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Depth-intensity relation for large depths in sea water derived from the primary-cosmic-ray energy spectrum

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Using the energy-loss parameters in water for muons according to Bezrukov and Bugaev the range-energy relation in sea water was derived and compared with previous results. For the first time the depth-intensity relation was derived dynamically from the latest primary spectrum in the range 1000–6000 $\mathrm{hg\,cm^{-2}}$ by considering the depth-energy relation and also accounting for the loss of muon intensity due to range fluctuations. The calculated result is in accordance with the latest depth-intensity measurements of the Soviet group in the Caribbean Sea.

INTRODUCTION

The investigation of cosmic rays deep underground and in the deep sea is of great importance because precise knowledge is necessary for underground experiments which are widely performed for the search of proton decay, ultrahigh-energy interactions, and neutrino astrophysics.

The rate-of-energy-loss parameters of muons in water due to ionization, pair production, bremsstrahlung, and nuclear interactions have been recently modified by Bezrukov and Bugaev,[1] and we have calculated those parameters along with their effective values by following the procedure of Kobayakawa.[2] The estimated muon energy-loss rate has been compared with the latest results for CERN experiments.[3] The estimated depth-energy curve has been compared with the results of Elbert[4] and Inazawa and Kobayakawa.[5] Finally from the muon energy spectrum at sea level[6] derived from the latest primary nucleon spectrum[7] the depth-intensity spectrum in the range 1000–6000 $\mathrm{hg\,cm^{-2}}$ has been calculated. This spectrum has been corrected for range fluctuations due to catastrophic processes by using the analytical procedure of Minorikawa, Kitamura, and Kobayakawa[8] and compared with the Caribbean Sea water measurements.[9–11]

DYNAMICAL METHOD

The energy spectrum of sea-level muons can be fitted by a power law of the form

$$I_{\mu 0}(>E)=KE^{-\gamma_\mu}\,. \tag{1}$$

The muon at energy E at sea level reaching depth h is losing energy due to electromagnetic interactions and nuclear interactions. The muon energy-loss rate in water is

$$-\frac{dE}{dh}=a_{\text{ion}}+(b_P+b_B+b_N)E\,, \tag{2}$$

where, according to Bezrukov and Bugaev,[1] the ionization loss parameter a_{ion} is given by

$$a_{\text{ion}}=[2.18+0.0853\ln(E_m/\mu)]\ \mathrm{MeV\,g^{-1}cm^2} \tag{3}$$

with $\mu=0.105\,66$ GeV, $E_m=e(E+eA)/A$, $e=2.718$, $A=11.3$ GeV. The coefficients of muon energy-loss parameters for pair production, bremsstrahlung, and nuclear interaction, viz., b_P, b_B, and b_N, have been found to follow the form

$$b_j=(C_{3j}\ln^3E+C_{2j}\ln^2E+C_{1j}\ln E+C_{0j})10^{-6}\ \mathrm{g^{-1}cm^2}\,, \tag{4}$$

where $j=P,B,N$, and C_{ij} are constants shown in Table I. The muon energy-loss relation (2) was solved by Kobayakawa[2] and is given by

$$h=\frac{1}{b_{\text{eff}}}\ln(1+b_{\text{eff}}E/a_{\text{eff}})\,, \tag{5}$$

where a_{eff} and b_{eff} are the effective values of a_{ion} and $b_T=b_P+b_B+b_N$, respectively. The values a_{ion} and b_T are energy dependent, but to simplify calculations their effective values were introduced by Kobayakawa;[2] these are slightly less than the exact values and do not change much with energy. On assuming that the muon energy losses in water are continuous the muon intensity at a depth h can be estimated as

TABLE I. The parametric values from Bezrukov and Bugaev (Ref. 1) for b_P,b_B,b_N estimation in sea water.

