Weinberg CP-violation model revisited

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Chiral properties of the weak amplitudes in the Weinberg model of CP violation are carefully treated. The CP-odd $K \rightarrow 2\pi$ amplitude is chiral-symmetry nonlinear-realization dependent, and so is the soft-pion relation. This realization dependence is canceled by the pole contribution arising from the combination of a strong $K\pi$ vertex and a K-vacuum tadpole, so that to lowest order in chiral symmetry the physical $K \rightarrow 2\pi$ amplitude is chiral-model independent. The vacuum-insertion method is valid only in a particular realization scheme. Various CP-violating effects are then reexamined in this model. When the η-η' mixing is taken into account, $\epsilon'/\epsilon \approx -0.007$ is obtained. The electric dipole moment of a neutron due to neutral-Higgs-boson exchange and the current experimental limit are used to set an upper bound for neutral-Higgs-boson mixing. Finally, CP violation in $K \rightarrow 3\pi$ decays is discussed.

I. INTRODUCTION

Recently there has been renewed interest in the Weinberg model of CP violation.[1] On the one hand, Donoghue and Holstein[2] showed how to correctly treat the chiral properties of the weak amplitudes within the framework of chiral perturbation theory. As a result, the CP-violating parameter ϵ'/ϵ is estimated to be -0.006 (see also Ref. 3), in contrast with the conventional result $\epsilon'/\epsilon = -\frac{1}{20}$ (Ref. 4). Since the new estimate is within the experimental limit, this model seems to be viable. On the other hand, Anselm *et al.*[5] showed that the dominant contribution to the neutron electric dipole moment (EDM) arises from neutral-Higgs-boson exchange rather than from the charged Higgs boson. This dominant contribution exceeds the present experimental upper bound by 2—3 orders of magnitude, which appears to be a serious argument against the Weinberg model of CP nonconservation unless the neutral-Higgs-boson mixing is smaller than the charged-Higgs-boson mixing.

Since a K-vacuum tadpole can be induced by the CP-odd Lagrangian in the Weinberg model, a pole contribution has to be included in the calculation of the $K \rightarrow 2\pi$ amplitude. Because the strong $K\pi$ scattering amplitude is chiral-symmetry nonlinear-realization dependent, the inclusion of an additional pole term seems to give a puzzle; the on-shell physical amplitude to lowest order in chiral symmetry ought to be chiral-model independent. After briefly reviewing the Weinberg model in Sec. II we study the chiral properties of the weak amplitudes and solve the aforementioned puzzle. Some constraints are obtained for the charged-Higgs-boson mixing angles and the vacuum expectation values.

Section III is devoted to the discussions of CP-odd effects which can be manifested in this model. Section IV gives the conclusions.

II. MODEL

In the Weinberg model of CP violation there are at least three Higgs doublets in order to implement CP violation and conservation of natural flavor at the tree level. The breakdown of CP symmetry can arise from the complex vacuum expectation values of Higgs fields and the complex Yukawa interactions. If CP nonconservation is realized spontaneously, the Kobayashi-Maskawa (KM) matrix of the quark mixing at the tree level is real[6] and CP violation comes solely from the Higgs sector. Higher-order radiative correction would yield a small CP-violating phase δ in the KM matrix in the gauge interaction. Since δ is calculable and naturally small, this has been regarded for some time as one of the attractive features of this type of CP-odd model. Nowadays we know that $\sin\delta$ is of order unity, inferred from the measurement of the b-quark lifetime and its semileptonic decay.[7]

The Yukawa interaction of the charged Higgs boson in the unitary gauge is given by[8,9]

$$\mathscr{L}_Y^{\pm} = (2\sqrt{2}G_F)^{1/2} \sum_{i=1}^{2} (\alpha_i \bar{U}_L K M_D D_R + \beta_i \bar{U}_R M_U K D_L + \gamma_i \bar{N}_L M_E E_R) H_i^{\dagger} + \text{H.c.} , \quad (1)$$

where K is a real KM matrix, M_U, M_D, and the M_E are diagonal mass matrices for the up quarks, down quarks, and leptons, respectively, and

$$\begin{aligned} \alpha_1 &= -\frac{\tilde{s}_1 \tilde{c}_3}{\tilde{c}_1}, \quad \beta_1 = \frac{-\tilde{c}_1 \tilde{c}_2 \tilde{c}_3 + \tilde{s}_2 \tilde{s}_3 e^{i\delta_H}}{\tilde{s}_1 \tilde{c}_2} , \\ \gamma_1 &= \frac{\tilde{c}_1 \tilde{s}_2 \tilde{c}_3 + \tilde{c}_2 \tilde{s}_3 e^{i\delta_H}}{\tilde{s}_1 \tilde{s}_2} , \\ \alpha_2 &= -\frac{\tilde{s}_1 \tilde{s}_3}{\tilde{c}_1}, \quad \beta_2 = \frac{-\tilde{c}_1 \tilde{c}_2 \tilde{s}_3 - \tilde{s}_2 \tilde{c}_3 e^{i\delta_H}}{\tilde{s}_1 \tilde{c}_2} , \\ \gamma_2 &= \frac{\tilde{c}_1 \tilde{s}_2 \tilde{s}_3 - \tilde{c}_2 \tilde{c}_3 e^{i\delta_H}}{\tilde{s}_1 \tilde{s}_2} , \end{aligned} \quad (2)$$

