Isospin as a spontaneously broken symmetry of the electroweak interactions

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(Received 5 September 1985)

We consider an electroweak model based on the gauge group $SU(2)_L \times U(1)_Y \times U(1)_X \times D$ with isospin as a spontaneously broken symmetry, and D a discrete symmetry which interchanges the two U(1)'s. The new physics is characterized by the vacuum expectation value of a Higgs triplet with Majorana couplings to the leptons, and an extra heavier neutral vector boson. The model is compatible with a global $SU(2)_L \times SU(2)_R$ symmetry which is extended to the Yukawa and Majorana sectors of the theory. The model also suggests the existence of a triplet of colored scalar particles with Majorana couplings to the quarks, which generate the u-d quark mass difference by radiative corrections in a strong-coupling regime. Such colored scalar particles would be observed in the $p\bar{p}$ collisions as dijets with large transverse missing energy, and the color singlets as like-sign isolated dimuon events. A dijet can simulate a monojet if one of the jets is hidden in the hadronic background.

I. INTRODUCTION

It was thought long ago that electromagnetic self-energies are at the origin of mass differences within isotopic multiplets and that the electron mass is of electromagnetic origin. However, all the attempts to actually compute a mass difference such as the neutron-proton mass splitting[1] have failed. Nowadays, it is no longer possible to deal with electromagnetism as an isolated phenomenon and one has to consider contributions from other interactions. At an energy scale of 100 GeV, for example, the weak contributions are as important as the electromagnetic effects, and this has suggested[2] an interesting mechanism for the cancellation of the electromagnetic self-energies by the weak interactions.

In renormalizable gauge models with spontaneously broken symmetries the only counterterms in the Lagrangian are those allowed by the gauge structure of the theory. Thus, if a mass difference, or a mass, vanishes to zeroth order of spontaneous symmetry breaking (SSB), the sum of the higher-order contributions is finite and calculable to each order of perturbation theory, as a consequence of the renormalizability of the theory.[2] This is a most interesting possibility, but no convincing model has been found to implement this idea in the framework of local gauge theories.[3,4] In the Glashow-Weinberg-Salam (GWS) model of the electroweak interactions,[5] based on the gauge group $SU(2)_L \times U(1)_Y$, each quark or lepton mass is generated by an independent Yukawa coupling with the scalar particles of the Higgs sector, and no relation among the particle masses exists. Infinite counterterms for each mass are introduced to compensate for the ultraviolet behavior of the self-energies. Quark and lepton masses appear as free parameters in the model. Thus, when computing a mass difference the electromagnetic divergence is not canceled by the weak interactions. Could this be symptomatic of a shortcoming of the standard model? Not necessarily. It would have been gratifying, however, to find a scheme for the computation of mass relations in the framework of a unified model of the weak and electromagnetic interactions.

How does one impose additional constraints in the theory in order to allow for the calculability of mass relations? The Higgs potential in the standard model is invariant under the global symmetry $O(4) \sim SU(2)_L \times SU(2)_R$. When the scalar field acquires a vacuum expectation value (VEV), O(4) spontaneously breaks down to $O(3) \sim SU(2)_{L+R}$ which is isospin.[6] Isospin is conserved by the self-interactions of the scalar bosons to all orders, and this guarantees that the zeroth-order relation $M_W = M_Z \cos\theta_W$ is not spoiled by higher-order effects. However, there is strong isospin violation in the fermion mass matrix, which is not understood within the standard model. Indeed, it is generally accepted that isospin is an accidental symmetry due to the smallness of the u and d quark masses with respect to the hadronic scale, rather than an exact symmetry.

In the model presented here, we extend the isospin symmetry to the whole theory by introducing a discrete symmetry, or permutation symmetry which interchanges the u and d quarks in the quark sector, and the neutrino and the electron in the leptonic sector. Furthermore, we require that this transformation leave unchanged the local SU(2). When applied to a left doublet the discrete symmetry acts as an element of SU(2). This is not the case for the hypercharge sector of the theory, which is not invariant under the discrete symmetry. We are thus led to introduce a new hypercharge, labeled here as the X hypercharge. It turns out that the sum of the X and Y quantum numbers corresponds to $B-L$ (baryon minus lepton), which is promoted to a local symmetry. Our model is thus based on the gauge group $SU(2)_L \times U(1)_Y \times U(1)_X \times D$, with D a discrete symmetry operation which interchanges the two U(1)'s. Models of the electroweak interaction with two massive neutral vector bosons have been studied extensively in the past,[7-21] and can be considered as descendents of the grand unified theory based on SO(10).[22,23] In our model isospin is an exact symmetry of the electroweak interaction, which is spontaneously broken by the presence of a new symmetry-breaking scale.[24] The isospin-breaking effects which are manifest at lower energies are sensitive to the spontaneous symmetry-violating scale and are calculable.

