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Terrestrially enhanced neutrino oscillations

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The effect of Wolfenstein neutrino-oscillation enhancement on atmospheric neutrinos is considered. The effect of these neutrino oscillations on data from proton-decay experiments is discussed. It is found that for $\Delta M^2 \sim 10^{-3}-10^{-4}$ eV^2 and $\sin^2(2\theta) \geq 3\times10^{-2}$, $\nu_e \leftrightarrow \nu_\mu$ oscillations might be detectable.

I. INTRODUCTION

Wolfenstein has pointed out that neutrino oscillations can be enhanced in the presence of matter.[1] In his original paper, he discussed the possibility of using 1000 km of rock to enhance these oscillations. Since that time, there has been much interest in solar oscillation enhancement.[2–4] This enhancement may well account for the solar-neutrino problem. The important effects of terrestrial enhancement, however, still remain to be studied.

I focus in particular on neutrinos generated by cosmic rays impinging on the atmosphere. Some of these neutrinos pass through the earth and, consequently, may undergo oscillations; this can change the ν_e/ν_μ ratio. Oscillations can be detected by comparing the ν_e/ν_μ ratios for upward- and downward-directed neutrinos.

I start by considering the motion of neutrinos through matter. The development for a highly relativistic plane wave is given by

$$i\, d\Psi/dt = (M^2/2E + \sqrt{2}G_F N_e P_e)\Psi \ , \tag{1}$$

where t is the distance traveled, or, for highly relativistic wave packets, the time ($c=\hbar=1$). G_F is the Fermi constant, N_e is the electron density, and P_e is the electron-neutrino projection operator. The mass matrix M can be written in terms of three parameters for two species of neutrinos,

$$M^2 = \begin{bmatrix} M_1{}^2c^2 + M_2{}^2s^2 & M_2{}^2cs - M_1{}^2cs \\ M_2{}^2cs - M_1{}^2cs & M_1{}^2s^2 + M_2{}^2c^2 \end{bmatrix} , \tag{2}$$

where M_1 and M_2 are the mass eigenvalues and $c \equiv \cos\theta$ and $s \equiv \sin\theta$, where θ is the vacuum mixing angle. The trace of M^2 contributes only to the overall phase of Ψ and is irrelevant for our purposes. When we remove the trace from the M^2 part of (1), we find

$$i\frac{d}{dt}\begin{bmatrix}\nu_e \\ \nu_\mu\end{bmatrix} = \begin{bmatrix} A & B \\ B & C\end{bmatrix}\begin{bmatrix}\nu_e \\ \nu_\mu\end{bmatrix} , \tag{3}$$

where

$$\begin{aligned} A &= -\Delta M^2/4E\cos(2\theta) + \sqrt{2}G_F N_e \ , \\ B &= \Delta M^2/4E\sin(2\theta) \ , \\ C &= \Delta M^2/4E\cos(2\theta) \ , \end{aligned} \tag{4}$$

and $\Delta M^2 = M_2{}^2 - M_1{}^2$. Resonant enhancement occurs when $A=C$ or

$$(\Delta M^2/2E)_{\text{res}}\cos(2\theta) = \sqrt{2}G_F N_e \ .$$

My calculations agree with the calculations of others[3–5] in that the oscillation enhancement occurs for neutrinos only if the lighter neutrino is primarily ν_e and the heavier neutrino is mostly ν_μ. The enhancement becomes a suppression if the ν_e is the heavier neutrino. The $\sqrt{2}G_F N_e$ term, however, has the opposite sign for antineutrino oscillations. If we cannot enhance oscillations in the neutrino sector, we can enhance oscillations in the antineutrino sector. Subsequently, I assume that the enhancement occurs for neutrinos; however, all arguments would apply equally well to antineutrinos if the sign of ΔM^2 were changed.

Equation (3) implies that I am considering only $\nu_e \leftrightarrow \nu_\mu$ oscillations. This is only partly true. Most of the following discussion is equally valid for $\nu_e \leftrightarrow \nu_\tau$ oscillations. Because ν_τ's do not have sufficient energy to be detected, however, the only detectable effect of $\nu_e \leftrightarrow \nu_\tau$ oscillations is a relatively small decrease in the ν_e count rate. In contrast, the $\nu_e \leftrightarrow \nu_\mu$ oscillations cause the numerically superior ν_μ's to convert to ν_e's, causing an anomalously high ν_e/ν_μ ratio for upward-going neutrinos.

