PHYSICAL REVIEW D VOLUME 34, NUMBER 5 1 SEPTEMBER 1986

Time-dependent *CP*-violation effects in B^0-$\bar{B}^0$ systems

Isard Dunietz and Jonathan L. Rosner
The Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637
(Received 9 May 1986)

Large time-dependent partial-rate asymmetries associated with *CP* violation are predicted for the decays of B_s ($\equiv\bar{b}s$) and B_d ($\equiv\bar{b}d$) to certain two-body final states (such as $D^+\pi^-$). Tagging on an accompanying charged b-flavored particle ($B_u^-=b\bar{u}$) or identified baryon ($\Lambda_b\equiv bud$) permits the identification of the initial flavor (B or $\bar{B}$) of the decaying particle. The asymmetries are contrasted with the $B_d\to\psi K_S$ asymmetry discussed earlier in the literature.

I. INTRODUCTION

For more than twenty years, the only observed evidence for *CP* violation has come from the kaon system. While heavy-quark (c,b) decays are expected to manifest *CP* violation as well, the expected asymmetries are generally small except in processes with low rates.[1]

Efforts are now under way to gain large samples of charmed and b-flavored particles by identifying their decays in flight. It appears that at least 10^7 $b\bar{b}$ pairs will be needed to address *any* question of *CP* violation for b quarks.[2] Here we would like to call attention to a type of asymmetry in the neutral B-$\bar{B}$ systems which will be spectacular once suitable rates have been attained. Lest the reader despair, we recall that less than twenty years passed between discussions of individual kaon decays and experiments involving more than 10^8 such events.

We find large time-dependent partial-rate asymmetries in suitably chosen exclusive B decays, such as states which are initially B_s ($\equiv\bar{b}s$) or B_d ($\equiv\bar{b}d$)$\to D^+\pi^-$. Since the identification of such decays will rely crucially on detection of proper lifetimes of order 10^{-12} s, it will not be difficult to follow the time evolution of these decays once they are seen at all.

We begin (Sec. II) with some general considerations and questions of notation. We then specialize (Sec. III) to the magnitudes of *CP*-violating effects expected in the six-quark Kobayashi-Maskawa[3] (KM) formalism, and show how these are manifested in time-integrated asymmetries (Sec. IV) and time dependences (Sec. V) of observed final states. The question of flavor tagging is discussed in Sec. VI. We shall find that the best method is to identify a charged $B_u^\pm$ or Λ_b opposite the neutral meson in question. A short comment on $B_c^\pm$ asymmetries is made in Sec. VII. Our conclusions are contained in Sec. VIII.

II. DEFINITIONS AND FORMALISM

An arbitrary neutral b-flavored state $a\,|B^0\rangle+b\,|\bar{B}^0\rangle$ is governed by the time-dependent Schrödinger equation

$$i\frac{d}{dt}\begin{bmatrix}a\\b\end{bmatrix}=H\begin{bmatrix}a\\b\end{bmatrix}\equiv\left[M-\frac{i}{2}\Gamma\right]\begin{bmatrix}a\\b\end{bmatrix}. \tag{1}$$

Here M and Γ are 2×2 matrices, with $M=M^\dagger$, $\Gamma=\Gamma^\dagger$. *CPT* invariance guarantees $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$. We assume *CPT* throughout to obtain the eigenstates of the neutral b-flavored mass matrix as

$$|B_L\rangle=p\,|B^0\rangle+q\,|\bar{B}^0\rangle\,, \tag{2a}$$

$$|B_H\rangle=p\,|B^0\rangle-q\,|\bar{B}^0\rangle\,, \tag{2b}$$

with eigenvalues ($L=$"light," $H=$"heavy")

$$\mu_{L,H}=m_{L,H}-\frac{i}{2}\gamma_{L,H}\,. \tag{3}$$

Here $m_{L,H}$ and $\gamma_{L,H}$ denote the masses and decay widths of $B_{L,H}$. Defining

$$\Delta\mu\equiv\mu_H-\mu_L\equiv\Delta m-\frac{i}{2}\Delta\gamma\,, \tag{4}$$

we have

$$\frac{q}{p}=\frac{-\Delta\mu}{2(M_{12}-i\Gamma_{12}/2)}=\pm\left[\frac{M_{12}^*-i\Gamma_{12}^*/2}{M_{12}-i\Gamma_{12}/2}\right]^{1/2}. \tag{5}$$

The time evolution of an initially pure $|B^0(t=0)\rangle\equiv|B^0\rangle$ or $|\bar{B}^0(t=0)\rangle\equiv|\bar{B}^0\rangle$ is

$$|B^0_{\rm phys}(t)\rangle=f_+(t)\,|B^0\rangle+\frac{q}{p}f_-(t)\,|\bar{B}^0\rangle\,, \tag{6a}$$

$$|\bar{B}^0_{\rm phys}(t)\rangle=\frac{p}{q}f_-(t)\,|B^0\rangle+f_+(t)\,|\bar{B}^0\rangle\,, \tag{6b}$$

where

$$f_+(t)\equiv e^{-i[(m_L+m_H)/2]t}e^{-(\gamma/2)t}\cos\left[\frac{\Delta\mu\,t}{2}\right], \tag{7a}$$

$$f_-(t)\equiv e^{-i[(m_L+m_H)/2]t}e^{-(\gamma/2)t}\,i\sin\left[\frac{\Delta\mu\,t}{2}\right], \tag{7b}$$

and we have defined

$$\gamma\equiv(\gamma_L+\gamma_H)/2\,. \tag{8}$$

We now consider nonleptonic final states f such that both a pure B^0 and a pure $\bar{B}^0$ can decay to them:[4]

$$\begin{tikzcd}[row sep=small] B^0 \arrow[dr] & \\ & f,\bar{f}\,, \\ \bar{B}^0 \arrow[ur] & \end{tikzcd} \tag{9}$$

where we define

$$|\bar{f}\rangle \equiv CP\,|f\rangle \ . \tag{10}$$

The decay amplitude of a time-evolved B^0 ($\bar{B}^0$) into the final state f ($\bar{f}$) is

$$\langle f\,|\,B^0_{\text{phys}}(t)\rangle = f_+(t)\langle f\,|\,B^0\rangle + \frac{q}{p} f_-(t)\langle f\,|\,\bar{B}^0\rangle \ , \tag{11a}$$

$$\langle \bar{f}\,|\,\bar{B}^0_{\text{phys}}(t)\rangle = \frac{p}{q} f_-(t)\langle \bar{f}\,|\,B^0\rangle + f_+(t)\langle \bar{f}\,|\,\bar{B}^0\rangle \ . \tag{11b}$$

We observe that interference occurs for $t>0$ due to mixing. Let us define

$$x \equiv \frac{\langle f\,|\,\bar{B}^0\rangle}{\langle f\,|\,B^0\rangle} \ , \tag{12a}$$

$$\bar{x} \equiv \frac{\langle \bar{f}\,|\,B^0\rangle}{\langle \bar{f}\,|\,\bar{B}^0\rangle} \ , \tag{12b}$$

$$\lambda \equiv \frac{q}{p} x, \quad \bar{\lambda} \equiv \frac{p}{q}\bar{x} \ . \tag{13}$$

It is important to realize that the phases of λ and $\bar{\lambda}$ do not depend on arbitrary phase conventions.[5] This may be visualized by noting that they are "rephasing invariants" in the sense of Ref. 6.

We then obtain the time-dependent rate for an initially pure B^0 to decay to a final state:

$$\Gamma(B^0_{\text{phys}}(t)\to f) = |\langle f\,|\,B^0\rangle|^2 e^{-\gamma t}\left[\left|\cos\frac{\Delta\mu\, t}{2}\right|^2 + |\lambda|^2\left|\sin\frac{\Delta\mu\, t}{2}\right|^2 - 2\,\mathrm{Im}\left[\lambda \sin\frac{\Delta\mu\, t}{2}\cos^*\frac{\Delta\mu\, t}{2}\right]\right] \tag{14a}$$

and, for the $\bar{B}^0$ into $\bar{f}$,

$$\Gamma(\bar{B}^0_{\text{phys}}(t)\to \bar{f}) = |\langle \bar{f}\,|\,\bar{B}^0\rangle|^2 e^{-\gamma t}\left[\left|\cos\frac{\Delta\mu\, t}{2}\right|^2 + |\bar{\lambda}|^2\left|\sin\frac{\Delta\mu\, t}{2}\right|^2 - 2\,\mathrm{Im}\left[\bar{\lambda} \sin\frac{\Delta\mu\, t}{2}\cos^*\frac{\Delta\mu\, t}{2}\right]\right] \ . \tag{14b}$$

Any difference between these two expressions signals CP violation.

We stress that the differences between Eqs. (14a) and (14b) can arise (for $t>0$) even in cases when the "pure" state (i.e., $t=0$) decays $B^0\to f$ and $\bar{B}^0\to\bar{f}$ do not exhibit CP violation. Thus, it may frequently happen that $|\langle f\,|\,B^0\rangle|^2 = |\langle \bar{f}\,|\,\bar{B}^0\rangle|^2$ (for example, as we will show below, when a single KM phase governs the decay). Nonetheless, such processes turn out to be useful for CP-violation studies when their time dependences are followed.

The absence of CP violation implies

$$|\langle f\,|\,B^0\rangle| = |\langle \bar{f}\,|\,\bar{B}^0\rangle| \ , \tag{15}$$

$$\left|\frac{q}{p}\right| = 1 \ , \tag{16}$$

$$\bar{\lambda} = \lambda \ . \tag{17}$$

Equation (15) holds whenever the pure-state decay amplitudes exhibit no CP violation (sometimes known as *direct* CP violation). Equation (16) follows from the fact that $|B_L\rangle, |B_H\rangle$ are CP eigenstates when CP is conserved, so, for example,

$$\begin{aligned} CP\,|B_L\rangle &= CP(p\,|B^0\rangle + q\,|\bar{B}^0\rangle) \\ &= p\,|\bar{B}^0\rangle + q\,|B^0\rangle \\ &= \pm(p\,|B^0\rangle + q\,|\bar{B}^0\rangle) \ , \end{aligned} \tag{18}$$

where here we shall define

$$CP\,|\,B^0\rangle = |\bar{B}^0\rangle, \quad CP\,|\bar{B}^0\rangle = |B^0\rangle \ . \tag{19}$$

Thus, with the convention (19), we would have $p=\pm q$. With any other convention, one still obtains (16). Deviations from Eq. (16) are sometimes known as *indirect CP* violation. Equation (17) is a direct consequence of Eqs. (10), (12), (13), (15), and (16) in the limit of CP conservation. Again, it is independent of the phase convention (19).

Equations (14) always reduce to exponential decay laws for $f=\pm\bar{f}$ a CP eigenstate, when CP is conserved.[7] In that case, $\bar{\lambda}=\lambda=\lambda^*$. If f and $\bar{f}$ are not CP eigenstates, Eqs. (14) need not describe exponential decays. The most striking familiar example is the time distribution of $\pi^+e^-\bar{\nu}_e$ events arising from an initial K^0 (in the absence of CP violation). At $t=0$, this final state cannot appear at all (as a result of the $\Delta S=\Delta Q$ rule). At later times, the K_S and K_L components evolve separately, and the $\pi^+e^-\bar{\nu}_e$ final state can appear. The $\bar{K}^0\to\pi^-e^+\nu_e$ time distribution will be exactly the same as that for $K^0\to\pi^+e^-\bar{\nu}_e$ when CP is conserved.

It is important to notice that $\mathrm{Im}\lambda\neq 0$ *by itself* is not evidence for CP violation, if f is not a CP eigenstate. In the convention of Eq. (19) where q/p is real when CP is conserved, the phase of λ or $\bar{\lambda}$ arises from the phase of x or $\bar{x}$ in Eqs. (12). There is no reason for these phases to vanish, in general, as a result of final-state interactions.

