Dirac quantization of the chiral superfield

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We extend the method of Dirac quantization in superspace to the case of chiral superfields. We obtain quantization conditions in superspace which are consistent with the conditions for the component fields. Furthermore, we show that with these modified Dirac brackets and the modified Hamiltonian the correct Heisenberg equations of motion are obtained.

I. INTRODUCTION

A Lagrangian field theory which is subject to constraints does not have a unique Hamiltonian. Canonical quantization of such a theory, therefore, requires modification of the naive Poisson brackets. One way to handle this problem was suggested by Dirac[1] and can be applied to many constrained field theories.[2]

Supersymmetric theories inherently contain constraints.[3] Such theories contain auxiliary fields whose relations to the dynamical fields are constraints in the theory. Additionally there may be other constraints one wants to impose on the fields such as in the case of the nonlinear σ model or the chiral superfields.

In a previous paper[4] it was shown how Dirac's method can be extended to superspace to give proper quantization conditions for supersymmetric quantum mechanics as well as the supersymmetric nonlinear σ model in $1+1$ dimensions. Chiral superfields are more subtle since they satisfy differential constraints in superspace rather than the usual algebraic ones.

In this paper we show that the extension of Dirac's method works well for the chiral superfields in superspace as well. We obtain quantization conditions for the superfields which reduce to the known quantization conditions for the component fields. As a spinoff we find a Hamiltonian formulation for chiral superfields.

The paper is organized as follows. In Sec. II we present the Lagrangian for the chiral superfield and discuss various constraints in this theory. Section III contains a discussion of Dirac's procedure as modified for superspace. This procedure is applied to the chiral superfield theory and the Dirac brackets are obtained for this theory. We show that these brackets are consistent with the corresponding component results. In Sec. IV we show that the Dirac brackets together with the modified Hamiltonian give the correct Heisenberg equations of motion.

II. THE CHIRAL LAGRANGIAN AND ITS CONSTRAINTS

The superspace action[3,5–8] for the massive, interacting chiral multiplet has the form

$$S=\int d^4x\, d^2\theta\, d^2\bar{\theta}\left[\bar{\Phi}\Phi+\delta^2(\bar{\theta})\frac{m}{2}\Phi^2+\delta^2(\theta)\frac{m}{2}\bar{\Phi}^2+\delta^2(\bar{\theta})\frac{g}{3}\Phi^3+\delta^2(\theta)\frac{g}{3}\bar{\Phi}^3\right], \quad (2.1)$$

where $\Phi(x,\theta,\bar{\theta})$ and $\bar{\Phi}(x,\theta,\bar{\theta})$ satisfy the chiral and antichiral constraints given by

$$\begin{aligned}\bar{D}_{\dot{\alpha}}\Phi&=[-\partial_{\dot{\alpha}}-i(\theta\sigma^m)_{\dot{\alpha}}\partial_m]\Phi=0\,,\\ D_\alpha\bar{\Phi}&=[\partial_\alpha+i(\sigma^m\bar{\theta})_\alpha\partial_m]\bar{\Phi}=0\,.\end{aligned} \quad (2.2)$$

Here and in what follows we use $\partial_m=\partial/\partial x^m$, $m=0,1,2,3$; $\partial_{\dot{\alpha}}=\partial/\partial\bar{\theta}^{\dot{\alpha}}$; $\partial_\alpha=\partial/\partial\theta^\alpha$; $\dot{\alpha},\alpha=1,2$. The conventions used throughout in this paper are the same as those of Refs. 3 and 4.

The solutions of Eq. (2.2) determine the form of the superfields to be

$$\begin{aligned}\Phi&=\exp(i\theta\sigma^m\bar{\theta}\partial_m)\Phi_+\,,\\ \bar{\Phi}&=\exp(-i\theta\sigma^m\bar{\theta}\partial_m)\Phi_-\,,\end{aligned} \quad (2.3)$$

where the superfields Φ_+ and Φ_- satisfy the constraints

$$\partial_{\dot{\alpha}}\Phi_+=0,\quad \partial_\alpha\Phi_-=0\,. \quad (2.4)$$

It is clear from the form of Φ and $\bar{\Phi}$ that they contain time derivatives of functions and, therefore, their brackets would necessarily involve brackets of velocities. In this case it turns out that the velocities cannot be expressed in terms of the canonical momenta and, therefore, we cannot evaluate these brackets. Consequently we choose to work with the superfields Φ_+ and Φ_- which satisfy the simple constraints of Eq. (2.4).

We can now write the action of Eq. (2.1) in terms of Φ_+ and Φ_- and in order to generate the constraints given in Eq. (2.4) from the Lagrangian we add these to the Lagrangian through appropriate Lagrange multiplier fields $\bar{\Lambda}_{\dot{\alpha}}$ and Λ_α.

The action in superspace thus takes the form

$$S=\int d^4x\, d^2\theta\, d^2\bar\theta \Bigg[\Phi_+\Phi_- + i\theta\sigma^m\bar\theta\Phi_-\overleftrightarrow{\partial}_m\Phi_+ - \delta^2(\theta)\delta^2(\bar\theta)\partial_m\Phi_-\partial^m\Phi_+ + \delta^2(\bar\theta)\frac{m}{2}\Phi_+{}^2 + \delta^2(\theta)\frac{m}{2}\Phi_-{}^2$$
$$+\delta^2(\bar\theta)\frac{g}{3}\Phi_+{}^3 + \delta^2(\theta)\frac{g}{3}\Phi_-{}^3 + \bar\Lambda\bar\partial\Phi_+ + \Lambda\partial\Phi_-\Bigg], \tag{2.5}$$

where we have used $\bar\Lambda\bar\partial=\bar\Lambda_{\dot\alpha}\bar\partial^{\dot\alpha}$ and $\Lambda\partial=\Lambda^\alpha\partial_\alpha$.

