Current algebra and the cloudy-bag model

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Two versions of the cloudy-bag model (surface and volume coupling) satisfy the commutation relations of current algebra. Perturbation-theory evaluations of various observables are compared for the two treatments. The electromagnetic properties are essentially the same in both versions. The volume-coupling model contains, in lowest order, a pion-quark contact interaction that contributes to the axial-vector current. Including the effect of this term leads to a computed value of g_A of about 1.25, if the radius of the nucleon bag is around 1.0 fm.

I. INTRODUCTION

In the cloudy-bag model (CBM) hadrons are treated as quarks confined in an MIT bag that is surrounded by a cloud of pions.[1] The CBM Lagrangian satisfies the $SU(2)\times SU(2)$ current algebra[2] and obeys chiral symmetry. Pionic effects treated perturbatively as quantum fluctuations around bag model solutions, provide significant corrections to baryonic properties. Results for nucleonic charge, magnetic,[3] and axial-vector-current distributions[4] as well as pion-nucleon scattering[5] are in very good agreement with experiment. In addition g_A has the reasonably accurate value of 1.09. The significance of the g_A value is reviewed by Faessler.[6]

The purpose of the present work is to explore the consequences of the current algebra in a more detailed fashion than has been done previously. We begin with a brief review of the previous impact of current algebra on the CBM.

In the original version[3,5] of the CBM, the pseudoscalar interaction between pions and quarks takes place only at the surface of the bag. Although the Lagrangian for this model has conserved axial-vector and vector currents and obeys the current algebra (see Appendix A) it was difficult to identify the explicit terms and dynamics which lead to the well-known current-algebra predictions for various observables. For example, it was not clear how to obtain s-wave pion-nucleon scattering. This problem was solved by Thomas,[7] see also Ref. 8, who obtained a transformation on the quark field operators which made the current-algebra results more explicit. In particular, the transformed Lagrangian explicitly contains the Weinberg-Tomozawa[9,10] (WT) term responsible for low-energy s-wave pion-nucleon scattering. Another consequence is that the pion-quark interaction is of the pseudovector form and takes place over the entire volume of the bag. The use of the WT term leads to a reasonably accurate evaluation of the width for the ρ meson to convert to two pions.[11] The original and new formalisms are reviewed briefly in Secs. II and III.

The previously mentioned[3–5] good results were obtained with the original formalism. Thus it is necessary to determine the implications of the new formalism for observables computed earlier. For baryons of the ground-state octet, the pion-baryon vertex functions are the same in the two formalisms. This is discussed in Ref. 12 and in Appendix B.

There are, however, some changes in the electromagnetic interaction that are expected from the new derivative form of the pion-quark coupling. The preservation of electromagnetic current conservation requires that there be an additional contribution to the electromagnetic current absent in the surface coupling model. This $\gamma+q\leftrightarrow\pi+q$ term is shown in Fig. 1(a). Its influence on baryon magnetic moments and nucleonic mean charge radii is evaluated in Sec. IV. In addition there is a term that plays the role of a virtual ρ meson. A $\pi^+\pi^-$ pair produced by a virtual photon can be absorbed at once by the WT term [see Fig. 1(b) and Sec. V]. The effects of the terms of Figs. 1(a) and 1(b) are very small and tend to cancel.

A more interesting observable, for our present purpose, is the axial-vector coupling constant g_A. In the original CBM it is only the quarks that contribute to g_A, even though both pions and quarks carry axial current. The introduction of the WT term raises the possibility that there

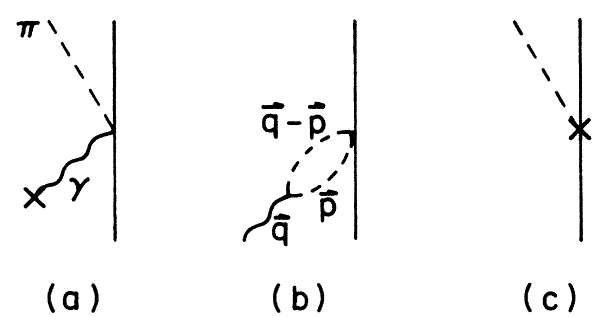

FIG. 1. Contact interactions in the volume-coupling form of the cloudy-bag model. In (b) $\mathbf{p}$ and $\mathbf{q}-\mathbf{p}$ are the momenta carried by the two virtual pions.