In addition to the usual Higgs doublet of the standard model, we introduce a singlet Higgs field which is a $SU(2)_L$ and Y hypercharge singlet, and whose VEV breaks $B-L$. The VEV of this new Higgs field contributes simultaneously to the mass of the new U(1) gauge boson and to the Majorana mass term of the neutrino. The smallness of the neutrino mass is thus naturally explained. The new singlet Higgs field is the color-singlet member of a larger family which includes a triplet of colored scalar fields with Majorana couplings to quarks. The large isospin violation in the quark mass matrix is generated by radiative corrections from the colored scalar particles in a strong-coupling limit. The model is compatible with the invariance of the Higgs and Majorana sectors of the theory under the global $SU(2)_L \times SU(2)_R$ symmetry.

The discrete symmetry is trivially realized in the local group $SU(2)_L \times SU(2)_R$, but this entails dealing with a rather complex Higgs sector. The model presented here is a minimal extension of $SU(2)_L \times U(1)_Y$. The only natural way to introduce a new mass scale in this model is through Majorana couplings since another Higgs doublet would make it difficult to keep the neutrino light.

In Sec. II we discuss the role of the discrete symmetry and some technical aspects of the construction of the model. In Sec. III we write the mass matrix of the vector bosons corresponding to the two VEV's of the model in terms of mass eigenstates, and the same is done with the mass matrix of the neutrino. In Sec. IV we discuss the new signatures in $p\bar{p}$ collisions at very high energy in terms of the scalar bosons incorporated into the model. The signatures correspond to dijets or dimuons, according to the type of scalar boson involved in the process. In Sec. V we perform a specific calculation to illustrate the use of the discrete symmetry. We compute the u-d quark mass difference, which is generated in this model by radiative corrections by colored scalar particles in a strong-coupling regime. Some final remarks and comments are given in the Conclusion.

II. THE MODEL

The divergent terms in the calculation of mass differences cancel due to the natural mass relation imposed by the discrete symmetry that interchanges neutrinos and electrons in the leptonic sector, and up and down quarks in the quark sector. An immediate problem arises here, since this would imply that the masses of the neutrino and the electron are degenerate. This problem is circumvented with the introduction of a new mass scale, as explained above.

We require that under the discrete symmetry the Lagrangian be invariant and the charge generators transform among themselves, so that the theory be closed. Under the discrete symmetry D the right fields transform according to

$$\nu_R \to e^{i\alpha} e_R^-, \quad e_R^- \to e^{i\beta}\nu_R , \tag{2.1}$$

for the leptons, and

$$u_R \to e^{i\gamma} d_R, \quad d_R \to e^{i\delta} u_R , \tag{2.2}$$

for the quarks. Similar relations hold for the left fermions but with different phases since the two chiral degrees of freedom are independent, although the phases within a doublet are not independent. Since the left fields are in $SU(2)_L$ doublets, the discrete symmetry enters as a global isospin rotation, that is, a particular SU(2) element. Thus, the left sector of the theory is invariant under D.

A minimal extension of the GWS model which contains the discrete symmetry requires a right neutrino and a new hypercharge generator (X hypercharge), since the usual Y hypercharge generator is not invariant under D. Our model is thus based on the gauge group $SU(2)_L \times U(1)_Y \times U(1)_X \times D$.

The generators of the X and Y Abelian hypercharges are

$$\begin{aligned} Q_X &= \int d^3x [-\tfrac{1}{2}(\nu_L^\dagger \nu_L + e_L^{-\dagger} e_L^-) - \nu_R^\dagger \nu_R] , \\ Q_Y &= \int d^3x [-\tfrac{1}{2}(\nu_L^\dagger \nu_L + e_L^{-\dagger} e_L^-) - e_R^{-\dagger} e_R^-] , \end{aligned} \tag{2.3}$$

and the generator of the $SU(2)_L$ gauge group is

$$Q_a = \int d^3x \, L_r^\dagger \left[\frac{\tau^a}{2} \right]_{rs} L_s, \quad L = \begin{bmatrix} \nu \\ e^- \end{bmatrix}_L , \tag{2.4}$$

where $r,s = 1,2$, $a = 1,2,3$, and the τ^a are the Pauli matrices. Under D the hypercharge generators transform into each other: $Q_X \leftrightarrow Q_Y$.

