Astrophysical axion bounds diminished by screening effects

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"Invisible axions" could be produced in stellar interiors through Compton- and Primakoff-type photoproduction and through bremsstrahlung processes. We point out that in a plasma screening effects lead to important reductions of these emission rates. Limits on the axion mass and interaction strength are thereby relaxed to values less restrictive than limits previously thought to be firm. For the case of the Sun the Primakoff rate is reduced by two orders of magnitude. This process is the dominant emission mechanism for Kim-Shifman-Vainshtein-Zakharov- (KSVZ) type axions which do not couple directly to electrons. The mass limit is then relaxed by an order of magnitude to $m_a < 20$ eV. Dine-Fischler-Srednicki (DFS) axions couple directly to electrons and the dominant solar emission process becomes bremsstrahlung from electron-nucleus and electron-electron collisions while the previously thought dominant Primakoff rate is now suppressed. The mass limit accidentally remains at $m_a < 3$ eV. Model-dependent bounds based on axion emission from red giants are $m_a < 1$ eV (KSVZ) and $m_a < 0.06$ eV (DFS). The mass limits for the DFS axions are understood for the special case where the free parameter $2\cos^2\beta$ of the model equals unity. Our results can be easily translated to other hypothetical pseudoscalar particles if they are light compared with typical stellar temperatures.

I. INTRODUCTION

Invisible axions,[1,2] if they exist, could carry away large amounts of energy from stellar interiors due to their enormous mean free path lengths in matter of typical astrophysical densities. Therefore their existence could dramatically alter the standard scenario of stellar evolution. Alternatively, the requirement that this should not be the case can be used to set upper limits to the mass and interaction parameters of this and similar hypothetical particles. Furthermore, an experiment to measure the hypothetical solar axion flux has been suggested.[3] This requires a reliable calculation of the expected axion flux from the Sun so that positive or negative results of this experiment, if ever performed, can be properly interpreted.

The method of constraining parameters of hypothetical light scalar particles through stellar energy emission goes back to an idea of Sato and Sato.[4] After it was recognized by Weinberg and by Wilczek[5] that an idea of Peccei and Quinn[6] to solve the strong *CP* problem led to the prediction of a relatively light, pseudoscalar particle, the method of Ref. 4 was applied to this particle by Dicus, Kolb, Teplitz, and Wagoner, by Mikaelian, by Sato, and by Vysotsskii, Zeldovich, Khlopov, and Chechetkin[7] as well as by Raffelt and Stodolsky.[8] Because of the relatively large mass of this particle compared with typical stellar temperatures, these authors[7] mainly considered thermal emission from hot, helium-burning stars mainly through the Compton photoproduction process [Fig. 1(a)] and production in nuclear radiative transitions in main sequence stars.[8]

After the "standard axion" of Weinberg and Wilczek had been excluded by overwhelming laboratory and astrophysical evidence, an alternative axion model was constructed by Kim and by Shifman, Vainshtein, and Zakharov[1] (KSVZ) such as to render the axion "invisible." This new axion couples only to heavy quarks, not to leptons or light quarks. Therefore, its coupling to regular matter is mainly through a two-photon vertex and its production mechanism in stellar heat baths is mainly photon conversion through a Primakoff amplitude [Fig. 1(b)]. The coupling of this KSVZ axion is proportional to its mass which is a free parameter of the model. Dicus *et al.*[9] applied their method to this particle and derived an upper limit to its mass of about 10^{-2} eV by considering axion emission from main sequence stars and red giants. For such a low-mass particle thermal energies of typical stellar plasmas are large.

Dine, Fischler, and Srednicki[2] (DFS) constructed another axion model, similar to that of KSVZ, where the axion couples to electrons and light quarks. On the basis of photoproduction of this particle through a Compton amplitude [Fig. 1(a)] in the interior of red giants, Barroso

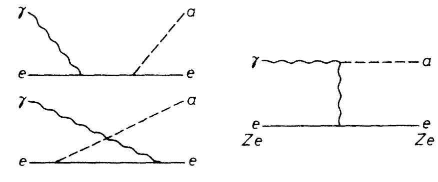

FIG. 1. Feynman graphs contributing to photoproduction of axions: $\gamma+(e^-,Ze)\rightarrow(e^-,Ze)+a$. (a) Compton-type amplitude on electrons. (b) Primakoff-type amplitude on electrons or nuclei.

and Branco and Ellis and Olive[10] derived a limit on its mass. The coupling of the DFS axion to electrons is proportional to its mass in analogy to the KSVZ axion.

Fukugita, Watamura, and Yoshimura[11] (FWY) took up the subject again and calculated in detail the energy loss from a plasma due to photoproduction of axions and other light, pseudoscalar particles through Compton- and Primakoff-type processes (Fig. 1). They carefully took into account plasma effects on transverse photons, i.e., the fact that photons in a plasma follow the dispersion relation $\omega^2=|\mathbf{k}|^2+\omega_{\rm pl}^2$ where the plasma frequency $\omega_{\rm pl}$ may be viewed as an effective photon mass. They showed that the effect of this dispersion relation for typical stellar plasmas is mainly to reduce the Primakoff emission rate by a factor of about 0.3–0.5. Other emission processes are essentially left unchanged. They showed, furthermore, that plasmon decay can always be neglected in comparison with other emission processes. They applied their results to the Sun and other stars and derived an upper limit to the DFS-axion mass of about 1 eV. In the case of the Sun, from which the most conservative bound ($m_a<3$ eV) was derived, their result depends dominantly on the Primakoff emission process.

Three-body processes, furthermore, can be very important for axion interaction with matter at stellar temperatures and densities as was first pointed out by Anselm and Uraltsev.[12] It is amusing to note that for solar conditions three-body absorption of axions, $a+e^-+(e^-,Ze)\to e^-+(e^-,Ze)$, yields a similar mean free path as two-body absorption, $a+(e^-,Ze)\to(e^-,Ze)+\gamma$. For an axion mass on the order of 1 eV it is on the order of 10^{23} cm. This is about the radius of the Milky Way galaxy; therefore stars can be considered transparent as far as axion propagation is concerned.

Krauss, Moody, and Wilczek[13] (KMW) discussed the reverse three-body process, i.e., the energy-loss rate from a nondegenerate, nonrelativistic plasma due to axion bremsstrahlung from electron-nucleus collisions, $e^-+Ze\to e^-+Ze+a$ [Fig. 2(a)]. They showed that for conditions of the solar interior this process is roughly equally important as the photon conversion processes considered by FWY.

Bremsstrahlung emission of axions from neutron stars has been considered by Iwamoto.[14] According to his work the relevant processes are neutron-neutron collisions in the interior and electron-nucleus collisions in the crust of the star.

The Primakoff effect and bremsstrahlung are processes which, at low energies, involve only the electrostatic field of the target particle. These processes would also occur in external, classical electric fields. The photon propagators appearing in the Feynman graphs Figs. 1(b), 2(a), and 3 then mainly represent the Coulomb fields of the target particles. Such fields will be screened in an environment of freely moving electric charges such as a stellar plasma (Debye-Hückel effect). Therefore the cross section of the relevant axion-producing reactions will be reduced. Apparently this effect was thought to be negligible by all authors who considered the Primakoff production process. This neglect, however, is not justified as can be seen by the following argument.

According to the theory of Debye and Hückel,[15] which was applied to the case of stellar plasmas by Salpeter,[16] the Coulomb potential of a charge Ze immersed in a medium of freely moving electric charges must be replaced—in the "weak screening limit"[16]—by what is now called a Yukawa potential:

$$\frac{Ze}{4\pi}\frac{1}{r}\to\frac{Ze}{4\pi}\frac{e^{-\kappa r}}{r}\,. \qquad (1)$$

The inverse of κ is now called the Debye-Hückel radius. In a plasma at temperature T this "Yukawa mass" of "static photons" is in natural units ($\hbar=c=k_B=1$)

$$\kappa=\left[\frac{4\pi\alpha\hat{n}}{T}\right]^{1/2}, \qquad (2)$$

where in Lorentz-Heaviside units $\alpha=e^2/4\pi\approx\frac{1}{137}$. $\hat{n}$ is a weighted number density of charged particles defined by Eq. (16c).