The action in Eq. (2.5) is completely equivalent to the original action with the constraints. In Eq. (2.5) the Lagrange multiplier fields $\bar\Lambda,\Lambda$ are general fermionic superfields. Note here that the action in Eq. (2.5) is invariant under the gauge transformations

$$\bar\Lambda_{\dot\alpha}\to\bar\Lambda'_{\dot\alpha}=\bar\Lambda_{\dot\alpha}+\bar\rho_{\dot\alpha}(x,\theta)$$
and
$$\Lambda_\alpha\to\Lambda'_\alpha=\Lambda_\alpha+\rho_\alpha(x,\bar\theta)\ . \tag{2.6}$$

It is therefore always possible to make a transformation such that the multiplier fields are of the form

$$\bar\Lambda_{\dot\alpha}=\bar\theta_{\dot\alpha}\bar\Lambda,\quad \Lambda_\alpha=\theta_\alpha\Lambda\ , \tag{2.7}$$

where $\bar\Lambda=\bar\Lambda(x,\theta,\bar\theta)$ and $\Lambda=\Lambda(x,\theta,\bar\theta)$ are so far general scalar superfields. The form of Eqs. (2.7) can be formulated equivalently through the gauge-fixing constraints:

$$\delta^2(\bar\theta)\bar\Lambda_{\dot\alpha}=0,\quad \delta^2(\theta)\Lambda_\alpha=0\ . \tag{2.8}$$

As it should be,[2] we will show that there are first-class constraints in the theory which reflect the gauge invariance and which can be made second class by incorporating Eq. (2.8) in the set of constraints.

The equations of motion are obtained from Eq. (2.5) by the usual variational principle:

$$\frac{\delta S}{\delta\Phi_+}=\Phi_- - 2i\theta\sigma^m\bar\theta\partial_m\Phi_- + \delta^2(\theta)\delta^2(\bar\theta)\Box\Phi_-$$
$$+\delta^2(\bar\theta)m\Phi_+ + \delta^2(\bar\theta)g\Phi_+{}^2 + \bar\partial\bar\Lambda=0\ ,$$
$$\frac{\delta S}{\delta\Phi_-}=\Phi_+ + 2i\theta\sigma^m\bar\theta\partial_m\Phi_+ + \delta^2(\theta)\delta^2(\bar\theta)\Box\Phi_+$$
$$+\delta^2(\theta)m\Phi_- + \delta^2(\theta)g\Phi_-{}^2 + \partial\Lambda=0\ ; \tag{2.9a}$$

$$\frac{\delta S}{\delta\bar\Lambda_{\dot\alpha}}=\bar\partial^{\dot\alpha}\Phi_+=0,\quad \frac{\delta S}{\delta\Lambda_\alpha}=\partial^\alpha\Phi_-=0\ . \tag{2.9b}$$

The equations of motion for the multiplier fields (2.9b) give the desired constraints of Eq. (2.4). Moreover, Eq. (2.9) gives rise to more constraints: for example, $\delta S/\delta\Phi_+=0=\delta S/\delta\Phi_-$ implies

$$\delta^2(\bar\theta)\left[\frac{\delta S}{\delta\Phi_+}\right]=0=\delta^2(\theta)\left[\frac{\delta S}{\delta\Phi_-}\right]$$

which has the equivalent form

$$\delta^2(\bar\theta)(\Phi_- + \bar\partial\bar\Lambda)=0=\delta^2(\theta)(\Phi_+ + \partial\Lambda)\ .$$

Several constraints are generated in this way.

The momenta are obtained from the Lagrangian in Eq. (2.5) as

$$\Pi_+(x,\theta,\bar\theta)=\frac{\partial L}{\partial\dot\Phi_+(x,\theta,\bar\theta)}$$
$$=i\theta\sigma^0\bar\theta\Phi_- + \delta^2(\theta)\delta^2(\bar\theta)\dot\Phi_-\ ,$$
$$\Pi_-(x,\theta,\bar\theta)=\frac{\partial L}{\partial\dot\Phi_-(x,\theta,\bar\theta)}$$
$$=-i\theta\sigma^0\Phi_+ + \delta^2(\theta)\delta^2(\bar\theta)\dot\Phi_+\ ; \tag{2.10a}$$

$$\bar p_{\dot\alpha}(x,\theta,\bar\theta)=\frac{\partial L}{\partial\dot{\bar\Lambda}{}^{\dot\alpha}(x,\theta,\bar\theta)}=0\ ,$$
$$p_\alpha(x,\theta,\bar\theta)=\frac{\partial L}{\partial\dot\Lambda{}^\alpha(x,\theta,\bar\theta)}=0 \tag{2.10b}$$