II. TERRESTRIAL ENHANCEMENT

The electron density of the earth is far from constant; it varies from 6.1 mol/cm^3 at the center to 1.6 mol/cm^3 near the surface. In particular, there is a sharp discontinuity at the core/mantle boundary at $R=3500$ km. I calculated these densities by taking polynomial fits for different regions for the mass density and multiplying by 0.472 and 0.495 for the core and mantle, respectively (see Fig. 1).[6] Since $\sqrt{2}G_F N_A = 0.387\times10^{-3}$ cm^3/km mol, oscillation enhancement is likely to occur on distances of 1000 km or more. Since 1000 km is comparable to distances over which the electron density changes significantly, we can approximate the density changes as neither "sudden" nor "adiabatic." (This is fortunate, since in both limits no net conversion takes place.) I therefore de-

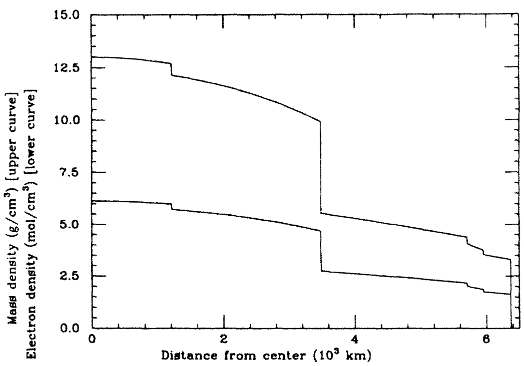

FIG. 1. Mass density and electron density of the earth. The lower curve was obtained by multiplying the upper curve by 0.478 and 0.494 for the core and mantle, respectively. The upper curve is based on polynomial fits from Stacey.

cided that the best way to get accurate results is by direct integration of Eq. (3) on a computer. Since 1 $\mathrm{eV}^2/\mathrm{GeV}=5.07/\mathrm{km}$, resonant enhancement occurs near $\Delta M^2/E \sim 10^{-3}$ $\mathrm{eV}^2/\mathrm{GeV}$.

I took the initial condition $\nu_e(0)=1$ and $\nu_\mu(0)=0$ and integrated Eq. (3) to find $|\nu_\mu(t)|^2$, the probability at an arbitrary position that the neutrino, if detected, would be a muon neutrino. Figure 2 shows $|\nu_\mu(t)|^2$ for $\Delta M^2/E=10^{-3}$ $\mathrm{eV}^2/\mathrm{GeV}$, near the core resonance value, and $\sin^2(2\theta)=0.04$, where the core is approximately half a wavelength long. Because of the small $\sin^2(2\theta)$ value, the resonance peak is relatively narrow, and very little oscillation occurs in the mantle, where the electron density is far from its resonance value. The neutrino is almost pure ν_e until it reaches the core, where $|\nu_\mu|^2$ climbs to nearly 0.8, and then the neutrino undergoes small oscillations, emerging at $|\nu_\mu|^2=0.67$. The probability for conversion is

$$P_{\nu_e\to\nu_\mu}=0.67\ . \tag{5}$$

By unitarity, $P_{\nu_\mu\to\nu_e}=P_{\nu_e\to\nu_\mu}$.

$P_{\nu_e\to\nu_\mu}$ is needed for several $\Delta M^2/E$ and $\sin^2(2\theta)$ values. I wrote several computer programs to calculate conversion probabilities. The first, called EARTH1, calculated $P_{\nu_e\to\nu_\mu}$ for $\Delta M^2/E=(0\text{–}2)\times 10^{-3}$ $\mathrm{eV}^2/\mathrm{GeV}$ and for $\sin^2(2\theta)=0.01\text{–}0.30$. Figure 3 shows some of the results. There are two clearly defined peaks; the left peak corresponds to $\Delta M^2/E$ appropriate for mantle resonance, and the right peak corresponds to core resonance. The smaller peaks at higher $\Delta M^2/E$ are due primarily to vacuum oscillations without enhancement.