For any particular reaction this screening effect is important if a typical momentum Δ transferred by the electrostatic field is on the order of or smaller than κ. In the coordinate picture this criterion can be visualized through the Heisenberg uncertainty relation: Δ^{-1} is a typical distance between the particles at which the reaction takes place. Therefore Debye-Hückel screening is important if this distance is of the same order or larger than the shielding radius κ^{-1}.

Following a detailed discussion by Stodolsky[17] of the classical Primakoff effect involving neutral pions we mention that the dominant contribution of this effect comes

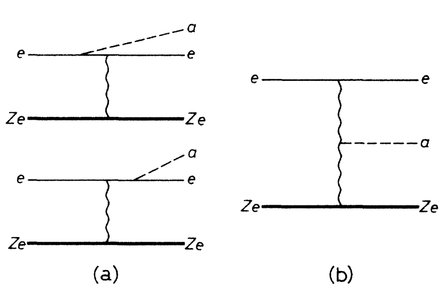

FIG. 2. Dominant Feynman amplitudes contributing to the production of axions in electron-nucleus collisions. (a) Bremsstrahlung amplitudes in analogy with photon emission. (b) Electro-Primakoff amplitude.

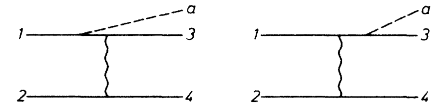

FIG. 3. Two representative Feynman graphs contributing to bremsstrahlung emission of axions in electron-electron collisions. There occur three more pairs of graphs which can be obtained by the following permutation of the labels of the electrons in the shown Feynman graphs: (1↔2,3↔4), (1↔2), and (3↔4).

from forward scattering. This means that most photons which transmute into axions in the electrostatic field of a nucleus or electron do so without changing much their direction of motion. Even in this case momentum must be transferred because the "photon mass" $\omega_{\rm pl}$ and the axion mass m_a are different. We assume that no energy is transferred because the relevant photon energies are small compared with nuclear or electron masses. This momentum transfer at zero degrees is given by

$$\Delta_0 \equiv |(\omega^2-\omega_{\rm pl}^2)^{1/2}-(\omega^2-m_a^2)^{1/2}| \approx \frac{|\omega_{\rm pl}^2-m_a^2|}{2\omega} , \quad (3)$$

where the latter approximation holds if $\omega_{\rm pl}$ and m_a are small compared with the photon energy ω.

In order to compare Δ_0 with κ for conditions of the solar center we neglect the axion mass and let approximately $\Delta_0 \approx \omega_{\rm pl}^2/2\omega$. For nondegenerate electrons the plasma frequency is given by[18]

$$\omega_{\rm pl} = \left[\frac{4\pi\alpha n_e}{m_e}\right]^{1/2} , \quad (4)$$

where n_e is the number density of electrons. According to a standard model of the Sun[19] the mass density at the solar center is $\rho = 156\ {\rm g\,cm^{-3}}$, the mass fraction of hydrogen is $X_{\rm H}=0.35$, and the temperature is $T=1.55\times 10^7$ K. Therefore $\omega_{\rm pl}=0.3$ keV. The temperature is, in energy units, $T=1.3$ keV which is representative for energies of thermal photons. Then we find $\Delta_0 \approx \omega_{\rm pl}^2/2T \approx 35$ eV for the case of a negligible axion mass. Comparing this value with $\kappa \approx 9$ keV clearly indicates that Debye-Hückel screening dramatically reduces the Primakoff production cross section of axions. For a helium-burning star (red giant) typical values are $\rho = 10^4\ {\rm g\,cm^{-3}}$ and $T=10^8$ K. Then $\omega_{\rm pl}=2.0$ keV, $T=8.6$ keV, $\kappa=27$ keV, and $\Delta_0=0.23$ keV. Therefore again $\Delta_0 \ll \kappa$ and the same conclusion holds.

We emphasize that this result is not in contradiction with the astrophysical standard picture that screening effects were relatively unimportant for nuclear reaction rates in stellar interiors.[18] For nuclear reactions a typical momentum transfer is on the order of the momenta of the reaction partners, i.e., $\Delta = O(\sqrt{Tm_{\rm nucleus}})$. For nuclei in the Sun this is in the MeV range, much in excess of κ.

The case of bremsstrahlung from electron-nucleus or electron-electron collisions is on the margin where screening effects become important. In this case a typical momentum transfer is on the order of the electron momenta, i.e., $\Delta = O(\sqrt{m_e T})$. For the solar center this is about 25 keV. Then κ/Δ is typically between 0.1 and 1.

In the bremsstrahlung calculations by KMW shielding effects have been taken into account. These authors, however, appear to be mislead by taking the notion of the plasma frequency as an effective photon mass too literally and use $\omega_{\rm pl}$ in place of κ as a scale for the exponential decrease of an electrostatic field. We emphasize that the dispersion relation $\omega^2=\omega_{\rm pl}^2+k^2$ applies only to *transverse* excitations of the electromagnetic field in a plasma. *Longitudinal* excitations follow a different dispersion relation which cannot be written in the simple form $\omega^2 = m_{\rm eff}^2 + k^2$ (Ref. 20). The Coulomb field should be viewed as the static limit of "longitudinal photons"; therefore the plasma frequency is not the correct scale for the decrease of electrostatic fields. Consequently KMW arrive at the conclusion that screening corrections were always below the 1% level for the case of stellar plasmas. We find, in contrast, that bremsstrahlung rates are typically reduced by a few tens of percent in comparison with the vacuum rates.

In the calculations of KMW, furthermore, only bremsstrahlung from electron-nucleus collisions has been considered. In a plasma of low-Z nuclei, however, such as is believed to exist in the interior of main sequence stars, one expects that electron-electron collisions are of similar importance. It turns out, in fact, that in a plasma of pure hydrogen the bremsstrahlung contribution of electron-electron collisions is about 70% that of electron-proton collisions.

Considering the above remarks we feel that a new calculation of axion emission rates from stellar plasmas is called for. Concentrating on the case of nonrelativistic, nondegenerate plasmas we have carried out such a calculation and presently wish to communicate the results of this work. To this end we collect in Sec. II the relevant effective Lagrangian densities and coupling constants for the interaction of axions with photons and electrons. In Sec. III we calculate the emission rates from photoproduction processes and emphasize the importance of Debye-Hückel screening for the Primakoff process in a plasma. In Sec. IV we calculate bremsstrahlung emission from electron-nucleus and electron-electron collisions and discuss corrections due to screening effects. In Sec. V we apply our results to the Sun and red giants and we derive new upper bounds on the axion mass and interaction strength. In Sec. VI, then, we give a brief summary of this work.

II. AXION COUPLING TO PHOTONS AND ELECTRONS

The effective coupling of axions to photons can be described by the Lagrangian density

$$\mathcal{L}_{a\gamma\gamma} = -\frac{G}{4} F_{\mu\nu}\tilde{F}^{\mu\nu}\varphi_a = G\mathbf{E}\cdot\mathbf{B}\varphi_a , \quad (5)$$

where G is a coupling constant of dimension $({\rm energy})^{-1}$, φ_a is the pseudoscalar axion field, and $F_{\mu\nu}$ is the electromagnetic field strength tensor. The dual tensor is defined as $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. FWY, in contrast, use a convention where this factor $\frac{1}{2}$ is absorbed into the coupling constant.[21] Srednicki[22] recently pointed out that the coupling constant G can be expressed in terms of the axion mass m_a in a model-independent fashion. Using his result we define a dimensionless coupling constant

$$\alpha_{a\gamma\gamma} = \frac{(Gm_e)^2}{4\pi} \approx 4.37\times 10^{-28}\left[\frac{m_a}{\rm eV}\right]^2 , \quad (6)$$

where m_e is the electron mass.

This two-photon vertex allows for a decay channel of the axions into two photons. We find for the relevant decay width

$$\Gamma=\frac{G^2}{64}\frac{m_a{}^3}{\pi} . \tag{7}$$

This corresponds to an axion lifetime of

$$\tau\approx 6.3\times 10^{24}\ \text{sec}\ (\text{eV}/m_a)^5 . \tag{8}$$

The coupling constant G is related to the energy scale v of the breaking of the Peccei-Quinn symmetry. Using the notation of FWY this relationship is expressed as

$$G=\frac{\alpha N 2z/(1+z)}{\pi v}\approx\frac{1.7\times 10^{-3}}{v/N} , \tag{9}$$

where N is an integer coefficient of the color anomaly of the current associated with the Peccei-Quinn symmetry. In the DFS model $N=3$ for three generations of quarks and leptons. In the notation of Srednicki[22] N stands for twice the number that is symbolized by N in the notation of FWY. $z\approx 0.56$ is the mass ratio of up and down quarks.

