Soft-pion corrections to the Skyrme soliton

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(Received 3 May 1985)

We consider soft-pion corrections to the Skyrme soliton to one-loop order. Starting from Weinberg's low-energy expansion, we construct a meaningful and consistent effective description to one loop, which embodies Skyrme's model in the classical limit. In this context the soft-pion contributions to the nucleon and Δ-isobar masses are evaluated, and their relevance to baryon dynamics discussed.

The renewed interest in effective chiral models to describe low-energy phenomena stems from the relative success of Skyrme-soliton phenomenology.[1-3] There seems to be an increasing indication that chiral symmetry rather than confinement is the pertinent constraint on low-energy observables, reminding us perhaps of the relevance of current algebra.[4] However, such a description will remain unsatisfactory in the absence of a quantum scheme that would account for soft-pion Skyrme-soliton dynamics in agreement with soft-pion threshold theorems.

Skyrme's approach[1] to hadron dynamics is rooted in the nonlinear σ model. The common wisdom is that the large-N_c version of QCD is a weakly interacting phase of mesons and glueballs decoupled to leading order, and out of which baryons emerge as solitons.[5,6] In this respect, Skyrme's model provides a simple realization of the large-N_c scenario consistent with current algebra. In yet another line of thought, Weinberg[7] argued some time ago for a systematic description of low-energy phenomena based on the sole assumptions of unitarity, locality, Lorentz invariance, and chiral symmetry in conformity with the general precepts of current algebra. Not surprisingly perhaps, it turns out that Skyrme's script provides the starting lines to Weinberg's general scenario.

It is well known that the long-wavelength properties of QCD are dominated by the u and d quarks whose current masses (~ 10 MeV) are small compared to the QCD cutoff $\Lambda_{\overline{\rm MS}}$ ($\sim 150\pm 80$ MeV), where $\overline{\rm MS}$ is the modified minimal-subtraction scheme. In this regime, massless QCD has an additional $SU(2)_L\otimes SU(2)_R$ symmetry which is believed to be spontaneously broken to $SU(2)_V$ with the appearance of three massless Goldstone bosons: π^0,π^+,π^-. Quantitatively, this symmetry translates into a set of anomaly-free chiral $SU(2)_L\otimes SU(2)_R$ Ward identities which, in return, constrain the different parameters of the various correlation functions in the QCD vacuum. At low energy, any sensible approximation to QCD must abide by these identities. A systematic procedure that keeps track of them makes use of an effective Lagrangian description in terms of the chiral excitations ($\pi^0,\pi^\pm$) (Ref. 8). In this spirit, Gasser and Leutwyler[9] have shown that to leading order in the pion momentum, the solution to the chiral Ward identities is the gauged $SU(2)_L\otimes SU(2)_R$ nonlinear σ model. They have established that the next-to-leading-order solution is consistent with Weinberg's expansion in the presence of source terms, assessing Weinberg's approach in the context of QCD. It is this approach that we seek in this paper to investigate soft-pion Skyrme-soliton dynamics beyond the tree level, on a basis which is consistent with the $\pi\pi$ data.

Consider a model with $SU(2)_L\otimes SU(2)_R$ symmetry spontaneously broken to the diagonal subgroup $SU(2)_V$. The vacuum state of this model contains massless Goldstone bosons ($\pi^0,\pi^\pm$) which are described by an SU(2)-valued field $U(\mathbf{x})$, i.e.,

$$U(\mathbf{x})=\exp\left[i\tau\cdot\frac{\pi}{F_\pi}\right] . \tag{1}$$

Here $F_\pi=93$ MeV is the pion-decay constant, and the τ's generate the Lie algebra of SU(2). At low energy, the dynamics is given by Weinberg's chiral expansion[10]

$$\begin{aligned}\mathscr{L}_0&=c_0\,{\rm Tr}(L_\mu{}^2)+c_1\,{\rm Tr}[L_\mu,L_\nu]^2\\&\quad+c_2\,{\rm Tr}\{L_\mu,L_\nu\}^2+\cdots ,\\L_\mu&=U^\dagger\partial_\mu U ,\end{aligned} \tag{2}$$

