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Phase transition induced by cosmic strings

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It is shown that gauge-theory strings can alter the nature of a cosmic phase transition. An otherwise first-order transition is converted to a smooth one and supercooling is precluded. String configurations with the full symmetry group is not restored in the core play an important role in this phenomenon. Bubbles of true vacuum nucleate on strings by a real-time evolution rather than quantum tunneling, and then begin to grow at the speed of light. Implications for cosmology are discussed, and conditions needed to produce the observed smooth Universe are derived.

Many interesting consequences of grand-unified-theory (GUT) vortices[1] in cosmology have been studied.[2–5] The nature of a phase transition in GUT's has been shown to have profound implications for the evolution of the Universe.[6] The role of SU(5) monopoles in a first-order cosmic phase transition has been investigated by Steinhardt[7] and by Hosotani.[8] In this paper I will show that if certain general conditions are satisfied, vortices can prevent supercooling in a cosmic phase transition. I shall consider a general grand-unified model involving two or more stages of symmetry breaking. Vortices, also referred to as strings in the following, are supposed to arise at some early stage and supercooling can potentially occur at one of the later stages of symmetry breaking. The two scalar fields, viz., the one involved in the vortex configuration and the one that signals the first-order transition, called χ and ϕ in the following, have mutual polynomial coupling terms consistent with renormalizability. The coefficients of the coupling terms are chosen to maintain the mass hierarchy. It will be shown that in some of the vortex sectors, the hierarchy arrangement can break down in the interior of the vortex and $\langle\phi\rangle$ can be forced to become as large as $\langle\chi\rangle$. This is sufficient to cause $\langle\phi\rangle$ to evolve into a configuration of lower energy without recourse to tunneling. Such a process may be referred to as a real-time rollover from a false vacuum to the true vacuum. The entire scenario is possible in the SO(10) GUT model

$$\mathrm{SO}(10)\underset{126}{\xrightarrow{M_1}}\mathrm{SU}(5)\underset{45}{\xrightarrow{M_2}}\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$$
$$\underset{10}{\xrightarrow{M_3}}\mathrm{SU}(3)\times\mathrm{U}(1)_{\mathrm{EM}}\ , \qquad (1)$$

where $M_1 \simeq 10^{16}$ GeV, $M_2 \simeq 10^{14}$ GeV, $M_3 \simeq 10^3$ GeV, and **126**, etc., refer to representations of spin(10), the covering group of SO(10). (This breaking scheme has been proposed[9] in the context of galaxy formation.) Also, aside from details of the mechanism, the process is completely analogous to seeding in a supersaturated or a supercooled system. The conclusions here have no direct relevance to the inflationary models.

Let us first consider the conditions needed to ensure that $\langle\phi\rangle$ becomes large in the vortex core. A typical *Ansatz* for a vortex formed in the breaking of a group G to a subgroup H_1 consists of[10,11]

$$\langle A_\theta\rangle(r,\theta)=\frac{1}{2e}K\frac{1+L(r)}{r}\ , \qquad (2)$$

with $L(0)=-1+O(r^2)$ and $L=O(\exp(-\lambda r))$ as $r\to\infty$; $\langle A_0\rangle, \langle A_r\rangle, \langle A_z\rangle=0$, and

$$\langle\chi\rangle(r,\theta)=f(r)M(\theta)\ , \qquad (3)$$

with $F(0)=0$ and $f\to f_0+O(\exp(-\xi r))$ as $r\to\infty$. Here λ and ξ are constants, K is a generator in G, and M takes values in the representation of G to which χ belongs. If $f_0M_0\equiv\langle\chi\rangle_0$ is a nontrivial minimum of $V(\chi)$ then we choose

$$M(\theta)=\exp(\tfrac{1}{2}iK\theta)\circ M_0\ , \qquad (4)$$

where $\circ$ signifies the action of the group appropriate to the representation to which χ belongs.

If another scalar ϕ signals the further breaking $H_1\to H_2$ by acquiring a vacuum expectation value $\langle\phi\rangle_0=g_0S_0$, and if it couples to the vortex, energy considerations dictate the *Ansatz*

$$\langle\phi\rangle(r,\theta)=g(r)S(\theta)\ , \qquad (5)$$

$$S(\theta)=\exp(\tfrac{1}{2}iK\theta)\circ S_0\ , \qquad (6)$$

with $\langle\phi\rangle$ approaching 0 or g_0S_0 at $r=\infty$ depending on whether H_1 is unbroken or broken.