The tree-level coupling of axions to electrons can be written as

$$\mathcal{L}_{aee}=ig\bar{\psi}_e\gamma_5\psi_e\varphi_a , \tag{10}$$

where ψ_e is the electron field and g is a dimensionless coupling constant related to the scale v through

$$g=\frac{m_e}{v}2\cos^2\beta . \tag{11}$$

$\cos\beta$ is a free parameter of the DFS model. Srednicki[22] speculates that $2\cos^2\beta>1$. In the KSVZ axion model no tree-level coupling to electrons exists. In our phenomenological approach we can account for this situation by letting $\cos\beta=0$.

The constant g can again be related to the axion mass. For three generations of quarks and leptons we write, using a result from Ref. 22,

$$\alpha_{aee}=\frac{g^2}{4\pi}\approx 1.60\times 10^{-23}\left[\frac{m_a}{\text{eV}}\right]^2(2\cos^2\beta)^2 . \tag{12}$$

FWY use the notation $2\cos^2\beta\equiv 2x^2/(x^2+1)$ where x is a free parameter which they set equal to unity.

III. CALCULATION OF PHOTOPRODUCTION RATES

A. General remarks

In the presence of electrons and nuclei photons can convert into axions:

$$\gamma+(e^-,Ze)\rightarrow(e^-,Ze)+a . \tag{13}$$

This physical process can occur via a Compton-type amplitude [Fig. 1(a)] by virtue of the interaction Lagrangian (10) or via a Primakoff-type amplitude [Fig. 1(b)] by virtue of (5). In both cases electrons and nuclei are possible targets for this reaction. The Compton process on nuclei, however, is suppressed relative to that on electrons by a factor $Z^2(m_e/m_{\text{nucleus}})^2$. Therefore this process is neglected.

In the case of electrons as targets both amplitudes, Primakoff and Compton, are important. Therefore interference between the two terms must be considered in addition to the pure "Compton process plus Primakoff process." We are interested, however, only in the limiting case of small photon energies, $\omega\ll m_e$. Then for the Primakoff amplitude only the charge (and not the magnetic moment) of the electron is relevant for the axion production. Therefore the electron spin is left unchanged in Primakoff events. The Compton amplitude, in contrast, always occurs through a spin flip of the target electron due to the pseudoscalar γ_5 coupling Eq. (10). Therefore in the limit of soft photons no interference occurs between the Compton and Primakoff terms which may, then, be visualized as distinct processes. Going beyond the lowest order in an ω/m_e expansion, however, would require consideration of the interference term.

In our low-frequency approximation the energy E_a of the outgoing axion is equal to the energy ω of the incident photon. Then the energy emission per unit volume at temperature T is

$$\epsilon^*(T)=\int_0^\infty \omega\sigma_{\text{tot}}(\omega)n_t n'(T,\omega)d\omega , \tag{14}$$

where n_t is the number density of targets (taken to be at rest) and

$$n'(T,\omega)=\frac{1}{\pi^2}\frac{\omega^2}{e^{\omega/T}-1} \tag{15}$$

is the blackbody number density (or flux) of photons per frequency band at temperature T and frequency ω. ϵ^* is measured in units power per volume, e.g., erg cm^{-3}sec^{-1}. The power output per unit mass (erg g^{-1}sec^{-1}) is then obtained through $\epsilon=\epsilon^*/\rho$ where ρ is the mass density.

For later reference we mention that the number densities of target particles are related to the mass density and the chemical composition of the plasma. For all cases of interest to us we may assume that the chemical composition is dominated by hydrogen and helium. Then the only relevant composition parameter is the mass fraction X_H of hydrogen. In this approximation we obtain for the relevant number densities

$$n_e\approx\frac{1+X_\text{H}}{2}\frac{\rho}{m_u} , \tag{16a}$$

$$\bar{n}\equiv\sum_z Z^2 n_z\approx n_p+4n_\alpha=\frac{\rho}{m_u} , \tag{16b}$$

$$\hat{n}\equiv n_e+\sum_z Z^2 n_z\approx n_e+n_p+4n_\alpha=\frac{3+X_\text{H}}{2}\frac{\rho}{m_u} , \tag{16c}$$

where $m_u=1.66\times 10^{-24}$ g is the atomic mass unit. n_e is the number density of electrons, n_p refers to protons, n_α to helium nuclei, and n_z to nuclei of charge Z.

B. Primakoff process

1. Primakoff cross section

We now consider a photon with energy ω incident on a localized charge distribution $\rho(\mathbf{r})$. This charge distribution creates an external electric field $\mathbf{E}(\mathbf{r})$ in which the photon can convert into axions according to the interaction Lagrangian Eq. (5). With "external field" we merely mean that the sources of this field will remain essentially undisturbed by the Primakoff reaction as will be the case for keV photons incident on a collection of electrons and nuclei. The differential cross section is then found as

$$\frac{d\sigma_p}{d\Omega}=\frac{1}{2}\left[\frac{Ge}{4\pi}\right]^2\frac{|\mathbf{k}\times\mathbf{p}_a|^2}{|\mathbf{\Delta}|^4}|F(\mathbf{\Delta})|^2\ , \tag{17}$$

where $\mathbf{\Delta}\equiv\mathbf{k}-\mathbf{p}_a$ is the transfer of three momentum. The factor $\frac{1}{2}$ comes from averaging over photon polarizations. The form factor F of the charge distribution is defined as

$$F(\mathbf{q})\equiv\frac{1}{e}\int d^3\mathbf{r}\,\rho(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\ , \tag{18}$$

where e is the elementary charge unit.

If the plasma frequency and axion mass are assumed small in comparison with the photon energy this cross section is to lowest order

$$\frac{d\sigma_p}{d\Omega}=\frac{1}{2}\left[\frac{Ge}{4\pi}\right]^2\frac{\sin^2\vartheta}{[2(1-\cos\vartheta)+(\Delta_0/\omega)^2\cos\vartheta]^2}\times|F(\mathbf{\Delta})|^2\ , \tag{19}$$

where the zero-degree momentum transfer is given by Eq. (3). In the present approximation obviously $\Delta_0/\omega \ll 1$.

If the target is a pointlike charge of magnitude Ze the form factor is $|F|^2=Z^2$, independently of the momentum transfer. Then at small scattering angles the cross section is proportional to $\vartheta^2/[\vartheta^2+(\Delta_0/\omega)^2]^2$ which is strongly peaked at $\vartheta=\Delta_0/\omega$. The total cross section

$$\sigma_p=Z^2\left[\frac{Ge}{4\pi}\right]^2\pi\left[\ln\left[\frac{2\omega}{\Delta_0}\right]-1\right] \tag{20}$$

is dominated by this peak. Since the reaction is assumed to take place in vacuum with only the target charge present the plasma frequency vanishes and $\Delta_0=m_a{}^2/2\omega$. Obviously $\Delta_0{}^{-1}$ plays the role of a cutoff parameter for the infinite range of the Coulomb field.

FWY have argued that Δ_0 should be set to $\Delta_0=|\omega_{\rm pl}{}^2-m_a{}^2|/2\omega$ if the reaction is assumed to take place in a plasma and that consequently the axion emission rate would be suppressed by about $\ln(\omega_{\rm pl}/\omega)/\ln(m_a/\omega)\approx 0.3$ for the relevant parameter values. We emphasize that this argument is not complete because the presence of the plasma will not only modify the dispersion relation of photons. It also constitutes a nontrivial charge distribution which will modify the above simple form factor which was calculated for an *isolated* point charge. Every charge in a plasma attracts charges of opposite sign and repels charges of its own sign. Therefore it is surrounded, on average, by a charge cloud of opposite sign which neutralizes its influence over large distances. We therefore proceed to discuss the changes in the form factor brought about by the presence of this statistical charge cloud.