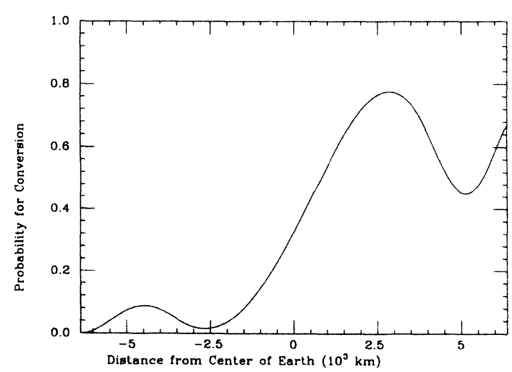

FIG. 2. Spatial dependence of conversion probability $|\nu_\mu(t)|^2$ for $\Delta M^2/E=1.0$ $\mathrm{eV}^2/\mathrm{GeV}$ and $\sin^2(2\theta)=0.04$.

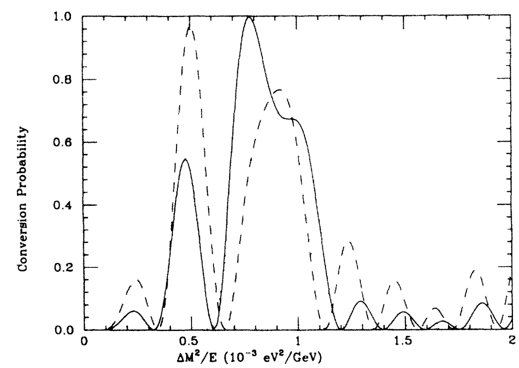

FIG. 3. Conversion probabilities for $\sin^2(2\theta)=0.01$ (dotted curve), $\sin^2(2\theta)=0.04$ (solid curve), and $\sin^2(2\theta)=0.10$ (dashed curve). These calculations are for $\phi=0°$.

All the results shown in Fig. 3 (and many that are not shown) are conversion probabilities for neutrinos following a path straight through the center of the earth, a path forming an angle $\phi=0°$ with the vertical (see Fig. 4). It is instructive (and necessary) to consider other angles as well. Figure 5 shows the results of EARTH1's calculations repeated at $\phi=40°$. At 40°, the path of the neutrinos misses the core entirely and no core enhancement occurs.

III. ATMOSPHERIC NEUTRINOS

I now consider the spectrum of atmospheric neutrinos. The primaries are cosmic rays, mostly protons, which collide with atmospheric nuclei to make pions and kaons. These decay primarily to $\mu\nu_\mu$, and then $\mu\to e\nu_e\nu_\mu$. If all these decays go to completion, then a naive calculation gives

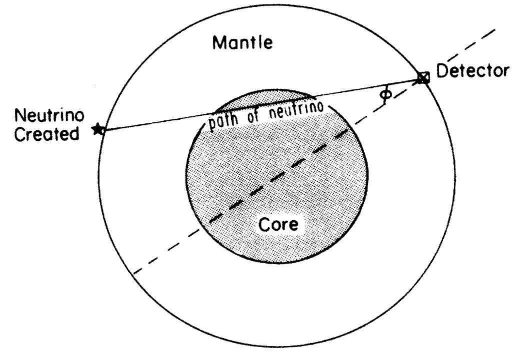

FIG. 4. Dependence of the path on the azimuth angle ϕ. Note that for $\phi > 33°$, no core effects are possible.

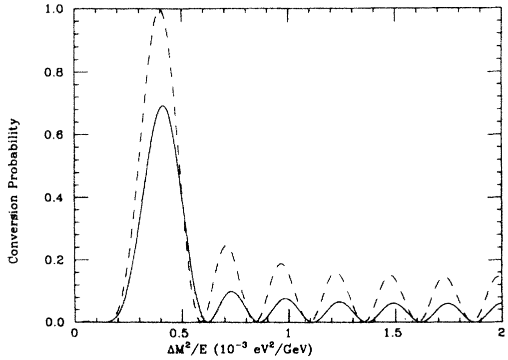

FIG. 5. Conversion probabilities for $\sin^2(2\theta)=0.01$ (dotted curve), $\sin^2(2\theta)=0.04$ (solid curve), and $\sin^2(2\theta)=0.10$ (dashed curve). These calculations are for $\phi=40°$.

$$\frac{\nu_e+\bar{\nu}_e}{\nu_\mu+\bar{\nu}_\mu}=0.5\ , \tag{6}$$

where ν_e stands for the number of ν_e, etc.