The electric charge generator is $Q_{em} = Q_3 + Q_Y$, as usual. It is noteworthy that the sum of the two Abelian hypercharges corresponds to the baryon minus lepton $(B-L)$ quantum number, which becomes a generator of the theory. Thus, for the leptons

$$Q_Y + Q_X = -\int d^3x (\nu^\dagger \nu + e^{-\dagger} e^-) = Q_{B-L} . \tag{2.5}$$

The corresponding generators for the quark sector are

$$\begin{aligned} Q_X &= \int d^3x [\tfrac{1}{6}(u_L^\dagger u_L + d_L^\dagger d_L) - \tfrac{1}{3} u_R^\dagger u_R + \tfrac{2}{3} d_R^\dagger d_R] , \\ Q_Y &= \int d^3x [\tfrac{1}{6}(u_L^\dagger u_L + d_L^\dagger d_L) + \tfrac{2}{3} u_R^\dagger u_R - \tfrac{1}{3} d_R^\dagger d_R] , \end{aligned} \tag{2.6}$$

$$Q_a = \int d^3x \, q_r^\dagger \left[\frac{\tau^a}{2} \right]_{rs} q_s, \quad q = \begin{bmatrix} u \\ d \end{bmatrix}_L ,$$

where $r,s = 1,2$, $a = 1,2,3$. It follows that

$$Q_X + Q_Y = \int d^3x \, \tfrac{1}{3}(u^\dagger u + d^\dagger d) = Q_{B-L} . \tag{2.7}$$

The quantum number assignments are indicated in Table I.

Let us consider the leptonic terms of the Lagrangian density

$$\begin{aligned} \mathcal{L}_l = \bar{L} \left[i\partial\!\!\!/ - g\mathbf{W}\cdot\frac{\tau}{2} - \frac{g'}{2} B\!\!\!/ - \frac{g'}{2} B\!\!\!/' \right] L \\ + \bar{e}_R^-(i\partial\!\!\!/ + g' B\!\!\!/) e_R + \bar{\nu}_R (i\partial\!\!\!/ + g' B\!\!\!/') \nu_R . \end{aligned} \tag{2.8}$$

TABLE I. Quantum numbers of $SU(2)\times U(1)\times U(1)\times D$.

Sector	Particle	I_3	X	Y	Q_{em}	$B-L$
Leptons	$\begin{bmatrix} \nu \\ e^- \end{bmatrix}_L$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	-1
		$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
	ν_R	0	-1	0	0	-1
	e_R	0	0	-1	-1	-1
Quarks	$\begin{bmatrix} u \\ d \end{bmatrix}_L$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{3}$
		$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$	$\frac{1}{3}$
	u_R	0	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$
	d_R	0	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$
Bosons	$\begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	1	0
		$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
	χ^0	0	2	0	0	2
	χ^+	0	1	1	1	2
	χ^{++}	0	0	2	2	2
	$\Xi^{-2/3}$	0	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$
	$\Xi^{1/3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
	$\Xi^{4/3}$	0	$-\frac{2}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{2}{3}$

Under D the last two terms transform into each other, with $e_R^- \leftrightarrow \nu_R$ and $B \leftrightarrow B'$. The discrete symmetry requires that the X and Y hypercharge couplings are identical. The doublet L in the first term transforms as $L \to UL$, where

$$U=\begin{bmatrix} 0 & e^{i\alpha} \\ e^{i\beta} & 0 \end{bmatrix}, \tag{2.9}$$

is a SU(2) element provided $\alpha=\pi-\beta$. We can choose $\alpha=0$ and have $U=i\tau_2$ as a representation for the global unitary transformation. The $\mathbf{W}$ term transforms as usual under SU(2): $\mathbf{W}\cdot\tau \to U\mathbf{W}\cdot\tau U^{-1}$. Thus, this term is left invariant by D.

Let us examine the Higgs structure of our model. The terms of the Lagrangian that involve the usual Higgs doublet are

$$\mathscr{L}_\phi=\left[\left[\partial_\mu+ig\mathbf{W}_\mu\cdot\frac{\tau}{2}+i\frac{g'}{2}B_\mu-i\frac{g'}{2}B'_\mu\right]\phi\right]$$

$$-\lambda_f(\bar{L}\phi e_R+\bar{L}\tilde{\phi}\nu_R)\ , \tag{2.10}$$

where $\tilde{\phi}=i\tau^2\phi$, and the scalar self-interaction terms have been omitted. The X hypercharge assignment of the Higgs doublet follows from the above equation.

How does ϕ transform under D? From the invariance of the Yukawa interaction terms in Eq. (2.10), it follows that ϕ transforms as

$$\phi \to Ui\tau^2\phi^*\ , \tag{2.11}$$

which is an antiunitary transformation. Notice that Eq. (2.11) reduces to $\phi \to -\phi^*$ for the particular phase $\alpha=0$. What is remarkable is that the first term of Eq. (2.10) is also invariant under the transformation given by Eq. (2.11). This is not apparent since ϕ has opposite sign X and Y hypercharge quantum numbers (see Table I), and thus under D the B and B' part of the Lagrangian of Eq. (2.10) changes sign. Compare this with the first term of Eq. (2.8), where the B and B' part of the Lagrangian does not change sign under D, since the X and Y quantum number assignment is identical for left quarks and leptons, respectively. The invariance of the first term of Eq. (2.10) follows from the rules of transformation under D that we have seen so far, and $\tau\tau^2=-\tau^2\tau^*$.