(where $\dot A=\partial_0 A$). The forms of the momenta give rise to additional constraints. Altogether we have the following set of constraints ($\approx$ means weakly equal[2]):

$$\Gamma_1=\delta^2(\bar\theta)\frac{\bar\partial\bar\partial}{4}\Phi_+\approx0,\quad \Gamma_2=\delta^2(\bar\theta)\Pi_+\approx0,\quad \Gamma_3=\delta^2(\theta)\frac{\partial\partial}{4}\Phi_-\approx0,\quad \Gamma_4=\delta^2(\theta)\Pi_-\approx0,\quad \Gamma_{5\dot\alpha}=\delta^2(\bar\theta)\bar\partial_{\dot\alpha}\Phi_+\approx0\ ,$$

$$\Gamma_{6\dot\alpha}=\bar\theta_{\dot\alpha}\Pi_+ + \frac{i}{2}\delta^2(\bar\theta)(\theta\sigma^0)_{\dot\alpha}\Phi_-\approx0,\quad \Gamma_{7\alpha}=\delta^2(\theta)\partial_\alpha\Phi_-\approx0,\quad \Gamma_{8\alpha}=\theta_\alpha\Pi_- - \frac{i}{2}\delta^2(\theta)(\sigma^0\bar\theta)_\alpha\Phi_+\approx0\ ,$$

$$\Gamma_9=\delta^2(\theta)\left[\frac{\bar\partial\bar\partial}{4}\Phi_- + m\Phi_+ + g\Phi_+{}^2\right]\approx0,\quad \Gamma_{10}=\delta^2(\bar\theta)\Pi_-\approx0,\quad \Gamma_{11}=\delta^2(\bar\theta)\left[\frac{\partial\partial}{4}\Phi_+ + m\Phi_- + g\Phi_-{}^2\right]\approx0\ ,$$

$$\Gamma_{12}=\delta^2(\theta)\Pi_+\approx0,\quad \Gamma_{13\alpha}=\theta_\alpha\Pi_+ + \frac{i}{2}\delta^2(\theta)(\sigma^0\bar\theta)_\alpha\Phi_-\approx0,\quad \Gamma_{14\dot\alpha}=\bar\theta_{\dot\alpha}\Pi_- - \frac{i}{2}\delta^2(\bar\theta)(\theta\sigma^0)_{\dot\alpha}\Phi_+\approx0,\quad \Gamma_{15\dot\alpha}=\delta^2(\bar\theta)\frac{\bar\partial\bar\partial}{4}\bar p_{\dot\alpha}\approx0\ ,$$

$$\Gamma_{16\dot\alpha}=\delta^2(\bar\theta)\bar\Lambda_{\dot\alpha}\approx0,\quad \Gamma_{17\alpha}=\delta^2(\theta)\frac{\partial\partial}{4}p_\alpha\approx0,\quad \Gamma_{18\alpha}=\delta^2(\theta)\Lambda_\alpha\approx0,\quad \Gamma_{19}=\delta^2(\bar\theta)\bar\partial\bar p\approx0,\quad \Gamma_{20}=\delta^2(\bar\theta)(\Phi_- + \bar\partial\bar\Lambda)\approx0\ , \tag{2.11}$$

$$\Gamma_{21}=\delta^2(\theta)\partial p\approx 0,\;\; \Gamma_{22}=\delta^2(\theta)(\Phi_+ +\partial\Lambda)\approx 0,\;\; \Gamma_{23\dot\alpha}=\delta^2(\theta)\delta^2(\bar\theta)\bar p_{\dot\alpha}\approx 0,\;\; \Gamma_{24\dot\alpha}=\bar\theta_{\dot\alpha}\delta^2(\theta)\frac{\partial\partial}{4}\bar\partial\bar\Lambda\approx 0\;,$$

$$\Gamma_{25\alpha}=\delta^2(\theta)\delta^2(\bar\theta)p_\alpha\approx 0,\;\; \Gamma_{26\alpha}=\theta_\alpha\delta^2(\bar\theta)\frac{\bar\partial\bar\partial}{4}\partial\Lambda\approx 0,\;\; \Gamma_{27\dot\alpha}=\delta^2(\theta)\delta^2(\bar\theta)\frac{\partial\partial}{4}\bar p_{\dot\alpha}\approx 0\;,$$

$$\Gamma_{28\dot\alpha}=\delta^2(\theta)\delta^2(\bar\theta)\frac{\bar\partial\bar\partial}{4}[\bar\theta_{\dot\alpha}(\Phi_- +\bar\partial\bar\Lambda)]\approx 0,\;\; \Gamma_{29\alpha}=\delta^2(\theta)\delta^2(\bar\theta)\frac{\bar\partial\bar\partial}{4}p_\alpha\approx 0,\;\; \Gamma_{30\alpha}=\delta^2(\theta)\delta^2(\bar\theta)\frac{\partial\partial}{4}[\theta_\alpha(\Phi_+ +\partial\Lambda)]\approx 0\;,$$

$$\Gamma_{31\alpha\dot\alpha}=\delta^2(\theta)\delta^2(\bar\theta)\partial_\alpha\bar p_{\dot\alpha}\approx 0,\;\; \Gamma_{32\alpha\dot\alpha}=2i\sigma^0_{\alpha\dot\alpha}(\Pi_+ -i\theta\sigma^0\bar\theta\Phi_-)+\delta^2(\theta)\delta^2(\bar\theta)(2i\sigma^k_{\alpha\dot\alpha}\partial_k\Phi_- +\partial_\alpha\partial_{\dot\alpha}\bar\partial\bar\Lambda)\approx 0\;\;(k=1,2,3)\;,$$

$$\Gamma_{33\alpha\dot\alpha}\delta^2(\theta)\delta^2(\bar\theta)\partial_{\dot\alpha}p_\alpha\approx 0,\;\; \Gamma_{34\alpha\dot\alpha}2i\sigma^0_{\alpha\dot\alpha}(\Pi_- +i\theta\sigma^0\bar\theta\Phi_+)+\delta^2(\theta)\delta^2(\bar\theta)(2i\sigma^k_{\alpha\dot\alpha}\partial_k\Phi_+ -\partial_\alpha\partial_{\dot\alpha}\partial\Lambda)\approx 0\;\;(k=1,2,3)\;.$$

