.

The hard-scattering amplitude is called T_{H5} to distinguish it from the electromagnetic case, to which it is very similar. There are 42 lowest-order diagrams, but only 14 are nonzero and only the 4 in Fig. 1(b) require separate calculation, the rest being obtained by symmetries. The result is

$$T_{H5}=\left[\frac{16\pi\alpha_s(Q^2)}{3Q^2}\right]^2\sum_{j=1}^{3}\mathrm{sgn}(\lambda_j)[I_{j-}T_j(x,y)+(x\leftrightarrow y)] ,$$

where $\mathrm{sgn}(\lambda_j)$ is the sign of the helicity for quark j, I_{j-} is the isospin-lowering operator for quark j, and[1,10]

$$T_1=\frac{1}{x_3(1-x_1)^2y_3(1-y_1)^2}+\frac{1}{x_2(1-x_1)^2y_2(1-y_1)^2}-\frac{1}{x_2x_3(1-x_3)y_2y_3(1-y_1)}=T_3(1\leftrightarrow 3) ,$$

$$T_2=\frac{1}{x_1x_3(1-x_1)y_1y_3(1-y_3)} .$$

We write the distribution amplitude in terms of spatial pieces $\phi_S(x)$ and $\phi_A(x)$ that are symmetric and antisymmetric under $x_1\leftrightarrow x_3$ (where 1 and 3 are the parallel-helicity quarks), then

$$\phi_p(x)=\phi_S(x)\frac{1}{\sqrt{6}}\,|2u_\uparrow d_\downarrow u_\uparrow-u_\uparrow u_\downarrow d_\uparrow-d_\uparrow u_\downarrow u_\uparrow\rangle+\phi_A(x)\frac{1}{\sqrt{2}}\,|u_\uparrow u_\downarrow d_\uparrow-d_\uparrow u_\downarrow u_\uparrow\rangle ,$$

$$\phi_n(x)=\phi_S(x)\frac{1}{\sqrt{6}}\,|d_\uparrow d_\downarrow u_\uparrow+u_\uparrow d_\downarrow d_\uparrow-2d_\uparrow u_\downarrow d_\uparrow\rangle+\phi_A(x)\frac{1}{\sqrt{2}}\,|u_\uparrow d_\downarrow d_\uparrow-d_\uparrow d_\downarrow u_\uparrow\rangle ,$$

and

$$g_A(Q^2)=\left[\frac{16\pi\alpha_s}{3Q^2}\right]^2\int[dx][dy]\left[\phi_S(x)\phi_S(y)(\tfrac{8}{3}T_1+\tfrac{2}{3}T_2)-2\phi_A(x)\phi_A(y)T_2(x,y)-\frac{4}{\sqrt{3}}T_1(x,y)[\phi_A(x)\phi_S(y)+\phi_S(x)\phi_A(y)]\right] .$$

This can be algebraically related to the electromagnetic form factors:

$$g_A(Q^2)=[G_{Mp}(Q^2)-G_{Mn}(Q^2)]_{T_1\to T_1,T_2\to -T_2} .$$

For symmetric wave functions we also have the relation

$$g_A(Q^2)=\tfrac{5}{3}G_{Mp}+G_{Mn} .$$

To obtain concrete results, we must choose some wave function. We will make two sets of choices.

(1) Simple symmetric distribution amplitude. This choice is popular.[1,6,8] We have

$$\phi(x)=N'(x_1x_2x_3)^\eta$$

and can give the resulting g_A as a ratio to the nucleon form factors (the integrals can all be evaluated analytically if $\eta>\frac{1}{2}$ so that they converge):

$$g_A:G_{Mp}:G_{Mn}=1+4\eta(1-\eta):6\eta(1-\eta):1-6\eta(1-\eta) .$$

A plot of g_A/G_{Mp} is given in Fig. 2 showing that this ratio is wave-function sensitive. However if we want G_{Mp} and G_{Mn} both to have the correct sign[11,14]

$$0.5<\eta<\frac{1}{2}+\frac{1}{\sqrt{12}}\simeq 0.79$$

so that

$$\frac{4}{3}<\frac{g_A}{G_{Mp}}<\frac{5}{3} .$$

The simple symmetric distribution amplitude does not allow values of G_{Mp}/G_{Mn} between -1 and -3; for the preferred range of η it is -3 or below.[11] However, the e-n elastic scattering data[15] interpreted in terms of G_{Mn}

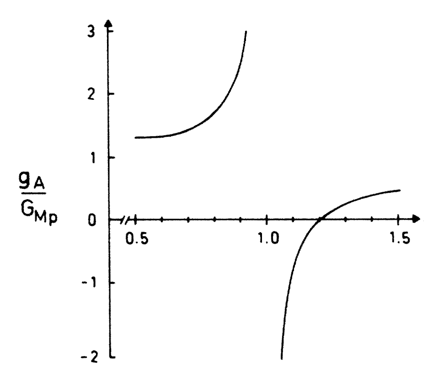

FIG. 2. The ratio g_A/G_{Mp} at large Q^2 for a distribution amplitude proportional to $(x_1x_2x_3)^\eta$.

suggest that G_{Mp}/G_{Mn} is about -2 at 10 GeV^2, the highest measured momentum transfer. This motivates examining other possible distribution amplitudes.