2. Statistical form factor

We begin with the idealized situation of a plane electromagnetic wave (or photon) incident on a stationary charge distribution consisting of N point charges of magnitude Z_ie, $i=1,\ldots,N$, at locations $\mathbf{r}_i$. Then the relevant squared form factor for the Primakoff cross section is

$$|F_N(\mathbf{q})|^2=\sum_{i=1}^{N}Z_i{}^2+\sum_{\substack{i,j=1\\ i\neq j}}^{N}Z_iZ_j\cos(\mathbf{q}\cdot\mathbf{r}_{ij})\ , \tag{21}$$

where $\mathbf{r}_{ij}\equiv\mathbf{r}_i-\mathbf{r}_j$ is the relative coordinate of particles i and j. The first term is the sum over the form factors of individual charges. The second term arises from the interference of the scattered waves from different target particles.

Since we are interested in a physical situation where many different photons interact with many different ensembles of targets which are assumed to be in thermal equilibrium at temperature T we need to consider the thermal average of this squared form factor. If all configurations of particles had equal probabilities—as would be the case of an ideal Boltzmann gas—then the interference term in Eq. (21) would average to zero. Physically this situation corresponds to very dilute or very hot plasmas. Then it would be indeed correct to use the single-particle cross section Eq. (20) for the calculation of the axion emission rate as was done by previous authors.

In a real plasma, however, the particles mutually interact through their Coulomb fields and their motion is slightly correlated. The pair correlation function for this case can be explicitly calculated from a Gibbs ensemble as outlined by Landau and Lifshitz.[23] Using their result we may write the probability of finding particle j at a distance r away from particle i as

$$p_{ij}(r)=\frac{1}{V}\left[1-\frac{Z_iZ_j\alpha}{T}\frac{e^{-\kappa r}}{r}\right],\ i\neq j\ , \tag{22}$$

where α is the fine-structure constant and

$$\kappa^2\equiv\frac{4\pi\alpha}{T}\sum_{i=1}^{N}\frac{Z_i{}^2}{V} \tag{23}$$

is recognized as the square of the inverse Debye-Hückel radius as defined in our introductory remarks. V is a normalization volume taken to be so large that the normalization of the pair correlation function is $\int d^3\mathbf{r}\,p_{ij}(r)=1$ with arbitrary precision. Furthermore $p_{ii}(r)=\delta^3(\mathbf{r})$.

The physical interpretation of p_{ij} is straightforward. Taking the position of particle i as the origin of a coordinate system there will be, on average, a charge cloud at and around this particle with charge density

$$\rho_i(r)=\sum_{j=1}^{N} p_{ij}(r)Z_j e = Z_i e \left[\delta^3(\mathbf{r})-\frac{\kappa^2}{4\pi}\frac{e^{-\kappa r}}{r}\right] . \quad (24)$$

In order to derive the latter equality we have made use of the definition of κ and of the overall charge neutrality of the plasma: $\sum_{i=1}^{N} Z_i=0$. The first term arises from the particle i itself while the second term is an average cloud of opposite charges attracted by the primary charge. This charge cloud modifies the Coulomb potential of the particle i, on average, to the Yukawa potential mentioned in our introductory discussion. Therefore the Coulomb field of any charge is, on average, screened over distances larger than about κ^{-1}.

Using the pair correlation function Eq. (22) in order to calculate the thermal average of the squared form factor Eq. (21) we find

$$\langle |F_N(\mathbf{q})|^2\rangle=\sum_{i=1}^{N} Z_i^2\left[1-\frac{\kappa^2}{\kappa^2+|\mathbf{q}|^2}\right] . \quad (25)$$

Then we may drop the summation and assign an effective form factor to every individual particle with charge Ze:

$$|F_{\text{eff}}(\mathbf{q})|^2=Z^2\frac{|\mathbf{q}|^2}{\kappa^2+|\mathbf{q}|^2} . \quad (26)$$

This "form factor" takes into account the contribution from scattering off the given particle and *half* of the interference term arising from this and all other particles. Summing eventually over all particles appropriately counts each of these interference terms twice and therefore no double counting occurs. Clearly only particles not much further away than κ^{-1} contribute to any given scattering event.

We emphasize that F_{eff} is *not* the form factor of the charge distribution Eq. (24) defined by a particle and its average Debye-Hückel cloud. This would lead to a form factor

$$|F_{\text{DH}}(\mathbf{q})|^2=Z^2\frac{|\mathbf{q}|^4}{(\kappa^2+|\mathbf{q}|^2)^2} \quad (27)$$

which is different from F_{eff}.

The justification for squaring the amplitude of the scattered waves first and averaging afterward over different target configurations lies in the stationarity of the target. The dominating time scale of the present problem is the time it takes a photon to cross a certain configuration of size κ^{-1}, i.e., $\delta t\approx\kappa^{-1}$. During this period the configuration will not change appreciably because the time scale of change is the time it takes an electron to cross such a region, i.e., $\kappa^{-1}/v_e \gg \kappa^{-1}$. The electromagnetic field will remain coherent for the time δt because the scale of temporal coherence is approximately given by the inverse bandwidth of the photon frequency spectrum,[24] i.e., $\delta\tau=O(T^{-1})$. In the cases of interest to us $T^{-1}\gg\kappa^{-1}$, which also implies that a typical target region is much smaller than a photon wavelength.

We conclude that the Primakoff cross section Eq. (17) must be supplemented by the effective form factor $F_{\text{eff}}(\Delta)$ as given by Eq. (26). For the case of bremsstrahlung, which will be discussed later, the situation is different because there the reaction partners move at similar speeds as all other particles in the plasma. Therefore any charge distribution may rearrange itself during a collision. Then we use a time-averaged charge distribution for estimating the effects of screening; i.e., we then use the form factor F_{DH} for every target charge. If we would use F_{DH} instead of F_{eff} for the Primakoff process the axion emission rate would be suppressed even more than indicated by our following discussion.

3. Energy emission from a plasma

We now proceed to calculate the energy emission from a plasma due to the Primakoff production of axions. To this end we note that the zero degree momentum transfer for the very light axions presently under consideration is given by $\Delta_0=\omega_{\text{pl}}^2/2\omega$. Comparing the definition of the plasma frequency with that of the Debye-Hückel radius reveals that

$$\frac{\Delta_0^2}{\kappa^2}=\frac{\pi\alpha n_e{}^2 T}{\hat{n} m_e{}^2\omega^2}\approx\frac{\pi\alpha}{8}\frac{\rho}{m_e{}^2 m_u T}=0.6\times10^{-5}\frac{\rho_2}{T_7} , \quad (28)$$

where we have used $\omega\approx 2T$, $\hat{n}/2\approx n_e\approx\rho/m_u$, $\rho_2\equiv\rho/10^2$ g cm^{-3}, and $T_7\equiv T/10^7$ K. Therefore in all cases of interest to us we may neglect Δ_0^2 relative to κ^2.

At scattering angles near the forward direction the cross section is then proportional to $\vartheta^2/[\vartheta^2-(\Delta_0/\omega)^2]$ which is *not* peaked anymore and which would not diverge for $\Delta_0=0$. Therefore we neglect Δ_0 altogether thereby introducing a small error at angles $0<\vartheta\lesssim\Delta_0/\omega$. Then we find for the differential cross section

$$\frac{d\sigma_p}{d\Omega}=Z^2\left[\frac{Ge}{4\pi}\right]^2\frac{1}{8}\frac{1+\cos\vartheta}{1+2(\kappa/2\omega)^2-\cos\vartheta} .$$

The previously dominant forward peak has now completely vanished. It was a typical feature of the infinite-range Coulomb field which was only cut off by the zero degree momentum transfer Δ_0. In the present situation the Coulomb field does not reach so far anymore because it is cut off at the much shorter-distance scale κ^{-1}.

Integration over the scattering angles yields the total cross section

$$\sigma_p(\omega)=Z^2\left[\frac{Ge}{4\pi}\right]^2\frac{\pi}{2}\left[\left[1+\frac{\kappa^2}{4\omega^2}\right]\ln\left[1+\frac{4\omega^2}{\kappa^2}\right]-1\right] . \quad (29)$$

For $\omega\ll\kappa$ the term in square brackets is simply $2\omega^2/\kappa^2$. Application of Eq. (14) yields the energy emission rate

$$\epsilon_p^*(T)=\frac{\alpha\alpha_{a\gamma\gamma}}{m_e{}^2}\hat{n}T^4 f(\xi^2) , \quad (30)$$

where $\hat{n}$ is given by (16c) and

$$\xi^2\equiv\left[\frac{\kappa}{2T}\right]^2=\frac{\pi\alpha\hat{n}}{T^3} . \quad (31)$$

The function f is defined as

$$f(\xi^2)\equiv\frac{1}{2\pi}\int_0^\infty [(x^2+\xi^2)\ln(1+x^2/\xi^2)-x^2]\frac{x}{e^x-1}dx \,, \tag{32}$$

where we have used $x\equiv\omega/T$ as a variable of integration. For numerical orientation we mention that in a standard solar model[19] the dimensionless parameter is $\xi^2\approx 12$ for the entire Sun with a variation of at the most 15%. For a pure helium red giant with $\rho=10^4$ g cm^{-3} and $T=10^8$ K it is $\xi^2=2.5$. In Table I we list a few typical values for $f(\xi^2)$.