We shall assume biquadratic coupling terms between ϕ

and χ,

$$V(\chi,\phi)=c_1\,\mathrm{Tr}\chi\phi\chi\phi+c_2\,\mathrm{Tr}\chi^2\phi^2 +c_3(\mathrm{Tr}\chi\phi)^2+c_4\,\mathrm{Tr}\chi^2\,\mathrm{Tr}\phi^2\ , \tag{7}$$

where Tr means contraction of all group indices. In general, terms linear and cubic in each of the fields may also exist, but they may not be possible for a general choice of representations for χ and ϕ.

If we now substitute the above *Ansatz* in the energy expression

$$E(\phi)=\int d^3x\,[\tfrac{1}{2}\mathrm{Tr}\,|D_i\phi|^2+V(\phi)+V(\chi,\phi)]\ , \tag{8}$$

we find the gauge-field contribution to be $[L(r)g(r)/r]^2$, thus improving the stability of the $g=0$ minimum. As for $V(\chi,\phi)$, first consider the biquadratic terms in (7). The c_i's have been arranged to make its contribution vanish when $\langle\phi\rangle=g_0S_0$ and $\langle\chi\rangle=f_0M_0$, as per the hierarchy requirement. We now find that this term continues to vanish in the presence of the θ-dependent *Ansätze*. Because of the identical similarity transformations (4) and (6), the θ dependence disappears when traces are taken. If ϕ and χ were in such representations as to permit coupling terms cubic and linear in the fields, hierarchy arrangement will indeed break down in the core of the string.

But even in the case of purely biquadratic terms, other possibilities exist. Vortices are formed as defect lines through a chaotic process during a phase transition. We do not expect them all to form the simplest *Ansatz*. The key to a vortex configuration playing a nontrivial role during subsequent supercooling is the scalar-field *Ansatz* requiring more than one independent radial function. This happens when the chosen M_0 has a piece carrying no K charge and gives rise to a θ-independent term in (4). The general *Ansatz* should then be written as

$$\langle\chi\rangle(r,\theta)=q_1(r)M_1+q_2(r)M_2(\theta)\ , \tag{9}$$

in which q_1, q_2 both go to f_0 as $r\to\infty$, but for regularity at the origin, we need $q_2(0)=0$ whereas $dq_1/dr(0)=0$. One then finds that $q_1(0)\neq 0$ and thus G remains broken in the core. A simple SO(3) example is lucidly described in Ref. 11. It has also been shown[12] that such string configurations behave like superconducting wires and could produce intriguing astrophysical effects. The details of setting up an *Ansatz* such as (9) will be discussed in a separate publication.

$\langle\phi\rangle$ could remain as in (5) or be modified to

$$\langle\phi\rangle(r,\theta)=p_1(r)S_1+p_2(r)S_2(\theta)\ , \tag{10}$$

with appropriate behaviors for p_1 and p_2. Now a hierarchy arrangement made with M_0 and S_0 breaks down near the origin and the interaction between the two scalars becomes nontrivial. If the c_i's are such as to make $\delta^2V(\chi,\phi)/\delta\phi^2<0$ for $\chi\simeq q_1(0)M_1$ and $p_2\simeq 0$, $\langle\phi\rangle$ will acquire values of the same order as $\langle\chi\rangle$ in the core of the vortex.

I shall now demonstrate the mechanism involved in the real-time rollover. We shall be concerned only with r greater than some typical r_0, which is meant to be a radius at which $q_1\simeq q_2\simeq f_0$ and if $\langle\phi\rangle$ is as in (10), $p_1(r)$, $p_2(r)$ are the same function $g(r)$, so that the hierarchy arrangement is in place, but $g(r_0)\sim f_0$ and falls off with radius over a characteristic length much longer than that of $\langle\chi\rangle$. We shall assume the problem to be cylindrically symmetric and 2+1 dimensional. Ignoring the z direction and including time dependence, we write the Lagrangian for g as

$$L_g=2\pi\int_0^\infty r\,dr\left[\frac{1}{2}\left(\frac{\partial g}{\partial t}\right)^2+\frac{1}{2}\left(\frac{\partial g}{\partial r}\right)^2-V^T(g)\right]\ ,$$

$$V^T=\frac{\sigma}{4!}g^4+\frac{\gamma}{3!}g^3+\tfrac{1}{2}(m^2+AT^2)g^2-\frac{\pi^2}{90}T^4+V_0\ . \tag{11}$$