In general one expects for decays of b-flavored mesons that $|\Delta\gamma| \ll |\Delta m|$. This is very different from the kaonic situation, where the difference between the 2π and 3π channels causes appreciable differences in the lifetimes of the mass eigenstates. Moreover, specific calculations[1,2,4] in the KM framework[3] based on the box diagram imply $|\Gamma_{12}| \ll |M_{12}|$. Henceforth we shall assume $\gamma_H\approx\gamma_L\approx\gamma$, $\Delta\gamma/\Delta m \ll 1$, and, as a result of (5), $|q/p|\approx 1$. Specifically, it is found that

$$\frac{q}{p} = \frac{\xi_t^*}{\xi_t} \ , \tag{20}$$

where $\xi_t = V_{tb} V_{t\alpha}^*$ and $\alpha = d$ or s for the B_d or B_s system. Moreover,

$$\Delta m \equiv m_H - m_L \approx 2 \mid M_{12} \mid \ . \tag{21}$$

Then the complex mass $\Delta\mu \approx \Delta m$ is almost a real number.

A measure of mixing is $\Delta m/\gamma$. Before the particle can decay away (within a time scale of γ^{-1}), some mixing of order $\sin(\Delta m/2\gamma)$, due to the term $f_-(t)$ in Eq. (7b), has occurred. Conventional estimates[5] lead to $(\Delta m/\gamma)_{B_s} \sim 0.5$–$2$ and $(\Delta m/\gamma)_{B_d} \sim 10^{-2}$–$10^{-1}$.

III. EXPECTATIONS IN KOBAYASHI-MASKAWA FRAMEWORK

Henceforth we shall assume that the KM matrix explains the observed CP violation in the kaon system. Throughout we use the following convention for the KM matrix:[3,8]

$$V = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix} . \tag{22}$$

In addition, from the recent b-quark lifetime and $\Gamma(b\to u)/\Gamma(b\to c)$ measurements, we have (in standard KM phase convention) $s_\delta \sim O(1)$, s_δ positive (for a positive parameter B describing the ratio of $\langle K^0 \mid [\bar{d}\gamma_\mu(1-\gamma_5)s]^2 \mid \bar{K}^0\rangle$ to its vacuum-insertion value); $0.04 > s_3 > 0.01$ (where the lower limit depends on B), and $s_2 \sim 0.05$ (from the b-quark lifetime).

When only a single KM combination contributes to $B^0 \to f$, and another to $\bar{B}^0 \to f$, we have

$$\mid \langle f \mid B^0 \rangle \mid = \mid \langle \bar{f} \mid \bar{B}^0 \rangle \mid , \quad \mid \langle \bar{f} \mid B^0 \rangle \mid = \mid \langle f \mid \bar{B}^0 \rangle \mid \ . \tag{23}$$

The proof is as follows.[9] A general amplitude is given by

$$\langle f \mid B^0 \rangle = \sum_i G_i a_i e^{i\alpha_i} \ , \tag{24a}$$

$$\langle \bar{f} \mid \bar{B}^0 \rangle = \sum_i G_i^* a_i e^{i\alpha_i} \ . \tag{24b}$$

Here G_i are KM combinations; a_i are real kinematic factors; and α_i are final-state phases. If all the G_i are equal they can be factored out and it is clear that Eq. (23) holds.

As a direct consequence of (23), we obtain

$$\mid \bar{x} \mid = \mid x \mid \ . \tag{25}$$

Equations (23) and (25) hold under the above circumstances even though different strong channels (rendering different final-state phases) exist. Thus, for example, $B_s \to D^0\phi, D^+\pi^-, F^+K^-$ are all governed by the single combination $V_{ub}^* V_{cs}$; $B_s \to \bar{D}^0\phi, D^-\pi^+, F^-K^+$ are all governed by $V_{cb}^* V_{us}$ in the absence of D^0-$\bar{D}^0$ mixing; $B_d \to D^+\pi^-, F^+K^-$ depend on $V_{ub}^* V_{cd}$; and $B_d \to D^-\pi^+$, F^-K^+ depend on $V_{cb}^* V_{ud}$. For all these processes, Eqs. (23) and (25) are exact.

For the process $B_d \to \psi K_S$ there is a highly dominant KM combination $V_{cb}^* V_{cs}$, the others being associated with a suppressed penguin graph and production of a $c\bar{c}$ or $t\bar{t}$ pair from the vacuum. Similarly, $B_d \to D^+D^-$ involves a dominant $V_{cb}^* V_{cs}$ contribution. For these last two processes, Eqs. (23) and (25) are an excellent approximation.

The processes $B_{s,d} \to \phi K_S$ are governed by penguin-graph amplitudes[10] for $\bar{b} \to \bar{d}$, $\bar{b} \to \bar{s}$, respectively. These involve one dominant KM contribution from a t-quark loop. The process $B_s \to \pi^0 K_S$ has a dominant KM combination $V_{ub}^* V_{ud}$ (the others being due to suppressed penguin graphs). Therefore, for these cases Eqs. (23) and (25) are only good approximations.

In this analysis, for simplicity, we shall neglect all differences among final-state phases (assuming that the decay proceeds via a single strong eigenchannel). Under these assumptions, $\bar{x} = x^*$. If, in addition, we assume $\mid q/p \mid = 1$, we obtain $\bar{\lambda} = \lambda^*$. Thus, under these simplifying assumptions, $\mathrm{Im}\lambda \neq 0$ will in fact be the signal for CP violation. [See Eq. (17).]

The time-integrated rate asymmetry is now[2]

$$C_f \equiv \frac{\Gamma(B^0_{\text{phys}} \to f) - \Gamma(\bar{B}^0_{\text{phys}} \to \bar{f})}{\Gamma(B^0_{\text{phys}} \to f) + \Gamma(\bar{B}^0_{\text{phys}} \to \bar{f})} = \frac{-2az\,\mathrm{Im}\lambda}{1 + a + \mid x \mid^2(1-a) + 2y\,\mathrm{Re}\lambda} \ . \tag{26}$$

Here B^0_{phys} and $\bar{B}^0_{\text{phys}}$ correspond to those states which were known initially to be B^0 and $\bar{B}^0$, respectively. (The way in which this information is obtained will be discussed further in Sec. VI.) The rates Γ correspond to integration of their decays (to f and $\bar{f}$, respectively) over all times. We have assumed the relation (23), and have defined

$$y \equiv \frac{\Delta\gamma}{2\gamma}, \quad z \equiv \frac{\Delta m}{\gamma}, \quad a \equiv \frac{1-y^2}{1+z^2} \ . \tag{27}$$

IV. TIME-INTEGRATED ASYMMETRY CALCULATIONS

We are now ready to display time-integrated asymmetries expected for a number of two-body neutral B decays. In this section we are recapitulating a number of results of Ref. 2, for the sake of a self-contained discussion.

An illustrative example is the $B_s \to D^+\pi^-$ process shown in Fig. 1. Here, as mentioned,

$$\langle D^+\pi^- \mid B_s \rangle \sim V_{cs} V_{ub}^* \ , \tag{28a}$$

$$\langle D^+\pi^- \mid \bar{B}_s \rangle \sim V_{us}^* V_{cb} \ . \tag{28b}$$

Therefore,

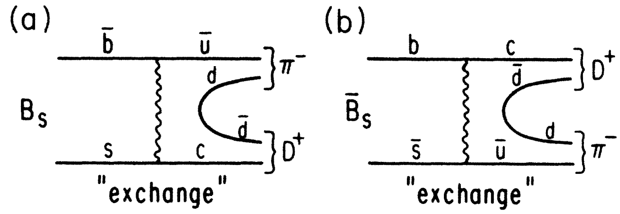

FIG. 1. Graphs governing decays to $D^+\pi^-$. (a) $B_s \to D^+\pi^-$; (b) $\bar{B}_s \to D^+\pi^-$.

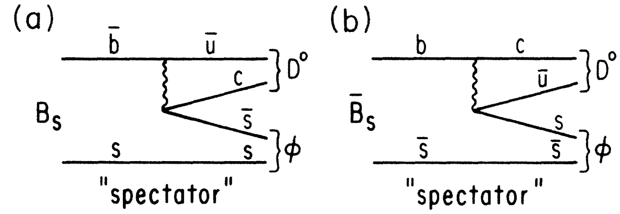

FIG. 2. Dominant graphs describing (a) $B_s \to D^0\phi$; (b) $\bar{B}_s \to D^0\phi$.

$$x = \frac{V_{us}^* V_{cb}}{V_{cs} V_{ub}^*} , \tag{29}$$

and we have, in this particular process,

$$|x|^2 = \frac{s_2^2 + s_3^2 + 2 s_2 s_3 c_\delta}{s_3^2}, \quad \mathrm{Im}\lambda = -\frac{s_2}{s_3} s_\delta . \tag{30}$$

In Eq. (30) we approximate cosines of the three KM angles by 1, but the phase δ is arbitrary.

The identical KM structures characterize the decays B_s (or $\bar{B}_s$)$\to D^+\pi^-, D^0\pi^0, D^0\phi, F^+K^-$ and corresponding processes involving D^*'s or F^*'s. However, the hadronic nature of these processes may be very different. For example, the decays $B_s, \bar{B}_s \to (D^+\pi^-$ or $D^0\pi^0)$ are expected to proceed primarily via an "exchange" graph (Fig. 1), while the $D^0\phi$ final state is generated mainly by what we shall call a "spectator" process (Fig. 2). The F^+K^- final state can receive contributions from both processes (Fig. 3). In all these processes we have neglected disconnected graphs (those forbidden by the Okubo-Zweig-Iizuka rule), which may be important for producing isosinglet pseudos-

TABLE I. KM estimates, for various quark subprocesses, of $x \equiv \langle f | \bar{B}^0 \rangle / \langle f | B^0 \rangle$ and of the parameter $\lambda \equiv (q/p)x$, whose phase governs *CP* violation.