Note here that the constraints $\Gamma_{16\dot\alpha}$ and $\Gamma_{18\alpha}$ are the gauge-fixing constraints of Eq. (2.7) which do not come from the Lagrangian. These gauge-fixing constraints make the initially first-class constraints $\Gamma_{15\dot\alpha}$ and $\Gamma_{17\alpha}$ second class.[2] Furthermore, as can be easily seen if one uses the fundamental Poisson brackets given in Sec. III, all Γ_i are second-class constraints.

Finally it is important to note that only Γ_1–Γ_{14} are constraints on the dynamic fields, whereas the remaining constraints simply determine the Lagrange multiplier fields in terms of the dynamic fields of the theory. This will become more obvious if one looks at the results in component form as we shall do in Sec. III.

III. THE DIRAC BRACKETS OF THE THEORY

Let us introduce the following notations: denote the superspace variable by $z=(x^m,\theta_\alpha,\bar\theta_{\dot\alpha})$ and define $d^7z=d^3x\,d^2\theta\,d^2\bar\theta$,

$$\delta^7(z-z')=\delta^3(x-x')\delta^2(\theta-\theta')\delta^2(\bar\theta-\bar\theta')$$

and the signature function S by

$$S(A)=\begin{cases}0 & \text{if } A \text{ is bosonic}\;,\\ 1 & \text{if } A \text{ is fermionic}\;,\end{cases}$$

such that $\theta A=(-1)^{S(A)}A\theta$ for all anticommuting variables θ. We define the fundamental Poisson brackets as in Refs. 4 and 9. These brackets satisfy the following set of identities:[9]

$$\begin{aligned}&\{A,B\}=-(-1)^{S(A)S(B)}\{B,A\}\;,\\ &\{A,B+C\}=\{A,B\}+\{A,C\}\;,\\ &\{A,BC\}=(-1)^{S(A)S(B)}B\{A,C\}+\{A,B\}C\;,\\ &\{AB,C\}=(-1)^{S(B)S(C)}\{A,C\}B+A\{B,C\}\;,\\ &(-1)^{S(A)S(C)}\{A,\{B,C\}\}+(-1)^{S(A)S(B)}\{B,\{C,A\}\}\\ &\qquad\qquad+(-1)^{S(B)S(C)}\{C,\{A,B\}\}=0\;.\end{aligned}\tag{3.1}$$

The only nonvanishing Poisson brackets of our fields are

$$\begin{aligned}&\{\Phi_+(z),\Pi_+(z')\}_{x_0=x_0'}=\delta^7(z-z')\\ &\qquad=\{\Phi_-(z),\Pi_-(z')\}_{x_0=x_0'}\;,\\ &\{\bar\Lambda_{\dot\alpha}(z),\bar p_{\dot\beta}(z')\}_{x_0=x_0'}=-\epsilon_{\dot\alpha\dot\beta}\delta^7(z-z')\;,\\ &\{\Lambda_\alpha(z),p_\beta(z')\}_{x_0=x_0'}=-\epsilon_{\alpha\beta}\delta^7(z-z')\;.\end{aligned}\tag{3.2}$$

Following Dirac's method, we have to make sure that $\dot\Gamma_i\approx 0$, $i=1,\ldots,34$. The canonical Hamiltonian for our system is

$$H_c=\int d^7z(\Pi_+\dot\Phi_+ +\Pi_-\dot\Phi_- +\dot\Lambda^\alpha p_\alpha+\dot{\bar\Lambda}_{\dot\alpha}\bar p^{\dot\alpha}-L)$$

with L obtained from Eq. (2.5). H_c can be cast in the form

$$\begin{aligned}H_c=\int d^7z[&\tfrac{1}{16}\partial\partial(\Pi_+ -i\theta\sigma^0\bar\theta\Phi_-)\bar\partial\bar\partial(\Pi_- +i\theta\sigma^0\bar\theta\Phi_+)-i\theta\sigma^k\bar\theta\Phi_-\overleftrightarrow{\partial}_k\Phi_+ +\delta^2(\theta)\delta^2(\bar\theta)\partial_k\Phi_-\partial^k\Phi_+\\ &-L_m-L_g+\dot\Lambda^\alpha p_\alpha+\dot{\bar\Lambda}_{\dot\alpha}\bar p^{\dot\alpha}-\Lambda\partial\Phi_- -\bar\Lambda\bar\partial\Phi_+]\;\;(k=1,2,3)\;,\end{aligned}\tag{3.3}$$

where

$$L_m=\frac{m}{2}[\delta^2(\bar\theta)\Phi_+{}^2+\delta^2(\theta)\Phi_-{}^2]\;,$$

$$L_g=\frac{g}{3}[\delta^2(\bar\theta)\Phi_+{}^3+\delta^2(\theta)\Phi_-{}^3]\;.$$

H_c is defined only on the constrained hypersurface $\Gamma_i=0$ and we assume that the velocities $\dot\Lambda,\dot{\bar\Lambda}$ can be expressed as functions of $\Phi_\pm,\Pi_\pm,\Lambda,\bar\Lambda,p,\bar p$.