As it stands, the model has three problems. The electron and neutrino masses are degenerate, and so are the masses of the two neutral vector bosons that arise in this model, which we will call Z_1 and Z_2, linear combinations of W_3, B, and B'. We also have an interacting right neutrino. However, we can solve all three problems simultaneously through a minimal extension of the Higgs sector, with the introduction of a neutral scalar boson χ^0 which is a $SU(2)_L$ singlet and whose VEV breaks $B-L$ by two units. The new field χ^0 couples inequivalently to the B and B' Abelian bosons lifting the mass degeneracy of the neutral vector bosons Z_1 and Z_2. It also couples with the right neutrino via a Majorana interaction term $\bar{\nu}_L^C\chi^0\nu_R$ to keep the physical neutrino almost massless, while the ν_R acquires a large Majorana mass. We expect the VEV of the χ^0 to be larger than the VEV of the ϕ^0 of the standard model, and it defines a new mass scale. Since $B-L$ is a generator of the theory, gauge invariance implies that the $B-L$ quantum number of the χ^0 is 2. Furthermore, it is neutral and a $SU(2)_L$ singlet, its Y hypercharge is zero, and it does not couple with the B. This is consistent with the requirements that the photon be a linear combination of the W_3 and B, and massless. The

X hypercharge assignment of χ^0 is 2.

To maintain the invariance under D of a Lagrangian with the Majorana term present, it is necessary to include also a doubly charged Higgs boson χ^{++}. These two scalar bosons with $B-L=2$ are reminiscent of a $SU(2)_R$ triplet found in the **126** representation of the grand unified model based on SO(10), which also contains a singly charged boson χ^+. We will include this third boson in our model, as seems reasonable. The Lagrangian for the new mass scale is then

$$\begin{aligned}\mathscr{L}_\chi=&[(\partial_\mu+2igB'_\mu)\chi^0]^2+[(\partial_\mu+ig'B_\mu+ig'B'_\mu)\chi^+]^2+[(\partial_\mu+2ig'B_\mu)\chi^{++}]^2-\lambda_1\bar{e}^c_L\chi^+\nu_R\\&-\lambda_1\bar{\nu}^c_L\chi^+e^-_R-\lambda_2\bar{e}^c_L\chi^{++}e^-_R-\lambda_2\bar{\nu}^c_L\chi^0\nu_R+\text{H.c.}\ .\end{aligned}\tag{2.12}$$

Let us remark that the global $SU(2)_L\times SU(2)_R$ symmetry of the Higgs potential $V(\phi,\phi^*)$ of the standard model, is extended in our model to the Yukawa interactions. Furthermore, if $\lambda_1=\lambda_2$ this symmetry is also present in the Majorana sector. This can be made evident by introducing a matrix formalism for the Higgs particles:

$$\mathscr{L}_Y=\lambda_f\bar{L}\Phi R-\lambda_1\bar{L}^c(i\tau^2)\chi R\ ,\tag{2.13}$$

where

$$\Phi=\begin{bmatrix}\phi^0 & \phi^+\\ -\phi^- & \phi^0\end{bmatrix},\quad \chi=\begin{bmatrix}\chi^+ & \sqrt{2}\chi^{++}\\ \sqrt{2}\chi^0 & \chi^+\end{bmatrix}\ .\tag{2.14}$$

III. MASS EIGENSTATES

In this section we examine in detail the symmetry breakdown of our model for the specific Higgs assignment discussed in the last section. We first find the gauge-boson masses that are the eigenvalues of the mass matrix obtained from the Lagrangian terms

$$\begin{aligned}\mathscr{L}_{M_Z}=&\left[\left[\partial_\mu+ig\mathbf{W}_\mu\cdot\frac{\tau}{2}+i\frac{g'}{2}B_\mu-i\frac{g'}{2}B'_\mu\right]\phi\right]^2\\&+[(\partial_\mu-i\,2B'_\mu)\chi^0]^2\ .\end{aligned}\tag{3.1}$$