Quark subprocess	$\lvert x \rvert^2$	$\mathrm{Im}\lambda$	$\mathrm{Re}\lambda$
		(a) B_d-$\bar{B}_d$ system	
$\bar{b} \to (\bar{u}u\bar{d}, \bar{u}u\bar{s})$	1	$-\sin 2\delta + 2 s_2 s_3 s_\delta$	$\cos 2\delta - 2 s_2 s_3 c_\delta$
$\bar{b} \to \bar{u}c\bar{d}$	$\frac{s_2^2+s_3^2+2s_2s_3c_\delta}{s_1^4 s_3^2}$	$\frac{(s_2+2s_3c_\delta)s_\delta}{s_1^2 s_3}$	$\frac{(-)(s_2c_\delta+s_3\cos 2\delta)}{s_1^2 s_3}$
$\bar{b} \to \bar{c}u\bar{d}$	$\frac{s_1^4 s_3^2}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_1^2 s_3(s_2+2s_3c_\delta)s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{(-)s_1^2 s_3(s_2c_\delta+s_3\cos 2\delta)}{s_2^2+s_3^2+2s_2s_3c_\delta}$
$\bar{b} \to (\bar{c}c\bar{s}, \bar{c}c\bar{d}, \bar{s})$	1	$\frac{(-)2s_3(s_2+s_3c_\delta)s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_2^2+s_3^2\cos 2\delta+2s_2s_3c_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$
$\bar{b} \to \bar{u}c\bar{s}$	$\frac{s_2^2+s_3^2+2s_2s_3c_\delta}{s_3^2}$	$(-)\left[\frac{s_2}{s_3}+2c_\delta\right]s_\delta$	$\frac{s_2}{s_3}c_\delta+\cos 2\delta$
$\bar{b} \to \bar{c}u\bar{s}$	$\frac{s_3^2}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$(-)\frac{s_3(s_2+2s_3c_\delta)s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_3(s_3\cos 2\delta+s_2c_\delta)}{s_2^2+s_3^2+2s_2s_3c_\delta}$
		(b) B_s-$\bar{B}_s$ system	
$\bar{b} \to (\bar{u}u\bar{d}, \bar{u}u\bar{s})$	1	$\frac{(-)2s_2s_\delta[s_3+s_2c_\delta]}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_2^2\cos 2\delta+s_3^2+2s_2s_3c_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$
$\bar{b} \to \bar{u}c\bar{s}$	$\frac{s_2^2+s_3^2+2s_2s_3c_\delta}{s_3^2}$	$(-)\frac{s_2}{s_3}s_\delta$	$\frac{s_2c_\delta+s_3}{s_3}$
$\bar{b} \to \bar{c}u\bar{s}$	$\frac{s_3^2}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{(-)s_2s_3s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_3(s_2c_\delta+s_3)}{s_2^2+s_3^2+2s_2s_3c_\delta}$
$\bar{b} \to (\bar{c}c\bar{s}, \bar{c}c\bar{d})$	1	$\frac{2s_1^2s_2s_3s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	1
$\bar{b} \to \bar{u}c\bar{d}$	$\frac{s_2^2+s_3^2+2s_2s_3c_\delta}{s_1^4 s_3^2}$	$\frac{s_2s_\delta}{s_1^2 s_3}$	$(-)\frac{s_3+s_2c_\delta}{s_1^2 s_3}$
$\bar{b} \to \bar{c}u\bar{d}$	$\frac{s_1^4 s_3^2}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_1^2s_2s_3s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$(-)\frac{s_1^2s_3(s_3+s_2c_\delta)}{s_2^2+s_3^2+2s_2s_3c_\delta}$
$\bar{b} \to \bar{d}$	1	$\frac{2s_3(s_2+s_3c_\delta)s_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$	$\frac{s_2^2+s_3^2\cos 2\delta+2s_2s_3c_\delta}{s_2^2+s_3^2+2s_2s_3c_\delta}$
$\bar{b} \to \bar{s}$	1	0	1

TABLE II. Numerical estimates, for various processes, of $x \equiv \langle f \mid \bar{B}^0 \rangle / \langle f \mid B^0 \rangle$ and of the parameter $\mathrm{Im}\lambda \equiv \mathrm{Im}[(q/p)x]$ governing CP violation. If the final state is a CP-even eigenstate or a PP (pseudoscalar$\times$pseudoscalar) one, λ is simply read off Table I. However, for CP-odd eigenstates and PV (pseudoscalar$\times$vector) final states the λ of this quark subprocess (Table I) must be multiplied by still another minus sign (consult the Appendix).

Quark subprocess	Final state	$\lvert x \rvert^2$	Imλ
	(a) B_d-$\bar{B}_d$ system		
$\bar{b}\to\bar{u}c\bar{d}$	$B_d\to D^+\pi^-, D^0\pi^0$, $F^+K^-, \ldots, (D^*\pi, \ldots)$	1756	37.5 (-37.5)
$\bar{b}\to\bar{c}u\bar{d}$	$B_d\to D^-\pi^+, \bar{D}^0\pi^0$, $F^-K^+, \ldots, (\bar{D}^*\pi, \ldots)$	5.7×10^{-4}	2×10^{-2} (-2×10^{-2})
$\bar{b}\to\bar{c}c\bar{s}, \bar{c}c\bar{d}, \bar{s}$	$B_d\to D^+D^-, (\psi K_s, \phi K_s)$	1	-0.8 (0.8)
	(b) B_s-$\bar{B}_s$ system		
$\bar{b}\to\bar{u}c\bar{s}$	$B_s\to D^+\pi^-, D^0\pi^0$, $F^+K^-, \ldots, (D^0\phi, D^*\pi, \ldots)$	5	-2 (2)
$\bar{b}\to\bar{c}u\bar{s}$	$B_s\to D^-\pi^+, \bar{D}^0\pi^0$, $F^-K^+, \ldots, (\bar{D}^0\phi, \bar{D}^*\pi, \ldots)$	0.2	-0.4 (0.4)
$\bar{b}\to\bar{u}u\bar{d}, \bar{u}u\bar{s}, \bar{d}$	$B_s\to\pi^0K_s, (\phi K_s)$	1	0.8 (-0.8)

calar mesons (like η and η') but are probably negligible elsewhere.

In comparing processes in Figs. 1(a), 2(a), and 3(a) (governed by $\bar{b}\to\bar{u}c\bar{s}$) with those in Figs. 1(b), 2(b), and 3(b) (governed by $b\to c\bar{u}s$) another point must be kept in mind. The incorporation of the final-state quarks into hadrons proceeds very similarly for Figs. 1(a) and 1(b), and for Figs. 2(a) and 2(b). However, for Figs. 3 this is only true for the exchange graphs. For the spectator graphs of Fig. 3, the hadronic behavior can be very different. The Fermi momenta of the constituent quarks of a b-flavored meson, in its rest frame, are small with respect to the meson's mass. Hence, when the spectator quark hadronizes into K^- [see Fig. 3(a)], it requires a much larger, and more unlikely, boost than when it hadronizes into F^+ [see Fig. 3(b)]. Notice that this "momentum mismatch" problem occurs only for those processes where both exchange and spectator contributions can occur.[11]

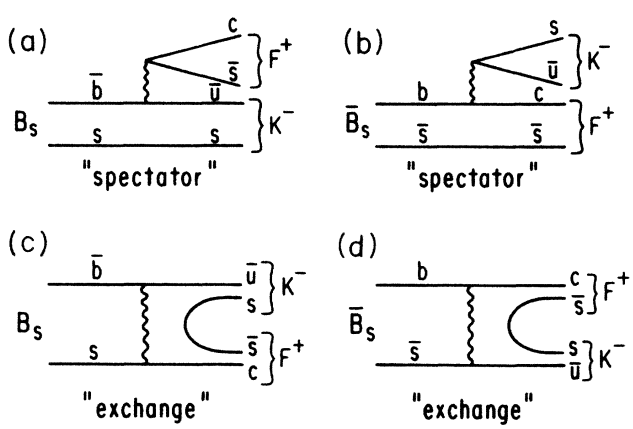

FIG. 3. Graphs describing (a) and (c) $B_s\to F^+K^-$; (b) and (d) $\bar{B}_s\to F^+K^-$.

Thus, in order to calculate x and λ for the processes in Figs. 1—3, we need varying amounts of hadronic information. The hadronization processes in Figs. 1(b) and 2(b) look very similar to those in Figs. 1(a) and 2(a), respectively. Thus, we expect that calculations of magnitudes and phases of x and λ based only on KM elements should be fairly reliable. This is not so for Figs. 3(a) and 3(b), for two reasons. First, two different types of contribution (with different *a priori* final-state phases) can occur. Second, the momentum mismatch problem just noted implies that a naive calculation of x and λ based only on KM elements may be far from the truth. We will nonetheless quote such results for illustrative purposes.

A lesser problem arises as a result of rescattering corrections. The $D^0\phi$ final state could, in principle, arise as a result of rescattering from F^+K^-. We expect this to be small in comparison with elastic $D^0\phi$ scattering only if the basic amplitude for $D^0\phi$ production is not much smaller than that for F^+K^-. This is a question which will be decided ultimately by experiment. We will ignore such rescattering corrections here.

In all our discussions, we shall ignore D^0-$\bar{D}^0$ mixing, for which experimental limits exist at the $\frac{1}{2}$—2% level.[12]

TABLE III. Assumed detection efficiencies (from Ref. 2).

Particle	Detection efficiency
$B^\pm$	50%
$D^\pm, D^0, \bar{D}^0$	10%
K^0	33%
$\pi^0, \pi^\pm$	100%
$K^\pm$	100%
η	40%
$F^\pm$	1%
ϕ	50%
ψ	14%

We shall point out where such effects could be important.

In Tables I and II we display, for various processes, the amplitude ratio x and the parameter $\mathrm{Im}\lambda$ governing *CP* violation. In the standard KM phase convention, when dealing with a final state that is an odd *CP* eigenstate, our tabulated values of λ must be multiplied by an additional minus sign,[13] as noted in the Appendix. The quark subprocesses in Tables I and II have wide applicability to other final states; we have indicated only a subset of them.

We now take representative values of the KM elements. Based on the discussion in Sec. III, we shall assume

$$s_1=0.231,\ s_2=0.05,\ s_3=0.025,\ s_\delta=1\ . \qquad (31)$$

We then obtain the values of $|x|^2$ and $\mathrm{Im}\lambda$ shown in Table II.

In order to evaluate the effectiveness of a given asymmetry measurement, we must estimate detection efficien-

TABLE IV. Predicted time-integrated partial-rate asymmetry C_f (Eq. 26) for various processes. The displayed signs of the asymmetry C_f (third column) are taken from the quark subprocess calculations for λ; for specific final states additional minus signs must be applied (consult the Appendix). The fourth and fifth columns contain an optimistic estimate of the pure branching ratio and number of $b\bar{b}$ events required ($N_{b\bar{b}}$).

Quark subprocess	Final state	C_f	$Br\ (B_{d_{\rm pure}}\to f)$	$N_{b\bar{b}}$
		(a) B_d-$\bar{B}_d$ system		
$\bar{b}\to\bar{u}u\bar{d}$	$B_d\to\pi^+\pi^-$	0.1	$<10^{-4}$	$>5.6\times10^7$
	$B_d\to K^+K^-$		$<10^{-4}$	$>5.6\times10^7$
$\bar{b}\to\bar{u}c\bar{d}$	$B_d\to D^0\pi^0$	-0.61	5×10^{-6}	7×10^7
	$B_d\to D^+\pi^-$		10^{-5}	3.5×10^7
	$B_d\to F^+K^-$		10^{-5}	3.5×10^8
	$B_d\to\psi D^0$		5×10^{-6}	4.9×10^8
$\bar{b}\to\bar{c}u\bar{d}$	$B_d\to\bar{D}^0\pi^0$	-2×10^{-3}	10^{-2}	1.4×10^{10}
	$B_d\to D^-\pi^+$		2×10^{-2}	7.0×10^9
	$B_d\to F^-K^+$		2×10^{-2}	7.0×10^{10}
	$B_d\to\psi\bar{D}^0$		10^{-2}	9.8×10^{10}
$\bar{b}\to\bar{c}c\bar{s},\bar{c}c\bar{d},\bar{s}$	$B_d\to D^+D^-$	0.08	10^{-3}	8.8×10^8
	$B_d\to F^+F^-$		10^{-4}	8.8×10^{11}
	$B_d\to\psi\phi$[a]		10^{-5}	1.3×10^{10}
	$B_d\to\psi K_s$		5×10^{-4}	1.9×10^8
	$B_d\to\phi K_s$		5×10^{-5}	5.3×10^8
$\bar{b}\to\bar{u}c\bar{s}$	$B_d\to D^0K_s$	0.19	10^{-4}	2.2×10^8
$\bar{b}\to\bar{c}u\bar{s}$	$B_d\to\bar{D}^0K_s$	0.039	5×10^{-4}	1.1×10^9
		(b) B_s-$\bar{B}_s$ system		
$\bar{b}\to(\bar{u}u\bar{d},\bar{u}u\bar{s})$	$B_s\to\pi^0K_S$	0.38	5×10^{-5}	2.0×10^7
	$B_s\to K^+K^-$		10^{-5}	6.7×10^7
	$B_s\to\pi^+\pi^-$		10^{-5}	6.7×10^7
$\bar{b}\to\bar{u}c\bar{s}$	$B_s\to D^0\phi$	0.56	2×10^{-4}	1.4×10^7
	$B_s\to F^+K^-$		2×10^{-4}	6.9×10^7
	$B_s\to D^+\pi^-$		2×10^{-4}	6.9×10^6
	$B_s\to D^0\pi^0$		10^{-4}	1.4×10^7
$\bar{b}\to\bar{c}u\bar{s}$	$B_s\to\bar{D}^0\phi$	0.23	10^{-3}	4.8×10^7
	$B_s\to F^-K^+$		10^{-3}	2.4×10^8
	$B_s\to D^-\pi^+$		10^{-3}	2.4×10^7
	$B_s\to\bar{D}^0\pi^0$		5×10^{-4}	4.8×10^7
$\bar{b}\to\bar{c}c\bar{s},\bar{c}c\bar{d}$	$B_s\to\psi\phi$[a]	-0.022	3×10^{-3}	1.1×10^9
	$B_s\to F^-F^+$		2×10^{-2}	1.2×10^{11}
	$B_s\to D^-D^+$		5×10^{-3}	4.7×10^9
	$B_s\to\psi K_s$		2.7×10^{-5}	9.0×10^{10}
$\bar{b}\to\bar{u}c\bar{d}$	$B_s\to D^0K_s$	-0.054	5.7×10^{-6}	3.0×10^8
$\bar{b}\to\bar{c}u\bar{d}$	$B_s\to\bar{D}^0K_s$	-0.013	10^{-2}	1.2×10^9
$\bar{b}\to\bar{d}$	$B_s\to\phi K_s$	-0.4	2.1×10^{-6}	8.4×10^8

[a]Reference 13.

cies for various final states. The results are shown in Table III.