Following Dirac we let the time development be governed by the Hamiltonian[2]

$$\tilde H=H_c+\int d^7z\,\lambda_i(z)\Gamma_i(z)\tag{3.4}$$

which coincides with H_c on the constrained hypersurface, namely $\tilde H\approx H_c$. The "velocities" $\dot\Gamma_i$ of the constraints must vanish weakly,[2] i.e.,

$$\dot\Gamma_i(z)=\{\Gamma_i(z),\tilde H\}\approx 0\;.\tag{3.5}$$

It is straightforward though tedious to verify that Eq. (3.5) can be satisfied for all Γ_i by a proper choice of the λ_i in terms of $\Phi_\pm,\Pi_\pm,\Lambda,\bar\Lambda,p,\bar p$ and their spatial and Grassmannian derivatives. We thus have found that the constraints $\Gamma_1,\ldots,\Gamma_{34}$ given in Eq. (2.11) are all the constraints in the theory and they are all second class.

In proceeding we use the fact that Dirac's method can be applied iteratively and we choose subsets of constraints for these iterations as follows: $\Gamma_1,\ldots,\Gamma_4$; $\Gamma_5,\ldots,\Gamma_8$; $\Gamma_9,\ldots,\Gamma_{12}$; Γ_{13},Γ_{14}; $\Gamma_{r,\ldots,r+3}$ for $r=15,19,23,27,31$.

Starting with $\Gamma_1,\ldots,\Gamma_4$ we find that the matrix of the Poisson brackets of the constraints is given by

$$\{\Gamma_i(z),\Gamma_j(z')\}_{x_0=x_0'} = \delta^3(x-x')\begin{pmatrix} 0 & \delta^2(\theta-\theta')\delta^2(\bar\theta)\delta^2(\bar\theta') & 0 & 0 \\ -\delta^2(\theta-\theta')\delta^2(\bar\theta)\delta^2(\bar\theta') & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta^2(\bar\theta-\bar\theta')\delta^2(\theta)\delta^2(\theta') \\ 0 & 0 & -\delta^2(\bar\theta-\bar\theta')\delta^2(\theta)\delta^2(\theta') & 0 \end{pmatrix} \quad (i,j=1,\ldots,4)\,. \tag{3.6}$$

This matrix is not invertible due to the Grassmannian nature of the variables. However, as already pointed out in Ref. 4, for consistent quantization conditions it is sufficient to find a matrix $D_{ij}(z'',z''')$ such that the newly defined brackets

$$\{A(z),B(z')\}^*_{x_0=x_0'} = \{A(z),B(z')\}_{x_0=x_0'} - \int d^7z''d^7z'''\{A(z),\Gamma_i(z'')\}D_{ij}(z'',z''')\{\Gamma_j(z'''),B(z')\} \tag{3.7}$$

satisfy

$$\{A(z),\Gamma_i(z')\}^*=0 \tag{3.8}$$

for any dynamical $A(z)$ and all constraints Γ_i. Since we consider here only $\Gamma_1,\ldots,\Gamma_4$ in this first iteration step we have to require that Eq. (3.8) be satisfied only for these constraints. Equations (3.8) determine the matrix $D_{ij}(z'',z''')$ which is effectively unique and has the form

$$D_{ij}(z'',z''') = \delta^2(x''-x''')\begin{pmatrix} 0 & -\delta^2(\theta''-\theta''') & 0 & 0 \\ \delta^2(\theta''-\theta''') & 0 & 0 & 0 \\ 0 & 0 & 0 & -\delta^2(\bar\theta''-\bar\theta''') \\ 0 & 0 & \delta^2(\bar\theta''-\bar\theta''') & 0 \end{pmatrix}. \tag{3.9}$$

The fundamental brackets now modify to

$$\{\Phi_+(z),\Pi_+(z')\}^*_{x_0=x_0'} = \delta^2(x-x')\delta^2(\theta-\theta')[\delta^2(\bar\theta')-2\bar\theta\bar\theta'] \,,$$

$$\{\Phi_-(z),\Pi_-(z')\}^*_{x_0=x_0'} = \delta^3(x-x')\delta^2(\bar\theta-\bar\theta')[\delta^2(\theta')-2\theta\theta'] \,.$$

The remaining brackets between the elementary fields are as the original ones.

For the next ıteration now calculate $\{\Gamma_i,\Gamma_j\}^*$ for $i,j=5,\ldots,8$ and repeat the procedure to find $\{A,B\}^{**}$, etc. After the ten iterations indicated earlier we find the following final Dirac brackets of the theory:

$$\{\Phi_+(z),\Pi_+(z')\}^D_{x_0=x_0'} = \delta^3(x-x')\delta^2(\bar\theta')[\delta^2(\theta')-\theta\theta'] \,,$$

$$\{\Phi_-(z),\Pi_-(z')\}^D_{x_0=x_0'} = \delta^3(x-x')\delta^2(\theta')[\delta^2(\bar\theta')-\bar\theta\bar\theta'] \,,$$