where $c_0=-F_\pi{}^2/4$, c_1 and c_2 are dimensionless parameters. The terms deleted are of higher order in derivatives. Aside from form-factor terms such as ${\rm Tr}[L_\mu\partial^2L^\mu]$, quartic terms of the form ${\rm Tr}[(\partial_\mu L_\mu)^2]$, ${\rm Tr}[(\partial_\mu L_\nu)^2],\ldots$ can be eliminated through the use of the *Cartan-Maurer* equation.[11] The Skyrme model is a particular case of (2) in which $c_2=0$. The topological character of the model allows for the existence of nontrivial field configurations *(Skyrme solitons)* that can be identified with QCD baryons at low energy. Their classical stability depends on the relative strength and sign of c_1 and c_2. In principle, these parameters are uniquely determined by the QCD inputs. However, the absence of a quantitative understanding of QCD in the long-wavelength approximation leaves them undetermined. They are to be fixed by experiment. For simplicity, we will be assuming later on that the renormalized couplings satisfy $|c_2^R|$

$\ll |c_1^R|$, and specialize to the $B=1$ *hedgehog* configuration, widely discussed in the literature,[1-3] i.e.,

$$U_0(\mathbf{x})=\exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}F(r)]\ ,\qquad F(0)=\pi,\ \ F(\infty)=0\ . \tag{3}$$

Finally, note that since the pion coupling is derivative in nature, i.e., involves $\partial_\mu\pi/F_\pi$, one can view (2) in the trivial sector as a soft-pion expansion (p^2), or a weak-coupling expansion ($1/F_\pi$), or a large color expansion ($1/N_c$) since $F_\pi{}^2=O(N_c)$, and use it as a starting point to understand the long-wavelength properties of QCD.

To investigate soft-pion Skyrme-soliton dynamics we need to consider (2) beyond the tree level. Unfortunately, the model as it stands suffers from severe ultraviolet and infrared divergences and turns out to be not renormalizable in 3 + 1 dimensions. What this means is that beyond the tree graphs the model requires counterterms of increasing complexity to cure the infinities arising from higher-loop calculations. This should come as no surprise since (2) is, after all, a truncated form in a chiral expansion that involves infinitely many higher-derivative terms. There is no reason to believe that the leading terms should account for all dynamical scales.

Weinberg[7] has shown that the leading term in (2) describes uniquely $\pi\pi$ scattering in the trivial topological sector, to order p^2 in the pion momentum. Consistency with unitarity requires the inclusion of terms of order p^4. Since loop contributions are suppressed in the sense of a low-energy limit by powers of p^2, one needs to consider the one-loop effect about the leading term in (2) along with the tree-level contributions from the quartic terms to order p^4 in the trivial sector. In this spirit, *it is possible* to make the model meaningful to one-loop order.

Consider the vacuum-to-vacuum amplitude associated with the Lagrangian (2) in the presence of the Skyrme soliton, i.e.,

$$\begin{aligned}Z&=\langle 0_{\rm out}\,|\,0_{\rm in}\rangle\\ &=\int d\mu[U]e^{iS[U]}\ ,\end{aligned} \tag{4}$$

where S is the classical action for the SU(2)-valued fields described by (2). To investigate soft-pion fluctuations, let us specialize to the following class of field configurations:

$$U(\mathbf{x},t)=U_0(\mathbf{x})R^\dagger(\phi) \tag{5a}$$

or

$$U(\mathbf{x},t)=L(\xi)U_0(\mathbf{x})\ . \tag{5b}$$

$L(\xi)$ and $R(\phi)$ are also SU(2)-valued fields which are to be treated as fluctuating left and right mesons about the Skyrme-soliton configuration $U_0(\mathbf{x})$. [If $U_0(\mathbf{x})$ is a solution of the equation of motion associated to (2), then both (5a) and (5b) lead to the same on-shell form of the soft-pion Lagrangian of Ref. 12.] Using (5a), one can linearize the *Haar* measure $d\mu[U(x)]$ on SU(2) to an ordinary *Lebesgue* measure, i.e.,

$$\prod_x d\mu[U(x)]=\prod_x d\phi(x)[1+O(\phi^2)] \tag{6}$$

and similarly for the ansatz (5b). Since $U_0(\mathbf{x})$ satisfies the saddle-point equation

$$\left[\frac{\delta S}{\delta\phi^a}\right]_{\phi=\xi=0}=\left[\frac{\delta S}{\delta\xi^a}\right]_{\xi=\phi=0}=0\ , \tag{7}$$

one deduces the one-loop contribution to the vacuum-to-vacuum amplitude in the form

$$\begin{aligned}Z_1&=\exp\left\{i\left[S[U_0]-\tfrac{1}{2}\,\mathrm{Tr}\ln\left[\frac{\delta^2S}{\delta\phi^a\delta\phi^b}\right]\right]\right\}\\ &=\exp\left\{i\left[S[U_0]-\tfrac{1}{2}\,\mathrm{Tr}\ln\left[\frac{\delta^2S}{\delta\xi^a\delta\xi^b}\right]\right]\right\}.\end{aligned} \tag{8}$$