Converting our result to a power output per unit mass and employing the approximate relation Eq. (16c) for $\hat{n}$ we find

$$\epsilon_p(\rho,T,X_{\rm H})\approx\frac{\alpha\alpha_{a\gamma\gamma}}{m_e^2 m_u}\frac{3+X_{\rm H}}{2}T^4 f\left[\frac{\pi\alpha\rho(3+X_{\rm H})}{2T^3}\right]$$
$$=4.95\times10^{-3}\ {\rm erg\,g^{-1}sec^{-1}}(3+X_{\rm H})T_7{}^4 f(8.28(3+X_{\rm H})\rho_2 T_7{}^{-3})(\alpha_{a\gamma\gamma}/4.37\times10^{-28})\,. \tag{33}$$

This result is valid for any pseudoscalar particle which couples to photons through Eq. (5) and whose mass is small compared with the relevant temperature scale.

Considering specifically invisible axions and using conditions of the solar center in a standard model[19] with $T_7=1.55$, $X_{\rm H}=0.3545$, and $\rho_2=1.563$ this is $\epsilon_p=0.047$ erg g^{-1}sec$^{-1}(m_a/{\rm eV})^2$. For such conditions and if all parameters except T are held fixed we find that in the neighborhood of $T=1.5\times10^7$ K approximately $f\sim T^2$, i.e., $\epsilon_p\sim T^6$.

TABLE I. Selected values for the function $f(\xi^2)$ as defined in Eq. (32) from a numerical calculation.

ξ^2	$f(\xi^2)$
0.5	2.424
1.0	1.864
1.5	1.567
2.0	1.371
2.5	1.229
3.0	1.119
3.5	1.031
4.0	0.958
4.5	0.896
5.0	0.843
6.0	0.756
7.0	0.688
8.0	0.632
9.0	0.586
10.0	0.546
11.0	0.512
12.0	0.482
13.0	0.456
14.0	0.433
15.0	0.412
16.0	0.393
17.0	0.376
18.0	0.361
19.0	0.347
20.0	0.333
22.0	0.310
24.0	0.290
26.0	0.273
28.0	0.257
30.0	0.243

We mention for comparison that in the solar center the nuclear energy generation rate is about 16 erg g^{-1}sec^{-1} from the *pp* chains and 4 erg g^{-1}sec^{-1} from the CNO cycle for an assumed abundance of carbon and nitrogen of $X_{\rm CN}=0.02$.

We note, furthermore, that the axion emission rate at the solar center would have been about $\epsilon_p=1.1$ erg g^{-1}sec$^{-1}(m_a/{\rm eV})^2$ if we had neglected screening effects as in the work of FWY, about 25 times as much as our result. Our result, on the other side, would have been further reduced to $\epsilon_p=0.015$ in the same units if we had used the form factor $F_{\rm DH}$ Eq. (27) for a time-averaged charge distribution instead of $F_{\rm eff}$ for an ensemble averaged, stationary charge distribution.

The energy distribution of the emitted axions is far from being a blackbody spectrum because the spectrum of the incident photons is modulated by the frequency dependence of the cross section. For low energies the Primakoff cross section varies as ω^2; therefore, the low-energy photon spectrum is strongly suppressed. For $\xi^2=12$, as is typical for the Sun, the maximum of the differential number flux of axions occurs at $E_a/T\approx3.5$. The average axion energy is then $\langle E_a/T\rangle\approx4.4$. This is to be compared with blackbody radiation where the maximum occurs at $\omega/T\approx1.6$ and the average photon energy is $\langle\omega/T\rangle\approx2.7$. We mention that in the case $\kappa=0$ as considered by FWY the axion spectrum *is* almost like a blackbody spectrum because in this case the cross section varies only as $\ln\omega$.

C. Compton process

The total cross section for the process depicted by the Feynman graphs of Fig. 1(a) can be calculated to be

$$\sigma_c(\omega)=\frac{4\pi}{3}\frac{\alpha\alpha_{aee}}{m_e^2}\left[\frac{\omega}{m_e}\right]^2 \tag{34}$$

to lowest order in ω/m_e. Application of (14) then yields

$$\epsilon_c^*(T)=\frac{160}{\pi}\zeta(6)\frac{\alpha\alpha_{aee}}{m_e^4}n_e T^6\,, \tag{35}$$

where the Riemann ζ function takes on the value $\zeta(6)\approx1.017$. This result is in perfect agreement with the relevant formula of FWY.

Using the approximate relation Eq. (16) we find for the corresponding emission rate per unit mass

$$\epsilon_c(T,X_{\rm H})\approx\frac{80}{\pi}\zeta(6)\frac{\alpha\alpha_{aee}}{m_e{}^4m_u}(1+X_{\rm H})T^6$$

$$=2.67\times10^{-2}{\rm erg\,g^{-1}sec^{-1}}(1+X_{\rm H})$$

$$\times T_7{}^6(\alpha_{aee}/1.60\times10^{-23})\ . \qquad (36)$$

At the solar center this is $\epsilon_c=0.50$ erg g^{-1}sec$^{-1}(m_a/{\rm eV})^2(2\cos^2\beta)^2$ for invisible axions.

The energy distribution of the axions is again far from being a blackbody spectrum at temperature T. The maximum of the differential axion number flux occurs at $E_a/T\approx3.9$ and the average axion energy is $\langle E_a/T\rangle\approx4.9$.

IV. CALCULATION OF BREMSSTRAHLUNG EMISSION RATES

A. General remarks

As was first pointed out by KMW axions may be efficiently produced in a stellar plasma through bremsstrahlung processes where the axion is emitted in place of a photon. Candidates for the incident reaction partners are electrons and nuclei. These particles are in thermal equilibrium. Therefore the nuclei move much slower than electrons and may be neglected as sources of bremsstrahlung in comparison with electrons. Then one expects nucleus-electron and electron-electron collisions to contribute significantly to this type of energy emission process where the axion is emitted by one of the scattered electrons. The relevant Feynman graphs are shown in Figs. 2(a) and 3.

We mention that usually electron-electron collisions are thought to be negligible in comparison with electron-nucleus collisions due to the Z^2 enhancement of the latter process. In the case at hand, however, the nuclei in question are mainly protons and alpha particles. Therefore this enhancement is only small. Consequently electron-electron collisions cannot be neglected in contrast with the treatment of KMW.

In addition to the usual bremsstrahlung Feynman amplitudes [Fig. 2(a)] one must consider the electro-Primakoff amplitude [Fig. 2(b) and a similar amplitude for the case of electron-electron collisions]. Both types of amplitudes contribute to the same physical process and must be added coherently. We shall show, however, that the contribution of the electro-Primakoff effect is much smaller than that of the bremsstrahlung term and may thus be neglected. This would not apply for the case of the KSVZ axion which does not couple directly to electrons and thus would not exhibit the bremsstrahlung amplitudes. Then only the electro-Primakoff effect is relevant. Therefore one needs to consider only the bremsstrahlung amplitudes for the DFS axion and only the electro-Primakoff amplitude for the KSVZ axion. The interference term never needs to be considered, and we treat the two cases as distinct physical processes.

In order to estimate the effect of screening on the bremsstrahlung emission rates we use the form factor $F_{\rm DH}$ for every charge in the plasma. This amounts to replacing the Coulomb denominator $|\Delta|^{-4}$ by $(|\Delta|^2+\kappa^2)^{-2}$ which is in the spirit of the effective electrostatic Yukawa potential Eq. (1) arising from the time-averaged Debye-Hückel charge cloud around each particle in the plasma. We follow this procedure because the radiating electrons move no faster than all the other electrons in the plasma such that a substantial rearrangement of charges may be expected during a collision. Therefore it appears appropriate to use a time-averaged target-charge distribution. We emphasize that this treatment may slightly overestimate the importance of screening. Since we find that these effects typically reduce the axion emission rate by a few tens of percent such an overestimate would not substantially change our final result. It leads to slightly more conservative bounds on the axion parameters.