b_i	C_3	C_2	C_1	C_0
b_P	0.002 27	−0.072 65	0.79	−1.2
b_B	−0.000 279	−0.005 63	0.197	0.078
b_N	0.001 16	−0.019 25	0.106	0.2285

$$I_{\mu 0}(>h)=K\left[\frac{a_{\text{eff}}}{b_{\text{eff}}}[\exp(b_{\text{eff}}h)-1]\right]^{-\gamma_\mu} . \quad (6)$$

But at a large depth, viz., comparable to the radiation length, the catastrophic process (bremsstrahlung and nuclear interactions) suffers. A correction factor $R_{\text{fl}}=I_{\mu 0}(>h)/I_{\text{fl}}(>h)$ is introduced to take into consideration range fluctuations. We have used the analytical solution for R_{fl} after Gurentsov, Zatsepin, and Mikhalčhi[12] modified by Minorikawa, Kitamura, and Kobayakawa[8] for simplicity. The correction factor R_{fl} is given by

$$R_{\text{fl}}=I_{\mu 0}(>h)/I_{\text{fl}}(>h)$$
$$=\left[\frac{b_P+b_f}{b_P+kb_f}\exp[b_f(\chi-k)h]\left[\frac{\exp[(b_P+kb_f)h]-1}{\exp[(b_P+b_f)h]-1}\right]\right]^{\gamma_\mu} , \quad (7)$$

where $b_f=b_B+b_N$, χ and k are independent of h or E but dependent on γ_μ. By using expressions (1)–(7) the muon flux at a great depth h under water can be calculated.

RESULTS AND DISCUSSION

In a recent survey[7] it is found that the latest primary nucleon spectrum from the directly estimated JACEE (Ref. 13) and DEIS (Ref. 14) results yield the primary nucleon spectrum of the form

$$N(E)dE=2.026E^{-2.7}dE\ (\text{cm}^2\text{s sr GeV})^{-1} . \quad (8)$$

Starting from this primary spectrum the sea-level muon energy spectrum in the vertical direction has been calculated by using the procedure described in Ref. 6 which gives

$$I_{\mu 0}(>E)=2.92E^{-2.6}\ (\text{cm}^2\text{s sr})^{-1} . \quad (9)$$

Figure 1 shows the comparison of our derived muon energy spectrum with the data of different authors.[15–20] It is also found that the calculated muon spectrum supports the latest measurements[19,20] for energies up to 50 TeV. The slight deviation from the result of Khalchukov *et al.*[18] is mainly due to the larger meson spectral index chosen by them.

The depth-intensity relation for muons can be calculated from the derived muon energy spectrum by using the quantum electrodynamical model developed by Bezrukov and Bugaev.[1] The muon energy-loss rate $a(E)$ from ionization and excitation losses in sea water has been estimated using relation (3) and is plotted along with its effective values in Fig. 2. The coefficient of muon energy-loss parameters for pair production, bremsstrahlung, and nuclear interactions, viz., b_P, b_B, and b_N, have been calculated by using the parameters C_{ij} from Table I and relation (4) and have been presented in Fig. 3. The total muon energy-loss coefficient $b(E)$ along with effective values b_{eff} are also plotted as functions of muon energy in the same figure.