is a contribution to g_A that was missing in the early evaluations in the CBM. This is because of the Adler-Weisberger[13] relation between g_A and pion-nucleon scattering cross sections. Any term that influences pion-nucleon scattering should have some effect on the values of g_A (Ref. 14). Here we identify the new term. It is of the $q \to q + \pi$ form shown in Fig. 1(c) and, tends to increase g_A by significant amounts depending on the bag radius. The correction to g_A is evaluated in Sec. VI.

II. THE ORIGINAL CLOUDY-BAG MODEL FORMALISM

The cloudy-bag model (CBM) in the absence of gluon fields is defined by its Lagrangian density

$$\begin{aligned}\mathscr{L}(x) &= [\bar{q}(x)(i\gamma\cdot\partial - m_q)q(x) - B]\theta_V \\ &\quad - \tfrac{1}{2}\bar{q}(x)e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma^5/f}q(x)\Delta_S \\ &\quad + \tfrac{1}{2}\mathscr{D}_\mu\boldsymbol{\pi}(x)\cdot\mathscr{D}^\mu\boldsymbol{\pi}(x) - \tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2(x) . \end{aligned} \tag{2.1}$$

Here $q(x)$ is the quark field, m_q is the quark mass matrix, $\boldsymbol{\pi}(x)$ is the pion field of mass m_π, B is the constant "vacuum pressure," and f is a constant. θ_V is a volume step function

$$\theta_V(x) = \begin{cases} 1, & x \in V , \\ 0 & \text{otherwise} \end{cases}$$

and $\Delta_S = -n\cdot\partial\theta_V$, a surface δ function in terms of the outward four-normal to V with $n_\mu n^\mu = -1$. The covariant derivative is defined as

$$\mathscr{D}_\mu\boldsymbol{\pi} \equiv (\partial_\mu\pi)\hat{\boldsymbol{\pi}} + f\sin(\pi/f)\partial_\mu\hat{\boldsymbol{\pi}} ,$$

where $\pi = |\boldsymbol{\pi}|$ and $\hat{\boldsymbol{\pi}} = \boldsymbol{\pi}/\pi$. The equations of motion appear in many places.[1,3,5] Here we only recall that

$$(i\gamma\cdot\partial - m_q)q(x) = 0, \quad x \in V , \tag{2.2a}$$

$$i\gamma\cdot n q(x) = e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma^5/f}q(x), \quad x \in S , \tag{2.2b}$$

$$(\partial^2 + m_\pi^2)\boldsymbol{\pi}(x) = -\frac{i}{2f}\bar{q}(x)\gamma^5\boldsymbol{\tau}q(x)\Delta_S , \tag{2.2c}$$

where (2.2a) is the free Dirac equation for the quarks inside the bag; (2.2b) is the linear surface boundary condition which guarantees that the quarks remain permanently confined inside the the bag volume; and (2.2c) is the Klein-Gordon equation for the pions with a highly nonlinear source term on the bag surface which is given only to the lowest order in f^{-1}. The Lagrangian density (2.1) is invariant under the infinitesimal vector transformation

$$q \to q + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\alpha}q, \quad \boldsymbol{\pi} \to \boldsymbol{\pi} - \boldsymbol{\alpha}\times\boldsymbol{\pi} ,$$

which, according to Noether's theorem, results in the conserved vector current

$$\mathbf{V}^\mu = \bar{q}\gamma^\mu\frac{\boldsymbol{\tau}}{2}q\theta_V + j_0{}^2(\pi/f)\boldsymbol{\pi}\times\partial^\mu\boldsymbol{\pi} , \tag{2.3}$$

where j_0 is the spherical Bessel function. In the limit of $m_q = m_\pi = 0$, (2.1) is also invariant under the infinitesimal chiral transformation

$$\begin{aligned} &q \to q - \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\beta}\gamma^5 q , \\ &\boldsymbol{\pi} \to \boldsymbol{\pi} + \boldsymbol{\beta}f + f(\boldsymbol{\beta}\times\hat{\boldsymbol{\pi}})\times\hat{\boldsymbol{\pi}}[1 - (\pi/f)\cot(\pi/f)] . \end{aligned}$$