$$\{\Pi_-(z),\Phi_+(z')\}^D_{x_0=x_0'} = \delta^2(x-x')\delta^2(\bar\theta)\delta^2(\theta)\delta^2(\theta')[m+2g\Phi_-(z)] \,,$$

$$\{\Pi_+(z),\Phi_-(z')\}^D_{x_0=x_0'} = \delta^3(x-x')\delta^2(\bar\theta)\delta^2(\theta)\delta^2(\bar\theta')[m+2g\Phi_+(z)] \,,$$

$$\{\Phi_+(z),\Phi_-(z')\}^D_{x_0=x_0'} = 2i\theta\sigma^0\bar\theta'\delta^3(x-x') \,,$$

$$\{\Pi_+(z),\Pi_-(z')\}^D_{x_0=x_0'} = i\delta^3(x-x')[\delta^2(\bar\theta)\delta^2(\theta)\theta'\sigma^0\bar\theta'+\delta^2(\bar\theta')\delta^2(\theta')\theta\sigma^0\bar\theta+\tfrac{1}{2}\delta^2(\theta')\delta^2(\bar\theta)\theta\sigma^0\bar\theta'] \,,$$

$$\{\Phi_+(z),\Phi_+(z')\}^D_{x_0=x_0'} = \{\Phi_-(z),\Phi_-(z')\}_{x_0=x_0'} = 0 \,,$$

$$\{\Pi_+(z),\Pi_+(z')\}^D_{x_0=x_0'} = \{\Pi_-(z),\Pi_-(z')\}_{x_0=x_0'} = 0 \,,$$

$$\begin{aligned} &\{\Phi_-(z),\bar\Lambda_{\dot\alpha}(z')\}^D_{x_0=x_0'} = i\delta^3(x-x')\delta^2(\bar\theta')(\theta'\sigma)_{\dot\alpha}\delta^2(\bar\theta)[m+2g\Phi_+(x,0,0)] \,, \\ &\{\Phi_+(z),\bar\Lambda_{\dot\alpha}(z')\}^D_{x_0=x_0'} = -i\delta^3(x-x')\delta^2(\bar\theta')[(\theta'+\theta)\sigma^0]_{\dot\alpha} \,, \\ &\{\Pi_-(z),\bar\Lambda_{\dot\alpha}(z')\}^D_{x_0=x_0'} = [-\tfrac{1}{2}\bar\theta_{\dot\alpha}\delta^2(\bar\theta')\delta^2(\theta)+\tfrac{1}{2}\bar\theta'_{\dot\alpha}\delta^2(\bar\theta)\delta^2(\theta)-\delta^2(\bar\theta')(\theta'\sigma^0)_{\dot\alpha}\theta\sigma^0\bar\theta \\ &\qquad +i\delta^2(\bar\theta')(\theta'\sigma^k)_{\dot\alpha}\delta^2(\theta)\delta^2(\bar\theta)\partial'_k]\delta^3(x-x') \quad (k=1,2,3)\,, \end{aligned} \tag{3.10}$$

$$\{\Pi_+(z),\bar{\Lambda}_{\dot{\alpha}}(z')\}^D_{x_0=x'_0}=0\ ,$$

$$\{\Phi_+(z),\Lambda_\alpha(z')\}^D_{x_0=x'_0}=-i\delta^3(x-x')\delta^2(\theta')\delta^2(\theta)(\sigma^0\bar{\theta}')_\alpha[m+2g\Phi_-(x,0,0)]\ ,$$

$$\{\Phi_-(z),\Lambda_\alpha(z')\}^D_{x_0=x'_0}=-i\delta^3(x-x')\delta^2(\theta')[\sigma^0(\bar{\theta}-\bar{\theta}')]_\alpha\ ,$$

$$\{\Pi_+(z'),\Lambda_\alpha(z')\}^D_{x_0=x'_0}=[-\tfrac{1}{2}\theta_\alpha\delta^2(\theta')\delta^2(\bar{\theta})+\tfrac{1}{2}\theta'_\alpha\delta^2(\theta)\delta^2(\bar{\theta})+\delta^2(\theta')(\sigma^0\bar{\theta}')_\alpha\theta\sigma^0\bar{\theta}$$
$$-i\delta^2(\theta')(\sigma^k\bar{\theta}')_\alpha\delta^2(\theta)\delta^2(\bar{\theta})\partial'_k]\delta^3(x-x')\quad(k=1,2,3)\ ,$$

$$\{\bar{\Lambda}_{\dot{\alpha}}(z),\Lambda_\alpha(z')\}^D_{x_0=x'_0}=\left[-\frac{i}{2}\sigma^0_{\alpha\dot{\alpha}}\delta^2(\bar{\theta})\delta^2(\theta')-\frac{i}{2}\delta^2(\bar{\theta})(\theta\sigma^0)_{\dot{\alpha}}\theta'_\alpha-\frac{i}{2}\delta^2(\theta')\bar{\theta}_{\dot{\alpha}}(\sigma^0\bar{\theta}')_\alpha\right.$$
$$\left.+\delta^2(\bar{\theta})\delta^2(\theta')\theta^\beta\bar{\theta}^{\dot{\beta}'}(\sigma^0_{\beta\dot{\alpha}}\sigma^k_{\alpha\dot{\beta}}+\sigma^k_{\beta\dot{\alpha}}\sigma^0_{\alpha\dot{\beta}})\partial_k\right]\delta^3(x-x')\ ,$$

$$\{\bar{p}_{\dot{\alpha}},\Phi_\pm\}=\{\bar{p}_{\dot{\alpha}},\Pi_\pm\}=\{\bar{p}_{\dot{\alpha}},\bar{\Lambda}_{\dot{\beta}}\}=\{\bar{p}_{\dot{\alpha}},\Lambda_\alpha\}=\{\bar{p}_{\dot{\alpha}},p_\beta\}=\{\bar{p}_{\dot{\alpha}},\bar{p}_{\dot{\beta}}\}=0\ ,$$

$$\{p_\alpha,\Phi_\pm\}=\{p_\alpha,\Pi_\pm\}=\{p_\alpha,\Lambda_\beta\}=\{p_\alpha,\bar{\Lambda}_{\dot{\alpha}}\}=\{p_\alpha,p_\beta\}=0\ .$$

