Brief Reports

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Dynamical breakdown of symmetries in three-dimensional QED

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We analyze the possibility of dynamical breakdown of parity in three-dimensional QED. We find that for an even number of fermions only chiral symmetry is broken dynamically while parity is conserved. For an odd number of fermions, where a parity-conserving mass generation is not possible, we find no consistent nontrivial solution for the Schwinger-Dyson equations, leading us to conjecture that at least one fermion remains massless and no mass is induced for the photon in this case.

Three-dimensional QED with a P- and T-violating mass term for the photon[1] has attracted considerable interest recently. In the presence of massive fermions, this mass term is induced by loop corrections, even if it is absent at the tree level. The case with massless fermions is slightly more subtle.[2] The photon remains massless in the physically relevant regime[3] to all orders in perturbation theory, but the infrared divergences of the theory could lead to a dynamical generation of mass for the fermions, which in turn could lead to an induced mass for the photon.

Pisarski[4] and Appelquist *et al.*,[5] studied QED_3 with $2N$ two-component massless fermions in the large-N limit. They looked for a dynamical generation of mass for the fermions, but in their formalism they enforced parity conservation by forming N four-component spinors, whose mass term translates to a positive mass for N of the two-component fermions and a negative mass for the other N. By solving a suitably approximated Schwinger-Dyson equation for the fermion self-energy, they obtained a (parity invariant) mass for the fermion and showed that chiral symmetry was dynamically broken.

In this Brief Report, we allow for parity-breaking mass terms for the fermions. Since this would induce a mass term for the photon, we allow for a massive photon as well. We then analyze numerically an approximated set of Schwinger-Dyson equations for the self-energies of the photon and the fermions. For an even number of fermions, we find that the only self-consistent solution is when half of the fermions acquire a positive mass, the remaining half acquire a negative mass, and the photon remains massless. Though our approximations are strictly valid only in the large-N limit, we should mention that our numerical analysis was confined to $N=1$, since considerable computer time was needed to go to larger values of N, and previous experience[5] indicated that there was no reason to expect qualitatively different behavior for larger N. Furthermore, our results agree with recent analytic analyses of Stam[6] and Appelquist *et al.*,[7] and also with the general arguments of Vafa and Witten.[8] However, unlike the analytic analyses, we improve the analysis by going beyond the drastic approximation of setting $\Sigma(0)=\Sigma(p)=m$, $m<p<\alpha$ and $\Sigma(p)=0$, $p>\alpha$.[7] For an odd number of fermions, we solve the set of Schwinger-Dyson equations for $N=\frac{1}{2}$, i.e., with a single fermion. We find that within our approximations the only consistent solution of the equations is to have the fermion and the photon remain massless.

We start with $2N$ flavors of two-component complex spinors ψ_i interacting with a U(1) gauge field A_μ. The Lagrangian is given by

$$L=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\sum_{i=1}^{2N}\bar{\psi}_i \not{D}\psi_i - m_i\bar{\psi}_i\psi_i+\frac{\mu}{4}\varepsilon_{\mu\nu\alpha}F^{\mu\nu}A^{\alpha}. \tag{1}$$

When the photon and the fermions are massless, this Lagrangian has a global (flavor) $U(2N)$ chiral symmetry. If N of the fermions acquire a (explicit or dynamical) mass m_i and the remaining N acquire a mass $-m_i$, the $U(2N)$ chiral symmetry breaks to $SU(N)\times U(1)\times U(1)$, but the Lagrangian can still be parity (and time reversal) invariant and the photon remains massless. Arbitrary masses m_i for the fermions break the parity symmetry as well, and for consistency (since P-odd mass fermions induce a photon mass), $\mu\neq 0$.

To examine the possibility of either parity and/or global (chiral) symmetry breaking, we investigate the Schwinger-Dyson equations when all the $m_i=0$, $\mu=0$, but we allow different self-energies for all the fermions and a nonzero induced photon mass. The complete set of Schwinger-Dyson equations for the system described in Eq. (1) with $m_i,\mu=0$ is given by

$$S_i^{-1}(p)=-i\{\not p[1+a_i(p)]-\Sigma_i(p)\}=-i\not p+e^2\int\frac{d^3k}{(2\pi)^3}\gamma_\mu\Delta_{\mu\nu}(p-k)\Gamma_\nu(p,k)S_i(k)\ , \tag{2}$$

$$\begin{aligned}\Gamma_\mu(p_1,p_2)&=\gamma_\mu+\Lambda_\mu(p_1,p_2)\\&=\gamma_\mu-ie^2\int\frac{d^3k}{(2\pi)^3}[\Gamma_\alpha(p_2+k,p_2)S_i(p_2+k)\gamma^\mu S_i(p_1+k)\Gamma_\alpha(p_1,p_1+k)\Delta_{\alpha\beta}(k)]\ ,\end{aligned} \tag{3}$$