This gives a conserved axial-vector current

$$\mathbf{A}^\mu = \bar{q}\gamma^\mu\gamma^5\frac{\boldsymbol{\tau}}{2}q\theta_V + f\hat{\boldsymbol{\pi}}\partial^\mu\pi + \frac{f^2}{2}\partial^\mu\hat{\boldsymbol{\pi}}\sin(2\pi/f) , \tag{2.4}$$

which when $m_\pi \neq 0$ becomes a partially conserved axial-vector current (PCAC),

$$\partial_\mu\mathbf{A}^\mu = -fm_\pi{}^2\boldsymbol{\pi}$$

from which we can identify f as $f_\pi = 93$ MeV, the pion decay constant. The time components of the currents in (2.3) and (2.4) are shown to satisfy the current-algebra commutation relations in Appendix A.

The reasonable approximation used when working with (2.1) is to linearize about the point $\boldsymbol{\pi} = 0$. Keeping only terms of order $(\boldsymbol{\pi}/f_\pi)$, (2.1) becomes

$$\mathscr{L} = \mathscr{L}_{\rm MIT} + \mathscr{L}_\pi + \mathscr{L}_I \tag{2.5}$$

with

$$\begin{aligned} \mathscr{L}_{\rm MIT} &= \bar{q}(i\gamma\cdot\partial - m_q)\theta_V - B\theta_V - \tfrac{1}{2}\bar{q}q\Delta_S , \\ \mathscr{L}_\pi &= \tfrac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \tfrac{1}{2}m_\pi{}^2\boldsymbol{\pi}^2 , \\ \mathscr{L}_I &= -\frac{i}{2f_\pi}\bar{q}\gamma^5\boldsymbol{\tau}\cdot\boldsymbol{\pi}q\Delta_S . \end{aligned}$$

A quark-pion Hamiltonian is derived in the canonical fashion. One then obtains

$$H = H_{\rm MIT} + H_\pi + H_I \tag{2.6}$$

with

$$\begin{aligned} H_{\rm MIT} &= \int [\bar{q}(-i\gamma\cdot\nabla + m_q)q + B]\theta_V d^3x , \\ H_\pi &= \tfrac{1}{2}\int [(\partial_t\boldsymbol{\pi})^2 + (\nabla\boldsymbol{\pi})^2 + m_\pi{}^2\boldsymbol{\pi}^2]d^3x , \\ H_I &= \frac{i}{2f_\pi}\int \mathbf{q}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma^5 q\Delta_S d^3x . \end{aligned}$$

After canonical quantization of the quark and pion fields (2.6) becomes an operator equation with $H \to \hat{H}$. The fields are then given by their mode sums

$$\pi_j(\mathbf{x};t) = \int \frac{d^3k}{[(2\pi)^3 2\omega_k]^{1/2}}a_{\mathbf{k},j}e^{-i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.} , \tag{2.7}$$

$$q_\alpha(\mathbf{x};t) = \sum_l [\psi_l(x)b_{\alpha l} + \phi_l(x)d^\dagger_{\alpha l}] , \tag{2.8}$$

where $\omega_k=(k^2+m_\pi{}^2)^{1/2}$ and $a_{\mathbf{k}j}$ destroys a pion with momentum $\mathbf{k}$ and isospin label j, and $b_{\alpha l}$ ($d_{\alpha l}^\dagger$) destroys (creates) a quark (antiquark) and flavor α in the bag model l. Here l represents all quantum numbers for a given mode and the creation and destruction operators satisfy the canonical commutation relations for bosons and fermions. The positive-energy bag modes are solutions to (2.2a) with the linear boundary condition given by (2.26) *neglecting the effect of the pion field.* The most general solution assuming a static spherical geometry and massless quarks is

$$\psi_{n\kappa jm}(x)=N_{n\kappa}\begin{bmatrix} j_l\left[\dfrac{\omega_{n\kappa}r}{R}\right] \\ -i\,\mathrm{sgn}(\kappa)j_{l'}\left[\dfrac{\omega_{n\kappa}r}{R}\right]\sigma\cdot\hat{\mathbf{r}}\end{bmatrix}\times\mathcal{Y}_\kappa^m e^{-iE_{n\kappa}t}\theta(R-r)\,, \quad (2.9)$$

