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Time variation of fundamental constants, primordial nucleosynthesis, and the size of extra dimensions

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In theories with extra dimensions, the dependence of fundamental constants on the volume of the compact space allows one to use primordial nucleosynthesis to probe the structure of compact dimensions during the first few minutes after the big bang. Requiring the yield of primordial ^{4}He to be within acceptable limits, we find that in ten-dimensional superstring models the size of the extra dimensions during primordial nucleosynthesis must have been within 0.5% of their current value, while in Kaluza-Klein models the extra dimensions must have been within 1% of their current value.

A remarkable feature of fundamental constants is that they are, in fact, constant. At present there is no evidence for variability of fundmental constants, although it must be noted that all limits are implicitly or explicitly model dependent. Almost all discussions of variability have considered only time dependence, and have further assumed that the scale for the time change is set by the cosmological time, ${H_0}^{-1}$, where H_0 is the Hubble constant (${H_0}^{-1}=9.8\times 10^9 h^{-1}$ yr; $H_0=100h$ km sec^{-1}Mpc^{-1}). It is also generally assumed that the variation of constants is a power law in cosmological time, which is not generally true in Kaluza-Klein theories. If the relevant time scale is instead the Planck time ($t_{\rm Pl}={G_N}^{1/2}=5.39\times 10^{-44}$ sec) it is easy to imagine large amplitude oscillations in constants, with the observed values the result of averages over many oscillations. A further model dependence is the assumption that variations of different constants do not conspire to cancel in consideration of any physical effect. For instance, the absorption spectra from distant quasars depends upon $|\alpha^2 m_e/m_p|$; changes in α may conspire to cancel changes in m_e/m_p. Limits obtained by several different methods will depend upon different combinations of constants, and lessen the likelihood of such cosmic conspiracies. It is therefore reasonable to consider variations in the constants to be independent.

In Table I we list some limits on $|\dot{\alpha}/\alpha|$, assuming changes in α are independent of changes in other constants. Although the best limit on $|\dot{\alpha}/\alpha|$ comes from the shortest "look-back" time, long look-back times are relevant if $|\dot{\alpha}/\alpha|$ does not follow a power-law dependence upon cosmological time. It is useful to know how soon after the big bang the fundamental constants had essentially the values they do today. The earliest reliable limit comes from primordial nucleosynthesis, or about three minutes after the big bang.

The possible variation of fundamental constants is of particular interest due to the recent work in theories with extra dimensions. In such theories the truly fundamental constants are defined in $4+D$ dimensions, and the observed constants in the four-dimensional world are the result of dimensional reduction of D compact dimensions. In such models the observed fundamental constants depend upon the volume (or radii) of the compact D space, and any variation in the physical size of the internal space

TABLE I. Constraints on the time variation of the fine-structure constant.

$\lvert\dot{\alpha}/\alpha\rvert$	Method	$\Delta\tau$[a]	Ref.
5×10^{-15} yr^{-1}	^{187}Re/^{187}Os	5×10^{9} yr	1
1×10^{-17} yr^{-1}	Oklo reactor	1.8×10^{9} yr	2
$13\times10^{-13}h$ yr^{-1}	Radio galaxies	$2\times10^{9}h^{-1}$ yr	3
$2\times10^{-14}h$ yr^{-1}	QSO[b]	$5\times10^{9}h^{-1}$ yr	4
$15\times10^{-15}h$ yr^{-1}	Primordial nucleosynthesis	$6.6\times10^{9}h^{-1}$ yr	This work

[a]$\Delta\tau$ is the look-back time. For the cosmological events we assumed an $\Omega=1$ cosmology for which $\Delta\tau=t_0[1-(1+z)^{-3/2}]$ and $t_0=2/3H_0\simeq6.6\times10^{9}h^{-1}$ yr.

[b]The QSO data actually measures $\Delta[\ln(\alpha^2 g_p m_e/m_p)]$ which we take as $\sim2\Delta\ln\alpha$ (see text for further discussion).

should result in a variation of α, G_N, G_F, ..., etc.[5] For simplicity we will assume that the volume of the internal space is determined by a single radius $V_D\propto R^D$. This radius represents a mean radius of the internal space. The observed constancy of fundamental constants is then related to the constant size of the extra dimensions. Static cosmological solutions are in general difficult to find, and it is quite reasonable to imagine cosmological models with the extra dimensions contracting, expanding, or oscillating.[6] We find that by the time of primordial nucleosynthesis any expansion or contraction of the extra dimensions must have been damped to give the extra dimensions the size they have today to an accuracy of better than 1%.