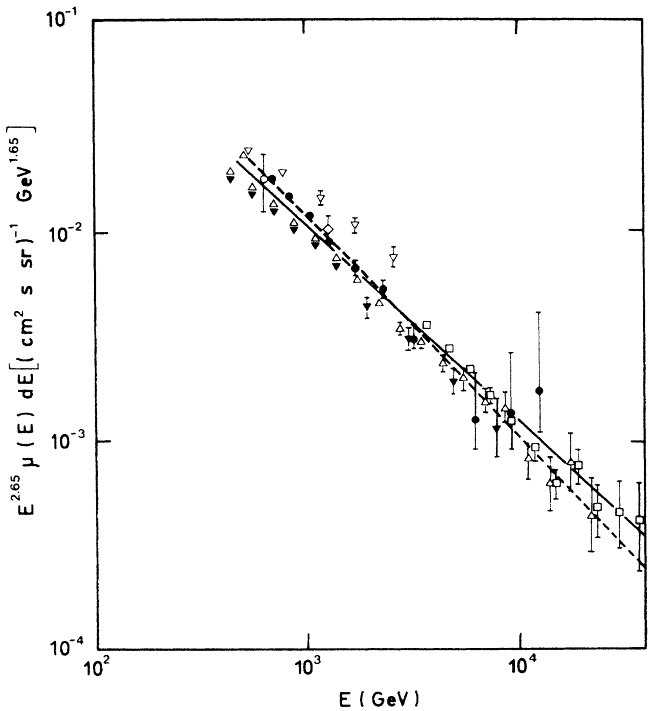

FIG. 1. Muon energy spectrum at sea level. Full line is the calculated result (Ref. 6) from the latest primary nucleon spectrum (Ref. 7). Broken line is the calculated result after Khalchukov *et al.* (Ref. 18). Experimental data shown from the compilation of Khalchukov *et al.* (Ref. 18): ○, Allkofer, Carstensen, and Dau (Ref. 15); ▽, Thompson, Thornley, Whalley, and Wolfendale (Ref. 16); ◇, Rastin (Ref. 17); ●, Khalchukov *et al.* (Ref. 18); □, Ivanenko *et al.* (Ref. 19); △, Matsuno (Ref. 20).

The calculated energy loss rate from relation (2) has been compared with the estimated values of Lohmann, Kopp, and Voss[3] in Fig. 4. The present estimated muon energy-loss rate in water is well in accord with their calculated results for muon energies above 2000 GeV.

Using the relation (5) and the effective values of $a(E)$

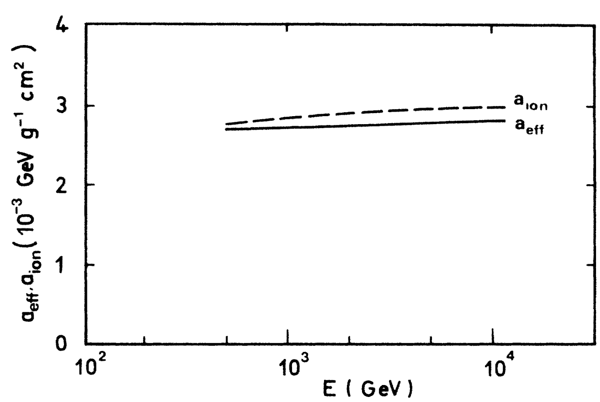

FIG. 2. Muon energy-loss parameters in sea water as a function of energy owing to collision, excitation, and knock-on electrons from Bezrukov and Bugaev (Ref. 1). Solid and dashed curves represent the effective and exact values, viz., a_{eff} and a_{ion}, respectively.

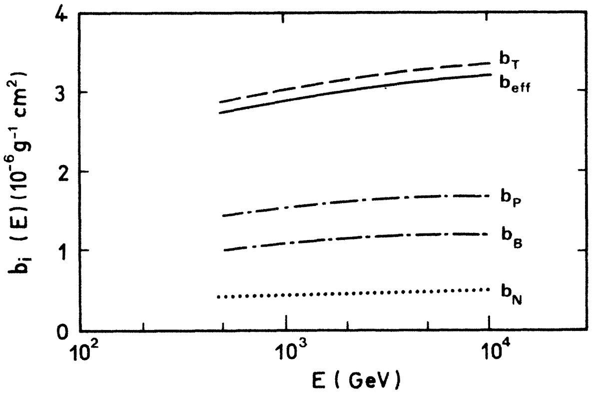

FIG. 3. The energy-loss coefficient of muons in sea water vs energy: curve b_P for pair production; curve b_B for bremsstrahlung; curve b_N for nuclear interaction calculated from Bezrukov and Bugaev (Ref. 1). The total energy-loss coefficient $b_T = b_P + b_B + b_N$ and its effective values are represented by dashed and solid curves, respectively.