B. Bremsstrahlung from electron-nucleus collisions

We first consider axion production in the collision of a nonrelativistic electron, initial velocity $\boldsymbol{\beta}_i$, $|\boldsymbol{\beta}_i|=\beta_i\ll1$, with an infinitely heavy nucleus of charge Ze which is taken to be at rest [Fig. 2(a)]:

$$e^-+Ze\to e^-+Ze+a\ . \qquad (37)$$

The axion is treated as a massless particle; i.e., its momentum four-vector is $\omega_a(1,\boldsymbol{\beta}_a)$, $|\boldsymbol{\beta}_a|=1$, where ω_a is the axion energy in units of the electron mass.

We find for the energy emission rate the following expression where all incident electrons are taken to have the same velocity β_i

$$\epsilon^*_{eZ}(\beta_i)=\frac{Z^2\alpha^2\alpha_{aee}}{m_e}n_en_z\int_0^{\beta_i}\beta_f{}^2d\beta_f\int\frac{d\Omega_f}{4\pi}\int\frac{d\Omega_a}{4\pi}4\omega_a{}^2\frac{(\boldsymbol{\beta}_i-\boldsymbol{\beta}_f)^2-[\boldsymbol{\beta}_a\cdot(\boldsymbol{\beta}_i-\boldsymbol{\beta}_f)]^2}{[(\boldsymbol{\beta}_i-\boldsymbol{\beta}_f)^2+(\kappa/m_e)^2]^2}\ . \qquad (38)$$

$\boldsymbol{\beta}_f$ is the velocity of the outgoing electron, Ω_f and Ω_a are the directions of motion of the outgoing electron and axion. Energy conservation yields $\omega_a=\frac{1}{2}(\beta_i{}^2-\beta_f{}^2)$.

Carrying out the integral over the axion angles we find

$$\int\frac{d\Omega_a}{4\pi}\{(\boldsymbol{\beta}_i-\boldsymbol{\beta}_f)^2-[\boldsymbol{\beta}_a\cdot(\boldsymbol{\beta}_i-\boldsymbol{\beta}_f)]^2\}=\tfrac{2}{3}(\boldsymbol{\beta}_i-\boldsymbol{\beta}_f)^2\ . \qquad (39)$$

Then integrating over $d\Omega_f$ we find, using $x \equiv \beta_f/\beta_i$ and $\mu \equiv \kappa/m_e\beta_i$

$$\epsilon^*_{eZ}(\beta_i)=\frac{2}{3}\frac{Z^2\alpha^2\alpha_{aee}}{4m_e}n_e n_z \beta_i^5 \int_0^1 dx\, x(1-x^2)^2 \left[\ln\left[\frac{(1+x)^2+\mu^2}{(1-x)^2+\mu^2}\right] - \frac{\mu^2}{(1-x)^2+\mu^2} + \frac{\mu^2}{(1+x)^2+\mu^2} \right] . \tag{40}$$

This result differs from the corresponding Eq. (1) of KMW by our factor $\frac{2}{3}$ and by our two terms proportional to μ^2.

In order to perform the remaining integral we expand this formula in powers of μ^2 and find, restoring the meaning of μ,

$$\epsilon^*_{eZ}(\beta_i)=\frac{8}{135}\frac{Z^2\alpha^2\alpha_{aee}}{m_e}n_e n_z \beta_i^5 \left[1-\frac{15}{2}\left[\frac{\kappa}{m_e\beta_i}\right]^2 + O((\kappa/m_e\beta_i)^4)\right] . \tag{41}$$

Averaging β_i^5 (for the term proportional to unity) and β_i^3 (for the term proportional to κ^2) over a Maxwellian velocity distribution at temperature T for the nondegenerate, nonrelativistic electrons we finally find

$$\epsilon^*_{eZ}(T)=\frac{128}{45}\left[\frac{2}{\pi}\right]^{1/2}\frac{\alpha^2\alpha_{aee}}{m_e}n_e\bar{n}\left[\frac{T}{m_e}\right]^{5/2}\left[1-\frac{5}{4}\frac{\kappa^2}{m_e T}+O((\kappa^2/m_e T)^2)\right] , \tag{42}$$

where we have summed over all species of nuclei by replacing $Z^2 n_z$ by $\bar{n}$ as defined by Eq. (16b). This result differs from that of KMW by the aforementioned factor $\frac{2}{3}$, by the coefficient of the correction term proportional to κ^2 and by the interpretation of κ. In our treatment κ is the inverse Debye-Hückel radius while KMW use the plasma frequency $\omega_{\rm pl}$ in place of κ.

Converting to an energy emission per unit mass and using the approximate expressions for number densities Eq. (16) we find

$$\epsilon_{eZ}(\rho,T,X_{\rm H}) \approx \left[\frac{68}{45}\left[\frac{2}{\pi}\right]^{1/2}\frac{\alpha^2\alpha_{aee}}{m_e}\frac{\rho}{m_u{}^2}\left[\frac{T}{m_e}\right]^{5/2}(1+X_{\rm H})\right]\left[1-\frac{5\pi\alpha\rho(3+X_{\rm H})}{2T^2 m_e m_u}\right]$$

$$=[0.150 \text{ erg}\,\text{g}^{-1}\text{sec}^{-1} T_7{}^{2.5}\rho_2(1+X_{\rm H})(\alpha_{aee}/1.60\times10^{-23})][1-6.98\times10^{-2}(3+X_{\rm H})\rho_2 T_7{}^{-2}] . \tag{43}$$

For invisible axions and conditions of the solar center this is $\epsilon_{eZ}=0.95$ $\text{erg}\,\text{g}^{-1}\text{sec}^{-1}(1-0.15)(m_a/\text{eV})^2(2\cos^2\beta)^2$. We conclude that Debye-Hückel shielding yields a nonnegligible 15% correction in the case at hand.

For bremsstrahlung the energy distribution of the axions is peaked at relatively low energies. For conditions of the Sun ($\kappa^2/m_e T \approx 0.12$) the maximum of the differential axion number flux occurs at $E_a/T \approx 0.82$ while the average axion energy would be $\langle E_a/T\rangle \approx 1.74$.

C. Electro-Primakoff effect in electron-nucleus collisions

Turning now to the electro-Primakoff effect [Fig. 2(b)] we find for the same physical situation as in the previous section

$$\epsilon^*_{e\rm P}(\beta_i)=\frac{2}{35}\frac{Z^2\alpha^2\alpha_{a\gamma\gamma}}{m_e}n_e n_z\beta_i^7 , \tag{44}$$

where we have not considered Debye-Hückel shielding. The relevant thermal average is

$$\epsilon^*_{e\rm P}(T)=\frac{768}{35}\left[\frac{2}{\pi}\right]^{1/2}\frac{\alpha^2\alpha_{a\gamma\gamma}}{m_e}n_e\bar{n}\left[\frac{T}{m_e}\right]^{7/2} . \tag{45}$$

Comparing the bremsstrahlung and electro-Primakoff rates yields the ratio

$$r=\frac{\epsilon^*_{e\rm P}(T)}{\epsilon^*_{eZ}(T)}=\frac{54}{7}\frac{\alpha_{a\gamma\gamma}}{\alpha_{aee}}\frac{T}{m_e} , \tag{46}$$

where the Debye-Hückel effect has been disregarded. For the solar center $T/m_e \approx 2.5\times10^{-3}$ and then $r \approx 5\times10^{-7}(2\cos^2\beta)^{-2}$. This large discrepancy between the two rates justifies to completely neglect the electro-Primakoff effect and the interference term between the two processes in comparison with the bremsstrahlung emission.

For the case of the KSVZ axion ($\cos\beta=0$) the bremsstrahlung amplitude would not contribute and the electro-Primakoff effect dominates this type of axion emission process. The photoproduction through the Primakoff effect as calculated in Sec. III C, however, would then dominate the axion emission for all temperatures and densities of interest. Therefore the electro-Primakoff effect can always be neglected.