Since our method preserves the relations in Eq. (3.1) these brackets $\{A,B\}^D$ also satisfy these properties. Note that all brackets with $\bar{p}_{\dot{\alpha}}$ or p_α vanish as a result of the constraints and, therefore, these can be set strongly equal to zero.

Equations (3.10) give the final Dirac brackets which are consistent with all constraints $\Gamma_i=0$, $i=1,\ldots,34$. These brackets give the consistent quantization conditions for the superfields. We have obtained these in superspace without making use of the component field language.

Having found the Dirac brackets we can now set all constraints strongly equal to zero $\Gamma_i=0$, $i=1,\ldots,34$. As a result the superfields have the following component expansion:

$$\Phi_+(z)=\Phi_+(x,\theta)=A(x)+\sqrt{2}\theta\psi(x)+\delta^2(\theta)F(x)\ ,\tag{3.11a}$$

$$\Phi_-(z)=\Phi_-(x,\bar{\theta})=A^*(x)+\sqrt{2}\bar{\theta}\bar{\psi}(x)+\delta^2(\bar{\theta})F^*(x)\ ,$$

$$\Lambda_\alpha(z)=-\theta_\alpha[\tfrac{1}{2}A(x)+\sqrt{2}\theta\psi(x)+2i\theta\sigma^m\bar{\theta}\partial_m A(x)]\ ,\tag{3.11b}$$

$$\bar{\Lambda}_{\dot{\alpha}}(z)=-\bar{\theta}_{\dot{\alpha}}[\tfrac{1}{2}A^*(x)+\sqrt{2}\bar{\theta}\bar{\psi}(x)-2i\theta\sigma^m\bar{\theta}\partial_m A^*(x)]\ ,$$

where $A(x)$ is a complex scalar field and $\psi(x)$ a complex Weyl spinor as usual.[3] Using the expansions of Eqs. (3.11a) in the definition of the momenta (2.10a) they take the form

$$\Pi_+(z)=i\theta\sigma^0\bar{\theta}A^*(x)-\frac{i}{\sqrt{2}}\delta^2(\bar{\theta})\theta\sigma^0\bar{\psi}(x)$$
$$+\delta^2(\theta)\delta^2(\bar{\theta})\dot{A}^*(x)\ ,$$
$$\tag{3.11c}$$
$$\Pi_-(z)=-i\theta\sigma^0\bar{\theta}A(x)+\frac{i}{\sqrt{2}}\delta^2(\theta)\psi(x)\sigma^0\bar{\theta}$$
$$+\delta^2(\theta)\delta^2(\bar{\theta})\dot{A}(x)\ .$$

These expressions for the superfields in terms of the component fields can be inserted into the Dirac brackets to obtain the quantization conditions for the component fields:

$$\{A(x),\dot{A}^*(x')\}^D_{x_0=x'_0}=\delta^3(x-x')\ ,$$
$$\{\psi_\alpha(x),\bar{\psi}_{\dot{\alpha}}(x')\}^D_{x_0=x'_0}=i\sigma^0_{\alpha\dot{\alpha}}\delta^3(x-x')\ ,$$
$$\tag{3.12}$$
$$\{F(x),\dot{A}(x')\}^D_{x_0=x'_0}=-\delta^3(x-x')[m+2gA^*(x')]\ ,$$
$$\{F^*(x),\dot{A}^*(x')\}^D_{x_0=x'_0}=-\delta^3(x-x')[m+2gA(x')]\ .$$

Equations (3.12) give the brackets which one would obtain if one had applied Dirac's method to each component individually. The results of the superspace quantization are thus consistent with the results of the component fields. Our method, however, allows us to quantize superfields directly.

IV. HEISENBERG EQUATIONS OF MOTION

We can now use the Dirac brackets derived in Sec. III and the Hamiltonian $\tilde{H}$ given by Eq. (3.4) in order to calculate the Heisenberg equations of motion:

$$\dot{\Phi}_\pm(z)=\{\Phi_\pm(z),\tilde{H}\}^D,\quad\dot{\Pi}_\pm(z)=\{\Pi_\pm(z),\tilde{H}\}^D\ .\tag{4.1}$$