The assumed efficiencies in Table III are obtained as follows. For $B^{\pm}$ we assume that the short track of a charged B can be identified with reasonable efficiency in a vertex detector. The D detection efficiences are based on reconstruction of exclusive final states (such as $K^-\pi^+, K^-\pi^+\pi^-\pi^+$ for D^0). The neutral kaon is K_S half of the time, and K_S decays to $\pi^+\pi^-$ $\frac{2}{3}$ of the time. We assume η is detected by its $\gamma\gamma$ decay. The $F^{\pm}$ is notably hard to reconstruct; 1% is only an estimate. We assume the ϕ is seen in its K^+K^- decay. The ψ can be seen in each of its leptonic modes e^+e^- and $\mu^+\mu^-$.

The expected asymmetries for various processes, and the number of events required to observe them, are shown in Table IV. $N_{b\bar{b}}$ denotes the number of $b\bar{b}$ pairs needed to observe a 3σ signature of the displayed asymmetry. To obtain $N_{b\bar{b}}$ we assume that the probability for the incoherent hadronization process $b\bar{b}\to B_{\bar{u}}B_qX$ ($q=d$ or s) is $\frac{4}{25}$ or $\frac{2}{25}$, respectively, since we have roughly $u\bar{u}:d\bar{d}:s\bar{s}\sim 2:2:1$ for the pair-creation probability of the light quarks. We note that $N_{b\bar{b}}$ is inversely proportional to the detection efficiency of the charged b-flavored hadron, used as our tagging device, and of the final state, all of which we took from Table III. Furthermore, the number of $b\bar{b}$ is inversely proportional to the branching ratio, an optimistic theoretical estimate of which is given in the fourth column of Table IV.

If experimentalists could increase the efficiency of observing $F^{\pm}$, some of the final states involving them could be viable alternatives to observe the asymmetries.

In Table IV we have assumed $(\Delta m/\gamma)_{B_d}=0.1$. The sole exception is the $\bar{b}\to\bar{u}c\bar{d}$ process where we take $(\Delta m/\gamma)=0.044$ for B_d. For all B_s quark subprocesses we assume $\Delta m/\gamma=0.8$. We shall explore the sensitivity of various processes to $\Delta m/\gamma$ presently.

For B_d decays, note the contrast between highly Cabibbo-suppressed modes $(D\pi, F^+K^0, \psi D^0)$ and Cabibbo-favored ones $(\bar{D}\pi, F^-K^+, \psi\bar{D}^0)$. Although the rates for the latter are much larger, the expected asymmetries are tiny, and so many more events are required to observe them. The highly Cabibbo-suppressed final states are particularly suited for observing the interference between two different KM combinations that is the signal of CP violation. Thus, the most favorable quark subprocess for observing CP asymmetry in B_d decays appears to be $\bar{b}\to\bar{u}c\bar{d}$. We first became aware of this fact through the work of Sachs, Ref. 4.

Certain B_d decay processes shown in Table IV involving final states which are eigenstates of CP are governed by the quark subprocess $\bar{b}\to\bar{c}c\bar{d}$, $\bar{b}\to\bar{c}c\bar{s}$, and $\bar{b}\to\bar{s}$. More events are required to observe a CP asymmetry for these cases. The case $\bar{b}\to\bar{c}c\bar{s},\bar{c}c\bar{d}$ has received considerable attention both in concrete studies of experimental possibilities,[15] and in many earlier theoretical studies.[4]

The asymmetries for B_d decays involving the quark subprocesses $\bar{b}\to\bar{u}c\bar{s}$ or $\bar{c}u\bar{s}$ involve final states D^0K_S or $\bar{D}^0K_S$, respectively. These appear to require no more events than the popular ψK_S decay in order to observe an asymmetry.

The B_d decays involving $\bar{b}\to\bar{u}u\bar{d}$ or $\bar{u}u\bar{s}$ could lead to asymmetries detectable with comparatively few events, in $\pi^+\pi^-$ or K^+K^- final states. Considerable uncertainty still exists in the possibility for observing such light mesons in the final state. If the decays $B_d\to\pi^+\pi^-$ or K^+K^- are observed at close to present upper limits, these processes could be quite promising. Note that the asymmetries for these processes would nearly vanish for $\delta=90°$; the values in Table IV are quoted for $\delta=45°$.

The best B_s decays for observing a CP asymmetry, in our opinion, are those dominated by the quark subprocess $\bar{b}\to\bar{u}c\bar{s}$, such as $B_s\to D^+\pi^-$. Here the contrast between this subprocess and that leading to $D^-\pi^+$, noted above for B_d, is less marked. The latter final state is produced via $\bar{b}\to\bar{c}u\bar{s}$, which also contains some Cabibbo suppression. As a result, appreciable asymmetries show up in both $D^+\pi^-$ and $D^-\pi^+$ final states.

The large (assumed) mixing for $B_s\leftrightarrow\bar{B}_s$ plays a role in generating detectable asymmetries. By contrast, for the $B_d\to D^+\pi^-$ case, both the direct amplitude and the $B_d\leftrightarrow\bar{B}_d$ mixing are small, but the large $\bar{B}_d\to D^+\pi^-$ amplitude leads to a detectable asymmetry (see Fig. 4).

The final states involving the subprocesses $\bar{b}\to\bar{c}c\bar{s}$ (such as $\bar{B}_d\to\psi\phi, D^-D^+$) are Cabibbo favored, so their expected rates could be appreciable. However, the expected asymmetries are extremely small.

The decay $B_s\to D^0K_S$ is the analogue of $B_d\to D^+\pi^-$. (Both involve the quark subprocesses $\bar{b}\to\bar{u}c\bar{d}$.) For the B_s, the large size of the assumed mixing amplitude actually suppresses the expected asymmetry. The decay $B_s\to\bar{D}^0K_S$ is Cabibbo favored and expected to have an appreciable branching ratio but small asymmetry.

The process $B_s\to\phi K_S$ would proceed via a penguin diagram ($b\to d$). Its expected asymmetry is large, and the main obstacle to its usefulness is the small expected branching ratio.

The subprocess $\bar{b}\to\bar{u}u\bar{d}$ leads to $B_s\to\pi^0K_s$ or ρ^0K_S, for which an appreciable asymmetry can be expected. A large part of the difficulty in studying CP violation in this decay will be seeing the final state at all. This comment in fact applies to many of the processes discussed here: Two-body decays, though expected to be rare, provide a wealth of information on CP violation, and should be searched for with high sensitivity.

The asymmetries in Table IV are quoted for values of

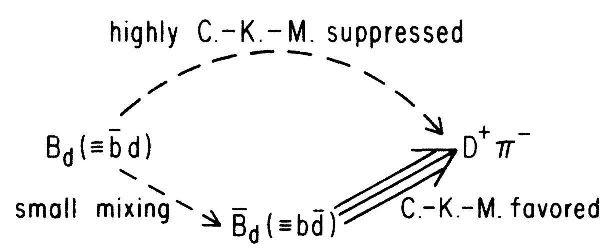

FIG. 4. Two decay routes of an initially pure B_d into $D^+\pi^-$ leading to a large CP asymmetry due to interference. B_d either decays directly, highly CKM suppressed, into $D^+\pi^-$, or B_d oscillates, with small mixing parameter $\Delta m/\gamma$ (the bottleneck), into $\bar{B}_d$ and then "rapidly" (CKM-allowed) decays into $D^+\pi^-$.

TABLE V. Maximal value of time-integrated asymmetry C_f and the corresponding mixing parameter z(max) for various processes. Here, $s_1=0.231$, $s_2=0.05$, $s_3=0.025$, and $s_\delta=1$; the sole exception is the $B_d, \bar{b}\to\bar{u}u\bar{d}, \bar{u}u\bar{s}$ processes, for which $\delta=45°$.

Process	z(max)	C_f(at max)
	(a) B_d-$\bar{B}_d$	
$\bar{b}\to\bar{u}u\bar{d},\bar{u}u\bar{s}$	1.00	0.500
$\bar{b}\to\bar{u}c\bar{d}$	3.3×10^{-2}	-0.635
$\bar{b}\to\bar{c}u\bar{d}$	1.41	-1.48×10^{-2}
$\bar{b}\to\bar{c}c\bar{s},\bar{c}c\bar{d},\bar{s}$	1.02	0.408
$\bar{b}\to\bar{u}c\bar{s}$	0.58	0.573
$\bar{b}\to\bar{c}u\bar{s}$	1.28	0.256
	(b) B_s-$\bar{B}_s$	
$\bar{b}\to\bar{u}u\bar{d},\bar{u}u\bar{s}$	0.98	0.393
$\bar{b}\to\bar{u}c\bar{s}$	0.58	0.583
$\bar{b}\to\bar{c}u\bar{s}$	1.30	0.260
$\bar{b}\to\bar{c}c\bar{s},\bar{c}c\bar{d}$	1.03	-2.22×10^{-2}
$\bar{b}\to\bar{u}c\bar{d}$	3.4×10^{-2}	-0.627
$\bar{b}\to\bar{c}u\bar{d}$	1.41	-1.48×10^{-2}
$\bar{b}\to\bar{d}$	1.02	-0.407

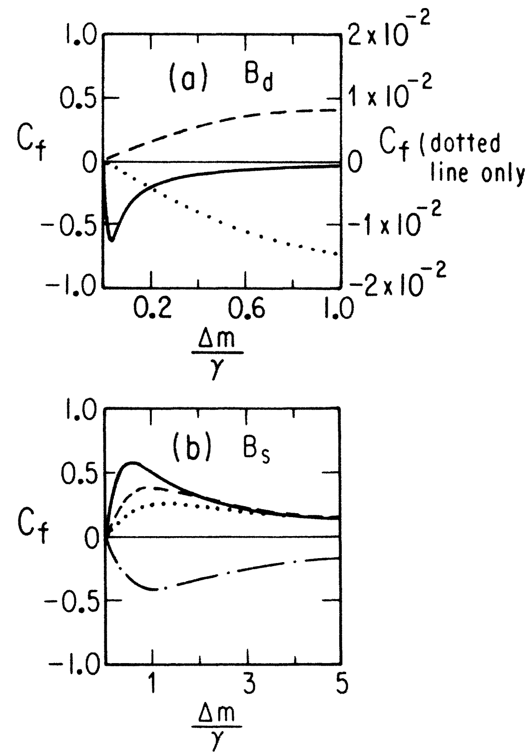

FIG. 5. Time-integrated asymmetries C_f [Eq. (26)] as functions of $\Delta m/\gamma$. (a) B_d decays are as follows. Solid curve: $\bar{b}\to\bar{u}c\bar{d}$ (left-hand scale); dotted curve: $\bar{b}\to\bar{c}u\bar{d}$ (right-hand scale); dashed curve: $\bar{b}\to(\bar{c}c\bar{s},\bar{c}c\bar{d},\bar{s})$ (left-hand scale). Most of the other processes have the slow behavior characteristic of $\bar{b}\to(\bar{c}c\bar{s},\bar{c}c\bar{d},\bar{s})$ (see also Table V). (b) B_s decays are as follows. Solid curve: $\bar{b}\to\bar{u}c\bar{s}$; dotted curve: $\bar{b}\to\bar{c}u\bar{s}$; dashed curve: $\bar{b}\to\bar{u}u\bar{d},\bar{u}u\bar{s}$; dash-dotted curve: $\bar{b}\to\bar{d}$. The highly suppressed process $\bar{b}\to\bar{u}c\bar{d}$ leads to a negative asymmetry which peaks at a value of $C_f=-0.63$ for $\Delta m/\gamma=3.4\times10^{-2}$ (see also Table V).