with $\tilde{s}_i$, $\tilde{c}_i$, and δ_H being the Higgs-boson mixing angles and phase defined in complete analogy to the KM matrix of quark mixing. From (2) this gives

$$\mathrm{Im}(\alpha_2\beta_2^*)=-\mathrm{Im}(\alpha_1\beta_1^*)\ ,$$
$$\mathrm{Im}(\alpha_2\gamma_2^*)=-\mathrm{Im}(\alpha_1\gamma_1^*)\ , \qquad (3)$$
$$\mathrm{Im}(\beta_2\gamma_2^*)=-\mathrm{Im}(\beta_1\gamma_1^*)\ .$$

The Yukawa interaction of the neutral Higgs boson in the diagonalized basis reads[9]

$$\mathcal{L}_Y^0=(\sqrt{2}\,G_F)^{1/2}\sum_{i=1}^{5}(\zeta_{1i}\bar{D}M_D D+i\zeta_{2i}\bar{D}M_D\gamma_5 D+\zeta_{3i}\bar{U}M_U U+i\zeta_{4i}\bar{U}M_U\gamma_5 U+\zeta_{5i}\bar{E}M_E E+i\zeta_{6i}\bar{E}M_E\gamma_5 E)H_i\ , \qquad (4)$$

where the coupling constants ζ_{ij} are real. Since $\bar{\psi}\psi$ and $i\bar{\psi}\gamma_5\psi$ have opposite P, T, and CP transformation properties, P and CP can be violated through the exchange of the neutral Higgs boson. In this model CP violation is usually characterized by the parameter

$$\mathrm{Im}\,A\equiv\mathrm{Im}(\langle\phi_1^{\dagger *}\phi_2^{\dagger}\rangle/v_1^*v_2)\equiv\frac{G_F}{m_0^2}=2\sqrt{2}\,G_F\sum_{i=1}^{2}\frac{\mathrm{Im}\alpha_i\beta_i^*}{m_{H_i}^2}\ . \qquad (5)$$

For later purposes we rewrite the equation for m_0^2:

$$\frac{1}{m_0^2}=\frac{\sqrt{2}\,vv_3}{v_1v_2}\sin 2\tilde{\theta}_3\sin\delta_H\left[\frac{1}{m_{H_1}^2}-\frac{1}{m_{H_2}^2}\right]\ , \qquad (6)$$

where v_1,v_2,v_3 are the vacuum expectation values of the Higgs fields ϕ_1, ϕ_2, and ϕ_3, respectively, $v=(v_1^2+v_2^2+v_3^2)^{1/2}=(2\sqrt{2}\,G_F)^{-1/2}$. In deriving (6), Eqs. (2) and (3.12) of Ref. 9 have been used.

A. Chiral properties of the weak amplitude and ϵ'/ϵ

In $K\to 2\pi$ decays the CP-violating parameters ϵ and ϵ' are given by Ref. 10:

$$\epsilon=\frac{1}{2\sqrt{2}}(\epsilon_m+2\xi_0)e^{i\pi/4}\ ,$$
$$\epsilon'=\frac{i}{\sqrt{2}}\frac{\mathrm{Re}a_2}{\mathrm{Re}a_0}(\xi_2-\xi_0)e^{i(\delta_2-\delta_0)}\ , \qquad (7)$$
$$\epsilon_m\equiv\mathrm{Im}M_{12}/\mathrm{Re}M_{12}\ ,$$

where $\xi_i=\mathrm{Im}a_i/\mathrm{Re}a_i$, $a_0(a_2)$ is the isospin-zero (-two) amplitude of $K\to 2\pi$. As in the KM model, $\mathrm{Im}a_2$ also vanishes in the Weinberg-model phase convention; hence

$$\frac{\epsilon'}{\epsilon}\cong-\frac{1}{20}\left[\frac{2\xi}{\epsilon_m+2\xi}\right]\ , \qquad (8)$$

where the subscript 0 of ξ has been dropped for convenience. The dominant contribution to the imaginary part of the $K\to 2\pi$ amplitude comes from the Higgs penguin diagram. The effective $\Delta S=1$ CP-odd bilinear operator[11,12] from $s\to d+$gluon is

$$\mathcal{L}_-^{\Delta S=1}=i\tilde{f}\,\bar{d}\sigma^{\mu\nu}(1+\gamma_5)\lambda^a s G_{\mu\nu}^a \qquad (9)$$

with[13]

$$\tilde{f}=\frac{G_F}{\sqrt{2}}\frac{g_s}{32\pi^2}m_s m_c^2\cos\theta_C\sin\theta_C\times\sum_{i=1}^{2}\frac{\mathrm{Im}\alpha_i\beta_i^*}{m_{H_i}^2}\left[\ln\frac{m_{H_i}^2}{m_c^2}-\frac{3}{2}\right]\ . \qquad (10)$$