The dependence of constants on the radius of the extra dimensions for several models is shown in Table II. In Kaluza-Klein theories, gauge symmetries arise from isometries of the extra dimensions, while in superstring theories the gauge symmetries are part of the fundamental theory. The different R_D dependence in Table II is a reflection of this difference in the origin of the gauge symmetries. The Fermi constant G_F is given in terms of the $SU(2)_L$ gauge coupling constant g and the mass of the W boson M_W: $G_F=g^2/8M_W{}^2$. The value of M_W is determined by the vacuum expectation value of the Higgs field responsible for $SU(2)_L$ breaking, which in turn depends upon parameters in the scalar sector which should change upon any change of the extra dimensions. Theories with extra dimensions are no exception to the rule that the Higgs sector is the least understood sector, and hence we will not consider changes in G_F due to variation of M_W, and we will assume only $\delta G_F\propto\delta g^2$.

TABLE II. Variation of fundamental constants with changes in compactified geometry.

Theory	α/α_0	G_N/G_N^0	G_F/G_F^0[a]
Kaluza-Klein (D compact dimensions)	$(R/R_0)^{-2}$	$(R/R_0)^{-D}$	$(R/R_0)^{-2}$
Superstrings ($D=10$)	$(R/R_0)^{-6}$	$(R/R_0)^{-6}$	$(R/R_0)^{-6}$

[a]Ignores possible changes in M_W.

To study the effect of changing fundamental constants, we have used the standard model of primordial nucleosynthesis to calculate the dependence of primordial ^{4}He production upon G_N, G_F, and Q, where Q is the neutron-proton mass difference:

$$Q=M_n-M_p\ . \tag{1}$$

The effect of independent variations of G_N, G_F, and Q is shown in Fig. 1. The amount of ^{4}He produced in the big bang is largely determined by the neutron-proton ratio at the freeze-out of $n\leftrightarrow p$ reactions:[7] Y_p = primordial ^{4}He by mass $\simeq2(n/p)_f/[1+(n/p)_f]$, $(n/p)_f=\exp(-Q/T_f)$, where a subscript f denotes the value at freeze-out. The temperature at freeze-out T_f is determined by the competition between the expansion rate of the Universe $[H=\dot{R}/R\propto(G_N\rho)^{1/2}]$ and the weak-interaction rate $(\Gamma_{wk}\propto G_F{}^2)$. The equality of the two rates defines the freeze-out temperature T_f. Increasing G_N results in an increase in the expansion rate which allows the weak interactions to freeze-out earlier and leads to an increase in T_f, hence an increase in primordial ^{4}He. Since the Fermi constant G_F is directly proportional to the weak-interaction rate, decreasing G_F will cause weak interactions to freeze-out at a higher temperature, again increasing primordial ^{4}He production. For a fixed freeze-out temperature, increasing the neutron-proton mass difference leads to an exponential decrease in the initial neutron-to-proton ratio and thus a decrease in primordial ^{4}He.

From Fig. 1 it is seen that Y_p is most sensitive to changes in Q. For small changes in Q, $Q=Q_0+\Delta Q$, where Q_0 is the present value (1.293 MeV), ΔY is roughly linear in ΔQ. This is because for small ΔQ ($\Delta Q<T_f$)

$$\begin{aligned}(n/p)_f&=\exp(-Q/T_f)\\&=\exp(-Q_0/T_f)\exp(-\Delta Q/T_f)\\&=(n/p)_{f0}(1-\Delta Q/T_f)\ ,\end{aligned} \tag{2}$$

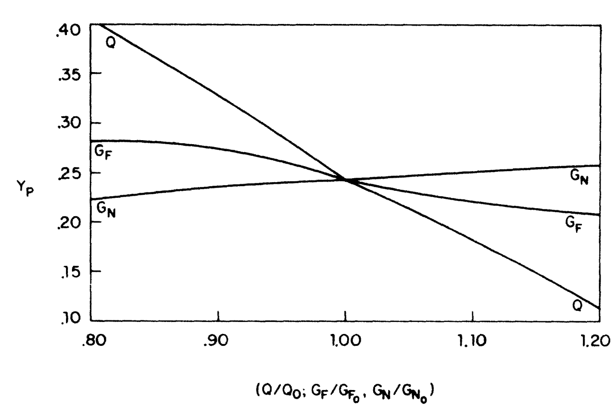

FIG. 1. The primordial ^{4}He mass fraction with independent changes in G_N, G_F, and Q. A zero subscript denotes the present value.

where $(n/p)_{f0}$ is the freeze-out value of n/p for $Q=Q_0$. Since $(n/p)_f$ is less than one, Y_p is roughly proportional to $(n/p)_f$, and from Eq. (2), the change in $(n/p)_f$ is proportional to ΔQ. Hence the linear dependence of ΔY and ΔQ is expected.