$\Delta m/\gamma$ which we believe to be reasonable, on the basis of our own[2] and other[4,5] calculations. Certain asymmetries behave in a very different manner from others as $\Delta m/\gamma$ is changed. This behavior is explored more fully in Ref. 2. Here we note that the asymmetries for the processes such as $B_d\to D^+\pi^-$ and $B_s\to D^0K_S$ dominated by $\bar{b}\to\bar{u}c\bar{d}$ peak at much lower values of $\Delta m/\gamma\approx0.033$ than others in B_d and B_s decays. These asymmetries, with the choice of a reasonable set of KM parameters [Eq. (31)] are shown as functions of $\Delta m/\gamma$ in Fig. 5.

In Table V we display, for various quark subprocesses, the mixing parameter z(max) which maximizes the asymmetry C_f defined in Eq. (26). Of course, the asymmetry is not the only quantity of interest with respect to the number of events required to perform a particular CP test. The branching ratio of a time-evolved B to a given final state also must be large enough to permit such a state to be observed. In Table VI we estimate the number of events required to see an asymmetry in selected processes as a function of the mixing parameter $\Delta m/\gamma$.

Clearly measurements of $\Delta m/\gamma$ for B_d and B_s systems will be an important first step toward identifying the best processes for seeing CP violation. Note that large mixing in the B_d-$\bar{B}_d$ system, while unlikely, could significantly enhance the prospects for useful experiments at the level of a few million $b\bar{b}$ pairs, for the ψK_S system discussed previously.[15]

TABLE VI. Event rate ($N_{b\bar{b}}$) as a function of mixing for selected final states.

Quark subprocess	Process	$\Delta m/\gamma$	$N_{b\bar{b}}$
$\bar{b}\to\bar{u}c\bar{d}$	$B_d\to D^+\pi^-$	3.3×10^{-2}	4.3×10^7
		4.4×10^{-2}	3.5×10^7
		0.1	3.4×10^7
		0.89	9.5×10^7
$\bar{b}\to(\bar{c}c\bar{s},\bar{c}c\bar{d},\bar{s})$	$B_d\to\psi K_S$	0.1	2×10^8
		0.54	9×10^6
		1.02	6×10^6
$\bar{b}\to\bar{u}c\bar{s}$	$B_d\to D^0K_S$	0.1	2.2×10^8
		0.58	1.2×10^7
		1.1	1.4×10^7

A final note about the neglect of D^0-$\bar{D}^0$ mixing is in order. The decay $B_{d,\mathrm{pure}}$ ($\equiv \bar{b}d$)$\to D^0\pi^0$ can proceed either via a highly KM-suppressed transition $\bar{b}\to\bar{u}c\bar{d}$, or via the favored $\bar{D}^0\pi^0$ final state, where the $\bar{D}^0$ then mixes with D^0. The same two routes affect the decay $B_{s,\mathrm{pure}}\equiv\bar{b}s\to D^0K_S$. The D^0-$\bar{D}^0$ mixing amplitude will then be important [and so the assumption (23), for example, will no longer be valid] if it exceeds (in magnitude) the value $|V_{ub}V_{cd}/V_{cb}V_{ud}|$, or $s_3 s_1{}^2/s_2 \approx (\text{few}\times 10^{-2})$. (Here we have assumed $s_2 > s_3$.) Present limits on the *square* of this quantity[12] are at the level of $\frac{1}{2}-1\,\%$, so that this possibility cannot yet be excluded. The conventional expectation for the short-distance contribution to this amplitude is $\approx 10^{-3}$, but long-distance contributions may be much more important. Similar cautionary remarks apply to the $D^0\phi$ system produced in B_s decays and D^0K_S in B_d decays, but only if D^0-$\bar{D}^0$ mixing is very close to present upper bounds.

V. TIME-DEPENDENT ASYMMETRIES

The observation of time-integrated asymmetries C_f [defined in Eq. (26)] is plagued with normalization problems, since the relative production rates of B and $\bar{B}$ cannot always be guaranteed to be measured carefully. (For example, they will differ in pp collisions.) Moreover, detection efficiencies for charge-conjugate final states may not always be identical (for example, when collisions occur in dense targets.) The measurement of time-*dependent* asymmetries[16] adds valuable information, such as different shapes of distributions, different slopes as a function of proper time, and so on, which can help to circumvent such normalization problems.

As examples of the kinds of behavior expected in the *absence* of *CP* violation, we remind the reader that when the final state is a *CP* eigenstate identical exponential decays are expected for B^0 and $\bar{B}^0$. For final states that are not *CP* eigenstates, we expect identical but nonexponential decay distributions for B^0 and $\bar{B}^0$, as illustrated in Fig. 6.

The signatures of *CP* violation in time-dependent asymmetries are varied and can be quite striking. A number of examples are shown in Figs. 7—12. Here all the parameters have been chosen as in Table II.

Perhaps the most spectacular behavior occurs for the decays $B_s\to D\pi$ (or $D^0\phi, D^*\pi, D\rho, F^+K^-,\ldots$), illustrated in Fig. 7, and for $B_d\to D\pi$ (or $D^*\pi, D\rho, F^+K^-,\ldots$), illustrated in Fig. 8. We have shown the patterns for various values of $\Delta m/\gamma$ to illustrate their sensitivity to this parameter. (We assume $\Delta m/\gamma$ will have been at least crudely measured by the time *CP* asymmetries are investigated.) The asymmetries are less spectacular for the slightly more Cabibbo-favored $B_s\to\bar{D}\pi$ (or $\bar{D}^0\phi, \bar{D}^*\pi, \bar{D}\rho, F^-K^+$) decays, as shown in Fig. 9, but they are still visible. For the Cabibbo-favored decays $B_d\to\bar{D}\pi$ (or $\bar{D}^*\pi, \bar{D}\rho, F^-K^+$) they are invisible (Fig. 10).

The $B_d\to\psi K_S$ decay, frequently discussed in the literature,[4,15] does not show a striking asymmetry for $\Delta m/\gamma = 0.1$ (Fig. 11). The time-dependent behavior is somewhat more interesting for $\Delta m/\gamma = 1.0$ (Fig. 12), but we regard such a value of $\Delta m/\gamma$ as an optimistic overestimate. On the other hand, for $B_s\to\pi^0K_S$ or ϕK_S, where

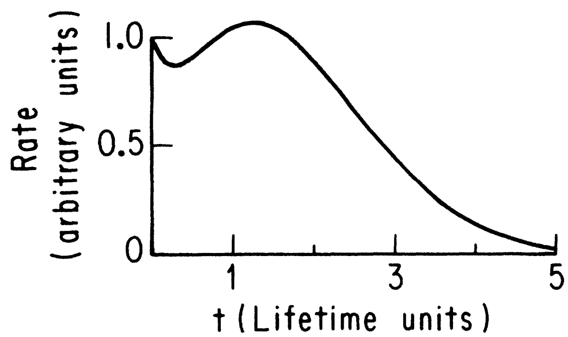

FIG. 6. Time distribution of $B^0_{s,\mathrm{phys}}(t)\to D^0\phi$ and $\bar{B}^0_{s,\mathrm{phys}}(t)\to\bar{D}^0\phi$ decays in the *CP*-conserving case $\mathrm{Im}\lambda = \mathrm{Im}\bar{\lambda} = 0$. Here we have assumed $\Delta m/\gamma = 1$ and $|x|^2 = 9$, as representative set of parameters in Eqs. (14).

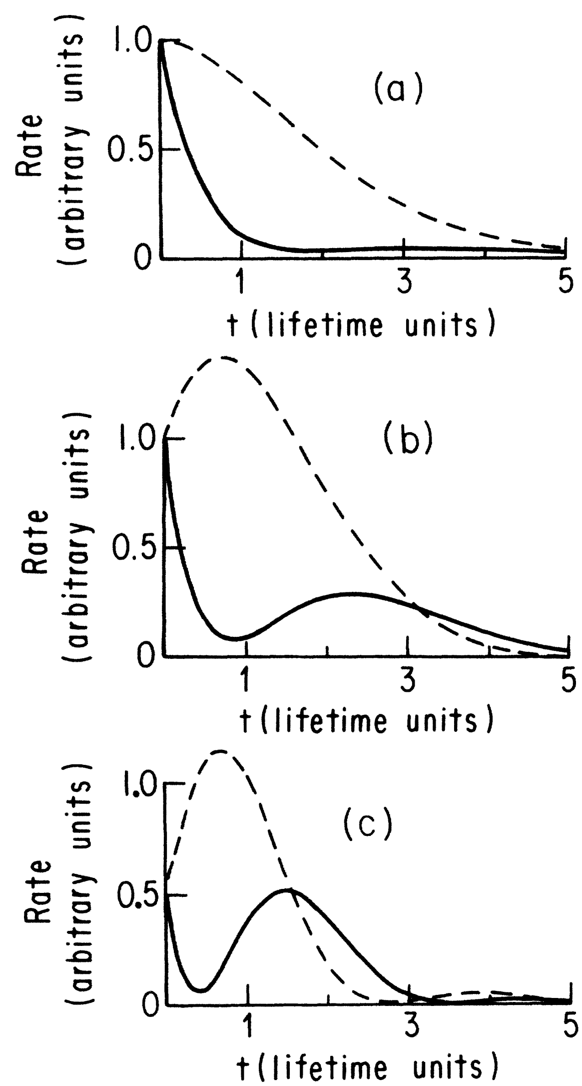

FIG. 7. Time distribution of $B^0_{s,\mathrm{phys}}(t)\to D^+\pi^-$ vs $\bar{B}^0_{s,\mathrm{phys}}(t)\to D^-\pi^+$. Dashed curve: $B^0_{s,\mathrm{phys}}(t)\to(D^+\pi^-, D^0\pi^0, F^+K^-,\ldots)$ or $\bar{B}^0_{s,\mathrm{phys}}(t)\to(\bar{D}^0\phi, \bar{D}^*\pi, F^{*-}K^+, F^-K^{*+},\ldots)$; solid curve: $\bar{B}^0_{s,\mathrm{phys}}(t)\to(D^-\pi^+, \bar{D}^0\pi^0, F^-K^+,\ldots)$ or $B^0_{s,\mathrm{phys}}(t)\to(D^0\phi, D^*\pi, F^{*+}K^-, F^+K^{*-},\ldots)$. (a) $\Delta m/\gamma = 0.5$, (b) $\Delta m/\gamma = 1$, (c) $\Delta m/\gamma = 2$.