The Higgs-boson penguin diagram does not correspond to a local four-quark operator since the loop integral does not give a factor of k^2 canceling the pole in the gluon propagator. However, using the vacuum-saturation approximation to evaluate the matrix element $\langle 2\pi|\mathcal{L}_-^{\Delta S=1}|K\rangle$ one encounters an effective four-quark operator of the form[4]

$$\tilde{\mathcal{L}}^{\Delta S=1}=\sum_{q}^{u,s,d}\bar{q}_R d_L\bar{s}_R q_L\ , \qquad (11)$$

where $q_L=(1-\gamma_5)q/2$.

It was noted by Donoghue and Holstein[2] that the amplitude $K\to 2\pi$ induced by the CP-odd Lagrangian $\Theta=\mathcal{L}_-^{\Delta S=1}$ or $\tilde{\mathcal{L}}^{\Delta S=1}$ involves an additional pole contribution which arises from the combination of a four-point strong K-K-π-π vertex and a weak K-vacuum vertex (i.e., a tadpole). More precisely,

$$P=A(K\pi\to K\pi)\langle 0|\Theta|K\rangle/m_K^2\ , \qquad (12)$$

where $A(K\pi\to K\pi)$ is the strong-interaction $K\pi$ scattering amplitude. The total CP-violating $K\to 2\pi$ amplitude is then given by

$$A(K^0\to\pi^+\pi^-)=\langle\pi^+\pi^-|\Theta|K^0\rangle+P\ . \qquad (13)$$

The kaon-pion scattering amplitude can be easily computed using chiral perturbation theory. The strong-interaction Lagrangian to lowest order in momenta including chiral-SU(3)$\times$SU(3)-symmetry breaking by the meson masses is

$$\mathcal{L}_s^{(2)}=\frac{f^2}{8}[\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger)+\mathrm{Tr}M(U+U^\dagger)]\ , \qquad (14)$$

where f is the common decay constant ~ 130 MeV, $M_{ij}=0$ $(i\neq j)$, $m_\pi^2=M_{11}=M_{22}$, $m_K^2=(M_{11}+M_{33})/2$, and the general form of the meson matrix U is[14]

$$U=1+2i\phi/f-2\phi^2/f^2-ia_3\phi^3/f^3+2(a_3-1)\phi^4/f^4+\cdots\ ,\qquad \phi\equiv\phi^a\lambda^a\ , \qquad (15)$$

with λ^a the Gell-Mann matrices. Up to order four in ϕ there is only one arbitrary constant a_3 in the expansion of U. For $a_3=\frac{4}{3}$ the meson matrix becomes the familiar form $U=\exp(2i\phi/f)$. From Eqs. (12)–(15), it follows that the pole term on shell is

$$P=\langle 0|\Theta|K\rangle a_3/(2f^2)\ . \qquad (16)$$

The matrix element $\langle 2\pi|\Theta|K\rangle$ can be evaluated either by current algebra or by the vacuum-insertion method. If we use the soft-pion relations

$$\langle \pi^+\pi^- | \Theta | K^0 \rangle = -i\sqrt{2}\langle \pi^0 | \Theta | K^0 \rangle / f \,,$$
$$\langle \pi^0 | \Theta | K^0 \rangle = -i\langle 0 | \Theta | K^0 \rangle / (\sqrt{2} f) \,, \tag{17}$$

we will encounter two related puzzles. From Eqs. (13), (16), and (17) we obtain

$$A(K^0 \to 2\pi) = -(1-a_3/2)\langle 0 | \Theta | K^0 \rangle / f^2 \,. \tag{18}$$

However, this is in contradiction with the theorem of Chisholm:[15] S-matrix elements (i.e., on-shell amplitudes) are independent of the value of a_3. Furthermore, as we will see later the Feinberg-Kabir-Weinberg (FKW) theorem[16] requires that no physical transition amplitudes be induced to lowest order in chiral perturbation theory. For $a_3 = \frac{1}{2}$, as originally chosen by Ref. 2, or $a_3 = \frac{4}{3}$, as often used in the literature, the CP-odd $K \to 2\pi$ amplitude does not vanish on shell. These puzzles are solved based on the observation that in the chiral limit the operator Θ transforms as $(\bar{3},3)$ under $\mathrm{SU}_L(3) \times \mathrm{SU}_R(3)$, and hence the corresponding effective Lagrangian is of the form

$$\Theta \sim \mathrm{Tr}(\lambda_6 U) + \mathrm{H.c.} \tag{19}$$

From Eq. (15) it becomes clear that the weak amplitude $\langle 2\pi | \Theta | K \rangle$ is a_3 dependent. Consequently, the soft-pion reduction of the $K \to 2\pi$ amplitude is nonlinear-realization dependent and should read

$$\langle \pi^+\pi^- | \Theta | K^0 \rangle = -i\frac{\sqrt{2}}{f}\left[\frac{a_3}{2}\right]\langle \pi^0 | \Theta | K^0 \rangle \,. \tag{20}$$