where $E_{n\kappa}=\omega_{n\kappa}/R$, $N_{n\kappa}$ is a normalization factor, $\mathcal{Y}_\kappa^m$ are the spin-spherical harmonics, and

$$j=|\kappa|-\tfrac{1}{2}\,,$$
$$l=j+\tfrac{1}{2}\mathrm{sgn}(\kappa)\,,$$
$$l'=j-\tfrac{1}{2}\mathrm{sgn}(\kappa)\,.$$

Here j and m label the mode's angular momentum and its $\hat{\mathbf{z}}$ component, κ is the Dirac quantum number which differentiates the two states of opposite parity for each value of j, and n labels the radial mode. Here we use only the mode, $1S_{1/2}$, corresponding to $\kappa=-1$. Then we have

$$\psi_{1-1(1/2)m}(x)=\frac{N_{1,-1}}{\sqrt{4\pi}}\begin{bmatrix} j_0\left[\dfrac{\omega r}{R}\right] \\ ij_1\left[\dfrac{\omega r}{R}\right]\sigma\cdot\hat{\mathbf{r}}\end{bmatrix}\chi_m e^{-i\omega t/R}\theta(R-r) \quad (2.10)$$

with the eigenfrequency $\omega\equiv\omega_{1,-1}$ determined by the linear boundary condition (2.2b)

$$j_0(\omega_{n\kappa})=-\kappa j_1(\omega_{n\kappa})$$

to be $\omega=2.04$. χ_m is a two-component Pauli spinor, and the normalization condition

$$\int q^\dagger(\mathbf{r},t)q(\mathbf{r},t)d^3r=1$$

sets

$$N_{n\kappa}{}^2=\frac{\omega_{n\kappa}}{2R^3(\omega_{n\kappa}+\kappa)j_0{}^2(\omega_{n\kappa})}\,. \quad (2.11)$$

The last step is to project $\hat{H}$ onto the subspace of colorless nonexotic baryonic bags,

$$H\equiv P\hat{H}P\,,$$
$$P=\sum_{A\in\{N,\Delta,\Lambda,\ldots\}}|A_Q\rangle\langle A_Q|\,. \quad (2.12)$$

Here $|A_Q\rangle$ are the SU(6) baryonic quark wave functions. The projection in (2.12) is restricted to the low-lying baryon octet and decuplet of physical particles which are members of the **56** representation of SU(6). Here we let the bag radius R be a parameter independent of B, and assume all baryons in (2.12) to have the same radius. Then the projected Hamiltonian in a second-quantized notation becomes

$$H=\sum_{A,B}A_0^\dagger B_0\langle A_Q|\hat{H}|B_Q\rangle=H_{\mathrm{MIT}}+H_\pi+H_I$$

with

$$H_{\mathrm{MIT}}=\sum_A m_{0A}A_0^\dagger A_0\,,$$
$$H_\pi=\sum_{j=1}^3\int d^3k\,\omega_k a_{\mathbf{k}j}^\dagger a_{\mathbf{k}j}\,, \quad (2.13)$$
$$H_I=\sum_{j=1}^3\int d^3k\sum_{A,B}A_0^\dagger B_0(v_{0\mathbf{k}j}^{AB}a_{\mathbf{k}j}+w_{0\mathbf{k}j}^{AB}a_{\mathbf{k}j}^\dagger)\,.$$