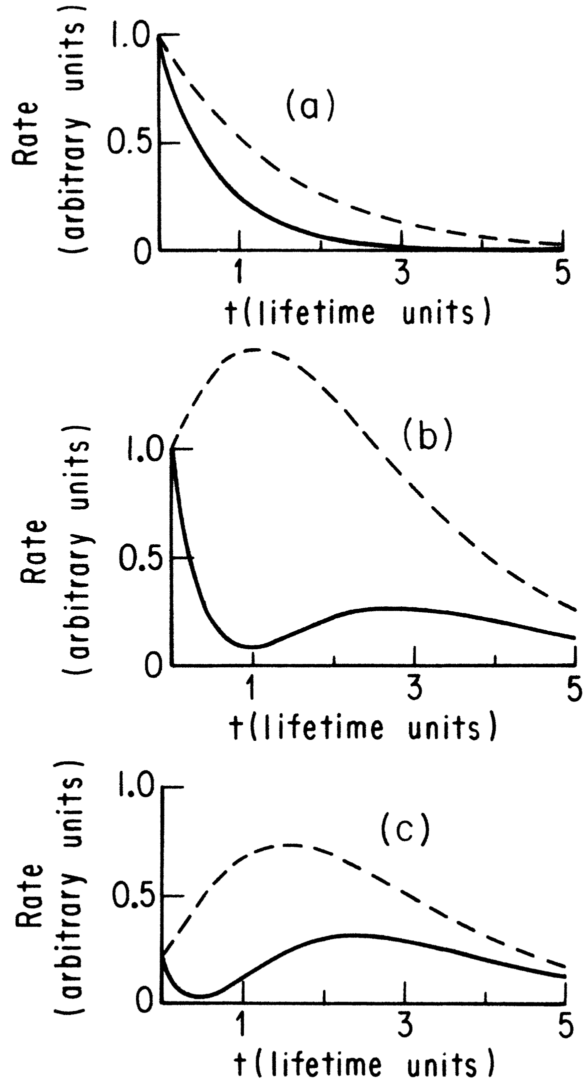

FIG. 8. Time distribution of $B^0_{d,\mathrm{phys}}(t)\to D^+\pi^-$ vs $\bar B^0_{d,\mathrm{phys}}(t)\to D^-\pi^+$. Dashed curve: $B^0_{d,\mathrm{phys}}(t)\to(D^+\pi^-, D^0\pi^0, F^+K^-, \dots)$ or $\bar B^0_{d,\mathrm{phys}}(t)\to(\bar D^*\pi, F^{*-}K^+, \bar D^0\omega, \bar D\rho, F^-K^{*+}, \dots)$; solid curve: $\bar B^0_{d,\mathrm{phys}}(t)\to(D^-\pi^+, \bar D^0\pi^0, F^-K^+, \dots)$ or $B^0_{d,\mathrm{phys}}(t)\to(D^*\pi, F^{*+}K^-, D^0\omega, D\rho, F^+K^{*-}, \dots)$. (a) $\Delta m/\gamma=10^{-2}$, (b) $\Delta m/\gamma=5\times 10^{-2}$, (c) $\Delta m/\gamma=0.1$.

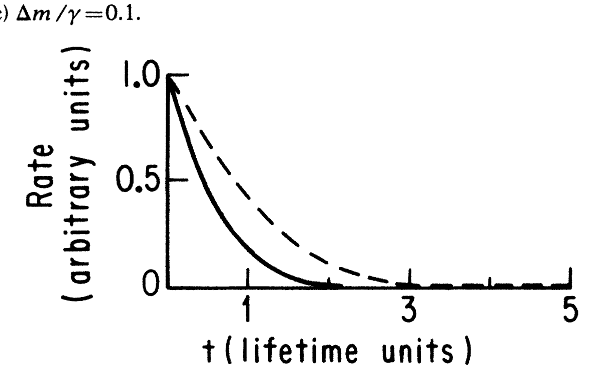

FIG. 9. Time distribution of $B^0_{s,\mathrm{phys}}(t)\to D^-\pi^+$ vs $\bar B^0_{s,\mathrm{phys}}(t)\to D^+\pi^-$, for $\Delta m/\gamma=1$. Dashed curve: $B^0_{s,\mathrm{phys}}(t)\to(D^-\pi^+, \bar D^0\pi^0, F^-K^+, \dots)$ or $\bar B^0_{s,\mathrm{phys}}(t)\to(D^0\phi, D^*\pi, F^{*+}K^-, F^+K^{*-}, \dots)$; solid curve: $\bar B^0_{s,\mathrm{phys}}(t)\to(D^+\pi^-, D^0\pi^0, F^+K^-, \dots)$ or $B^0_{s,\mathrm{phys}}(t)\to(\bar D^0\phi, \bar D^*\pi, F^{*-}K^+, F^-K^{*+}, \dots)$.

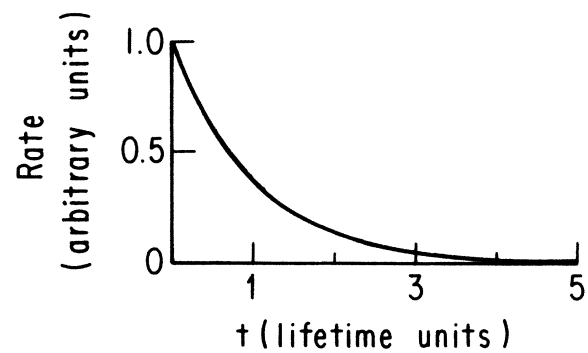

FIG. 10. Time distribution of $B_{d,\mathrm{phys}}(t)\to D^-\pi^+$ vs $\bar B_{d,\mathrm{phys}}(t)\to D^+\pi^-$ for $\Delta m/\gamma=0.1$. The single curve corresponds both to $B_{d,\mathrm{phys}}(t)\to(D^-\pi^+, \bar D^0\pi^0, F^-K^+, \dots)$ or $\bar B_{d,\mathrm{phys}}(t)\to(D^*\pi, D^0\omega, D\rho, F^{*+}K^-, F^+K^{*-}, \dots)$ and to $\bar B_{d,\mathrm{phys}}(t)\to(D^+\pi^-, D^0\pi^0, F^+K^-, \dots)$ or $B_{d,\mathrm{phys}}(t)\to(\bar D^*\pi, \bar D^0\omega, \bar D\rho, F^{*-}K^+, F^-K^{*+}, \dots)$.

we expect $\Delta m/\gamma=1$, Fig. 12 *also* describes the anticipated asymmetry. From Table IV we expect the observation of an asymmetry in $B_s\to\pi^0K_S$ to require fewer events than many of the other processes we discuss. This is because the expected asymmetry is large, and even though the branching ratio is likely to be low, the expected detection efficiency (Table III) is one of the highest for any two-body final state.

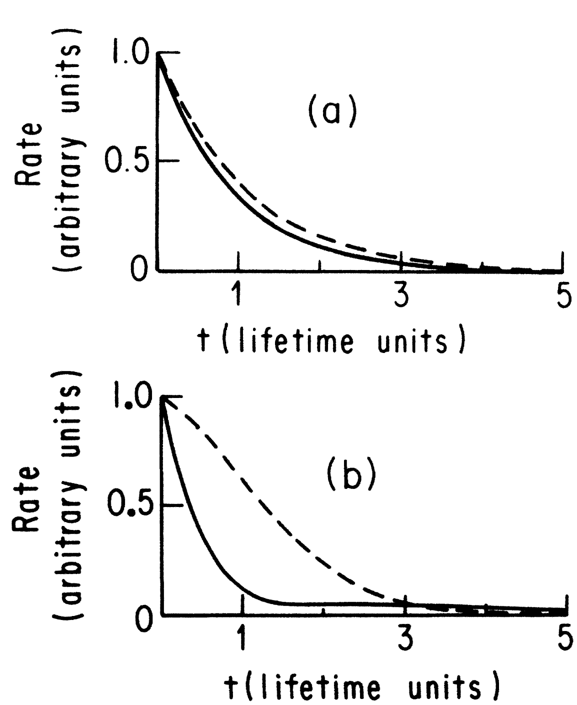

FIG. 11. Time distribution for CP eigenstates. (a) $\Delta m/\gamma=0.1$. Dashed curve: $B_{d,\mathrm{phys}}(t)\to(D^+D^-, \dots)$ or $\bar B_{d,\mathrm{phys}}(t)\to(\psi K_s, \phi K_s)$; solid curve: $\bar B_{d,\mathrm{phys}}(t)\to(D^+D^-, \dots)$ or $B_{d,\mathrm{phys}}(t)\to(\psi K_s, \phi K_s)$. (b) $\Delta m/\gamma=1$. Dashed curve: $B_{d,\mathrm{phys}}(t)\to(D^+D^-, \dots)$ or $\bar B_{d,\mathrm{phys}}(t)\to(\psi K_s, \phi K_s)$ or $B_{s,\mathrm{phys}}(t)\to(\phi K_s, \rho^0K_s, \omega K_s, \dots)$ or $\bar B_{s,\mathrm{phys}}(t)\to\pi^0K_s$; solid curve: $\bar B_{d,\mathrm{phys}}(t)\to(D^+D^-, \dots)$ or $B_{d,\mathrm{phys}}(t)\to(\psi K_s, \phi K_s)$ or $\bar B_{s,\mathrm{phys}}(t)\to(\phi K_s, \rho^0K_s, \omega K_s, \dots)$ or $B_{s,\mathrm{phys}}(t)\to\pi^0K_s$.

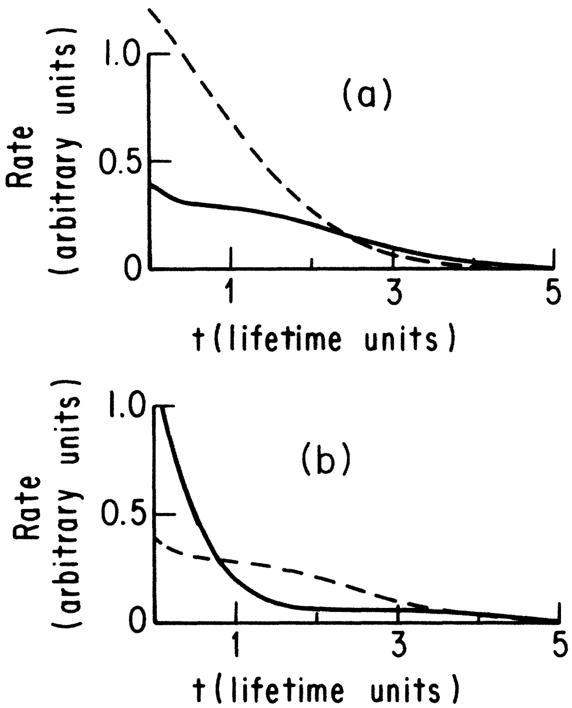

FIG. 12. Time distribution of $B^0_{s,\text{phys}}(t) \to D^+\pi^-$ vs $\overline{B}^0_{s,\text{phys}}(t) \to D^-\pi^+$ with leptonic tagging, for $\Delta m/\gamma = 1$. Dashed curve: $N([B_s\overline{B}_s]_\zeta \to l^-_{0\to\infty}(D^+\pi^-)_t)$ [Eq. (35a)]; solid curve: $N([B_s\overline{B}_s]_\zeta \to l^+_{0\to\infty}(D^-\pi^+)_t)$ [Eq. (35b)]. (a) $\zeta = 1$, charge-conjugation even. (b) $\zeta = -1$, charge-conjugation odd.

VI. FLAVOR TAGGING

The time-dependent asymmetries we have suggested measuring require that we know the flavor of the initial state. We wish to ensure that at $t=0$ we have a pure B ($=\overline{b}d$ or $\overline{b}s$) or pure $\overline{B}$ ($=b\overline{d}$ or $b\overline{s}$). In any hadronic or electromagnetic process, such a state must be produced in association with another b-flavored hadron so that $N(b)+N(\overline{b})=0$. Another possible source of $b\overline{b}$ pairs is $W \to t\overline{b}$, with $t \to b$. (In high-energy hadron reactions, multiple $b\overline{b}$ production is possible, and one must be careful to guard against this effect. Kinematic selection may reduce the background from such processes.)