So, if we use the usual soft-pion relations (17), the values of a_3 for the strong $K\pi$ scattering amplitude has to be 2 rather than $\frac{1}{2}$ or $\frac{4}{3}$. Thus,

$$A(K^0 \to \pi^+\pi^-) = \left[-i\frac{\sqrt{2}}{f}\langle \pi^0 | \Theta | K^0 \rangle + \frac{1}{f}\langle 0 | \Theta | K^0 \rangle\right]\frac{a_3}{2} = 0 \,, \tag{21}$$

in accordance with both Chisholm and FKW theorems. The fundamental reason for the physical CP-odd $K \to 2\pi$ amplitude vanishing to lowest order in chiral symmetry is that the effective chiral representation for the $\Delta S = 1$ operator $\mathcal{L}^{\Delta S=1}$ [Eq. (19)] is similar to the mass terms in the strong-interaction Lagrangian (14). As a result, its effect can be rotated away by an appropriate chiral transformation of the on-shell amplitudes.[17] We also evaluate $A(K^0 \to \pi^+\pi^-)$ using the vacuum-insertion approximation:

$$\begin{aligned}\langle \pi^+\pi^- | \tilde{\mathcal{L}}^{\Delta S=1} | K^0 \rangle &= \langle \pi^+\pi^- | \bar{d}_R d_L | 0 \rangle \langle 0 | \bar{s}_R d_L | K^0 \rangle \\ &+ \langle \pi^+ | \bar{u}_R d_L | 0 \rangle \langle \pi^- | \bar{s}_R u_L | K^0 \rangle \\ &+ \langle 0 | \bar{d}_R d_L + \bar{s}_R s_L | 0 \rangle \langle \pi^+\pi^- | \bar{s}_R d_L | K^0 \rangle\end{aligned} \tag{22}$$

and

$$\langle 0 | \tilde{\mathcal{L}}^{\Delta S=1} | K^0 \rangle = \langle 0 | \bar{d}_R d_L + \bar{s}_R s_L | 0 \rangle \langle 0 | \bar{s}_R d_L | K^0 \rangle \,, \tag{23}$$

where the third term in (22) and the tadpole contribution (23) are missing in the original computation in Ref. 4. Keeping the vertices independent of the momentum

$$\begin{aligned}\langle 0 | \bar{q}q | 0 \rangle &= -f^2 v/2 \,, \\ \langle 0 | \bar{s}_L d_R | K^0 \rangle &= -\langle \pi^+ | \bar{u}_R d_L | 0 \rangle \\ &= ifv/2 \,, \\ \langle \pi^+\pi^- | \bar{d}_R d_L | 0 \rangle &= \langle \pi^- | \bar{s}_L u_R | K^0 \rangle = v/2 \,,\end{aligned} \tag{24}$$

with

$$v = \frac{{m_{\pi^+}}^2}{m_u + m_d} = \frac{{m_{K^+}}^2}{m_u + m_s} = \frac{3{m_\eta}^2}{m_u + m_d + 4m_s} \,,$$

and noting that the matrix element $\langle \pi^+\pi^- | \bar{s}(1-\gamma_5)d | K \rangle$ can be evaluated either by current algebra or by the effective Lagrangian $\mathrm{Tr}[(\lambda_6 - i\lambda_7)U]$ for the bilinear operator $\bar{s}(1-\gamma_5)d$ (Ref. 18), so that

$$\langle \pi^+\pi^- | \bar{s}(1-\gamma_5)d | K^0 \rangle = i\frac{v}{f}\left[\frac{a_3}{2}\right] \,, \tag{25}$$

the reader can then check that the vacuum-insertion approximation yields the same results as the current algebra only for $a_3 = 2$. Indeed, Eq. (20) is respected by the vacuum-insertion method only if $a_3 = 2$.

We have thus demonstrated that in the Weinberg model of CP violation the physical CP-violating $K \to 2\pi$ amplitude vanishes to lowest order in chiral symmetry in both current-algebra and vacuum-insertion calculations. It is striking that the KM model also shares this important chiral property though for a different reason: the matrix element of the penguin operator, and hence ϵ'/ϵ, vanishes to lowest order in the low-energy expansion of chiral perturbation theory.[19]

Beyond the lowest-order chiral limit, $\mathrm{Im}a_0$ and ϵ'/ϵ may receive contributions from the following two sources.