Here, $m^2>0$, $\gamma<0$, $|\gamma|\sim m$. T is the temperature in the same units as m, and V^T is a typical effective potential for a scalar field at nonzero temperature,[2,13] to leading order in T. A turns out to be a dimensionless constant between 1 and 10 in a typical GUT; $N(T)$ accounts for the number of particle species present at T. Because $\gamma<0$ and $m^2>0$, such a potential typically has two minima: one at $g=0$ and the other at some $g>0$. These will be referred to as g_1 and g_2, respectively. $V_0>0$ makes $V^T(g_2)=0$ at $T=0$. Let $T_{\rm cr}$ be the critical temperature at which $V^T(g_1)=V^T(g_2)$. It is convenient to define $(\sigma/4!)m_{\rm cr}{}^2\equiv(m^2+AT_{\rm cr}{}^2)/2\equiv(1/3!)(\gamma^2/\sigma)$ and for small variations ΔT around $T_{\rm cr}$, $\varepsilon\equiv 4!AT_{\rm cr}\Delta T/\sigma m_{\rm cr}{}^2\sim\Delta T/T_{\rm cr}$. Finally, rescaling $t\to m_{\rm cr}t$, $r\to m_{\rm cr}r$, and $g\to g/m_{\rm cr}$, we find

$$V^T=\frac{\sigma}{4!}g^2(g-1)^2+\frac{\sigma}{4!}\varepsilon g^2\ , \tag{12}$$

and the equation satisfied by g is

$$\frac{\partial^2 g}{\partial t^2}-\frac{\partial^2 g}{\partial r^2}-\frac{1}{r}\frac{\partial g}{\partial r}+\frac{\sigma}{6}g^3-\tfrac{1}{4}\sigma g^2+\frac{\sigma}{12}(1+\varepsilon)g=0\ . \tag{13}$$

For $T>T_{\rm cr}$ we assume an essentially time-independent solution approaching $g_\infty\equiv g(r=\infty)=0$ as $r\to\infty$. It changes only adiabatically as V^T itself changes. As T drops below $T_{\rm cr}$, a solution with $g_\infty=g_2$ becomes energetically favorable; but in the absence of a vortex, a transition to such a configuration can come about only via quantum tunneling. However, in some region near the vortex, $g(r)$ passes through the value g_2. The question then is whether this region can begin to grow in such a way that the configuration evolves into one with $g_\infty=g_2$. We answer this in two parts, first asking whether a solution with $g_\infty=g_1$ exists for $T<T_{\rm cr}$, i.e., for $\varepsilon<0$, and if not, what kind of time dependence sets in at the point when no time-independent solution can be found.

To answer the first question, we look at the equivalent problem of a point particle moving in a potential $-V$ with time $\tau=r$. This potential has two maxima x_1 and x_2 (corresponding to g_1 and g_2). The particle is described by the equation

$$\frac{d^2x}{d\tau^2}+\frac{1}{\tau}\frac{dx}{d\tau}=-\frac{d(-V(x))}{dx}\ . \tag{14}$$

The initial condition at $\tau=\tau_0\gtrsim 0$ is a large value $x(\tau_0)$ and a large negative $dx/d\tau(\tau_0)$. The particle receives a big kick at τ_0 and the question is whether the particle will shoot over the hump at x_2 but then take infinite time to reach x_1. If the particle does clear the hump at x_2 at sufficiently small τ, the dissipative term $(1/\tau)(dx/d\tau)$ will still be effective and can, in principle, just exactly dissipate the excess potential energy and all the kinetic energy. The simultaneous nonlinear equations resulting from descretizing r in the time-independent part of (15) were solved using the IMSL routine ZSPOW.[14] Initial trial configurations with all $g_i = 0$ for grid points with large r were provided to ZSPOW to implement Newton's method. For σ between 0.1 and 1.0, solutions with $g_\infty = 0$ were found for $\varepsilon \gtrsim -0.08$. For ε more negative, ZSPOW does generate configurations similar in values to the solutions found earlier, but declares that no solution could be found close to the given trial. (See Fig. 1.) The configuration generated for ε less than the critical value is labeled unstable. In such a case, of course, a solution is always found when a trial with all $g_i = g_2$ for large r is provided. The large value of g near the origin was arbitrarily chosen to be 10 at $r_0 = 0.1$ in $m_{\mathrm{cr}} = 1$ units. Recalling that $\varepsilon \sim \Delta T/T_{\mathrm{cr}}$, and in fact $\Delta T/T_{\mathrm{cr}} < \varepsilon/2$ for $\gamma^2 > 3\sigma^2/8$, we conclude that soon after T reaches T_{cr}, the $g_\infty = 0$ solution ceases to exist. The critical value of ε was also determined for $r_0 = 0.01$, 0.05, and 0.1, $g(r_0) = 5$, 10, 15, 25, and 40, and $\sigma = 0.12$, 0.6, and 0.90. For all the possible combinations of these input values, the critical value of ε stays between -0.07 and -0.09. The persistence of this value can be explained by analytic arguments.[15]