$$\begin{aligned}\Delta_{\mu\nu}{}^{-1}&=i\{(p^2g_{\mu\nu}-p_\mu p_\nu)[1+\pi_{\rm even}(p)]+iM(p)\varepsilon_{\mu\nu\alpha}p^\alpha\}\\&=i(p^2g_{\mu\nu}-p_\mu p_\nu)-\sum_{i=1}^{2N}e^2\int\frac{d^3k}{(2\pi)^3}{\rm Tr}[\gamma_\mu S_i(p+k)\Gamma_\nu(k,p+k)S_i(k)]\ ,\end{aligned} \tag{4}$$

where $a_i(p)$ and $\Sigma_i(p)$ are the wave-function and mass renormalizations for the fermion, $\pi_{\rm even}(p)$ and $M(p)$ are the wave-function and mass renormalizations for the photon, and $\Gamma_\mu(p_1,p_2)$ is the vertex renormalization. Since we cannot solve the full set of five coupled equations for $a_i(p)$, $\Sigma_i(p)$, $\pi_{\rm even}(p)$, $M(p)$, and $\Gamma_\mu(p_1,p_2)$, we make the following simplification. We assume that $a_i(p)$ and $\Delta_\mu(p_1,p_2)$ are small and can be neglected; and for $\pi_{\rm even}(p)$, we use the asymptotic one-loop values—as a step-function ansatz:

$$\begin{aligned}\pi_{\rm even}(p)&=\alpha/16|p|\ ,\ p>\Sigma\\&=\alpha/12\pi|\Sigma|\ ,\ p<\Sigma\ ,\end{aligned} \tag{5}$$

where $\alpha=2e^2N$ and assume that there are no further corrections. These approximations are strictly valid in the large-N limit. For small values of N, an argument can be made to justify the retention of the $O(e^2)$ contribution of $\pi_{\rm even}(p)$, versus the $O(e^2)$ contributions of $a(p)$ and $\Lambda_\mu(p_1,p_2)$ which are dropped, by noting that the one-loop contribution of $\pi_{\rm even}(p)$ is needed to cure the infrared divergences that appear in higher orders of perturbation theory.[9] A test of our approximation is to check the values of $a_i(p)$ and $\Delta_\mu(p_1,p_2)$ numerically after computing $\Sigma_i(p)$ and $M(p)$. Such checks have not been made so far, but we think that our results may be *qualitatively* valid.

Under this approximation, the fermion and gauge field propagators are (in Euclidean space)

$$S_i(p)=\frac{i[\not p+\Sigma_i(p)]}{p^2+\Sigma_i{}^2(p)}\ , \tag{6}$$

$$\begin{aligned}\Delta_{\mu\nu}(p)&=\frac{-i}{[1+\pi_{\rm even}(p)][p^2-M_R{}^2(p)]}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}-M_R(p)\varepsilon_{\mu\nu\alpha}\frac{p^\alpha}{p^2}\right)\ ,\\M_R(p)&=M(p)/[1+\pi_{\rm even}(p)]\ .\end{aligned} \tag{7}$$

Using Eqs. (6) and (7) in Eqs. (2) and (4), we get (in Euclidean space)

$$\Sigma_i(p)=\frac{\alpha}{8\pi^2N}\int\frac{d^3k[\Sigma_i(k)-M_R(p-k)(k^2-p\cdot k)/(p-k)^2]}{[1+\pi_{\rm even}(p-k)][k^2+\Sigma_i{}^2(k)][(p-k)^2+M_R{}^2(p-k)]}\ , \tag{8}$$

$$M(p)p_\alpha=\sum_{i=1}^{2N}\frac{\alpha}{4\pi^3}\int\frac{d^3k[\Sigma_i(k)(p+k)_\alpha-\Sigma_i(p+k)k_\alpha]}{[k^2+\Sigma_i{}^2(k)][(p+k)^2+\Sigma_i{}^2(p+k)]}\ , \tag{9}$$

when $\pi_{\rm even}(p)$ is given by Eq. (5).

The calculations were done numerically for two cases: $N=1$, which is the simplest case for an even number of fermions, and $N=\frac{1}{2}$, which is the simplest case for an odd number of fermions. As explained earlier, due to the time needed for the computation, we were limited to these values only.