In terms of the VEV's of the Higgs fields ϕ and χ^0,

$$\langle\phi\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\ \rho_0\end{bmatrix},\quad \langle\chi^0\rangle=\sigma_0\ ,\tag{3.2}$$

we obtain for the neutral vector bosons the matrix

$$\mu^2=M_{Z^0}{}^2\begin{bmatrix}\cos^2\theta & -\sin\theta\cos\theta & \sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin^2\theta & -\sin^2\theta\\ \sin\theta\cos\theta & -\sin^2\theta & \delta\sin^2\theta\end{bmatrix},\tag{3.3}$$

where θ is the Weinberg angle, M_{Z^0} is the mass of the Z^0 boson of the standard model,

$$M_{Z^0}=(g^2+g'^2)^{1/2}\rho_0/2\tag{3.4}$$

and

$$\delta=1+32(\sigma_0/\rho_0)^2\ .\tag{3.5}$$

Following the crafty parametrization of the secular equation of μ^2, found in Ref. 22, we can write this matrix as follows:

$$\mu^2=M_{Z^0}{}^2\begin{bmatrix}\cos^2\theta & -\sin\theta\cos\theta & \epsilon\cos\theta\cot\phi\\ -\sin\theta\cos\theta & \sin^2\theta & -\epsilon\sin\theta\cot\phi\\ \eta\cos\theta\tan\phi & -\eta\sin\theta\tan\phi & \eta-\epsilon+1\end{bmatrix},\tag{3.6}$$

with

$$\begin{aligned}&\epsilon\cot\phi=\eta\tan\phi=\sin\theta\ ,\\&\eta-\epsilon+1=\delta\sin^2\theta\ .\end{aligned}\tag{3.7}$$

It is then straightforward to find the eigenvectors of the mass matrix μ^2, which are

$$\begin{aligned}&\mathbf{V}_1=\begin{bmatrix}\cos\theta\cos\phi\\ -\sin\theta\cos\phi\\ \sin\phi\end{bmatrix},\quad \mathbf{V}_2=\begin{bmatrix}\sin\theta\\ \cos\theta\\ 0\end{bmatrix},\\&\mathbf{V}_3=\begin{bmatrix}\cos\theta\sin\phi\\ -\sin\theta\sin\phi\\ \cos\phi\end{bmatrix},\end{aligned}\tag{3.8}$$

with eigenvalues $1-\epsilon$, 0, and $1+\eta$, respectively. We identify the $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_3$ with the mass eigenstates Z_1, γ, Z_2, respectively:

$$\begin{aligned}&Z_1=W^3\cos\theta\cos\phi-B\sin\theta\sin\phi-B'\sin\phi\ ,\\&\gamma=W^3\sin\theta+B\cos\theta\ ,\\&Z_2=W^3\cos\theta\sin\phi-B\sin\theta\sin\phi+B'\cos\phi\ .\end{aligned}\tag{3.9}$$

The procedure followed to calculate the values of ϵ, η, and the new mixing angle ϕ is as follows: we eliminate ϵ and η by using (3.7), and are left with an equation for $\tan\phi$,

$$\tan^2\phi+(\delta\sin\theta-1/\sin\theta)\tan\phi-1=0\ .\tag{3.10}$$

Setting $\sin\theta$ to its accepted value, $\sin\theta=0.47$, we approximate

$$\delta\sin\theta-1/\sin\theta\simeq 15.04(\sigma_0/\rho_0)^2\ ,\tag{3.11}$$

which is valid to one part in 50. To this same precision we obtain the solution of the quadratic equation:

$$\tan\phi=\frac{1}{15.04}(\rho_0/\sigma_0)^2\ .\tag{3.12}$$

(We chose the solution of the equation that makes ϵ small.) From (3.12) and assuming that $\rho_0\leq\sigma_0$, it follows that $\phi \leq \frac{1}{15}$ rad, thus giving the Z_1 a very small B' admixture, and making Z_2 basically a B' state. The values of ϵ and η are

$$\epsilon=0.031(\rho_0/\sigma_0)^2, \quad \eta=7.07(\sigma_0/\rho_0)^2 . \tag{3.13}$$

From here we obtain the masses of the neutral vector bosons:

$$\begin{aligned} M_{Z_1}&=M_{Z^0}\sqrt{1-\epsilon}=0.98M_{Z^0}(\rho_0/\sigma_0)^2 , \\ M_{Z_2}&=M_{Z^0}\sqrt{1+\eta}=2.66M_{Z^0}(\sigma_0/\rho_0)^2 . \end{aligned} \tag{3.14}$$

Thus, the mass of the Z_1 is very close to, and that of the Z_2 is at least three times larger than, the mass of the Z^0.