Several calculations exist in the literature for neutrino spectra.[7,8] These all agree approximately that the neutrino spectrum falls off as a power law:

$$dN/dE=CE^{-n}\ , \tag{7}$$

where $n\approx 2.7$. This simple formula completely ignores effects of geomagnetic latitude, azimuth angle, solar activity, etc. Geomagnetic effects are particularly troublesome because an up/down asymmetry in the ν_e/ν_μ ratio may be a signal, not of neutrino oscillations, but of geomagnetic effects. A complete analysis of any search for a ν_e/ν_μ asymmetry must take these effects into account.

The ν_e/ν_μ ratio is also a function of energy.[8] At high energies, some of the muons survive to reach the surface of the earth, where they quickly stop and decay into neutrinos too low in energy to detect. As energy increases, the ν_e/ν_μ ratio decreases (see Fig. 6), and at a typical energy of 0.4 GeV it is nearer to

$$\frac{\nu_e+\bar{\nu}_e}{\nu_\mu+\bar{\nu}_\mu}=0.4\ . \tag{8}$$

This ratio is used both for ν_e/ν_μ and $\bar{\nu}_e/\bar{\nu}_\mu$; this is appropriate because the ratios are approximately equal.[9] I ignore the energy dependence of these ratios.

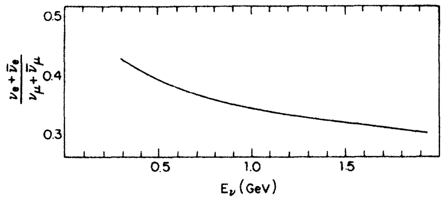

FIG. 6. The energy dependence of $(\nu_e+\bar{\nu}_e)/(\nu_\mu+\bar{\nu}_\mu)$ from Gaisser. Though the ratio is not constant, it does stay near the value 0.4.

IV. DETECTORS

The detectors used to search for terrestrial-length neutrino oscillations are intended as proton-decay detectors. Two types of events might be used to search for neutrino oscillations.

First, muon neutrinos can interact with the rock immediately beneath the detectors to produce muons, which then pass through the detectors. Unfortunately, these events are spread over a wide range of energies, which covers 2 or 3 orders of magnitude.[10,11] Because oscillation enhancement occurs only over a range of a factor of 2 or 3 and because the muon energies are not measured, these oscillations are difficult to detect. Furthermore, the oscillations can cause only a relatively small decrease in the ν_μ's (compared to a large increase in ν_e's), and statistics are poor. Finally, because only the $\nu_\mu+\bar{\nu}_\mu$ flux is measured, systematic errors in the total neutrino flux at different latitudes cannot be eliminated.

The other method for searching involves contained events, in which ν_e's and ν_μ's interact within the detector itself, primarily through weak-current events. These events produce electrons and muons which are detected by Cherenkov radiation. The cross sections (and the detector efficiencies) are roughly proportional to energy.[9] The detector efficiency drops drastically at low energies, approaching zero near 0.2 GeV. A reasonable approximation for the effective cross section is

$$\begin{aligned}&\sigma(E)=C(E-0.2\text{ GeV})\ ,\\&\sigma(E)dN/dE=(E-0.2\text{ GeV})E^{-2.7}\ .\end{aligned} \tag{9}$$

This approximation is imprecise because it implies that detection efficiency is the same for ν_e and ν_μ. This is inaccurate at low energies, because kinematics makes muons from ν_μ's less energetic than electrons from ν_e's of comparable energy. The detection efficiency for the ν_e's is much better at low energies, and the detected ν_e/ν_μ ratio may exceed 0.5 (Ref. 9). The relative suppression of ν_μ's, however, should be the same for upward- and downward-going neutrinos, and the errors in this approximation should largely cancel out.

V. RESULTS

Because of the poor statistics involved (approximately one event/day), it is highly desirable to integrate neutrino oscillations over as wide an angle as possible. I nonetheless want to stop the integration at a point where the oscillations are disappearing. For $\sin^2(2\theta)\leq 0.30$, the effects are quite small for $\phi\geq 70°$ (but still large at 60°) and 70° is a reasonable cutoff. With a cutoff of 70°, a downward-directed cone covers a solid angle of $4\pi\times 0.342$, and more than a third of the neutrinos passing through a given volume can be included.