It is therefore sufficient to use either left or right fluctuations to describe soft-pion dynamics to one-loop order. In this spirit, consider

$$\begin{aligned}&U(\mathbf{x},t)=U_0(\mathbf{x})R^\dagger(\phi(\mathbf{x},t))\ ,\\ &R^\dagger(\phi)=\exp\left[-i\frac{\phi}{F_\pi}\right]\ ,\end{aligned} \tag{9}$$

in terms of which the left-handed currents on S^3 read

$$\begin{aligned}L_\mu^R&=R(L_\mu+r_\mu)R^\dagger\ ,\\ r_\mu&=(\partial_\mu R^\dagger)R\\ &=-\frac{i}{F_\pi}\partial_\mu\phi-\frac{1}{2F_\pi{}^2}[\phi,\partial_\mu\phi]+O\left[\frac{1}{F_\pi{}^3}\right]\ .\end{aligned} \tag{10}$$

The last expression constitutes a $1/F_\pi$ expansion about the Skyrme soliton *(weak coupling)* consistent with the present semiclassical description. Injecting (10) into (2) and neglecting pion Skyrme-soliton correlations with large momentum transfer, one obtains a soft-pion Skyrme-soliton dynamics of the form *(massless chiral fluctuations)*

$$\begin{aligned}\mathscr{L}_1=\mathscr{L}_0&-\frac{c_0}{F_\pi{}^2}\mathrm{Tr}[(\partial_\mu\phi)^2]+\frac{2ic_0}{F_\pi}\mathrm{Tr}(\phi\partial^\mu\Lambda_\mu)\\ &-\frac{c_0}{F_\pi{}^2}\mathrm{Tr}([\phi,\partial_\mu\phi]\Lambda^\mu)\ ,\end{aligned} \tag{11}$$

where Λ_μ is an SU(2)-valued field defined through

$$\Lambda_\mu=L_\mu+\frac{2c_1}{c_0}[L_\nu,[L_\mu,L_\nu]]+\frac{2c_2}{c_0}\{L_\nu,\{L_\mu,L_\nu\}\}\ . \tag{12}$$

The semiclassical contribution to Z is given by the saddle-point equation

$$\partial^\mu\Lambda_\mu=0 \tag{13}$$

which is just the classical equation of motion for the hedgehog configuration in the $B=1$ sector, under the announced assumption that $|c_2|\ll|c_1|$. In Euclidean space, the soft-pion contribution follows from the one-loop correction through

$$Z=\exp(S_0^E)\int\prod_x d\phi(x)\exp\left[-\tfrac{1}{2}\int d^4x\,\phi^a(x)\hat{D}_E^{ab}\phi^b(x)\right]$$
$$=(\det\hat{D}_E)^{-1/2}\exp(S_0^E)\,, \tag{14}$$

the operator $\hat{D}_E$ is given by

$$\hat{D}_E^{ab}=\delta^{ab}\partial^2+\mathrm{Tr}(\tau^a\Lambda_E\cdot\partial\tau^b)\,. \tag{15}$$

In Euclidean space our conventions are

$$g_{\mu\nu}=\mathrm{diag}(+1,+1,+1,+1)$$

and $\Lambda_E^k=(-i\Lambda^0,\mathbf{\Lambda})$, etc. From (14), we conclude that

$$S_1^E=S_0^E-\tfrac{1}{2}\,\mathrm{Tr}\ln\hat{D}_E\,. \tag{16}$$

As it stands, the Tr ln term in (16) suffers from both ultraviolet and infrared divergences and needs proper regularization. The UV divergences can be handled by dimensional regularization in a chiral-invariant way. Unfortunately, the infrared divergences can only be removed via a finite infrared cutoff which breaks explicitly chiral symmetry as shown later on. In d-dimensional Euclidean space ($d=4-\epsilon$), we have