A change in the neutron-proton mass difference should arise from any change in the fine-structure constant α, since Q should receive an electromagnetic contribution. We parametrize Q by

$$Q=\alpha Q_\alpha+\beta Q_\beta \,, \tag{3}$$

where αQ_α is the electromagnetic contribution and βQ_β accounts for any nonelectromagnetic contribution. In (3), α and β are dimensionless parameters which may vary; we assume Q_α, Q_β are constant. It is reasonable to expect $\alpha Q_\alpha=O(Q)$; i.e., the magnitude of the electromagnetic contribution is roughly the size of the total contribution. Therefore we expect

$$\frac{Q}{Q_0}=\frac{\alpha}{\alpha_0}\left[\frac{1+\beta Q_\beta/\alpha Q_\alpha}{1+\beta Q_\beta/\alpha_0 Q_\alpha}\right]\simeq\frac{\alpha}{\alpha_0}\,. \tag{4}$$

With the assumptions $\alpha/\alpha_0=Q/Q_0$ and the scaling of constants in Table II, we have calculated the primordial helium abundance as a function of R_D/R_0, where R_D is the radius of the extra dimensions[8] during primordial nucleosynthesis and R_0 is the present value. In Fig. 2 we give the results for three models—a superstring model and two Kaluza-Klein models with 2 and 7 extra dimensions. Also shown Fig. 2 is the observationally restricted region for the primordial helium mass fraction, $Y_p=0.24\pm0.01$.[9] If we require the ^{2}H and ^{3}He abundance to be in agreement with observation, it is not possible to change the baryon-to-photon ratio to compensate for changes in α, G_F, and G_N.

For the superstring model, $Y_p=0.24\pm0.01$, only if $1.005\geq R_D/R_0\geq0.995$. The two Kaluza-Klein models result in a less stringent result $1.01\geq R_D/R_0\geq0.99$.

In conclusion, primordial nucleosynthesis provides a probe of the Universe seconds to minutes after the big bang. If the observed fundamental constants depend upon the volume of an internal space, any change in the internal space would result in a change in the fundamental constants, which would affect the outcome of primordial nucleosynthesis. Previous limits from primordial nucleosynthesis have considered changes in G_N and G_F. We have shown that if the neutron-proton mass difference has a large electromagnetic contribution, primordial nucleosynthesis places a good limit on changes in the fine-structure constant. We have also related limits on changes in α, G_N, and G_F to limits on changes of the volume of the internal space.

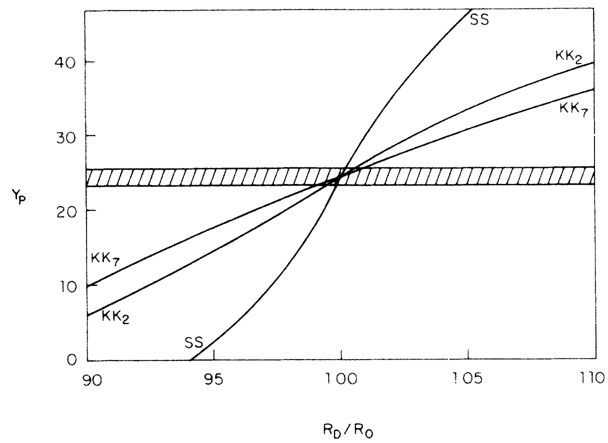

FIG. 2. The primordial ^{4}He mass fraction as a function of R_D/R_0 assuming G_N, G_F, and Q depend upon R_D/R_0 as in Table II. The three curves are for a superstring model ($D=6$) and two Kaluza-Klein models ($D=2$ and 7). The hatched region represents the observationally restricted primordial ^{4}He mass fraction, $Y_p=0.24\pm0.01$.

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[7]See, e.g., S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 15.

[8]In Kaluza-Klein models gauge couplings depend upon different radii, and in principle, it is possible to distort the space to give different changes in different gauge couplings. For simplicity, we will assume all radii are the same.

[9]See, e.g., Y. Yang, M. S. Turner, G. Steigman, D. N. Schramm, and K. A. Olive, Astrophys. J. **281**, 493 (1984).