Here A_0,B_0 destroy bare baryonic bags, m_{0A} is the corresponding bare MIT baryon mass, and the sums run over all baryons in the octet and decuplet representations of SU(6). The vertex function $v_{0\mathbf{k}j}^{AB}$ is associated with each unrenormalized vertex which turns a pion $(\mathbf{k},j)$ and bag B into a new bag A and $w_{0\mathbf{k}j}^{AB}\equiv v_{0\mathbf{k}j}^{BA*}$ is the vertex function associated with the creation of a pion. We have

$$v_{0\mathbf{k}j}^{AB}=\frac{i}{2f_\pi}\int\frac{d^3x}{[(2\pi)^3 2\omega_k]^{1/2}}e^{i\mathbf{k}\cdot\mathbf{x}}\langle A_Q|\bar{q}(x)\tau_j\gamma^5 q(x)|B_Q\rangle\delta(r-R)$$
$$=-i\frac{f_0^{AB}}{m_\pi}\frac{u(kR)}{[(2\pi)^3 2\omega_k]^{1/2}}\langle S_B s_B 1m|S_A s_A\rangle\langle T_B t_B 1n|T_A t_A\rangle k_m^* e_{j,n}^*\,, \quad (2.14)$$

where (2.10) is used for the spatial part of the quark wave functions so that $u(kR)\equiv 3j_1(kR)/kR$. f_0^{AB} are the transition coupling constants listed in Table I (from Ref. 15). They are proportional to the reduced matrix elements of the rank-1 irreducible spin-isospin tensor belonging to the **56** representation of SU(6). The Clebsch-Gordan coefficients depend on the spin S and isospin T and their third components s,t of the initial and final baryons. k_m and $e_{j,n}$ are the complex spherical components of the pion momentum $\mathbf{k}$ and the Cartesian unit vector $\hat{\mathbf{e}}_j$, respectively. For the case $A=B=N$ we find from Table I $f_0^{NN}=5f_Q$ with

$$f_Q\equiv\frac{1}{6}\frac{m_\pi}{f_\pi}\frac{\omega}{\omega-1}\,. \quad (2.15)$$

TABLE I. The $A\leftrightarrow\pi+B$ bare coupling constants f_0^{AB}/f_Q, where f_Q is defined in Eq. (2.15). All blank entries are zero.

A \ B	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*
N	5	$4\sqrt{2}$					
Δ	$2\sqrt{2}$	5					
Λ			0	$2\sqrt{3}$	$2\sqrt{6}$		
Σ			-2	$4\sqrt{6}/3$	$-4\sqrt{3}/3$		
Σ^*			2	$2\sqrt{6}/3$	$2\sqrt{30}/3$		
Ξ						-1	$-2\sqrt{2}$
Ξ^*						2	$\sqrt{5}/3$

Then comparing the vertex (2.14) at zero pion momentum one can show that f_0^{NN} is related to the usual pseudoscalar pion-nucleon coupling constant by

$$f_0^{NN}=3\sqrt{4\pi}fNN_\pi \; .$$

This results in a value of $f_{NN\pi}=0.23$ in comparison to the experimentally determined physical value of 0.28. A correction that brings these numbers closer together is discussed in Sec. VI.

It is worthwhile to discuss the specific values of the bag radius, R, used to compute observables. Nadkarni, Nielson, and Zahed[16] have shown that, in certain idealized (1 + 1)-dimensional models, shifting the position of the bag wall has no effect. The possible implication of this is that predictions in bag models should be insensitive to the value of R. As reviewed in Ref. 1, the perturbative treatment of the cloudy-bag model is valid only for bag radii greater than about 0.8 fm. In this work, we present results for a variety of radii. These demonstrate insensitivity to the specific value of R in the model's region of validity.

III. VOLUME-COUPLING FORM OF THE CBM

As mentioned in the Introduction, the cloudy-bag Lagrangian (2.1) has nonlinear conserved vector and axial-vector currents which satisfy the $SU(2)\times SU(2)$ current algebra of Gell-Mann. The proof of this to $O(f_\pi^{-2})$ is given in Appendix A. This means that all the old current-algebra results for soft pions[13] should also hold in the CBM. In particular the s-wave pion-nucleon low-energy scattering lengths of Weinberg-Tomozawa[9,10] should be derivable. However, in the original cloudy-bag model where quarks are confined to the ground state and recoil of the nucleon is neglected, there can only be p-wave pion scattering. This is because the interaction between quarks and pions conserves angular momentum and parity.

Clearly what one needs to get s-wave scattering in the CBM is to allow quarks into odd-parity bag states, such as encountered when including Z graphs at the quark level.