$$S_1^E=S_0^E-\frac{\mu^{d-4}}{2}\int d^dx\,\mathrm{Tr}_I\langle x\,|\ln\hat{D}_E\,|x\rangle\,, \tag{17}$$

where μ stands for the renormalization scale and the remaining trace is over internal indices. To evaluate the Tr ln term in (17), we will use the adiabatic approximation of Goldstone and Wilczek.[13] The latter is consistent with Weinberg's expansion[7] in the trivial sector. For that we assume that the variations in the Skyrme-soliton field are small compared to its *Compton* wavelength. In this spirit, it is straightforward to show that the commutator and anticommutator terms in (17) yield higher-order terms in the field gradients to one-loop order and should be dropped. This result agrees with Weinberg's argument that loop contributions are suppressed by powers of p^2 with respect to the tree-level terms in the trivial topological sector (p_μ is the pion momentum). At one-loop order, it is therefore sufficient and consistent to consider

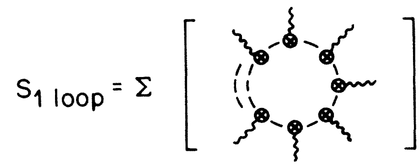

FIG. 1. General contribution to the effective action to one-loop order. The dashed line corresponds to a soft meson, while the wiggly lines are soft-meson Skyrme-soliton interactions as described by the soft-pion Lagrangian (11) with $\Lambda_\mu=L_\mu+O(L^3)$.

$$\Lambda_\mu=L_\mu+O(L^3) \tag{18}$$

in terms of which we have

$$\begin{aligned}\hat{D}&=\partial^2+\hat{L}\cdot\partial+O(L^3)\,,\\ \hat{L}_\mu^{ab}&=\mathrm{Tr}(\tau^aL_\mu\tau^b)=2\epsilon^{abc}A_\mu^c\,,\end{aligned} \tag{19}$$

where we have used the fact that $L_\mu=i\boldsymbol{\tau}\cdot\mathbf{A}_\mu$, which follows from the fact that $\det U=1$. Injecting (19) into (17) yields

$$\begin{aligned}\mathrm{Tr}\ln\hat{D}&=\mathrm{Tr}\ln(\partial^2)+\mathrm{Tr}\ln[1+(\partial^2)^{-1}\hat{L}\cdot\partial]\\ &=\mathrm{Tr}\ln(\partial^2)+\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k}\mathrm{Tr}[(\partial^2)^{-1}\hat{L}\cdot\partial]^k\,.\end{aligned} \tag{20}$$

The general contribution to (20) is diagrammatically shown in Fig. 1. The first term in (20) is the free space divergence and can be subtracted away since it does not affect the present dynamics. Moreover, consistency with the tree-level expansion (2) shows that only $k=1,2,3,4$ contribute to S_{soft} as shown in Fig. 2. Higher-order terms yield higher-order field gradients which are suppressed at low energy. Straightforward calculations show that the tadpole diagram in Fig. 2(b) vanishes identically. Moreover, the $k=2$ and $k=3$ terms sum up to $O(p^6)$, so that the soft-pion contribution to the effective action is exclusively given by the box diagram of Fig. 2(d). Using the pion mass m_π as an infrared cutoff, we obtain ($\epsilon\to 0_+$)

$$\tfrac{1}{2}\mathrm{Tr}\ln\hat{D}=-\frac{\Sigma}{12}\int d^4x\{\mathrm{Tr}[(\partial_\mu\hat{L}_\alpha-\partial_\alpha\hat{L}_\mu)^2]-\mathrm{Tr}[(\partial_\alpha\hat{L}_\beta-\partial_\beta\hat{L}_\alpha)[\hat{L}_\beta,\hat{L}_\alpha]]+\mathrm{Tr}(\hat{L}_\mu{}^2\hat{L}_\nu{}^2)+\tfrac{1}{2}\mathrm{Tr}(\hat{L}_\mu\hat{L}_\nu)^2\}+O(\hat{L}^6)\,, \tag{21}$$

where Σ is a cutoff-dependent expression of the form[14]

$$\Sigma=\frac{1}{64\pi^2}\left[\frac{2}{\epsilon}-\ln\frac{m_\pi{}^2}{\mu^2}+\ln(4\pi)-\gamma\right]. \tag{22}$$