We now examine the effect of the new mass scale on the right and left neutrinos. The only terms that will give mass to the neutrino, that are allowed by gauge invariance, are (2.10) and (2.12),

$$\mathcal{L}_{M_\nu}=\lambda_f\bar{L}\phi\nu_R-\lambda_2\bar{\nu}_L^c\chi^0\nu_R+\text{H.c.} . \tag{3.15}$$

The neutrino mass matrix follows[25] from this equation. Diagonalizing it one obtains two eigenstates: a heavy fermion,

$$N=\nu_R+\nu_L^c , \tag{3.16}$$

with mass $m_N=\lambda_2\sigma_0$, and a light one

$$n=\nu_L+\nu_R^c , \tag{3.17}$$

with mass $m_n={m_e}^2/2m_N$.

If we set $m_n<10$ eV, we obtain a lower bound for m_N: $m_N>5$ GeV.

IV. SIGNATURES IN $p\bar{p}$ COLLISIONS

The χ bosons we have incorporated into the model couple to leptons with Majorana couplings, since they have $B-L=2$. We describe here the signatures that the scalar bosons χ and the related colored bosons Ξ, that we introduce below, will have in $p\bar{p}$ collision experiments at high energy, such as the one performed at CERN.

The χ bosons have a very well-defined signature which comes from the process $q\bar{q}\to Z^0\chi^{++}\chi^{--}Z^0(\chi^{\pm\pm}\to\mu^\pm\mu^\pm)$. It is one or two pairs of like-sign isolated muons, as shown in Fig. 1. The like-sign muons at these energies behave as if massless and are therefore created going parallel to each other, when the χ disintegrates. Such events have in fact been reported by UA1.[26] The background comes from heavy-flavor decays, although it seems insufficient to explain the observed events.

The scalar bosons χ are found in the **126** irreducible representation of SO(10).[23] Under $SU(2)_L\times SU(2)_R\times SU(4)_C$ the **126** transforms as

$$\mathbf{126}=(\mathbf{1},\mathbf{1},\mathbf{6})+(\mathbf{3},\mathbf{1},\overline{\mathbf{10}})+(\mathbf{1},\mathbf{3},\mathbf{10})+(\mathbf{2},\mathbf{2},\mathbf{15}) . \tag{4.1}$$

We can further decompose $SU(4)_C$ under its maximal subgroup $SU(3)_C\times U(1)_{B-L}$. We are interested in the $(\mathbf{1},\mathbf{3},\mathbf{10})$ which decomposes as

$$(\mathbf{1},\mathbf{3},\mathbf{10})=(\mathbf{1},\mathbf{3},\mathbf{1})(2)+(\mathbf{1},\mathbf{3},\mathbf{3})(\tfrac{2}{3})+(\mathbf{1},\mathbf{3},\mathbf{6})(-\tfrac{2}{3}) \tag{4.2}$$

under the color subgroup, and where the number in

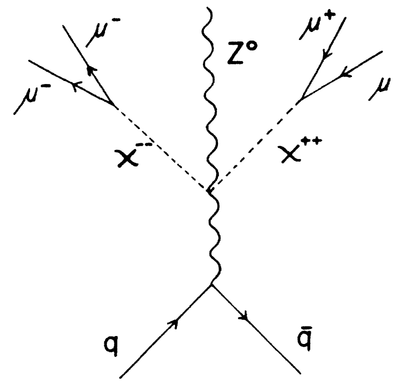

FIG. 1. Process that produces like-sign isolated dimuons.

parentheses corresponds to $B-L$. We recognize the three scalar bosons with $B-L=2$ in the $(\mathbf{1},\mathbf{3},\mathbf{1})$ triplet, and also find a larger family of colored scalar bosons with $B-L=\frac{2}{3}$ which has Majorana couplings with the quarks. These colored scalar representations are a **6** and a **3**, but since they involve the same kind of coupling, and it is likely that the **3** is lighter, we will simply ignore the **6**. The quantum numbers of the **3**, which we call Ξ are listed in Table I.

The symmetries involved, including the discrete one, dictate the following Lagrangian:

$$\begin{aligned} \mathcal{L}_C=&\sum_{i=1}^{3}[(\partial_\mu-igA_aT^{a*}+ig'y_iB_\mu+ig'x_iB'_\mu)\Xi_i]^2 \\ &-\lambda_3\bar{u}_L^c\Xi^{*-4/3}u_R-\lambda_3\bar{d}_L^c\Xi^{*2/3}d_R \\ &-\lambda_4\bar{d}_L^c\Xi^{*-1/3}u_R-\lambda_4\bar{u}_L^c\Xi^{*-1/3}d_R+\text{H.c.} , \end{aligned} \tag{4.3}$$

where the repeated index a indicates a sum over $a=1,\ldots,8$, and x_i and y_i are the appropriate X and Y hypercharge number for the particular Ξ_i boson. The interaction terms in this Lagrangian involve weak, strong, and Yukawa couplings in various combinations.