An easier way around this problem was discovered by Thomas[7] (also see Bartelski and Szymacha[8]) who showed that if one redefines the quark fields in a way similar to Weinberg,[9] a new effective chiral bag Lagrangian is obtained which leads directly to the s-wave scattering formula of Tomozawa and Weinberg.[9,10] We now outline the steps used by Thomas to arrive at his modified Lagrangian.

Start with the usual surface-coupling CBM Lagrangian (2.1), and define new quark fields,

$$q(x)\rightarrow q'(x)=U(x)q(x)\; , \tag{3.1}$$

where

$$U(x)=\exp[i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma^5/2f_\pi]\; . \tag{3.2}$$

Then equation (2.1) becomes

$$\begin{aligned}\mathscr{L}(x)&=(i\bar{q}'U\gamma\cdot\partial U^\dagger q'-B)\theta_V-\tfrac{1}{2}\bar{q}'q'\Delta_S+\tfrac{1}{2}(\mathscr{D}_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi{}^2\boldsymbol{\pi}^2\\&=(i\bar{q}'\gamma\cdot\partial q'-B)\theta_V-\tfrac{1}{2}\bar{q}'q'\Delta_S+\tfrac{1}{2}(\mathscr{D}_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi{}^2\boldsymbol{\pi}^2+i\bar{q}'\gamma^\mu(U\partial_\mu U^\dagger)q'\theta_V\; .\end{aligned} \tag{3.3}$$

If we now reinterpret the primed fields as the physical quarks, then the effect of this transformation is to completely eliminate the surface interaction between quarks and pions and replace it by a new volume coupling interaction. One can show that[7]

$$iU\partial_\mu U^\dagger=\frac{\gamma^5}{2f}\boldsymbol{\tau}\cdot\mathscr{D}_\mu\boldsymbol{\pi}+\left[\frac{\cos(\pi/f)-1}{2}\right]\boldsymbol{\tau}\cdot(\hat{\boldsymbol{\pi}}\times\partial_\mu\hat{\boldsymbol{\pi}})\; , \tag{3.4}$$

which upon substitution into (3.3) results in the modified cloudy-bag Lagrangian

$$\mathscr{L}'_{\rm CBM}=(i\bar{q}\gamma\cdot\partial q-B)\theta_V-\tfrac{1}{2}\bar{q}q\Delta_S+\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi{}^2\boldsymbol{\pi}^2+\frac{1}{2f}\bar{q}\gamma^\mu\gamma^5\boldsymbol{\tau}q\cdot\partial_\mu\boldsymbol{\pi}\theta_V-\frac{1}{4f^2}\bar{q}\boldsymbol{\tau}\gamma^\mu\cdot(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})\theta_V\; , \tag{3.5}$$

where we have dropped the primes on the quark fields and kept only the lowest-order terms in the pion field. The last term in (3.5) gives the Tomozawa-Weinberg result for zero-energy s-wave πN scattering.[9,10]

Equations (2.1) (surface coupling) and (3.5) (volume coupling) are related by a change of variable and differ by a canonical transformation. Thus the two theories must describe the same physics. However, the theories are to be evaluated in perturbation theory, and the result is two different approximations to the same theory. In certain cases[17,18] what is lowest order in one language may be obtained by an infinite class of diagrams in the other. Thus, similar approximation to the different Lagrangians can give different results. The surface-coupling model predicts magnetic moments and charge radii in fairly close agreement with experiment.[3-5] These predictions

TABLE II. A comparison of the contributions to the magnetic moments of the nucleon octet in the surface and volume versions of the CBM. The total pionic, baryonic, and center-of-mass contributions ($\mu_\pi^B, \mu_Q^B, \mu_{\rm c.m.}^B$, respectively) in the surface version are taken from Refs. 3 and 5. The two new contributions arising in volume coupling which are calculated here are $\mu_{Q\pi}^B$ (Fig. 2) and $\delta\mu$ [Fig. 1(b)]. The bag radius $R=1.0$ fm. All values are in nucleon magnetons.