The chiral logarithm in (22) signals the smooth infrared divergences inherent to chiral perturbation expansion for massless pions. Using the explicit form of $\hat{L}$ as given by (19), we can rewrite (17) in terms of the left current on S^3, i.e.,

$$S_1^E=S_0^E-\frac{\Sigma}{12}\int d^4x(\mathrm{Tr}[L_\mu,L_\nu]^2-3\,\mathrm{Tr}\{L_\mu,L_\nu\}^2)+O(L^6)\,. \tag{23}$$

The nature of the divergences occurring at one loop is similar to the tree-level terms retained in the chiral expansion, making the model renormalizable to this order. In the perturbative sector, the divergent terms in Σ agree with Weinberg's calculation[7] based on the renormalization-group equations. The explicit analysis performed here provides the finite terms as well. To get a meaningful action to one-loop order, we adopt a "minimal" subtraction procedure, and choose to fit the renormalized couplings to the D-wave $\pi\pi$ data at η threshold, as discussed by Gasser and Leutwyler.[9] This procedure is consistent with the soliton philosophy. In this spirit, consider

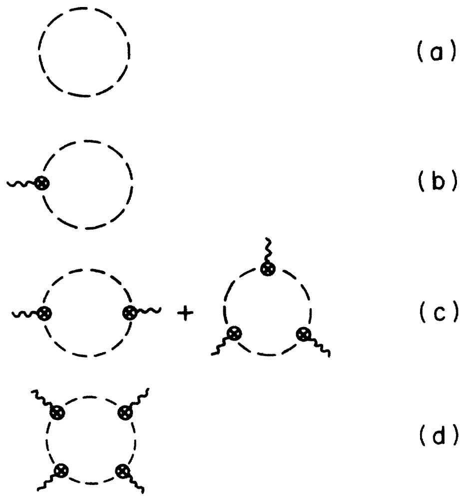

FIG. 2. Leading contribution to the effective action in the soft-pion limit. (a) is the free-space contribution. The tadpole contribution (b) vanishes identically, while the two- and three-point insertions in (c) sum up to $O(p^6)$. (d) shows the exclusive contribution to the effective action to $O(p^4)$.

$$\mathscr{L}_0=\mathscr{L}_0^R+(Z_0-1)c_0^R\,\mathrm{Tr}(L_\mu{}^2)+(Z_1-1)c_1^R\,\mathrm{Tr}[L_\mu,L_\nu]^2 +(Z_2-1)c_2^R\,\mathrm{Tr}\{L_\mu,L_\nu\}^2 , \tag{24}$$

where $\mathscr{L}^R$ is the renormalized version of (2), and the Z factors relate the bare couplings (c_k^0) to the renormalized ones (c_k^R) through $c_k^0=Z_k c_k^R$, $k=0,1,2$. The pion wave-function renormalization Z_π is fixed by unitarity, i.e., $Z_\pi=\sqrt{Z_0}$. To one-loop order, the minimal-subtraction prescription reads

$$\begin{aligned} &Z_k=1+\frac{\gamma_k}{64\pi^2}Z ,\quad k=0,1,2 ,\\ &\gamma_0=0,\quad \gamma_1=\frac{1}{12c_1^R},\quad \gamma_2=-\frac{1}{4c_2^R} ,\\ &Z=\frac{2}{\epsilon}+\gamma-\ln(4\pi)-1 . \end{aligned} \tag{25}$$

We have subtracted a finite piece as well for later convenience. This should be of no concern since c_0^R, c_1^R, and c_2^R are yet to be fitted. Through this procedure, the renormalized couplings c_k^R depend on the scale parameters μ and m_π. Using (24) to one-loop order, one can determine the D-wave $\pi\pi$-scattering lengths in the trivial sector and use them to evaluate phenomenologically the couplings c_k^R. Following Gasser and Leutwyler we obtain

$$\begin{aligned} &c_1^R=\tfrac{5}{8}\pi F_\pi{}^4(a_2^0-4a_2^2)-\frac{1}{768\pi^2}\left[\ln\frac{m_\pi{}^2}{\mu^2}+\frac{49}{40}\right] ,\\ &c_1^R+c_2^R=\tfrac{5}{4}\pi F_\pi{}^4(a_2^0-a_2^2)+\frac{1}{384\pi^2}\left[\ln\frac{m_\pi{}^2}{\mu^2}+\frac{27}{20}\right] , \end{aligned} \tag{26}$$

where a_2^0 and a_2^2 are the D-wave $\pi\pi$ scattering lengths, i.e.,

$$\begin{aligned} &a_2^0=(17.0\pm3)10^{-4}m_\pi{}^{-4} ,\\ &a_2^2=(1.3\pm3)10^{-4}m_\pi{}^{-4} . \end{aligned} \tag{27}$$