As can be seen from the constraints in Eq. (2.11) or more explicitly from Eqs. (3.11b) the multiplier fields are given in terms of the dynamical fields $\Phi_\pm$. The equations of motion for Λ and $\bar{\Lambda}$ therefore do not give us any new information and $\{\Lambda,\tilde{H}\}^D$ and $\{\bar{\Lambda},\tilde{H}\}^D$ need not be computed. Before we evaluate the brackets in Eqs. (4.1) we should note that by our construction of the Dirac brackets $\{A(\Phi_\pm,\Pi_\pm,\Lambda,p,\bar{\Lambda},\bar{p}),\Gamma_i\}^D=0$ and thus

$$\{A(\Phi_\pm,\Pi_\pm,\Lambda,p,\bar{\Lambda},\bar{p}),\tilde{H}\}^D=\{A(\Phi_\pm,\Pi_\pm,\Lambda,p,\bar{\Lambda},\bar{p}),H\}^D\ ,$$

where H is the final Hamiltonian of our theory:

$$H=\int d^7z\left[\tfrac{1}{16}(\partial\partial\Pi_+)(\bar\partial\bar\partial\Pi_-)-\Phi_+\Phi_- - i\theta\sigma^k\bar\theta\Phi_-\overleftrightarrow{\partial}_k\Phi_+ +\delta^2(\theta)\delta^2(\bar\theta)\partial_k\Phi_-\partial^k\Phi_+ - L_m - L_g\right]\quad (k=1,2,3)\,. \tag{4.2}$$

Consequently the Heisenberg equations of motion (4.1) can be written as

$$\dot\Phi_\pm(z)=\{\Phi_\pm(z),H\}^D,\quad \dot\Pi_\pm(z)=\{\Pi_\pm(z),H\}^D\,. \tag{4.3}$$

Evaluating the brackets in Eqs. (4.3) we find the following equations of motion:

$$\begin{aligned}
\dot\Phi_+(z)&=\dot\Phi_+(x,0)+(\theta\sigma^0\bar\sigma^k\partial_k)^\alpha\frac{\partial\partial}{2}[\theta_\alpha\Phi_+(z)]+\frac{\bar\partial\bar\partial}{2i}\{\theta\sigma^0\bar\theta[m\Phi_-(z)+g\Phi_-{}^2(z)]\}-\delta^2(\theta)\dot\Phi_-(x,0)[m+2g\Phi_-(x,0)]\,,\\
\dot\Phi_-(z)&=\dot\Phi_-(x,0)+(\bar\theta\bar\sigma^0\sigma^k\partial_k)_{\dot\alpha}\frac{\partial\partial}{2}[\bar\theta^{\dot\alpha}\Phi_-(z)]+\frac{\partial\partial}{2i}\{\bar\theta\bar\sigma^0\theta[m\Phi_+(z)+g\Phi_+{}^2(z)]\}-\delta^2(\bar\theta)\dot\Phi_+(x,0)[m+2g\Phi_+(x,0)]\,,\\
\dot\Pi_+(z)&=i\theta\sigma^0\bar\theta\,\dot\Phi_-(x,0)-\delta^2(\bar\theta)\theta^\alpha\left[\frac{\bar\partial\bar\partial}{4}[i(\sigma^k\bar\theta)_\alpha\partial_k\Phi_-(z)]+\frac{\partial\partial}{4}\{\theta_\alpha[m\Phi_+(z)+g\Phi_+{}^2(z)]\}\right]\\
&\quad+\delta^2(\theta)\delta^2(\bar\theta)\{\partial_k\partial^k\Phi_-(z)+\tfrac{1}{2}\partial\Phi_+(z)\partial\Phi_+(z)-[m\Phi_-(z)+g\Phi_-{}^2(z)][m+2g\Phi_+(z)]\}\,,\\
\dot\Pi_-(z)&=-i\theta\sigma^0\bar\theta\,\dot\Phi_+(x,0)-\delta^2(\theta)\bar\theta_{\dot\alpha}\left[\frac{\partial\partial}{4}[-i(\bar\sigma^k\theta)^{\dot\alpha}\partial_k\Phi_+(z)]+\frac{\bar\partial\bar\partial}{4}\{\bar\theta^{\dot\alpha}[m\Phi_-(z)+g\Phi_-{}^2(z)]\}\right]\\
&\quad+\delta^2(\theta)\delta^2(\bar\theta)\{\partial_k\partial^k\Phi_+(z)+\tfrac{1}{2}\partial\Phi_-(z)\partial\Phi_-(z)-[m\Phi_+(z)+g\Phi_+{}^2(z)][m+2g\Phi_-(z)]\}\quad(k=1,2,3)\,.
\end{aligned} \tag{4.4}$$

We thus obtain the Heisenberg equations of motion for the component fields which can be written in the following form:

$$\begin{aligned}
&i\sigma^m\partial_m\bar\psi+m\psi+-2gA\psi\,,\\
&i\bar\sigma^m\partial_m\psi+m\bar\psi=-2gA^*\bar\psi\,,\\
&(\Box-m^2)A=g\bar\psi\bar\psi+mgA^2+2gA^*(mA+gA^2)\,,\\
&(\Box-m^2)A^*=g\psi\psi+mgA^{*2}+2gA(mA^*+gA^{*2})\,.
\end{aligned} \tag{4.5}$$

These are the well-known field equations for the component fields.[3,5] As in Sec. III our results for the superfields, which were derived in superspace without making use of component fields, yield the correct equations for the components.

V. CONCLUSION

We have applied a modification of Dirac's method for constrained systems to chiral superfields. With this we derived quantization conditions, which are consistent with the constraints in the theory. We found a Hamiltonian for our system and showed that it generates the known equations of motion for the component fields. We have, therefore, presented a complete scheme for the canonical quantization of chiral superfields whose quantization previously[3] employed path integral methods.

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