My simple spectrum and detector efficiency given by (9) has no angular dependence, and the results from EARTH1 can be averaged simply over solid angle. This task is performed by the program EARTH2 (Ref. 12). Some of the results are shown in Fig. 7.

For $\Delta M^2/E\rightarrow\infty$, the earth's effect on neutrinos is

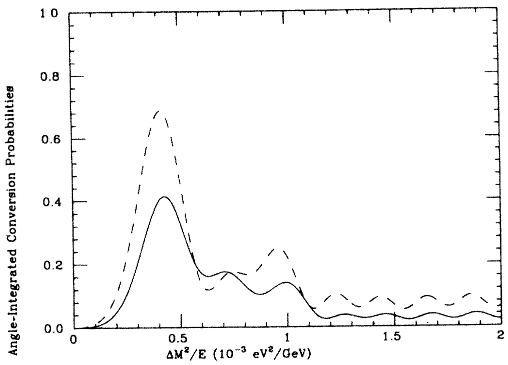

FIG. 7. Conversion probabilities averaged over solid angle from 0° to 70° for $\sin^2(2\theta)=0.01$ (dotted curve), $\sin^2(2\theta)=0.04$ (solid curve), and $\sin^2(2\theta)=0.10$ (dashed curve).

negligible, and conversion probabilities approach the vacuum-wavelength-averaged value of $\sin^2(2\theta)/2$. It is clear from Fig. 7 that these probabilities are near this value for $\Delta M^2/E \sim 2\times10^{-3}$ eV^2/GeV, and this vacuum conversion probability was used by subsequent programs whenever $\Delta M^2/E > 2\times10^{-3}$ eV^2/GeV. This leads to conservative estimates because terrestrial enhancement tends to increase this value. Because the core subtends a small solid angle as viewed from the surface, core effects are small compared to mantle effects.

Program EARTH3 integrates the results of EARTH2 with the spectrum and efficiency given by (9), as shown in Fig. 8. Because the conversion for the high-energy tail of the spectrum is effectively set to zero by the nature of the computer integration, these estimates tend to be a little conservative, especially for large ΔM^2. If all the initial neutrinos were ν_μ's, then Fig. 8 would represent the measured value for $\nu_e/(\nu_\mu+\nu_e)$.

Now I discuss whether it is necessary to specialize to $\nu_e \leftrightarrow \nu_\mu$ oscillations. If the oscillations are $\nu_e \leftrightarrow \nu_\tau$, then Fig. 8 shows the fraction of electron neutrinos that would fail to be detected; ν_μ's would not be affected at all. Thus, $(\nu_e/\nu_\mu)_{\rm up}/(\nu_e/\nu_\mu)_{\rm down}$ would vary from its null result by 20% or less. Random errors probably can be brought as low as 10% or less, but it is questionable whether systematic errors can be brought this low. Actual neutrino spectra are quite difficult to use to determine this ratio much better than 20%, and it may be difficult to detect $\nu_e \leftrightarrow \nu_\tau$ oscillations by this method.

$\nu_e \leftrightarrow \nu_\mu$ oscillations are much easier to detect. Because the ν_μ's predominate, a conversion of this type can cause a huge increase in the number of ν_e's and also a small decrease in the number of ν_μ's, greatly increasing the $(\nu_e/\nu_\mu)_{\rm up}$ ratio. If the created ratio is $\nu_e/\nu_\mu=\frac{2}{5}$ and the spectrum- and angle-averaged conversion probability is P, then the measured ratio is

$$\left[\frac{\nu_e}{\nu_\mu}\right]_{\rm up}=\frac{2(1-P)+5P}{5(1-P)+2P},\quad \left[\frac{\nu_e}{\nu_\mu}\right]_{\rm down}=\frac{2}{5}\,. \tag{10}$$

By comparing the results of EARTH3 with the formulas above, we can find how the $(\nu_e/\nu_\mu)_{\rm up}$ and $(\nu_e/\nu_\mu)_{\rm down}$ ratios compare for arbitrary $\sin^2(2\theta)$ and ΔM^2 values. If experimental limits can be placed on these ratios then limits can be placed on the $\sin^2(2\theta)$-ΔM^2 plane. The results are shown in Fig. 9.