(i) Next-order tree-graph and one-loop corrections to the weak amplitudes $\langle 2\pi | \Theta | K \rangle$, $\langle 0 | \Theta | K \rangle$, and the strong amplitude of $K\pi$ scattering. For example, the next order strong-interaction Lagrangian is of the form

$$\mathcal{L}_s^{(4)} = \frac{f^2}{\Lambda^2}(3 \text{ four-derivative terms} + \text{mass terms}) \,,$$

where $\Lambda \approx 1$ GeV is the scale of chiral symmetry.[23] Since the counterterms necessary to renormalize one-loop graphs are of the same structures as $\mathcal{L}_s^{(2)}$ (Ref. 24), both four-derivative and one-loop corrections are suppressed by a factor q^2/Λ^2 at low energies.

(ii) Current-quark masses can induce an (8,1) term to the CP-odd Lagrangian, which cannot be transformed away by any chiral rotation. This SU(3)-braking contribution to ϵ'/ϵ was first discussed by Dupont and Pham.[3] Lacking reliable techniques for handling the higher-order chiral terms and loop corrections to the weak amplitudes, we follow Donoghue and Holstein[2] in giving a crude estimate:

$$A(K^0 \to 2\pi^0) = \langle 2\pi^0 | \mathscr{L}_-^{\Delta S=1} | K^0 \rangle m_K^2/\Lambda^2$$
$$= -i\frac{\sqrt{2}}{f}\langle \pi^0 | \mathscr{L}_-^{\Delta S=1} | K^0 \rangle \frac{m_K^2}{\Lambda^2} . \quad (26)$$

The dispersive effect on the kaon mass matrix has been discussed in Refs. 3 and 25. The long-distance contribution to the matrix element $\text{Im}M_{12}$ due to the π,η,η' poles are approximately given by

$$2m_K(\text{Im}M_{12})_{\text{LD}} = \sum_i^{\pi,\eta,\eta'} \frac{\text{Im}(\langle K^0 | H_W | i \rangle \langle i | H_W | \bar{K}^0 \rangle)}{m_K^2 - m_i^2} . \quad (27)$$

Following Refs. 2 and 3, and taking into account the $\eta-\eta'$ mixing, we obtain

$$2m_K(\text{Im}M_{12})_{\text{LD}} \approx -3\langle K^0 | H_- | \pi \rangle \langle \pi | H_+ | \bar{K}^0 \rangle , \quad (28)$$

where use of the relation

$$\langle \eta_1 | H_+ | K \rangle = -2\sqrt{2/3}\rho \langle \pi | H_+ | K \rangle$$

[η_1 being the SU(3)-singlet state] has been made. The parameter ρ can be determined from $K_L \to 2\gamma$ decays and is found to be 0.96 (with 30% error) (Ref. 26)], which is close to unity as expected from the quark model. Without the $\eta-\eta'$ mixing, it has been noted that the η and π poles cancel each other due to the FKW theorem.[2] However, the inclusion of the $\eta-\eta'$ mixing alters the situation. An almost complete cancellation occurs between π and η' poles, so the η' dispersive pole is not the dominant contribution of long-distance effects. Since the long-distance contribution to $\text{Im}M_{12}$ is quite large compared to the short-distance one,[2] it is clear that $2\xi/\epsilon_m$, and ϵ'/ϵ are Higgs-boson-mixing and *CP*-violating angle independent. Using the experimental value $\langle 2\pi^0 | H_+ | K \rangle = 2.63 \times 10^{-7}$ GeV, we find[27]

$$2\xi/\epsilon_m \approx 0.16, \quad \epsilon'/\epsilon \approx -0.007 . \quad (29)$$

Because of the crude estimate made in (26), the result (29) should be regarded as order-of-magnitude estimates. At any rate, ϵ'/ϵ obtained here is consistent with the recent measurements[29]

$$\epsilon'/\epsilon = -0.0046 \pm 0.0053 \pm 0.0024 \text{ (Chicago-Saclay)}$$
$$= 0.0017 \pm 0.0082 \text{ (Yale-BNL)} .$$

Finally, we would like to remark that the relation $(\text{Im}M_{12}/\text{Re}M_{12})_{\text{LD}} = -2\xi$ does not hold in the Weinberg model of *CP* violation even for the π,η intermediate states because of the pole contribution in Eq. (13). Accordingly, for ϵ the long-distance contribution to $\text{Im}M_{12}$ is not canceled by the one occurring in $2\xi\,\text{Re}M_{12}$. In fact, if the aforementioned relation were true, then from Eq. (8) together with $(\text{Im}M_{12})_{\text{LD}} > (\text{Im}M_{12})_{\text{box}}$, one would have[30]

$$\frac{\epsilon'}{\epsilon} = -\frac{1}{20}\frac{\Delta m}{(2\,\text{Re}M_{12})_{\text{box}}} .$$

However, ϵ'/ϵ is now Higgs-boson-mixing-angle dependent through the $(\text{Re}M_{12})_{\text{box}}$ term, as it should not be.