D. Bremsstrahlung from electron-electron collisions

We turn now to bremsstrahlung from electron-electron collisions

$$e^- + e^- \rightarrow e^- + e^- + a . \tag{47}$$

The relevant Feynman graphs are those of Fig. 3, plus three more pairs of graphs which are obtained by exchange of the electron labels ($1\leftrightarrow2, 3\leftrightarrow4$), ($1\leftrightarrow2$), and ($3\leftrightarrow4$).

It can be shown in a rather laborious calculation that, to second order in the small electron velocities $\beta_1, \ldots, \beta_4$, interference occurs only between the graphs shown in Fig. 3 and between the members of the other three pairs of graphs. Each pair of graphs then gives the same result (41) as in the case of electron-nucleus scattering with the following obvious modifications.

(a) $Z^2 \rightarrow 1$.

(b) $n_e n_z \rightarrow n_e^2/2$, where the factor $\frac{1}{2}$ compensates for the overcounting of ingoing electrons.

(c) An extra factor $\frac{1}{2}$ accounts for the reduction of phase space due to the identity of the outgoing electrons.

(d) The initial velocity β_i refers to the c.m. frame of the initial electrons: $\boldsymbol{\beta}_i \equiv \boldsymbol{\beta}_1 = -\boldsymbol{\beta}_2$.

(e) $\boldsymbol{\beta}_f \equiv \boldsymbol{\beta}_3$ and because of momentum conservation $\boldsymbol{\beta}_4 = -\boldsymbol{\beta}_f - \omega_a \boldsymbol{\beta}_a$. Since from energy conservation $\omega_a = O(\beta^2)$, we have to lowest order $\boldsymbol{\beta}_4 = -\boldsymbol{\beta}_f$. This means that the contribution of the axion to the momentum balance is neglected.

(f) Energy conservation yields $\omega_a = \frac{1}{2}(\beta_1^2 + \beta_2^2 - \beta_3^2 - \beta_4^2) = (\beta_i^2 - \beta_f^2)$. This is twice as much as in the case of electron-nucleus scattering, and since ω_a appears as a square, we gain a factor of 4.

Collecting all these factors and inserting a factor of 4 for the four pairs of Feynman graphs contributing to this process we find

$$\epsilon_{ee}^{*}(\beta_i) = \frac{32}{135} \frac{\alpha^2 \alpha_{aee}}{m_e} n_e^2 \beta_i^5 \left[1 - \frac{15}{2} \left[\frac{\kappa}{m_e \beta_i} \right]^2 \right] . \qquad (48)$$

Averaging over the velocity distribution of both initial electrons considerably reduces this expression because, on average, a large fraction of the energy of the two electrons is "wasted" for the c.m. motion. Then we find for the thermally averaged energy-loss rate

$$\epsilon_{ee}^{*}(T) = \frac{128}{45} \frac{1}{\sqrt{\pi}} \frac{\alpha^2 \alpha_{aee}}{m_e} n_e^2 \left[\frac{T}{m_e} \right]^{5/2} \left[1 - \frac{5}{2} \frac{\kappa^2}{m_e T} \right] . \qquad (49)$$

Considering a plasma of pure hydrogen and disregarding the terms proportional to κ^2, the contribution of electron-electron collisions is $1/\sqrt{2} = 71\%$ of that of electron-proton collisions.

For electron-electron collisions we expect a similar relation to hold between bremsstrahlung and electro-Primakoff effect as in the case of electron-nucleus collisions. Therefore we neglect the electro-Primakoff process.

Translating our result to an emission rate per unit mass we find

$$\epsilon_{ee}(\rho, T, X_{\mathrm{H}}) \approx \left[\frac{68}{45} \left[\frac{2}{\pi} \right]^{1/2} \frac{\alpha^2 \alpha_{aee}}{m_e} \frac{\rho}{m_u^2} \left[\frac{T}{m_e} \right]^{5/2} (1 + X_{\mathrm{H}}) \right] \frac{1 + X_{\mathrm{H}}}{2\sqrt{2}} \left[1 - \frac{5\pi\alpha\rho(3 + X_{\mathrm{H}})}{T^2 m_e m_u} \right]$$

$$= [0.150 \text{ erg g}^{-1}\text{sec}^{-1} T_7^{2.5} \rho_2 (1 + X_{\mathrm{H}})(\alpha_{aee}/1.60 \times 10^{-23})] \frac{1 + X_{\mathrm{H}}}{2\sqrt{2}} [1 - 0.140(3 + X_{\mathrm{H}})\rho_2 T_7^{-2}] , \qquad (50)$$

where the term in square brackets is the same factor as in the case of electron-nucleus scattering Eq. (43). For invisible axions this is for conditions of the solar center $\epsilon_{ee} = 0.46 \text{ erg g}^{-1}\text{sec}^{-1}(1 - 0.30)(2\cos^2\beta)^2$. In this case Debye-Hückel shielding amounts to a 30% reduction of the rate, twice as much as in the case of electron-nucleus scattering.

The energy spectrum of the axions is now concentrated at even lower energies than in the case of electron-nucleus scattering. For the same conditions the maximum occurs at $E_a/T \approx 0.45$ and the average energy is $\langle E_a/T \rangle \approx 0.90$.

V. AXION EMISSION FROM STARS

A. The Sun

We may now use our results to calculate the axion emission from stars and begin with the star known best to us, i.e., the Sun. As long as axion emission is small compared with nuclear energy generation we may use a standard model of the Sun obtained from evolutionary calculations[19] in order to compute the axion luminosity of this star. Using our Eqs. (33), (36), (43), and (50) and integrating over the Sun we find for the solar axion luminosity in units of the solar photon luminosity $L_{\odot} = 3.85 \times 10^{33}$ erg sec^{-1}

$$L_P / L_{\odot} = 3.6 \times 10^{-3} (\alpha_{a\gamma\gamma}/4.37 \times 10^{-28}) , \qquad (51a)$$

$$(L_C + L_{eZ} + L_{ee})/L_{\odot} = (0.27 + 0.50 + 0.24) \times 9.54 \times 10^{-2} \times (\alpha_{aee}/1.60 \times 10^{-23}) , \qquad (51b)$$

where L_P refers to the Primakoff process, L_C to the Compton process, L_{eZ} to bremsstrahlung from electron-nucleus collisions, and L_{ee} to bremsstrahlung from electron-electron collisions.

If this axion flux is to be measured in a Sikivie-type experiment[3] it should be noted that the axion spectrum from the various processes is peaked at rather different energies. For KSVZ-type axions, where only the Primakoff production mechanism is relevant, the peak would be at around $E_a \approx 4$ keV. For DFS-type axions, where the bremsstrahlung processes dominate, the peak would be at around 0.8 keV.

The results Eq. (51a) and (51b) may now be used to constrain the interaction parameters $\alpha_{a\gamma\gamma}$ and α_{aee} by requiring that the axion luminosity of the Sun should not exceed its photon luminosity. This seemingly arbitrary requirement can be justified by noting that in the standard model of the Sun[19] the central abundance of hydrogen has decreased to less than half of its primordial value (from about 75% by mass to about 35%). Therefore the Sun has completed about half of its hydrogen burning phase. Therefore any energy-loss mechanism which rivals the solar photon luminosity would render the Sun younger than its assumed age of 4.5×10^9 yr. This number, however, is a minimum age indicated by the observation of old meteorites in the solar system.[19] This indicates that any unknown energy-loss mechanism of the Sun is constrained by its photon luminosity.

In the case of the DFS axion the Primakoff rate is a negligible 4% fraction of the other emission processes while for the KSVZ axion only the Primakoff rate contributes. This motivates us to consider the limits from Eqs. (51a) and (51b) separately. If for some hypothetical particle the coupling strengths α_{aee} and $\alpha_{a\gamma\gamma}$ were such that both types of emission processes would contribute on a similar level, Eqs. (51a) and (51b) would have to be summed and the following bounds would be slightly improved.

From Eq. (51a) we find the general result

$$\alpha_{a\gamma\gamma}<1.2\times10^{-25} \quad \text{or} \quad G<2.4\times10^{-18}\ \text{eV}^{-1} \tag{52}$$

which translates for the KSVZ axion into

$$\begin{aligned} &m_a<17\ \text{eV},\quad \tau_a>4.9\times10^{18}\ \text{sec}\ ,\\ &v/N>0.7\times10^{6}\ \text{GeV}\ . \end{aligned} \tag{53}$$

The limit on the mass is about an order of magnitude weaker than it would be without Debye-Hückel screening. The lifetime limit is reduced by about 4 orders of magnitude and is now not much larger than the age of the Universe. Some of the axions would decay on their way from the solar surface to Earth and therefore contribute to the solar x-ray luminosity. This effect, however, would not be observable because the x-ray background from the solar corona is several orders of magnitude stronger than this expected photon flux.