Let us assume, for definiteness, that we wish to follow the time evolution of a state which initially contains a $\overline{b}$. In a hadronic or electromagnetic process, this state must have been produced in association with a b. We want to identify this associated b.

One method suggested previously[15] for identifying an associated b relies on the energetic "primary" lepton l^- produced in the decay

$$b \to c l^- \overline{\nu}_l \ . \tag{32}$$

The subsequent semileptonic decay of c, produced either in this reaction or in nonleptonic $b \to c$ transitions, can produce a "secondary" lepton of the opposite sign, which can be distinguished from the primary one (at least in e^+e^- interactions near $b\overline{b}$ threshold) by kinematic selection. We refer to this method as "leptonic tagging." We shall show that serious problems of normalization, detection efficiency, and quantum-mechanical ambiguities can affect this method. We shall advocate instead tagging on the accompanying charged b-flavored meson $B_u^\pm$ or b-flavored baryon (Λ_b or $\overline{\Lambda}_b$) as an unambiguous indication of flavor at $t=0$.

Let us imagine that an initial b quark (accompanying the $\overline{b}$ whose time evolution we wish to follow) is incorporated in a hadron. In a time short compared with decay or mixing times, this b will end up in one of the following weakly decaying particles:

$$B_u^- = b\overline{u} \ , \tag{33a}$$

$$\overline{B}_d = b\overline{d} \ , \tag{33b}$$

$$\overline{B}_s = b\overline{s} \ , \tag{33c}$$

or

$$\Lambda_b = bud \ . \tag{33d}$$

The $\overline{B}_d$ and $\overline{B}_s$ can evolve into their charge conjugates as a result of mixing, so that any leptonic tagging will require an estimate of this effect. In contrast, we can imagine identifying a large fraction (we have assumed 50% in Table III) of charged-B decays, simply by seeing a short charged track with proper lifetime 10^{-12} s which then decays with the proper kinematics. Full reconstruction of the charged-B final state will not be necessary, but an estimate of the transverse momentum of the decay products will avoid contamination from charm or τ decays. The decay products of Λ_b will contain a baryon, which we would hope to distinguish from an antibaryon much of the time.

To see examples of the care that must be taken when employing leptonic tagging, let us consider the example of the decay B_s or $\overline{B}_s$ to $D^+\pi^-$ (assuming we can eliminate the B_d or $\overline{B}_d$ source of $D^+\pi^-$ by energy-momentum conservation). Then a $b\overline{b}$ final state can give rise to $D^+\pi^-$ through the following routes (as well as others, except close to threshold in e^+e^- annihilations):

$$\begin{array}{cccccr} & \nearrow & B_u\overline{B}_sX & \searrow & & (34\text{a}) \\ b\overline{b} & \longrightarrow & B_s\overline{B}_s+\zeta\overline{B}_sB_s & \longrightarrow & l^- + (D^+\pi^-)_t & (34\text{b}) \\ & \searrow & B_d\overline{B}_sX+\zeta\overline{B}_dB_s\overline{X} & \nearrow & & (34\text{c}) \end{array}$$

The factor $\zeta = +1$ or -1 denotes the assumed charge-conjugation eigenvalue of the $B\overline{B}$ state. Here we integrate over all times of l^- and look at the number of events of $D^+\pi^-$ as a function of time. In actual practice the l^- can be identified as coming from a B meson only if it is produced at a minimum time t_0 after associated $b\overline{b}$ production, where t_0 is determined by the spatial resolution of the detector for identifying short tracks.[17] Thus, the integration over l^- times should proceed from t_0 to ∞.

Problems already occur when we look at the contribution of (34b) to the l^- $(D^+\pi^-)_t$ final state. The time-evolved final state corresponding to (34b) may be constructed and the time dependence of any $l^-(f)_t$ final state may be evaluated in a manner very similar to that presented above. We integrate over the poorly determined semileptonic (l^-X) decay time from $t_0 > 0$ to ∞ to obtain the time dependent rate of f as

$$N([B_s\overline{B}_s]_\zeta \to l^-_{t_0\to\infty} f_t) = |\langle f|B^0\rangle|^2 |\langle l^-|\overline{B}{}^0\rangle|^2 \frac{e^{-\gamma(t_0+t)}}{2\gamma(1+z^2)}$$
$$\times\{(1+z^2)(1+|\lambda|^2) - \cos\Delta m\, t_0[(\cos\Delta m\, t - \zeta z \sin\Delta m\, t)(|\lambda|^2-1)$$
$$+2\,\mathrm{Im}\lambda(\sin\Delta m\, t + \zeta z\cos\Delta m\, t)]$$
$$+\sin\Delta m\, t_0[(z\cos\Delta m\, t+\zeta\sin\Delta m\, t)(|\lambda|^2-1)+2\,\mathrm{Im}\lambda(z\sin\Delta m\, t-\zeta\cos\Delta m\, t)]\}\ . \quad (35a)$$

Integrating over l^+ times (from t_0 to ∞) to observe the $\overline{f}$ time-dependent rate yields

$$N([B_s\overline{B}_s]_\zeta \to l^+_{t_0\to\infty} \overline{f}_t) = |\langle \overline{f}|\overline{B}{}^0\rangle|^2 |\langle l^+|B^0\rangle|^2 \frac{e^{-\gamma(t_0+t)}}{2\gamma(1+z^2)}$$
$$\times\{(1+z^2)(1+|\overline{\lambda}|^2) - \cos\Delta m\, t_0[(\cos\Delta m\, t - \zeta z \sin\Delta m\, t)(|\overline{\lambda}|^2-1)$$
$$+2\,\mathrm{Im}\overline{\lambda}(\sin\Delta m\, t + \zeta z\cos\Delta m\, t)]$$
$$+\sin\Delta m\, t_0[(z\cos\Delta m\, t+\zeta\sin\Delta m\, t)(|\overline{\lambda}|^2-1)+2\,\mathrm{Im}\overline{\lambda}(z\sin\Delta m\, t-\zeta\cos\Delta m\, t)]\}\ . \quad (35b)$$

We find that this rate (35a) and (35b) depends crucially on the charge-conjugation eigenvalue ζ. Because modern techniques allow us to anticipate vertex resolution of order of $t_0 \approx 10^{-13}$ s, we display in Fig. (12) the semileptonic tagged rates for $t_0=0$.

We must know the relative probabilities for states with $\zeta=\pm1$ in order to evaluate the contribution of (34b) to any time-dependent asymmetry.[18] Moreover, we would have to know the relative production ratios of $B_u\overline{B}_s$:$B_d\overline{B}_s$:$B_s\overline{B}_s$ and the branching ratios of $B_q,\overline{B}_q \to l^-$ (which can be different for $q=d,s,u$).

In brief, leptonic flavor tagging does not appear feasible for measurement of the time-dependent asymmetries of B mesons suggested here. We must tag on the charged b-flavored meson (or a b-flavored baryon) to know the flavor of the initial state. We have shown that this can be a very efficient method.

Note added in proof. When mixing amplitudes are small, many of the problems associated with leptonic tagging, for example in connection with Eq. (34b), do not appear to be as serious if one can select decay times appropriately. We intend to study this question further.

VII. A COMMENT ON $B_c^\pm$ ASYMMETRIES

The surprisingly long-lived b-flavored mesons ($\tau_B \sim 10^{-12}$ s), which in the spectator model imply $|V_{cb}| \ll 1$, could give rise to exotic *CP* violation in $B_c^\pm$ decays. Because the b lifetime is of the same order as the c lifetime, a B_c^+ ($\equiv \overline{b}c$) could have $B_c^+ \to \pi^+ B_s$ + (neutral system) as a substantial fraction. This idea could serve as an alternative B_s tagging device. A more interesting consequence of this exotic decay would be large partial-rate asymmetries in particular final states. Those asymmetries, indicative of *CP* violation, could arise as a result of the large B_s-$\overline{B}_s$ mixing as discussed above. In addition, asymmetries due to final-state strong-interaction phases can arise, as has been discussed in the literature.[19]

VIII. CONCLUSIONS

We have shown that time-dependent *CP*-violating effects can be quite spectacular in the B-$\overline{B}$ system if any *CP* violation can be observed at all for this system. A detector with fine spatial and time resolution seems ideal for observing these effects.

The largest asymmetries are found for processes which are suppressed from the Cabibbo-Kobayashi-Maskawa (CKM) standpoint, such as B_s ($\equiv \overline{b}s$)$\to D^+\pi^-$, $B_s \to D^0\phi$, or any decay with the basic quark transition $\overline{b}\to\overline{u}c\overline{s}$ in the B_s-$\overline{B}_s$ complex. The decay $B_d(\equiv \overline{b}d)\to D^+\pi^-$ also seems promising. We find that these asymmetries are expected to be large enough in the standard three-generation model to more than compensate for the rarity of the decay. Typically, one will need 10^6–10^7 $b\overline{b}$ pairs to detect an asymmetry in these "best cases." By contrast, the CKM-favored decays (such as $B_d \to D^-\pi^+$) or those for which the expected asymmetry peaks only for large mixing (such as $B_d\to\psi K_S$) are expected to require more events ($\sim 10^8$–10^9) in the standard model if an asymmetry is to be observed. Studies of these more rate-favored processes, of course, will provide an important test of the standard model even at reduced levels of statistics, since there is always the potential for surprises.

Note added in proof. It has been pointed out in Refs. 19 and 22 that $\overline{B}_d \to K^-\pi^+$ occurs both via penguin and spectator graphs, and can display considerable *CP* asymmetry with respect to $B_d \to K^+\pi^-$. Moreover, the $K^-\pi^+$ final state must have come from $\overline{B}_d$ and not B_d, so one is spared the need for independent tagging of the associated b-flavored particle. Thus, this final state, expected to occur with a branching ratio[23] of a few $\times 10^{-15}$, could also be promising for *CP*-violation studies.

ACKNOWLEDGMENTS

We are grateful to J. D. Bjorken for the initial suggestion that one study time-dependent effects, and to L. L. Chau, N. G. Deshpande, D. Du, C. Jarlskog, J. Pilcher, A. Soni, B. Winstein, and R. Winston for illuminating discussions. It is a pleasure for one of us (I.D.) to thank R. G. Sachs and Dan-di Wu for their patience, guidance, and sharing of physical intuition and experience. They have been of invaluable assistance during a time of crisis. This work was supported in part by the United States Department of Energy under Contracts Nos. DE AC02-80ER 10587 and DE AC02-82ER 40073.

APPENDIX

When dealing with final states that are CP eigenstates, special care must be taken. Assume for simplicity that only one weak phase contributes to our decay process. Then the claim is[4] that for CP-odd states we obtain $-\lambda$ (not λ) in Tables I and II. For CP-even states we obtain λ.