If the discrepancy between the up and down ratios can be determined to an accuracy of 50% or better, and no anomaly is found, then a large region of the $\sin^2(2\theta)$-ΔM^2 plane can be excluded. Unfortunately, even at the 30% level, very little of the region is excluded for $\sin^2(2\theta)<0.05$. It is clear from Figs. 3 and 5 why this happens. At small $\sin^2(2\theta)$ values, core enhancement is very important, more important than mantle effects, and when EARTH2 integrates over such a large angle these core effects are washed out.

The only reason that I chose such a large angle is to decrease statistical errors. If sufficient data are available, it might be advisable to choose an angle cutoff of 30°, which is the size of the core. The results, as shown in Fig. 10, show that smaller values for $\sin^2(2\theta)$ can be investigated.

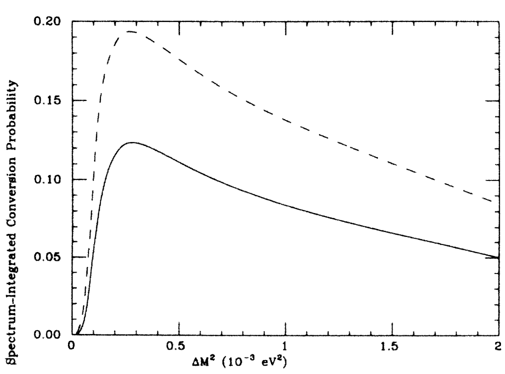

FIG. 8. Conversion probabilities averaged over solid angle from 0° to 70° and over the spectrum of Eq. (8) for $\sin^2(2\theta)=0.01$ (dotted curve), $\sin^2(2\theta)=0.04$ (solid curve), and $\sin^2(2\theta)=0.10$ (dashed curve).

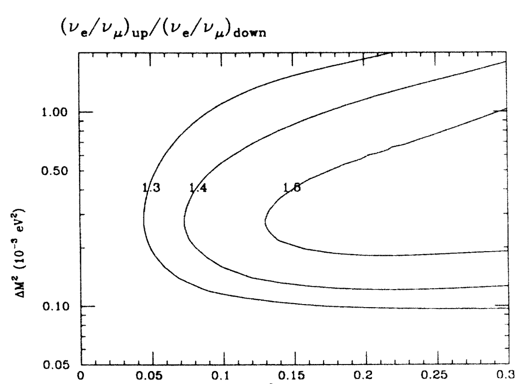

FIG. 9. Contours of constant $(\nu_e/\nu_\mu)_{\rm up}/(\nu_e/\nu_\mu)_{\rm down}$. Oscillations were averaged from 0° to 70°.

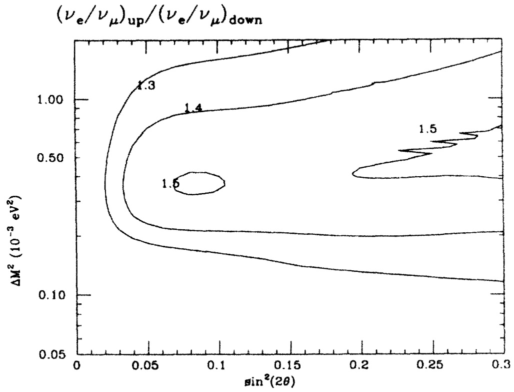

FIG. 10. Contours of constant $(\nu_e/\nu_\mu)_{up}/(\nu_e/\nu_\mu)_{down}$. Oscillations were averaged from 0° to 30°. The small fluctuations are an artifact of the calculations.

Such a cutoff tends to probe larger ΔM^2 values, because of the high core density.

I have been implying throughout that the oscillation enhancement occurs in the neutrino (rather than antineutrino) sector and that the ν_e and ν_μ rates are accounted for separately from the $\bar{\nu}_e$ and $\bar{\nu}_\mu$ rates. If the oscillation enhancements occur in the antineutrino sector (as they would if $M_{\nu_e} > M_{\nu_\mu}$), then all the calculations give the same results as before when we replace ν's with $\bar{\nu}$'s. These effects will be slightly more difficult to detect because the $\bar{\nu}$ count rate is about a factor of 2.5 lower than the ν count rate (primarily due to the lower cross section).[9]