In the case of Eq. (51b) the dominant emission process is bremsstrahlung (73%) followed by the Compton process (27%). We find the general limit

$$\alpha_{aee}<1.7\times10^{-22} \quad \text{or} \quad g<4.6\times10^{-11}\ . \tag{54}$$

This translates for the DFS axion into

$$\begin{aligned} &m_a<3.2\ \ \text{eV}/2\cos^2\beta\ ,\\ &\tau_a>1.8\times10^{22}\ \text{sec}(2\cos^2\beta)^5\ ,\\ &v/2\cos^2\beta>1\times10^{7}\ \text{GeV}\ . \end{aligned} \tag{55}$$

These results are virtually identical to those of FWY. It should be noted, however, that this is a numerical accident because the present results depend dominantly on the bremsstrahlung processes while the results of FWY depend dominantly on the Primakoff process which is now suppressed by the Debye-Hückel effect.

The requirement that the solar axion luminosity should not exceed its photon luminosity is somewhat arguable because the nominal nuclear energy resources of the Sun would suffice to maintain its present photon luminosity for more than ten times its present age. Therefore one might expect that it could be possible to construct a solar model incorporating axion losses even larger than the presently assumed nuclear energy generation rate without running into conflicts with the observed solar age and photon luminosity.

FWY have argued, however, that this was not possible and that the above requirement yielded a firm, conservative bound. Their argument is based on the observation that the temperature dependence of nuclear energy generation for the dominant solar *pp* chains is rather soft, viz., roughly proportional to T^4. Therefore the *effective* nuclear energy generation rate which is obtained by subtracting the axion losses is actually *decreasing* with temperature if the axion losses are on the same order as nuclear energy generation and if the dominant axion-loss mechanism has a temperature dependence steeper than approximately T^4. Then an increase of the presently assumed solar central temperature in order to compensate for axion losses would decrease the energy available for the solar photon luminosity and the temperature would have to be raised so much until the CNO cycle with a much steeper temperature dependence dominates. Then the Sun would burn faster than allowed by its observed age.

This argument is applicable to the case when the Primakoff process dominates, with or without Debye-Hückel screening, and this process was thought to dominate for both KSVZ- and DFS-type axions in the discussion of FWY. We have shown, however, that for DFS-type axions the dominant axion-loss mechanism is bremsstrahlung with its soft $T^{2.5}$ dependence. Therefore we expect that in this case axion losses on the order of the solar photon luminosity could be accommodated in a stable solar model.

We mention in passing that a measurement of the low-energy solar neutrino flux such as the proposed solar gallium experiment would directly measure the total rate of solar nuclear energy generation and would thus much more reliably constrain axion parameters than is possible from the solar photon luminosity.

B. Helium-burning stars

Several authors[7,9–11] have constrained axion parameters by requiring that the energy loss of helium-burning stars (red giants) with $\rho=10^4\ \text{g cm}^{-3}$ and $T=10^8$ K be less than the nuclear energy generation rate which was taken to be 100 $\text{erg g}^{-1}\text{sec}^{-1}$. This argument cannot be made as precise as in the case of the Sun and the resulting bounds are rather model dependent. For a discussion of these matters we refer to the work of previous authors.[7,9–11]

We merely follow this argument and begin by writing our energy-loss rates in terms of $\rho_4\equiv\rho/10^4\ \text{g cm}^{-3}$ and $T_8\equiv10^8$ K for a chemical composition of pure helium:

$$\epsilon_P=1.83\times10^{2}\ \text{erg g}^{-1}\text{sec}^{-1}T_8{}^4\frac{f(2.5\rho_4T_8{}^{-3})}{1.23}\,\frac{\alpha_{a\gamma\gamma}}{4.37\times10^{-28}}\ , \tag{56a}$$

$$\epsilon_C = 2.67\times 10^4 \text{ erg g}^{-1}\text{sec}^{-1} T_8{}^6(\alpha_{aee}/1.60\times 10^{-23}) , \quad (56b)$$

$$\epsilon_b = 0.47\times 10^4 \text{ erg g}^{-1}\text{sec}^{-1} T_8{}^{2.5}\rho_4(1.35-0.35\rho_4 T_8{}^{-2})(\alpha_{aee}/1.60\times 10^{-23}) . \quad (56c)$$

We remark that bremsstrahlung (ϵ_b) still contributes significantly although the Compton process now dominates. Applying the above requirement separately to the two cases yields for KSVZ-type axions

$$\alpha_{a\gamma\gamma} < 2.4\times 10^{-28} \text{ or } G < 1.1\times 10^{-19} \text{ eV}^{-1} \quad (57)$$

which translates into

$$m_a < 0.7 \text{ eV} ,$$
$$\tau_a > 2.9\times 10^{25} \text{ sec} , \quad (58)$$
$$v/N > 2\times 10^7 \text{ GeV} .$$

The limit on $\alpha_{a\gamma\gamma}$ is about 2 orders of magnitude less restrictive than in the case when Debye-Hückel screening is not taken into account. The mass limit is 1 order of magnitude less restrictive.

For DFS-type axions the limit is

$$\alpha_{aee} < 5.1\times 10^{-26} \text{ or } g < 8.0\times 10^{-13} . \quad (59)$$

This translates into

$$m_a < 0.06 \text{ eV}/2\cos^2\beta ,$$
$$\tau_a > 1.1\times 10^{31} \text{ sec}(2\cos^2\beta)^5 , \quad (60)$$
$$v/2\cos^2\beta > 0.6\times 10^9 \text{ GeV} .$$

This result is in basic agreement with the relevant discussion of FWY because in their work as well as in ours the emission rate for DFS axions depends dominantly on the Compton process.

We mention that FWY have shown that for the region of temperature and density presently under discussion the Compton rate must be slightly corrected for relativistic and electron degeneracy effects. The correction factor is about 0.7 for $\rho = 10^4$ g cm^{-3} and $T = 10^8$ K. Applying this correction would slightly relax the above bounds.

VI. SUMMARY

We have calculated the energy loss from a nonrelativistic, nondegenerate plasma due to the emission of pseudoscalar particles which are light compared with the temperature of the plasma. If these particles couple to electrons they are emitted in Compton-type photoproduction and in bremsstrahlung processes (electron-nucleus and electron-electron collisions). The relevant emission rates are given by Eq. (36) for the Compton process and by Eqs. (43) and (50) for the bremsstrahlung processes. If these particles couple to photons they are emitted through a Primakoff photoproduction process. In this case Debye-Hückel screening of electric charges in a plasma is the dominant effect which cuts off the Primakoff cross section which would be infinite for a vanishing axion mass and vanishing plasma frequency. The emission rate including the Debye-Hückel effect is given in Eq. (33).

We have applied these results to the Sun and low-luminosity red giants and have required that the axionic energy loss of these stars be less than their photon luminosity. In the case of the Sun this requirement translates into the limits

$$\alpha_{a\gamma\gamma} > 1.2\times 10^{-25} \text{ and } \alpha_{aee} < 1.7\times 10^{-22} . \quad (61)$$

The limit on the photon coupling derived from the Primakoff rate is about 2 orders of magnitude less restrictive than was indicated by the results of previous authors which did not take into account Debye-Hückel screening.

Considering specifically invisible axions of KSVZ type (mainly photon coupling) and of DFS type (mainly electron coupling) allows to translate the bounds on the coupling strengths into bounds on the mass and the lifetime of these particles and on the scale v of breaking of the Peccei-Quinn symmetry. For the case of the Sun, which yields the most conservative bounds, these limits are given by Eqs. (53) and (55).

Note added in proof. After this work was completed I became aware that bounds on DFS axions can also be derived from white dwarf cooling times. This is analogous to the bounds from neutron stars of Ref. 14. The dominant emission process is then bremsstrahlung from electron-nucleus collisions. Since the relevant considerations differ substantially from the present discussion, they will appear as a separate article [Phys. Lett. B (to be published)]. The results are numerically similar to Eq. (60).

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