	B					
	p	n	Σ^+	Σ^-	Ξ^0	Ξ^-
$\mu_{\rm expt}$	2.79	−1.91	2.38±0.02	−1.10±0.05	−1.25±0.01	−0.69±0.04
μ_π^B	0.60	−0.60	0.34	−0.34	−0.02	0.02
μ_Q^B	1.74	−1.22	1.73	−0.62	−1.16	−0.54
$\mu_{\rm c.m.}^B$	0.31	−0.22	0.27	−0.09	−0.18	−0.09
$\mu_{\rm CBM}^B$	2.65	−2.04	2.34	−1.05	−1.36	−0.61
$\mu_{Q\pi}^B$	−0.209	0.209	−0.111	0.111	0.0150	−0.0150
$\delta\mu$	0.120	−0.120	0.096	−0.096	−0.02	0.02
$\mu_{\rm CBM}^B+\delta_\mu+\mu_{Q\pi}^B$	2.55	−1.95	2.33	−1.04	−1.36	−0.61

and the experimental values for the magnetic moments are listed in Table II for a bag radius of 1 fm. One would hope that the volume coupling model gives results not too much different from these. Before going on to show that this is indeed the case, we discuss some general features of the modified cloudy-bag Lagrangian given in (3.5).

First notice that in the absence of the pion field, the quark terms are that of the MIT bag. The pionic interactions occur over the entire volume of the bag and are to be treated in a perturbation expansion based on MIT-bag quark wave functions. The absence of surface interactions means that the complicated boundary condition of Eq. (2.2b) is replaced by the simple MIT linear boundary condition $i\hat{\mathbf{n}}\cdot\gamma q=q$. It is therefore legitimate to neglect the influence of the pionic interactions on the quark eigenfrequencies, and the perturbation procedure is greatly simplified. Indeed, the truncation procedure of keeping all three quarks in the ($\kappa=-1$) state seems to be far better justified in the volume coupling version, since the WT term appears immediately in Eq. (3.5). To obtain this term in the surface coupling version one must break the truncation by including intermediate quarks in negative-energy states (Z graphs at the quark level), as found by Guichon,[18] and by Jennings and Maxwell.[17]

Another nice feature of (3.5) is that the lowest-order quark-pion interaction term results in the same πAB vertex [Eq. (2.14)] as the surface coupling model if the quarks are kept in their ground-state orbitals. For a proof of this see Appendix B. This means that the physical productions of both versions of the CBM that depend only on the lowest oder pion-baryon vertex are the same. So, for example, the physical baryon wave function and the baryon self-energies are identical to lowest order in both models. However, due to the presence of the derivative coupling in the interaction term of the modified Lagrangian, there is an additional contribution to the electromagnetic current absent from the surface-coupling model.

IV. THE CORRECTION TO THE BARYON MAGNETIC MOMENTS

Introduction of the electromagnetic field $A^\mu(x)$ into the cloudy-bag model is accomplished by minimal substitution into Eq. (3.5):

$$\mathscr{L}'_{\rm CBM,EM}(x)=[\overline{iq}\gamma\cdot(\partial-ieQA)q-B]\theta_V-\tfrac{1}{2}\bar{q}q\Delta_S-\tfrac{1}{2}m_\pi{}^2\pi_3{}^2-m_\pi{}^2\phi^\dagger\phi+\tfrac{1}{2}(\partial_\mu\pi_3)^2+[(\partial^\mu-ieA^\mu)\phi^\dagger][(\partial_\mu+ieA_\mu)\phi]$$
$$+\frac{\theta_V}{2f_\pi}[\bar{q}\gamma^\mu\gamma^5\tau_3 q\partial_\mu\pi_3-\bar{q}\gamma^\mu\gamma^5\tau_+q(\partial_\mu-ieA_\mu)\phi^\dagger+\bar{q}\gamma^\mu\gamma^5\tau_-q(\partial_\mu+ieA_\mu)\phi]-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \qquad (4.1)$$

where $F_{\mu\nu}$ is the electromagnetic field tensor,

$$\phi\equiv\frac{1}{\sqrt{2}}(\pi_1(x)+i\pi_2(x))$$

is the field that destroys negatively charged pions, Q is the quark charge matrix, and e the charge on the proton. Also, the subscript ± stands for the usual spherical components of a three-vector, $V_\pm\equiv\mp(V_1\pm iV_2)/\sqrt{2}$. The Lagrangian is invariant under the infinitesimal local gauge transformation

$$q(x)\to q'(x)=q(x)-i\delta\theta(x)eQq(x)\,,$$
$$\phi(x)\to\phi'(x)=\phi(x)+i\delta\theta(x)e\phi(x)\,, \qquad (4.2)$$
$$A^\mu(x)\to A^{\mu\prime}(x)=A^\mu(x)-\frac{1}{e}\partial^\mu\delta\theta(x)\,.$$