B. Constraints on Im *A* and the vacuum expectation values

From the experimental limit for the mass of the charged Higgs boson and the value of ϵ, we can obtain some constraints on $\text{Im}A$ [Eq. (5)] and the vacuum expectation values. The matrix element

$$\langle \pi^0 | \mathscr{L}_-^{\Delta S=1} | K^0 \rangle \equiv \tilde{f} A_{K\pi}$$

has been computed in the MIT bag model and was found to be $A_{K\pi} = 0.4$ GeV^3 (for $\alpha_s = 1$) (Ref. 11). From Eqs. (3), (7), (26), (27), and the experimental value for ϵ, we obtain

$$\text{Im}(\alpha_1\beta_1^*)\left[\frac{1}{m_{H_1}^2}\left[\ln\frac{m_{H_1}^2}{m_c^2} - \frac{3}{2}\right] - \frac{1}{m_{H_2}^2}\left[\ln\frac{m_{H_2}^2}{m_c^2} - \frac{3}{2}\right]\right] = \frac{1}{2\sqrt{2}\tilde{m}_0^2} \quad (30)$$

with $\tilde{m}_0 = 3.3$ GeV. The present experimental lower bound for the mass of the charged Higgs boson is[32]

$$m_H \geq 16 \text{ GeV} . \quad (31)$$

With our convention that $m_{H_2} > m_{H_1}$, the term in large square brackets in (30) is $\leq 1.3 \times 10^{-2}$; thus we have a lower limit on $\text{Im}(\alpha_1\beta_1^*)$:

$$\text{Im}(\alpha_1\beta_1^*) > 2.6 . \quad (32)$$

From Eq. (6) we also have a lower bound on $(1/m_{H_1}^2 - 1/m_{H_2}^2)$:

$$\left[\frac{1}{m_{H_1}^2} - \frac{1}{m_{H_2}^2}\right] \geq \frac{v_1 v_2}{\sqrt{2}\,v v_3 m_0^2} . \quad (33)$$

This together with Eq. (5) yields

$$\frac{v v_3}{v_1 v_2} \geq 2\,\text{Im}(\alpha_1\beta_1^*) = 5.2 \quad (34)$$

and

$$m_0 > 5.9 \text{ GeV} . \quad (35)$$

For a given m_0 (typically 10 GeV), the existence of a light-charged Higgs boson is in general required in the Weinberg model unless v_3 is unexpectedly high. We remark that without the Higgs-boson penguin contribution (hence $\xi = 0$) a small value $m_0 = 2$ GeV was obtained in Ref. 33. It is of interest to note that the constraint (34) on the vacuum expectation values (VEV's) is satisfied only if $v_3 > v_1, v_2$. In other words not all three VEV's are of the same order of magnitude. Indeed, the hierarchy of the VEV's $v_3 > v_2 \gg v_1$ is assumed in Ref. 31 in order to obtain an ϵ'/ϵ consistent with the experimental value. While in our case, ϵ'/ϵ is small because of the small *CP* violation in the $K \to 2\pi$ decay amplitude due to the chiral properties of the *CP*-odd Lagrangian, and because of the large long-distance contribution to the imaginary part of M_{12}. For later purposes of calculations, we choose $m_{H_1} = 20$ GeV, $m_{H_2} = 100$ GeV, which corresponds to

$$m_0 = 6.2 \text{ GeV}, \quad \text{Im}(\alpha_1\beta_1^*) = 3.8 . \quad (36)$$

III. APPLICATIONS

In this section we reexamine and update the discussions of other *CP*-violating effects in the Weinberg model of *CP* violation. The reason for doing this is attributed to the fact that some parameters in this model are changed because of the chiral properties of the *CP*-odd Lagrangian. Hence it is worthwhile to update this model's predictions by using the new determined constraints on the fundamental parameters in this theory, Eqs. (32)—(36).

A. Electric dipole moment of the neutron

The complete expression of the EDM of a quark due to the charged-Higgs-boson exchange is given in Ref. 34. Using $m_d=7.6$ MeV, $m_c=1.5$ GeV, $m_t=45$ GeV, $K_{cd}=0.227$, $K_{td}=0.0174$, and Eq. (36), we find

$$D_n \sim 1\times 10^{-25}\ e\,\text{cm}\ , \tag{37}$$

which is to be compared with the current experimental limit $D_n < 5\times 10^{-25}\ e$ cm (Ref. 35).

The contributions from the neutral Higgs boson are proportional to the cube of the small light-quark masses and thus are negligible.[36] Nevertheless, as pointed out by Anselm *et al.*,[5] the correct estimate of the nucleon coupling with the neutral Higgs boson at low momenta leads to a value which is proportional to the nucleon mass and does not vanish in the chiral limit m_u, $m_d \rightarrow 0$. The number $D_n=10^{-22}\ e$ cm is obtained by Anselm *et al.*, with $m_0=2$ GeV under the assumption that the neutral- and charged-Higgs-boson mixing are of the same order of magnitude (see also Ref. 37):

$$\frac{\langle Hx\rangle}{v^2} \sim \text{Im}\frac{\langle \phi_1^{\dagger *}\phi_2^{\dagger}\rangle}{v_1^* v_2} \tag{38}$$