The second part of the question is whether such a configuration is unstable toward evolving into one with $g_\infty = g_2$. This was checked numerically by finding a time-dependent solution to (13) using the IMSL routine DPDES. The initial data used were an unstable configuration ($\varepsilon < = 0.08$) found by ZSPOW and initial time derivative zero. Figure 2 shows an example. The abscissa being descretized into steps of 0.5 makes the curves jagged in places. An initial period of real-time rollover is seen to be followed by "vacuum burning," in which a wall interpolating between the two minima moves out, asymptotically approaching the speed of light. The latter is the same process that follows quantum tunneling.[16]

I shall now turn to the implications of such a phenomenon for cosmology. Assuming a Robertson-Walker metric for a spatially flat universe, we have, for the Hubble "constant,"

$$H^2(T) \equiv \left(\frac{\dot{T}}{T}\right)^2 = \frac{8\pi}{3} G \left(\frac{\pi^2}{30} N(T) T^4 + V_0\right) , \qquad (15)$$

where $N(T)$ accounts for the number of particle species. In our case $V_0 \sim m^4 \sim T_{\mathrm{cr}}{}^4$. We first note that $\Delta T \sim 0.1 T_{\mathrm{cr}}$

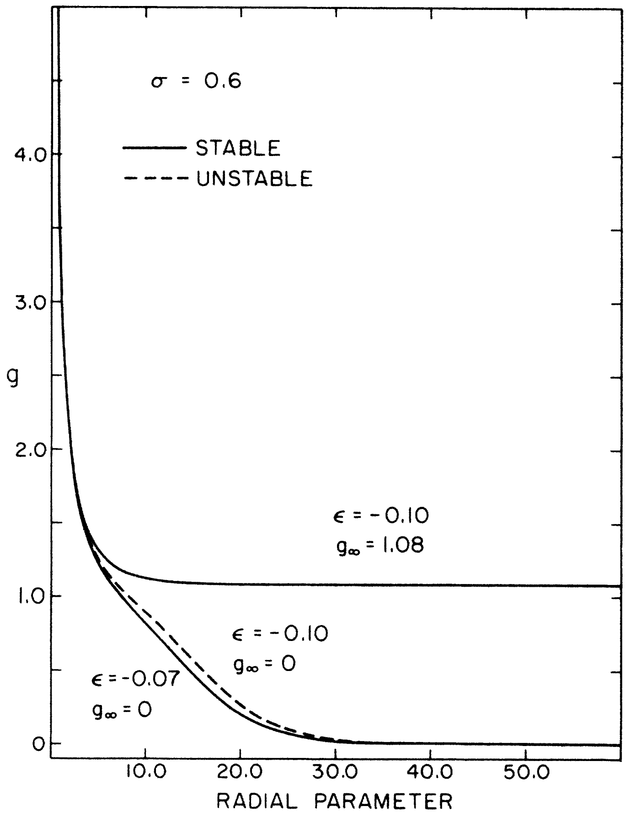

FIG. 1. Looking for time-independent solutions.

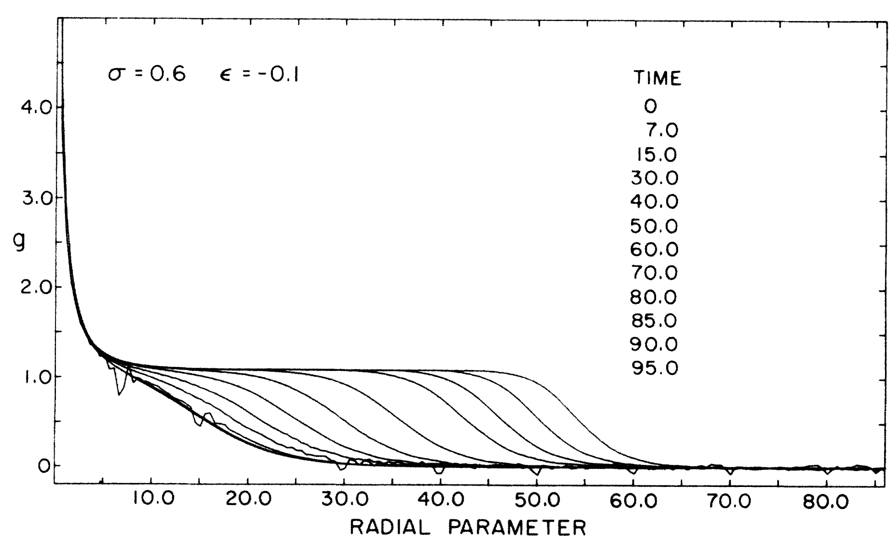

FIG. 2. Real-time rollover.