A very important point that has to be made is that these color bosons allow no new decay channels for the proton. This comes about from the fact that there are no leptoquark vertices, which are the ones that cause proton decay in grand unified theories. Such vertices are forbidden by color gauge invariance.

The Ξ color bosons have monojets and dijets with large missing transverse energy as a signature. These events cannot be explained within the standard model if the transverse energy is higher than a few GeV's. The reason for this is that if the transverse energy is high, the Drell-Yan process in the $p\bar{p}$ collision must involve the constituent quarks, and then the jet must be due to gluon bremsstrahlung, which cannot be the case since bremsstrahlung is longitudinal. The dijets are due to the process $q\bar{q}\to Z^0\to\Xi\bar{\Xi}Z^0(Z^0\to\nu\bar{\nu})$. (See Fig. 2.) This same process can be interpreted as a monojet if one of the Ξ's has

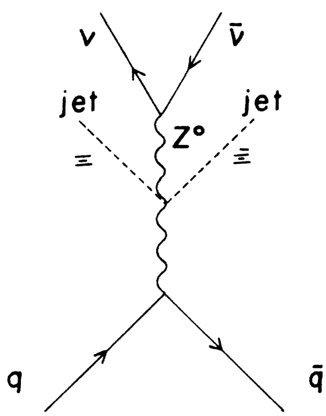

FIG. 2. Process that produces dijets (and simulates monojets), and large transverse missing energy.

low energy and remains hidden as part of the hadronic background. Our ignorance of the values of the mass and couplings of the Ξ prevents us from making a numerical prediction, but since it is clear that a four-prong process can give high transverse energies, there is no kinematical constraint here. It is possible that monojets have been seen at CERN,[27] and this has prompted several theoretical explanations for monojets.[28] If monojets have actually been observed at CERN then good statistics and a careful analysis will be necessary to clarify the issue.

V. SELF-ENERGY CALCULATION

As a specific example of the use of the discrete symmetry in the cancellation of infinities, we compute the u-d mass difference. We first consider the lowest-order radiative corrections from the gauge bosons of the theory, in terms of the mass eigenstates found in Sec. III. The Higgs triplet χ of the model with $B-L=2$ does not couple to quarks, and the discrete symmetry ensures that the tadpole and loop corrections from the Higgs doublet ϕ cancel out from the mass difference. The role of the colored scalar particles is discussed at the end of the section.

All the divergent quantities cancel as expected from the mass relation and we obtain the following expression for the mass difference renormalized at the scale $\mu^2 \ll M_Z{}^2$:

$$\begin{aligned}\Delta m(\mu^2) &= m_u(\mu^2) - m_d(\mu^2)\\ &= \frac{g'^2}{(4\pi)^2}\frac{\lambda}{2}\langle\phi_0\rangle\left\{\ln\left[\frac{M_{Z_1}{}^2}{\mu^2}\right]+\ln\left[\frac{M_{Z_2}{}^2}{\mu^2}\right]-2\sin^2\theta\left[\cos^2\phi\ln\left[\frac{M_{Z_1}{}^2}{\mu^2}\right]+\sin^2\phi\ln\left[\frac{M_{Z_2}{}^2}{\mu^2}\right]\right]\right.\\ &\qquad\left.-\cot^2\theta\sin\theta\sin\phi\cos\phi\ln(M_{Z_1}{}^2/M_{Z_2}{}^2)\right\}, \end{aligned} \tag{5.1}$$

where λ is the Yukawa coupling in the isospin limit $\lambda_u=\lambda_d=\lambda$.

If $M_{Z_1}\sim M_{Z_2}\sim M$, the above expression reduces to

$$\Delta m(\mu^2)=\frac{1}{4\pi}\alpha_{\rm em}\lambda\langle\phi_0\rangle\ln\left[\frac{M_Z{}^2}{\mu^2}\right], \tag{5.2}$$

which has the wrong sign and incorrect order of magnitude to account for the u-d mass difference, since $(m_d-m_u)/m_u\sim 1$. In the presence of strong interactions the above results remain unchanged, since we are computing the lowest-order correction to a natural zeroth-order symmetry.[29,30]

Finally, we evaluate the contribution to Δm from the colored scalar sector of the model in a strong-coupling regime, using the Dyson equation as a starting point for nonperturbative calculations. The effect of the geometric sum of bubbles in the scalar-boson propagator from its scalar self-interactions is given to lowest order by the renormalization-group scalar function $d(p^2/\mu^2,b(\mu^2))$

$$d(p^2/\mu^2,b(\mu^2))=\frac{1}{1-b\ln(-p^2/\mu^2)}, \tag{5.3}$$

where b is an effective coupling parameter, which depends on the unknown coupling of the scalar self-interactions. The above result for the function $d(p^2/\mu^2,b(\mu^2))$, is the first term of a series expansion in powers of b. We exponentiate the above result to extrapolate to very short distances (strong-coupling limit) the long-range perturbative results. Explicitly,

$$d(p^2/\mu^2,b(\mu^2))=\exp[b\ln(-p^2/\mu^2)]. \tag{5.4}$$

We have no rigorous justification for this choice, but it is a convenient ansatz, and it is also useful for analytical computation.