[i.e., $g^{(1)}g^{(2)}$ is of the same order as $\text{Im}(\alpha\beta^*)$ in our notation]. With Eq. (36) their result is modified to

$$D_n \leq 1.1\times 10^{-23}\ e\,\text{cm}\ . \tag{39}$$

However, no definite conclusion can be drawn about whether the Weinberg model is ruled or not, since nothing is known about the neutral-Higgs-boson mixing,[38] this can be interpreted as an equation for the upper bound on $g^{(1)}g^{(2)}$. The current experimental limit sets

$$g^{(1)}g^{(2)} \leq 4.5\times 10^{-2}\ . \tag{40}$$

B. Strong *CP*

The theoretical constraint for the effective strong *CP*-violating parameter $\bar{\theta}=\theta_{\text{QCD}}+\theta_{\text{QFD}}$ (QFD denotes quantum flavor dynamics) inferred from the upper bound on D_n is $\bar{\theta}<10^{-9}$ (Ref. 39). In models in which *CP* symmetry is spontaneously broken, $\theta_{\text{QCD}}=0$ appears naturally since the initial Lagrangian is *CP* invariant. Also

$$\theta_{\text{QFD}}(=\arg\det M_R=\arg\det(M_U)_R+\arg\det(M_D)_R)$$

is finite and calculable to all orders in the perturbation theory. At the tree level

$$\bar{\theta}_{\text{tree}}=N(\theta_1-\theta_2)\ , \tag{41}$$

where θ_1 and θ_2 are the phases of the VEV of ϕ_1 and ϕ_2, respectively, and N is the number of generations. To have weak *CP* violation, $(\theta_1-\theta_2)$ cannot be set to $m\pi/2$, with m integer.[6] Thus even at the zeroth order, $\bar{\theta}$ in the Weinberg model is of order unity. Even if one imposes a discrete symmetry on the Higgs potential so that $N(\theta_1-\theta_2)=n\pi(n\neq 0)$ and thus strong *CP* is conserved, one still has trouble with the radiative corrections to $\bar{\theta}$. At the one-loop level, the exact expression of the radiative correction from the charged Higgs bosons is given in Ref. 42. From Eq. (36), it follows that numerically

$$\bar{\theta}_{1\ \text{loop}} \sim 3\times 10^{-3} \tag{42}$$

which is also too large at the one-loop order. In fact, this is a general feature for spontaneous *CP*-violating models, though $\theta_{\text{QCD}}=0$ and θ_{QFD} is finite and calculable, $\bar{\theta}$ usually turns out too large unless one imposes an additional *ad hoc* discrete symmetry (for instance, the Peccei-Quinn symmetry, as done in Ref. 43) or enlarges the gauge group to protect a small $\bar{\theta}$. Before proceeding, we comment that the contribution of the neutral Higgs bosons to $\bar{\theta}_{1\ \text{loop}}$ is roughly of the same order as that of the charged one.

C. *T*-odd correlation in $K_{\mu3}$ decays

A test of the *T*-odd correction is the transverse polarization of the muon in $K_{\mu3}$ decays. The degree of the muon transverse polarization is usually expressed in terms of the parameter $\xi=f_-/f_+$, where f_+ and f_- are the form factors of the vector and scalar parts, respectively, of the $K_{\mu3}$ amplitude. The imaginary part of ξ governs the size of the *T*-odd polarization. In the Weinberg model it is given by[9,43]

$$\text{Im}\xi=-\frac{m_K{}^2}{2\sqrt{2}\,m_0{}^2}\frac{v_2{}^2}{v_3{}^2}\ , \tag{43}$$

where v_2 and v_3 are the vacuum expectation values of the Higgs fields ϕ_2 and ϕ_3, whose neutral components couple only to down quarks and leptons, respectively. From Eqs. (31), (33), and with $v_3 > v_1 \sim v_2$ we obtain an upper bound

$$\text{Im}\xi \leq 5\times 10^{-4} \tag{44}$$

while experimentally[44]

$$K_{\mu3}^{+}:\ \text{Im}\xi=-0.017\pm 0.025\ ,$$

$$K_{\mu3}^{0}:\ \text{Im}\xi=-0.020\pm 0.022\ .$$

This *T*-violating polarization is a unique prediction of the Weinberg model; all other known viable models of *CP* nonconservation predict a null result at the tree level.[36]

D. $K\rightarrow 3\pi$

Effects of *CP* nonconservation in $K\rightarrow 3\pi$ decays can be studied using the standard current-algebra technique to relate to the better known *CP*-violating $K\rightarrow 2\pi$ decays. It has been shown that[45,46] as long as the commutation relation