required for the onset of rollover translates to $\Delta t H(T_{cr}) \sim 0.1$, t being the coordinate time. The additional time $\sim 100 m_{cr}$ needed for completing rollover (Fig. 2) is a fraction ($\sim 100 m_{cr}\sqrt{G}$) of H^{-1}, insignificant in GUT models. This justifies the conclusion that rollover occurs soon after T_{cr} is reached. The important question to ask then is whether regions with string-induced transition can fill the whole Universe. Since strings are extended objects and the vacuum bubbles expand at the speed of light, we expect that this should be possible as long as we have sufficient string length per horizon volume. For the volume occupied by true vacuum bubbles originating on strings, we write the formula

$$V_B(t) = \pi l(t) \left[R(t) \int_{t_1}^{t} \frac{dt'}{R(t')} \right]^2 , \tag{16}$$

where R is the scale factor of the Robertson-Walker metric, and t_1 is a time at which the initial phase of rollover has been completed and the bubbles have begun to expand at the speed of light. $l(t)$ is a relevant length yet to be specified. We want to find what fraction of the horizon volume $h^3(t)$ comes to be occupied by $V_B(t)$. Strings are expected to mutually interact and continuously reduce their length[2,17] so two cases of interest are when this interaction process has really not started and when this process has reduced strings essentially to a horizon length of string per horizon volume. Thus

$$l(t) = d(t_0)[R(t)/R(t_0)]^2 h^3(t)$$

or $l(t) = h(t)$, where $d(t_0)$ is the length per unit volume of string formed originally in a phase transition at time t_0. It is a good estimate[2] to take $d(t_0) = T^{-2}(t_0)$. Then with Q denoting the relative abundance of the vortices capable of serving as seeds, we get two alternative formulas for large t:

$$V_B(t)/h^3(t) = \pi Q \frac{[T(t_1) - T(t)]^2}{H_0^2} , \tag{17}$$

$$V_B(t)/h^3(t) = \pi Q \frac{\left[\dfrac{T(t_1)}{T_{cr}} \right]^2}{\left[1 + 2\dfrac{T(t)}{T_{cr}} \left[\dfrac{30}{\pi^2 N} \dfrac{V_0}{T_{cr}{}^4} \right]^{1/2} \right]^2} . \tag{18}$$

In deriving these, pure radiation-dominated evolution [ignoring V_0 in (15)] has been assumed above T_{cr} and pure de Sitter [ignoring the T^4 term in (15)] evolution below T_{cr}. Also, we have $H_0{}^2 = (8\pi/3)GV_0$. No estimate is available for Q but we do not expect it to be smaller than 1 by many orders of magnitude. In such a case, from (17) we see that if strings are as abundant as when they were formed, the string-induced bubbles will easily fill the Universe. In (18), $T(t_1)/T_{cr} \simeq 0.9$ and the second term in the denominator grows exponentially small at large t. Also we have ignored string loops, which unlike the long strings are expected to persist.[18] Spherical bubbles of a true vacuum can originate on them, and this contribution will complement the above estimate. Even so, Q as well as other overall factors of order unity may prevent the right-hand side (RHS) from becoming 1. If too many regions of size $H_0{}^{-1}$ escape string bubbles, the resulting Universe will be inhomogeneous and unacceptable. On the other hand, if the phase transition precipitated by vortices is successfully completed, it will leave no characteristic signature in the cosmic evolution.

For strings occurring at $T(t_0) = 10^{16}$ GeV such as in the model (1), the time they take to deplete to one per horizon volume is estimated[2] to be 10^{-27} sec, the temperature then being 10^{11} GeV. Hence (17) would be relevant if such strings prevented supercooling at 10^{14} GeV and (18) would be relevant if they prevented supercooling during the electroweak transition at 10^3 GeV. Of course, for supercooling to occur at the Weinberg-Salam scale, the relevant Higgs boson cannot be in **10** but will have to be in some tensor representation.

In summary, vortices can serve as seeds particularly if the full symmetry group is not restored in their cores and if the couplings between the scalars make $\delta^2 V(\chi,\phi)/\delta\phi^2 < 0$ in the core. Vortex-induced bubbles of a true vacuum fill the Universe if the RHS's of (17) and (18), complemented by the contribution of string loops, become 1. Two important requirements for this to happen are that there be a sufficient abundance of strings that can serve as seeds and $\gamma^2 \gtrsim 3\sigma m^2/8$ so that the rollover is completed in a time short compared to H_0^{-1}.

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