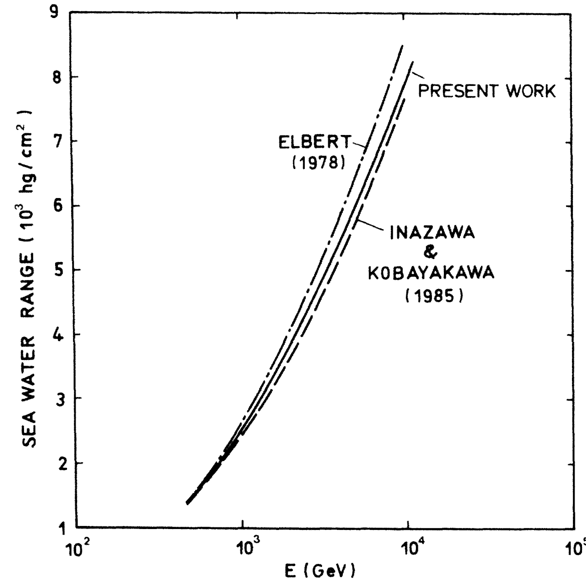

FIG. 5. Range-energy curves for muons in sea water: solid curve, present work from Ref. 1; dashed curve, from Ref. 5; dotted-dashed curve, from Ref. 4.

and $b(E)$ from Figs. 2 and 3 the depth-energy relation in sea water has been calculated and is displayed in Fig. 5 along with the estimated results of Elbert[4] and Inazawa and Kobayakawa.[5] The present calculated depth-energy result from Bezrukov and Bugaev[1] lies between the derived results of Elbert[4] and Inazawa and Kobayakawa.[5]

The difference in the calculated results occur due to different values for a_{eff} and b_{eff} which are based on different dynamical considerations.

The muon makes frequent encounters with target atoms losing in each a very small fraction of its incident energy. Muon energy loss in a target because of ionization, excitation of atoms, and production of knock-on electrons is a continuous process. The muon loses a large but random fraction of its energy by catastrophic collisions of muons with target atoms. The presently derived sea-water depth-energy relation is based on the energy-loss formulas for a continuous process in ionization and excitation, pair production and the catastrophic processes, viz., bremsstrahlung and nuclear interactions. These catastrophic processes suffer from fluctuations at great depths. Thus the calculated depth-intensity relation has been corrected for the loss of muon intensity due to range fluctuations in

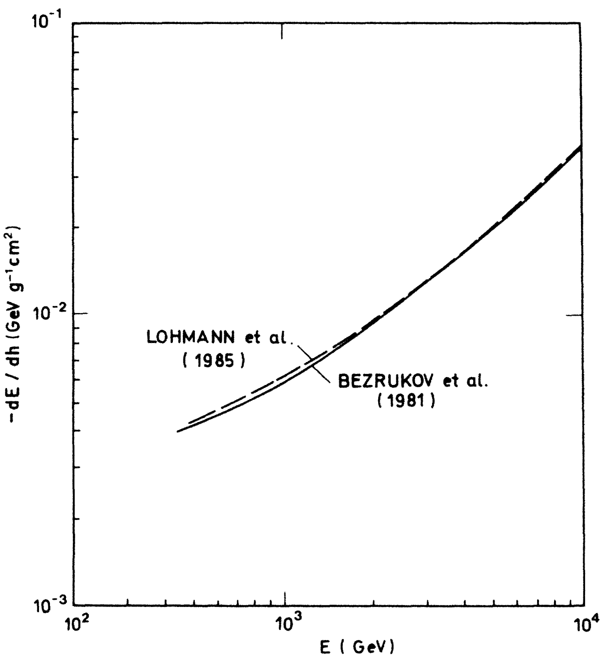

FIG. 4. The total muon energy-loss rate in sea water: solid curve is the present calculated result after Bezrukov and Bugaev (Ref. 1); dashed curve is the latest calculated results for CERN experiments (Ref. 3).