Proof. Suppose the different strong eigenchannels are labeled by α. Define

$$_{\rm out}\langle f,\alpha \mid B^0\rangle_{\rm in}=a_\alpha e^{i\delta_\alpha}\,, \tag{A1a}$$

$$_{\rm out}\langle f,\alpha \mid \bar{B}^0\rangle_{\rm in}=\bar{a}_\alpha e^{i\delta_\alpha}\,, \tag{A1b}$$

$$CP\mid f\rangle=\pm\mid f\rangle \tag{A2}$$

[e.g., if $f=\psi K_S$ we have $(-)$; if $f=D^+D^-$ we have $(+)$]. Choose the phase convention

$$CP\mid B^0\rangle=+\mid \bar{B}^0\rangle\,. \tag{A3}$$

Apply CPT onto (A1a) to obtain

$$\bar{a}_\alpha=\pm a_\alpha^*\,. \tag{A4}$$

In the above the $\pm$ merely reflects whether we deal with CP-even $(+)$ or CP-odd $(-)$ eigenstates. In reality our decay may proceed via several strong eigenchannels with unknown final state strong-interaction phases. Now

$$x=\frac{_{\rm out}\langle f\mid \bar{B}^0\rangle_{\rm in}}{_{\rm out}\langle f\mid B^0\rangle_{\rm in}}=\frac{\sum_\alpha {}_{\rm out}\langle f\mid f,\alpha\rangle_{\rm out}\bar{a}_\alpha e^{i\delta_\alpha}}{\sum_\alpha {}_{\rm out}\langle f\mid f,\alpha\rangle_{\rm out}a_\alpha e^{i\delta_\alpha}}\,. \tag{A5}$$

Now the assumption of only one weak phase enters as

$$a_\alpha=\mid a_\alpha\mid e^{i\psi_{wk}}\,, \tag{A6}$$

where ψ_{wk} does not depend on α. Then

$$x=\frac{\pm e^{-i\psi_{wk}}\sum_\alpha {}_{\rm out}\langle f\mid f,\alpha\rangle_{\rm out}\mid a_\alpha\mid e^{i\delta a}}{e^{i\psi_{wk}}\sum_\alpha {}_{\rm out}\langle f\mid f,\alpha\rangle_{\rm out}\mid a_\alpha\mid e^{i\delta a}}=\pm\frac{e^{-i\psi_{wk}}}{e^{i\psi_{wk}}}\,. \tag{A7}$$

In essence, x will be a ratio of KM combinations, and, in the above, the $\pm$ sign reflects what CP eigenstate we deal with.

For final states that are not CP eigenstates, i.e., $D^+\pi^-, D^{*+}\pi^-, \ldots,$ we have also a sign ambiguity in λ. We will compare the asymmetries arising from $B^0\to P_1P_2$ (for example, $D^+\pi^-$, F^+K^-, etc.) and $B^0\to V_1P_2$, where the vector meson V_1 is just the excited counterpart of P_1 (for example $D^{*+}\pi^-$, $F^{*+}K^-$, etc., respectively). In the neglect of kinematical considerations we obtain[20]

$$x_{P_1P_2}\equiv\frac{\langle P_1P_2\mid \bar{B}^0\rangle}{\langle P_1P_2\mid B^0\rangle}=-x_{V_1P_2}\equiv-\frac{\langle V_1P_2\mid \bar{B}^0\rangle}{\langle V_1P_2\mid B^0\rangle}\,, \tag{A8}$$

which leads to

$$\lambda_{P_1P_2}=-\lambda_{V_1P_2}\,. \tag{A9}$$

To show what assumptions are involved, we will discuss the $B_d\to D^+\pi^-$ asymmetry and the $B_d\to D^{*+}\pi^-$ one. As is commonly done[21] we define "reduced" amplitudes a_I and $\bar{a}_I$ that do not involve the final-state interactions:

$$\begin{aligned}\langle D^+\pi^-,I\mid B_d\rangle&=a_I(D^+)e^{i\delta_I}\,,\\ \langle D^+\pi^-,I\mid \bar{B}_d\rangle&=\bar{a}_I(D^+)e^{i\delta_I}\,,\\ \langle D^-\pi^+,I\mid B_d\rangle&=a_I(D^-)e^{i\delta_I}\,,\\ \langle D^-\pi^+,I\mid \bar{B}_d\rangle&=\bar{a}_I(D^-)e^{i\delta_I}\,.\end{aligned} \tag{A10}$$

δ_I is the (π,D) scattering phase shift.

We define

$$\begin{aligned}CPT\mid D\rangle&=-\mid \bar{D}\rangle\,,\\ CPT\mid \pi\rangle&=-\mid \bar{\pi}\rangle\,,\end{aligned} \tag{A11}$$

to obtain

$$CPT\mid D^+\pi^-,I\rangle_{\rm out}=(-1)^{I+1/2}\mid D^-\pi^+,I\rangle_{\rm in} \tag{A12a}$$

and

$$CPT\mid D^{*+}\pi^-,I\rangle_{\rm out}=-(-1)^{I+1/2}\mid D^{*-}\pi^+,I\rangle_{\rm in}\,. \tag{A12b}$$

We note, in advance, that the relative minus sign between Eq. (A12b) and Eq. (A12a) leads to the minus sign in Eq. (A9). By the CPT theorem we obtain

$$a_I^*(D^\pm)=(-1)^{I+1/2}\bar{a}_I(D^\mp)\,, \tag{A13a}$$

$$a_I^*(D^{*\pm})=-(-1)^{I+1/2}\bar{a}_I(D^{*\mp})\,. \tag{A13b}$$

Therefore,

$$x_{D\pi,I}\equiv\frac{\langle D^+\pi^-,I\mid \bar{B}_d\rangle}{\langle D^+\pi^-,I\mid B_d\rangle}=(-1)^{I+1/2}\frac{a_I^*(D^-)}{a_I(D^+)}\,, \tag{A14a}$$

whereas

$$x_{D^*\pi,I}\equiv\frac{\langle D^{*+}\pi^-,I\mid \bar{B}_d\rangle}{\langle D^{*+}\pi^-,I\mid B_d\rangle}=-(-1)^{I+1/2}\frac{a_I^*(D^{*-})}{a_I(D^{*+})}\,. \tag{A14b}$$

Assuming

$$\frac{a_I^*(D^-)}{a_I(D^+)}=\frac{a_I^*(D^{*-})}{a_I(D^{*+})} \tag{A15}$$

[which is not guaranteed at all, since $P\to PP$ and $P\to VP$

may have different matrix element structure], we obtain

$$\lambda_{I,D\pi} = -\lambda_{I,D^*\pi} , \tag{A16}$$

resembling Eq. (A9).

We can summarize our result as follows. Equation (A9) holds naively. However, (a) when kinematical considerations are taken into account and (b) given that the final state phases can differ for the $B^0 \to P_1 P_2$ and $B^0 \to V_1 P_2$ decays, then Eq. (A9) will not hold.

[1]For reviews see Ling-Lie Chau, Phys. Rep. **95**, 1 (1983); Andrzej J. Buras, in *Proceedings of the 19th Rencontre de Moriond on Electroweak Interactions and Unified Theories, La Plagne, France, 1984,* edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1984), p. 511; Andrzej J. Buras, in *Proceedings of the Workshop on the Future of Intermediate Energy Physics in Europe, Freiburg, West Germany, 1984,* edited by H. Koch and F. Scheck (Kernforschungzentrum Karlsruhe, Karlsruhe, Germany, 1984), p. 53; Lincoln Wolfenstein, in *Proceedings of the Fifth Moriond Workshop on Flavour Mixing and CP Violation, La Plagne, France, 1985,* edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1985), p. 239; I. I. Bigi and A. I. Sanda, Comments Nucl. Part. Phys. **14**, 149 (1985); P. Langacker, in *First Aspen Winter Physics Conference, 1985,* edited by M. M. Block (Ann. N.Y. Acad. Sci. No. 461) (New York Academy of Sciences, New York, 1986); Lincoln Wolfenstein, Ann. Rev. Nucl. Part. Sci. (to be published); T. Brown and S. Pakvasa, Phys. Rev. D **31**, 1661 (1985), and references therein.

[2]For a systematic and detailed discussion see D. Du, I. Dunietz, and Dan-di Wu, Enrico Fermi Institute Report No. EFI 86-9 (unpublished), and references therein.

[3]M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[4]A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981); I. I. Bigi and A. I. Sanda, Nucl. Phys. **B193**, 85 (1981); J. S. Hagelin, *ibid.* **B193**, 123 (1981); H. Y. Cheng, Phys. Rev. D **26**, 143 (1982); E. A. Paschos and U. Türke, *ibid.* **B243**, 29 (1984); A. J. Buras, W. Slominski, and H. Steger, Nucl. Phys. **B245**, 369 (1984); L. Wolfenstein, *ibid.* **B246**, 45 (1984); R. G. Sachs, Enrico Fermi Institute Report No. EFI 85-22 (unpublished); L. L. Chau and H. Y. Cheng, Phys. Lett. **165B**, 429 (1985).

[5]I. I. Bigi and A. I. Sanda, Phys. Rev. D **29**, 1393 (1984).

[6]O. W. Greenberg, Phys. Rev. D **32**, 1841 (1985); Dan-di Wu, *ibid.* **33**, 860 (1986); C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); Z. Phys. C **29**, 491 (1985).

[7]D. Loveless, K. Nishikawa, F. E. Paige, D. D. Reeder, W. Wenzel, and B. Winstein, in *Proceedings of DPF Workshop, $\bar{p}p$ Options for the Supercollider, Chicago, 1984,* edited by J. E. Pilcher and A. R. White (Argonne National Laboratory, Argonne, II, 1984), p. 294; N. W. Reay, in Proceedings of the SSC Fixed Target Workshop, The Woodlands, Texas, 1984 (unpublished), p. 53.

[8]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[9]A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981); C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); Z. Phys. C **29**, 491 (1985).

[10]B. Guberina, R. D. Peccei, and R. Rückl, Phys. Lett. **90B**, 169 (1980); M. B. Gavela *et al., ibid.* **154B**, 425 (1985). For an excellent review, see R. Rückl, Habilitationsschrift, University of Munich, 1983, and references therein.

[11]We thank A. I. Sanda for a discussion of the momentum mismatch problem in $B_s \to F^+ K^-$.

[12]G. E. Gladding, in *Physics in Collision V,* proceedings of the Fifth International Conference, Autun, France, 1985, edited by B. Aubert and L. Montanet (Editions Frontières, Gif-sur-Yvette, France, 1985); W. C. Louis *et al.*, Phys. Rev. Lett. **56**, 1027 (1986).

[13]Carter and Sanda (Ref. 4); Wolfenstein (Ref. 4); Bigi and Sanda (Ref. 1).

[14]$B_\alpha \to \psi\phi$ is a pseudoscalar$\to$vector$\times$vector, $P \to VV$, decay and hence the CP signature of the final CP eigenstate is uncertain, because of the possibility of mixtures of different orbital angular momenta. [See, e.g., Wolfenstein (Ref. 4).] Such mixtures, however, are rather unlikely, because $\psi\phi$ is a heavy final state, so the dominant orbital angular momentum should be $l=0$. Therefore, the sign of $C_{f=\psi\phi}$ is the one shown in Table IV.

[15]J. W. Cronin *et al.*, in *Design and Utilization of the SSC, Snowmass, 1984,* proceedings of the Snowmass Summer Study, edited by Rene Donaldson and Jorge G. Morfin (Fermilab, Batavia, IL, 1984), p. 161; Bigi and Sanda (Ref. 1) and (Ref. 4); Wolfenstein (Ref. 4); T. Brown, S. Pakvasa, and S. F. Tuan, Phys. Lett. **136B**, 117 (1984).

[16]Wolfenstein (Ref. 4); Reay (Ref. 7); E. A. Paschos and R. A. Zacher, Z. Phys. C **28**, 521 (1985) have discussed the time-dependent asymmetries into CP eigenstates and into primary leptons.

[17]Cronin *et al.* (Ref. 15).

[18]For a careful job the mixing of B_d-$\bar{B}_d$ must be taken into account, since not only $\bar{B}_d$ $(=b\bar{d})B_s$ but also $B_d(=\bar{b}d)\bar{B}_s$ can lead to $l^-(D^+\pi^-)$ via the presumably small but still postulated B_d-$\bar{B}_d$ mixing. As a first approximation this last effect can be neglected.

[19]M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979); J. Bernabeau and C. Jarlskog, Z. Phys. C **8**, 233 (1981); L. L. Chau and H. Y. Cheng, Phys. Rev. Lett. **53**, 1037 (1984).

[20]We thank I. I. Bigi for pointing this idea out to us.

[21]Sachs (Ref. 4).

[22]Chau and Cheng (Ref. 4).

[23]Gavela *et al* (Ref. 10).