(2) Polynomial expansion. We can naturally expand the distribution amplitudes in terms of a weight factor $x_1x_2x_3$ times a sum of Appel polynomials. "Naturally" means that the Appel polynomials are eigensolutions of the evolution equation[1] so that as Q^2 changes the coefficients of the Appel polynomials change logarithmically in a calculable way, although the starting values of those coefficients are not calculable. We have

$$\phi(x)=\phi_S(x)+\phi_A(x)=x_1x_2x_3\sum N_i\tilde{\phi}_i(x)\ ,$$

where

$$\tilde{\phi}_0(x)=1\ ,$$
$$\tilde{\phi}_1(x)=x_1-x_3\ ,$$
$$\tilde{\phi}_2(x)=2-3(x_1+x_3)\ ,$$
$$\tilde{\phi}_3(x)=2-7(x_1+x_3)+8x_1{}^2+4x_1x_3+8x_3{}^2\ ,$$
$$\tilde{\phi}_4(x)=x_1-x_3-\tfrac{4}{3}(x_1{}^2-x_3{}^2)\ ,$$
$$\tilde{\phi}_5(x)=2-7(x_1+x_3)+\tfrac{14}{3}x_1{}^2+14x_1x_3+\tfrac{14}{3}x_3{}^2\ ,$$
$$\dots\ .$$

The form factor g_A can be straightforwardly calculated. Keeping only the first six polynomials,

$$\begin{aligned}Q^4g_A(Q^2)=\left[\frac{4\pi\alpha_s}{27}\right]^2\Bigg[&54N_0{}^2+2N_1{}^2-84\sqrt{3}N_0N_1+54N_2{}^2+56\sqrt{3}N_1N_2-36N_0N_2\\&+\tfrac{770}{3}N_3{}^2-180N_2N_3-\frac{440}{\sqrt{3}}N_1N_3+300N_0N_3+0\times N_4{}^2+\frac{92}{\sqrt{3}}N_3N_4\\&-\frac{44}{\sqrt{3}}N_2N_4+0\times N_1N_4+20\sqrt{3}N_0N_4+\tfrac{625}{54}N_5{}^2-\frac{82}{9\sqrt{3}}N_4N_5\\&-\tfrac{230}{3}N_3N_5+\tfrac{50}{3}N_2N_5+\frac{70}{\sqrt{3}}N_1N_5-50N_0N_5\Bigg]\ .\end{aligned}$$

(a) Simple mixed-symmetry distribution amplitude. It seems that to fit the sign and magnitude of both G_{Mp} and G_{Mn} requires a mixed-symmetry spatial wave function. One simple example that works well is[11]

$$\phi_M=(0.383\ \text{GeV}^2)x_1x_2x_3(\tilde{\phi}_3-\tilde{\phi}_1)\ .$$

For this wave function one has

$$\frac{g_A}{G_{Mp}}=1.53$$

(as well as the right asymptotic magnitude for G_{Mp} and a ratio $G_{Mp}/G_{Mn}=-2.07$, in accord with observation[15] at $Q^2=10\ \text{GeV}^2$ where the neutron data run out).

(b) Chernyak-Zhitnitsky distribution amplitude. Chernyak and Zhitnitsky[3] have calculated some moments of the distribution amplitude using QCD sum rules, and if the distribution amplitude can be expanded in the first six Appel polynomials they get

$$\begin{aligned}\phi(x)=x_1x_2x_3(&0.111\tilde{\phi}_0-0.274\tilde{\phi}_1-0.212\tilde{\phi}_2\\&+0.248\tilde{\phi}_3+0.221\tilde{\phi}_4\\&+0.002\tilde{\phi}_5)\ \text{GeV}^2\ .\end{aligned}$$

This ϕ gives a good account of the nucleon form factors (with $G_{Mp}/G_{Mn}=-2.05$) and also gives

$$\frac{g_A}{G_{Mp}}=1.53\ .$$

III. COMPARISON TO EXPERIMENTAL DATA

The measurements of $g_A(Q^2)$ at the highest Q^2 were done at Fermilab by Kitagaki *et al.*[16] using $\nu_\mu n\rightarrow\mu^-p$ in a deuterium target. They parametrize g_A as

$$g_A(Q^2)=1.23/(1+Q^2/M_A{}^2)^2$$

and best fit their data with

$$M_A=1.05^{+0.12}_{-0.16}\ \text{GeV}\ .$$

The highest data point they have is at $Q^2\simeq3\ \text{GeV}^2$, which is a bit low for our purposes and this might be kept in mind as we proceed. At large Q^2, which should be understood as Q^2 larger than hadronic mass scales but not yet $\ln\ln Q^2\rightarrow\infty$, we extrapolate

$$Q^4g_A(Q^2)=1.23M_A{}^4\simeq1.5\ \text{GeV}^4\ .$$

The proton data at $Q^2\simeq5-10\ \text{GeV}^2$, where the plateau in Q^4G_{Mp} seems to begin, give

$$Q^4G_{Mp}\simeq1.1\ \text{GeV}$$

or

$$\frac{g_A}{G_{Mp}}\simeq1.35\ .$$

This is not in bad agreement with any of the wave functions that can describe the electromagnetic form-factor data. However, the errors in $M_A{}^4$ are not small and more accurate and/or higher-Q^2 data would be welcome.

IV. CONCLUSIONS

We have calculated $g_A(Q^2)$ using perturbative QCD to lowest order in α_s and leading twist and for a variety of nucleon distribution amplitudes. If the available high-Q^2 nucleon electromagnetic form-factor data can be described by PQCD, then so can g_A and it should fall like $1/Q^4$ and be roughly 50% larger than G_{Mp}. This is not in bad agreement with the g_A data, although (as always, seemingly) more accurate or higher-Q^2 data would be useful.

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