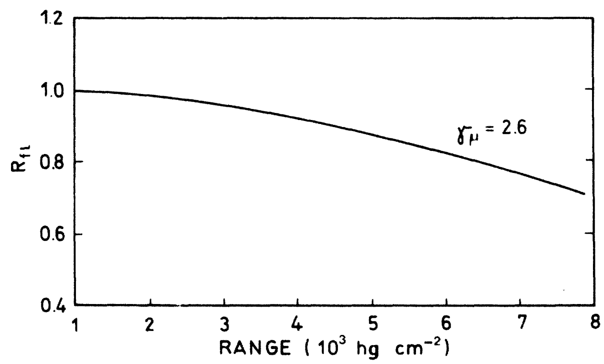

FIG. 6. Correction factor R_{fl} for range fluctuations in sea water due to catastrophic processes plotted as a function of depth after Ref. 11 for integral muon energy spectral index $\gamma_\mu = 2.6$.

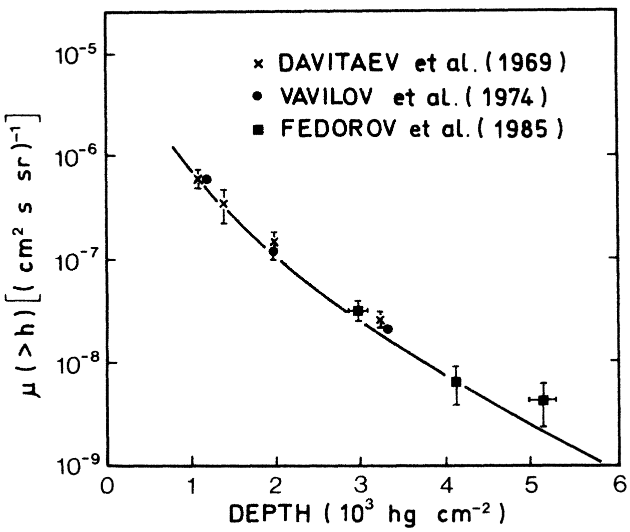

FIG. 7. Vertical muon intensity vs depth under sea water. Solid curve is the present calculated result for $\gamma_\mu=2.6$. Experimental data: ×, Davitaev, Fedorov, Trubkin, and Vavilov (Ref. 9); ●, Vavilov, Trubkin, and Fedorov (Ref. 10); □, Fedorov, Pustovetov, Trubkin, and Kirilenkov (Ref. 11).

the catastrophic processes. We followed the procedure of Minorikawa, Kitamura, and Kobayakawa[8] who modified the method of Gurentsov, Zatsepin, and Mikhalčhi[12] which is based on the analytic solution of the kinetic equation for muons through thick layers of matter. Using (7) and b_P, b_B, b_N from Fig. 3 we have estimated the correction factor R_{fl} for $k=0.36$, $\chi=0.69$, and $\gamma_\mu=2.6$ which is displayed in Fig. 6.

We have corrected our depth-intensity results using the relations (6) and (7) and plotted them in Fig. 7 along with the muon flux data in the range 1000—6000 hg cm^{-2} in sea water measured by Soviet groups.[9–11] The present calculated depth-intensity spectrum agrees with the experimental data.

CONCLUSION

Starting from a muon energy spectrum which was derived from the latest primary spectrum, the muon depth-intensity relation in sea water has been calculated. The muon energy-loss rate after Bezrukov and Bugaev[1] and the correction procedure for accounting for the muon intensity loss due to the range fluctuations due to catastrophic processes after Minorikawa, Kitamura, and Kobayakawa[8] have been used in this investigation. The calculated depth-intensity relation in the sea-water range 1000—6000 hg cm^{-2} is well in accord with the Caribbean Sea water depth-intensity data of Soviet groups.[9–11]

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