As already mentioned, Eq. (26) shows that the couplings depend explicitly on the renormalization scale μ, and the pion mass m_π. For definiteness, we choose to evaluate c_1^R and c_2^R at $\mu=m_\eta$, i.e.,[15]

$$\begin{aligned} &c_1^R=(0.7\pm0.5)10^{-3} ,\\ &c_1^R+c_2^R=(0.9\pm0.3)10^{-3} . \end{aligned} \tag{28}$$

In the chiral limit, the pion-decay constant does not renormalize at one-loop order, $c_0^R=c_0$. The renormalized action to one-loop order reads

$$S_1=\int d^4x\left\{c_0^R\,\mathrm{Tr}(L_\mu{}^2)+\left[c_1^R+\frac{1}{768\pi^2}\left[1+\ln\frac{m_\pi{}^2}{m_\eta{}^2}\right]\right]\mathrm{Tr}[L_\mu,L_\nu]^2+\left[c_2^R-\frac{1}{256\pi^2}\left[1+\ln\frac{m_\pi{}^2}{m_\eta{}^2}\right]\right]\mathrm{Tr}\{L_\mu,L_\nu\}^2\right\} . \tag{29}$$

As indicated by Rho,[16] the Skyrme-soliton parameters used by Jackson and Rho[3] to reproduce the pion-decay constant F_π and the axial-vector coupling g_A at the tree level, i.e., $c_1=1.4\times10^{-3}$ and $c_2=0$, are consistent with the renormalized couplings (28) at η threshold. Finally notice that the soft-pion corrections are attractive to one-loop order.

For the hedgehog configuration, it is straightforward to deduce the soft-pion correction to the mass term in the form

$$\Delta M_S=\frac{1}{24\pi}\left[1+\ln\frac{m_\pi{}^2}{m_\eta{}^2}\right]\times\int_0^\infty dr\,r^2\left[3F'^4+4F'^2\frac{\sin^2F}{r^2}+8\frac{\sin^4F}{r^4}\right] \tag{30}$$

which pushes down the Skyrme-soliton mass. This effect is expected from naïve second-order perturbation theory. The results quoted in Table I show a 20% effect due to the soft-pion correction on the Skyrme-soliton mass.

To investigate the soft-pion effects on the N and Δ isobar we follow the projection method advocated by Adkins, Nappi, and Witten.[2] To disentangle the slow rotation of the Skyrme soliton from the chiral fluctuations $R(\mathbf{x},t)$, we will make the plausible assumption that the collective rotation in isospace satisfies

$$|A^\dagger\dot{A}|\ll|\dot{R}^\dagger R| . \tag{31}$$

In this picture, the Skyrme soliton resembles a large but slow wheel coupled to a small but quick wheel representing the chiral fluctuations $R(\mathbf{x},t)$. In other words,

TABLE I. (a) Soft-pion corrections to the ground-state energies M^π for the hedgehog Skyrme soliton, nucleon, and Δ isobar are quoted along with the moment of inertia λ. For the parameters of the model (F_π and ϵ^2) we have considered two cases: (i) the parameters of Adkins, Nappi, and Witten (Ref. 2) which were fitted to reproduce the mass of the nucleon and Δ isobar: $F_\pi=64.5$ MeV, $\epsilon^2=0.004\,21$; (ii) the parameters of Jackson and Rho (Ref. 3) which reproduce the pion-decay constant F_π and the axial-vector coupling g_A using the Goldberger-Treiman relation: $F_\pi=93$ MeV, $\epsilon^2=0.005\,52$. (b) The results in Refs. 2 and 3 are summarized.