The discrete symmetry simplifies again the calculation of the mass differences, and the only surviving terms are those due to the $\Xi^{4/3}$ radiative corrections to the self-energy of the u quark, and the $\Xi^{-2/3}$ for the d quark. If the masses of the $\Xi^{4/3}$ and $\Xi^{2/3}$ (that we shall label M_u and M_d, respectively) were identical, we would obtain an absolute cancellation; otherwise we obtain a finite contribution to the quark mass difference. Even if the masses are equal at the tree level, there are important radiative corrections that split their values.

In the large p^2 limit, Δm is given by the renormalized Dyson equation (after performing a Feynman parametrization)

$$\Delta m(\mu^2)=\frac{m}{8\pi^4}\int_{M_u{}^2}^{M_d{}^2}dt\int_0^1 dx\int i\,d^4q\,\lambda_M(-q^2/\mu^2)\times\frac{(2-x)(1-x)}{[q^2-t(1-x)]^3}\,, \tag{5.5}$$

where $\lambda_M(p^2/\mu^2)=\lambda_3 d(p^2/\mu^2,b(\mu^2))$, and λ_3 is the Majorana coupling constant defined in Eq. (4.3). The above expression for Δm is finite for $b<1$ (for $b=0$ we obtain the usual logarithmic result) and can be expressed analytically as follows:

$$\Delta m(\mu^2)=\frac{\lambda_3{}^2 m}{16\pi^2}\frac{(2b+3)}{b(1-b^2)(b+2)}\times\Gamma(2+b)\Gamma(2-b)\frac{M_u^{2b}-M_d^{2b}}{\mu^{2b}}\,. \tag{5.6}$$

To illustrate our results, let us choose a particular value for the coupling parameter b. If we choose $b=\frac{1}{2}$ we find

$$\Delta m=\frac{\lambda_3{}^2}{10\pi}m(M_u-M_d)/\mu\,, \tag{5.7}$$

which could account for the u-d mass difference given the right values of λ_3 and M_u-M_d. We have thus translated our ignorance of the origin of the u-d mass difference to another scale.

VI. CONCLUSION

We have constructed a model based upon the requirements that it should have isospin as a good symmetry, have calculable self-energies, and be a simple extension of the GWS model. (Of course it should have this model as some kind of limit.) It has turned out that an appropriate way of achieving our purposes has been through the introduction of a discrete symmetry. We then accepted this discrete symmetry completely, and built the model around it. The result of following this logic was a model that incorporates an extra U(1) local group, and another mass scale apart from the usual one in the GWS model. It is a distinctive feature of this model (made necessary by our implicit belief in the discrete symmetry), that the new mass scale is due to the SSB of the new U(1) symmetry, and that the new leptonic mass terms in the Lagrangian have to be of the Majorana type. The GWS model is obtained in the limit of a large VEV of the χ^0 boson. Incorporating the new leptonic mass terms requires the introduction of other Majorana interaction terms that involve new particles and phenomenology.

The enforcement of the discrete symmetry in the complete theory led to some interesting self-consistent technical manipulations. In the final Lagrangian, global $SU(2)_L\times SU(2)_R$ is a symmetry of the Higgs sector, and the Majorana sector is compatible with this symmetry, as seen is Sec. II. The self-energy contribution to the mass difference of the up and down quarks was calculated, and all the terms either cancel or are negligible, except for the ones due to the colored bosons. Here the calculation was done using Dyson's equation and exponentiating the boson propagator. The result comes out to be in the right order of magnitude.

The new phenomenological predictions of the model are a new Z^0 particle at least three times as heavy as the usual one, like-sign isolated dimuon events, and dijets with large transverse missing energy. The dijets can simulate monojets if one is hidden in the low-energy hadronic background.

If isolated like-sign dimuon events continue to be observed in the laboratories, this model will gain in interest. At present the authors have no insight as to whether the discrete symmetry employed is just a useful calculational device, or if it really corresponds to something found in nature.

Note added in proof: It should be stressed that the jets resulting from the colored Higgs bosons are in fact composed of two overlapping quark jets. These jets originate in the decay of the massive scalar colored charged bosons and are thus emitted preferentially parallel. These pairs differ from the jets of the standard model which are preferentially emitted back to back in the laboratory frame.

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