Unfortunately, current experiments do not differentiate the charge of the lepton in the detector, so that only the $(\nu_e+\bar{\nu}_e)/(\nu_\mu+\bar{\nu}_\mu)$ ratio is measured. Because oscillations are enhanced in one sector and suppressed in the other, this makes the ratio much less sensitive to matter-enhanced neutrino oscillations. Assuming $\nu_e/\bar{\nu}_e=\nu_\mu/\bar{\nu}_\mu=\frac{5}{2}$ prior to oscillations, Eq. (10) must be modified to

$$\left[\frac{\nu_e+\bar{\nu}_e}{\nu_\mu+\bar{\nu}_\mu}\right]_{up}=\frac{2(1-P)+5P}{5(1-P)+2P}, \quad \left[\frac{\nu_e+\bar{\nu}_e}{\nu_\mu+\bar{\nu}_\mu}\right]_{down}=\frac{2}{5}, \tag{11}$$
$$P=\tfrac{5}{7}P_\nu+\tfrac{2}{7}P_{\bar{\nu}},$$

where P_ν and $P_{\bar{\nu}}$ are oscillation probabilities for neutrinos and antineutrinos, respectively.

If oscillations are enhanced for neutrinos, then we can approximate $P_{\bar{\nu}}=0$, and the curves labeled 1.3, 1.4, and 1.5 in Figs. 9 and 10 should be relabeled 1.209, 1.277, and 1.343, respectively. If they occur for antineutrinos and we set $P_\nu=0$, the curves should be relabeled 1.081, 1.106, and 1.130, respectively. If charges are not measured, it is still possible to detect oscillation enhancements, but only if they occur for neutrinos.

VI. OTHER TERRESTRIAL EFFECTS AND CONCLUSION

I have considered atmospheric neutrinos exclusively. It is reasonable to ask if any other neutrino experiments could be affected by terrestrial enhancement. I now consider the effects on solar neutrinos. Because the effects are large for $\Delta M^2/E \sim 10^{-6}$ eV^2/MeV, and solar neutrinos have energies of a few MeV, oscillation enhancement is likely to occur for $\Delta M^2=1-5\times10^{-6}$ eV^2. According to Ref. 4, this region may have large solar effects which would overwhelm the terrestrial effects.

The terrestrial oscillation-enhancement effect is small because the neutrinos arrive in a mixed state from the sun, and oscillation has less effect on a mixed state than on a pure state. Also, the earth affects only half the solar neutrinos, because only neutrinos detected at night pass through the earth. Furthermore, a small value of $\sin^2(2\theta)$ is required to obtain the necessary suppression of $\frac{1}{2}$ to $\frac{1}{4}$ in the ν_e count rate.[4] This value for $\sin^2(2\theta)$ is too small for substantial terrestrial oscillation enhancement.

Exploration of the earth by artificially generated neutrinos is another way to learn about terrestrially enhanced neutrino oscillations. This proposal is very similar to the original suggestion by Wolfenstein,[1] and also resembles other proposals along similar lines.[13] The engineering difficulties of building and transporting the necessary equipment are substantial. If it could be built, such a system would offer several advantages over atmospheric neutrinos; for example, such an instrument would not need to have its effects integrated over angle but could be set to an optimal angle to enhance the oscillations, perhaps by sitting on the core resonance. The spectrum would also be more sharply peaked, which would allow larger oscillation effects, and the energy could be tuned.

I would like to suggest what should be done to improve the search for oscillations in atmospheric neutrinos. Knowing the spectral and angular dependence of resulting neutrinos would be helpful to us in the study of both the earth and neutrinos. The determination of the charge of the leptons in the proton-decay experiments also would be helpful. In summary, terrestrial neutrino oscillations have many fascinating facets which deserve continued interest and study.

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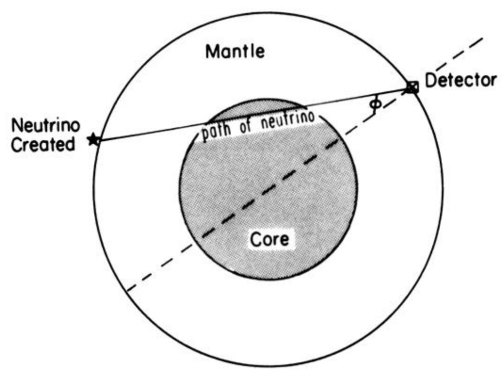

FIG. 4. Dependence of the path on the azimuth angle ϕ. Note that for $\phi > 33°$, no core effects are possible.