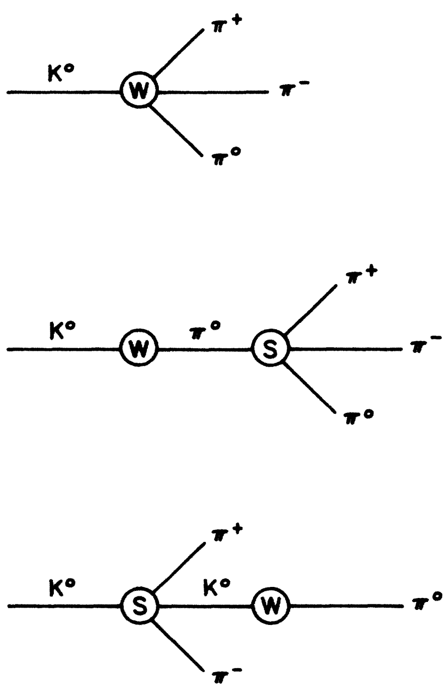

FIG. 1. $K^0 \rightarrow \pi^+\pi^-\pi^0$ decay due to the Lagrangian (9). The $S(W)$ denotes a strong (weak) vertex.

$$[Q_5^3, H^{\Delta S=1}] = [Q^3, H^{\Delta S=1}] \tag{45}$$

holds for both *CP*-conserving and -violating interactions, then at the center of the Dalitz plot (i.e., $s_1 = s_2 = s_3$)

$$\eta_{+-0}(0) - \epsilon = -2\epsilon' e^{i(\delta_2 - \delta_0)} . \tag{46}$$

The above result holds in the KM model since only the left-handed quarks are involved in weak interactions. In the Weinberg model *CP* violation in the decay amplitude $K \rightarrow 3\pi$ also vanishes to lowest order in chiral symmetry (i.e., for vertices independent of the momentum) as in the case of $K \rightarrow 2\pi$ decays due to the FKW theorem. A direct calculation of Fig. 1 using Eqs. (14), (15), and (19) shows

$$A(K^0(k) \rightarrow \pi^+(k_1)\pi^-(k_2)\pi^0(k_3)) \propto k\cdot(k_3 - k_2)/(m_K{}^2 - m_\pi{}^2) . \tag{47}$$

The momentum-independent terms are thus canceled out. Again, higher-order chiral terms and SU(3) breaking can generate *CP* violation in the decay amplitude at the center of the Dalitz plot. Since the *CP*-violating Lagrangian (9) does respect the commutation relation (45) to lowest order in chiral perturbation theory, the result (46) is valid in the Weinberg model of *CP* nonconservation. Of course, ϵ' and hence η_{+-0} in Eq. (46) is model dependent.

IV. CONCLUSIONS

Motivated by the recent work on the reanalyses of ϵ'/ϵ in the Weinberg model of *CP* violation, we have reexamined the chiral properties of the *CP*-violating weak amplitudes. In $K \rightarrow 2\pi$ decays there is a new pole contribution which arises from the combination of a strong $K\pi$ vertex and a K-vacuum tadpole. The strong $K\pi$ scattering amplitude depends on the chiral-symmetry nonlinear realization. However, the weak $K \rightarrow 2\pi$ transition is also realization dependent (so is the soft-pion relation), which renders the physical $K \rightarrow 2\pi$ amplitude nonlinear-realization independent, consistent with the Chisholm theorem. Consequently, the *CP*-odd amplitude vanishes to lowest order in the low-energy expansion of chiral perturbation theory. Although this is well known to experts in the field, the realistic calculation of the lowest-order amplitude has not been treated correctly in the literature. This is the primary purpose of this paper to clarify this problem and to show how to obtain the correct results. It is also pointed out that the vacuum-insertion method works only in the realization scheme in which $a_3 = 2$.

From the experimental value of ϵ, some constraints can be set on the Higgs-boson mixing and *CP*-violating angles and on the relative magnitudes of the vacuum expectation values. It turns out not all three vacuum expectation values are of the same order of magnitude. In fact, this is the starting assumption of Ref. 31 to obtain a small ϵ'/ϵ.

We also have reexamined various *CP*-violating effects which can be manifested in this model, these are the electric dipole moment of a neutron, the strong *CP*-violating parameter θ, the transverse polarization of the muon in $K_{\mu 3}$ decays, *CP* violation in the decay amplitude of $K \rightarrow 3\pi$ decays, and the charge asymmetry in $K_L \rightarrow 3\pi$ decays. The result $\epsilon'/\epsilon \approx -0.007$ is barely compatible with the present experimental measurements. The electric dipole moment of the neutron due to the neutral-Higgs-boson exchange and the current experimental limit are used to set an upper bound for the neutral-Higgs-boson mixing. The strong *CP*-violating parameter θ is too large (of order unity) at the tree level and does not get improved at the one-loop correction. As in the KM model, this difficulty may be avoided by imposing the Peccei-Quinn symmetry (so there would be four Higgs doublets), as already done in Ref. 43. A direct calculation also shows that no on-shell $K \rightarrow 3\pi$ transition amplitude can be induced by the *CP*-odd Lagrangian to lowest order in chiral symmetry, in accordance with the Feinberg-Kabir-Weinberg theorem.

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