For $N=1$ the three equations for $\Sigma_1(p)$, $\Sigma_2(p)$, and $M(p)$ in Eqs. (8) and (9) were solved by starting with arbitrary constant values of Σ_1 and Σ_2 and $M=0$ and allowing them to evolve to self-consistent values. The integrals were calculated with an ultraviolet cutoff of $(p/\alpha')=10^{10}$ (where $\alpha'=\alpha/4\pi$), and we checked that the results are not sensitive to the cutoff at that range. Besides the trivial solution, we found that the only other solution is $\Sigma_1(p)=-\Sigma_2(p)$ and $M(p)\rightarrow 0$ for all p. The results for $\Sigma_1(p)$ [and $\Sigma_2(p)$] are plotted in Figs. 1 and 2. For $p\rightarrow\infty$, $\Sigma_i(p)\sim 1/p^2$, as we would expect for a dynamical mass. For $p\rightarrow 0$, $\Sigma_1(p)=-\Sigma_2(0)\approx 0.4(e^2/4\pi)$, but considering our approximations, the actual value of $\Sigma_i(p)$ at $p=0$ should be suspected; it is the general behavior of $\Sigma_i(p)$ as $p\rightarrow 0$ [i.e., $\Sigma_i(0)\neq 0$] that can be trusted. But that is sufficient to conclude that a mass term is dynamically generated for the fermions, and moreover, *only* in such a way that parity is conserved and the global U($2N$) symmetry is the only one which is broken.

The reason for the convergence of the solution of the equations to the above result can be traced to the negative

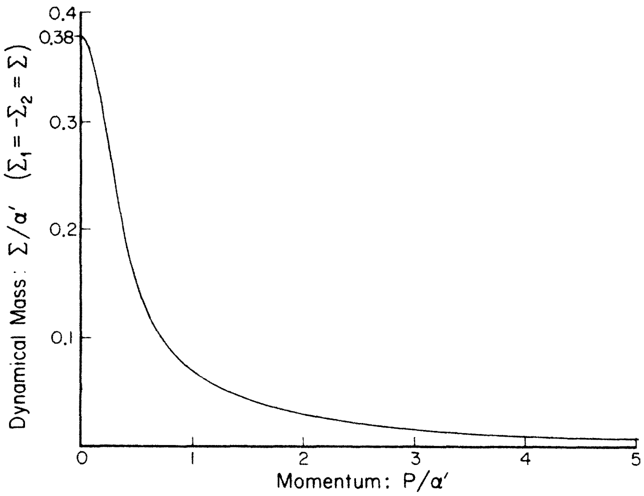

FIG. 1. The fermion self-energy $|\Sigma(p)|$ for $N=1$.

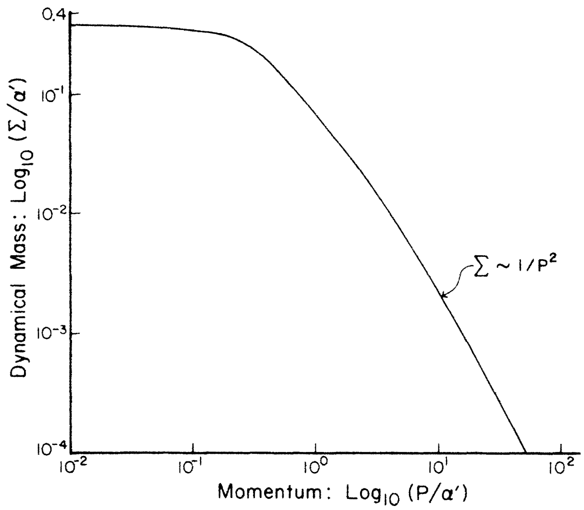

FIG. 2. The fermion self-energy for $N=1$, logarithmic scale.

sign of $M_R(p-k)$ in Eq. (8). This gives negative contributions to the integral which drives $\Sigma_i(p)$ negative, which in turn contributes to Eq. (9) and drives $M(p)$ negative. Hence the solutions oscillate, and the only consistent solution is to have $M(p)=0$ and $\Sigma_1(p)=-\Sigma_2(p)$.

For $N=\frac{1}{2}$ we could not find any nontrivial solutions to Eqs. (8) and (9), and the reason is again that the solutions oscillate due to the negative sign of $M_R(p-k)$ in Eq. (8). However, unlike the previous case, $M_{(p)}=0$ with a nonzero value of $\Sigma(p)$ is not possible. Therefore, the only solution is $\Sigma(p)=M(p)=0$, and hence, it is our conclusion that parity cannot be broken dynamically even for an odd number of fermions. We conjecture that this conclusion will not be changed by the corrections to $a_i(p)$ and $\Lambda_\mu(p_1,p_2)$.

To conclude, let us recapitulate our main results. Starting with QED_3 with an even number of massless fermions, we have shown that the fermions dynamically acquire a parity-invariant mass, and the photon remains massless, while for an odd number of fermions, no mass is generated either for the fermions or the photons. These results are supported by similar results of Ref. 7. However, a more detailed analysis which takes into acount the vertex and wave-function renormalization is needed to verify this conclusion for small N values.

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