(a)

ϵ^2	F_π (MeV)	M_H^π (MeV)	λ^π (fm)	M_N^π (MeV)	M_Δ^π (MeV)	$(M_\Delta^\pi - M_N^\pi)$ (MeV)	$g_A^{H,\pi}$
0.004 21	64.5	605	0.82	695	1057	361	0.22
0.005 52	93.0	1101	0.90	1183	1511	328	0.41

(b)

ϵ^2	F_π (MeV)	M_H (MeV)	λ (fm)	M_N (MeV)	M_Δ (MeV)	$(M_\Delta - M_N)$ (MeV)	g_A^H
0.004 21	64.5	863	1.01	937	1231	294	0.61
0.005 52	93.0	1425	1.05	1496	1778	282	0.80

$$U(\mathbf{x},t)=A(t)U_0(\mathbf{x})A^\dagger(t)R^\dagger(\mathbf{x},t) \tag{32}$$

and all our previous arguments follow through for the projected N and Δ states. Equation (32) shows that rigid chiral rotations do not affect the trivial vacuum, as of course expected. The resulting correction to the moment of inertia due to the soft-pion fluctuations reads

$$\Delta\lambda=\frac{1}{9\pi}\left[1+\ln\frac{m_\pi{}^2}{m_\eta{}^2}\right]\int_0^\infty dr\, r^2\sin^2F\left[F'^2+4\frac{\sin^2F}{r^2}\right] \tag{33}$$

which amounts to a net decrease in λ by about 15% as shown in Table I. The soft-pion fluctuations produce an overall attraction which tend to push down the N and Δ mass.

The soft-pion correction to the hedgehog axial form factor g_A^H is given by

$$\Delta g_A^H=-\frac{1}{18\pi}\left[1+\ln\frac{m_\pi{}^2}{m_\eta{}^2}\right]\times\int_0^\infty dr\, r^2\left[3F'^3+F'^2\frac{\sin(2F)}{r}+2F'\frac{\sin^2F}{r^2}+4\frac{\sin^2F}{r^3}\sin(2F)\right]. \tag{34}$$

This corresponds to a 50% decrease in g_A^H as shown in Table I. This result should be interpreted with care, since the calculation is done in the absence of pseudoscalar and axial-vector sources. A systematic calculation of g_A^H necessitates an elaborate renormalization scheme in the *presence* of these sources, and will be reported elsewhere.

We have presented an explicit and systematic approach for investigating soft-pion corrections to the Skyrme soliton. In the absence of a QCD script, Weinberg's general chiral expansion seems to be the reliable framework for discussing Skyrme-type phenomenology. We have emphasized that the urge to go beyond the tree level in this effective description is not only needed for pion Skyrme-soliton dynamics but also required by the unitarity of the S matrix in the trivial sector. Using the soft-pion limit, we have explicitly constructed the effective action to one-loop order, and carried it through beyond the standard effective potential. A more detailed account of this procedure will be given elsewhere.[17] In the light of the $\pi\pi$ data, our results show that the soft-pion corrections to the *nucleon* and Δ-*isobar* masses are attractive and substantial. This might suggest a mechanism based on soft-pion fluctuations to restore the missing medium-range attraction in the central channel of the NN potential in the Skyrme model. They also produce a 50% decrease in the axial form factor g_A^H of the hedgehog configuration. However, this negative result should be interpreted with care since we have disregarded possible source renormalizations. These soft-pion corrections to the Skyrme soliton are of order 1. Although sizable, these effects are only directional in the absence of the relevant vector-meson resonances in the 1-GeV scale. The conjugate effects of the vector mesons ω, ρ, and A_1 on the Skyrme soliton will be discussed elsewhere.[18]

Note added. In a recent work, Schnitzer[19] has investigated similar issues using a constrained Hamiltonian formalism. The character of his approximation does not rely on a gradient expansion, but rather on an explicit truncation of the pion propagator viewed as an infinite series in the interaction kernel. As a result, his soft-pion correction to the Skyrmion mass is finite and positive. While finiteness can be perhaps justified on the basis of the approximation used, his argument that the zero-point energy must be definite positive is not correct because of the possible counterterms. The standard example for this is the zero-point correction to the kink mass in the sine-Gordon model.[20] We will elaborate further on Schnitzer's work and the correct handling of the projection technique in the soft-pion regime in a further study.

We would like to thank G. E. Brown and T. H. Hansson for helpful comments and discussions, and E. Braaten for a useful remark. One of us (U.-G.M.) acknowledges discussions with J. Gasser and H. Leutwyler and support by Schweizerische Nationalfonds. This work was supported by the U.S. Department of Energy under Contract No. DE-AC02-76ER13001 with the State University of New York at Stony Brook.

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