The global version of this transformation results in a conserved electromagnetic current:

$$j^{\mu}_{CBM'}(x)=\sum_{\{\phi_r\}}\frac{\partial\mathscr{L}'\delta\phi_r}{\partial(\partial_\mu\phi_r)\delta\theta}$$

$$=j^{\mu}_{\rm CBM}(x)+j^{\mu}_{Q\pi}(x)\ . \qquad (4.3)$$

Here $j^{\mu}_{\rm CBM}$ has the same form as that of the surface-coupling model, namely,

$$j^{\mu}_{\rm CBM}(x)=j^{\mu}_{Q}(x)+j^{\mu}_{\pi}(x)\ , \qquad (4.4)$$

where the quark and pion currents are

$$j^{\mu}_{Q}(x)=e\bar{q}(x)\gamma^{\mu}Qq(x)\theta_V\ , \qquad (4.4a)$$

$$j^{\mu}_{\pi}(x)=-ie[\phi^{\dagger}(x)\partial^{\mu}\phi(x)-\phi(x)\partial^{\mu}\phi^{\dagger}(x)]\ . \qquad (4.4b)$$

The additional current involves both the quark and pion fields:

$$j^{\mu}_{Q\pi}(x)=\left[\frac{-e}{2if_\pi}\right]\bar{q}(x)\gamma^{\mu}\gamma^{5}(\tau_+\phi^{\dagger}(x)+\tau_-\phi(x))q(x)\theta_V\ . \qquad (4.5)$$

The lowest-order pion-nucleon interaction is the same in both models so we can use the same interaction Hamiltonian, and therefore the same physical nucleon expansion to calculate the lowest-order contributions to the magnetic moment and charge radii in the volume coupling model. Hence contributions due to $j^{\mu}_{\rm CBM}$ are identical to those calculated by Théberge, Thomas, and Miller.[3,5] All that remains is to calculate the additional contribution due to the new current in Eq. (4.5).

First consider the contribution of $j^{\mu}_{Q\pi}$ to the nucleon charge radius:

$$\langle r^2\rangle_{N,Q\pi}=\left\langle N\left|\int d^3r\, r^2 j^{0}_{Q\pi}(\mathbf{r})\right|N\right\rangle\ .$$

From (4.5) we find we need $q_0^{\dagger}(x)\gamma^5\tau_{\pm}q_0(x)$, where q_0 is the quark ground-state orbital given in Eq. (2.10). The γ^5 operator is odd under the parity transformation, so this quantity *vanishes* and hence the baryonic charge radii are identical in the two models to lowest order.

Turning now to the magnitude moments, we first note that the quantities $q_0^{\dagger}(x)\gamma^0\gamma\gamma^5\tau_{\pm}q_0(x)$ are not zero, and

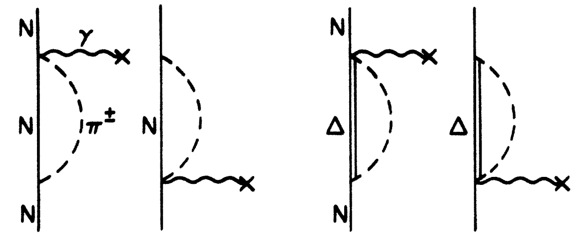

FIG. 2. Graphical representation of the new contributions to the nucleon magnetic moment in the volume-coupling CBM.

so there is a calculation to do. The new contribution to the nucleon magnetic moment is

$$\mu^{N}_{Q\pi}=\langle N\uparrow|\boldsymbol{\mu}_{Q\pi}\cdot\hat{\mathbf{z}}|N\uparrow\rangle\ , \qquad (4.6)$$

where

$$\boldsymbol{\mu}_{Q\pi}=\tfrac{1}{2}\int d^3r\,\mathbf{r}\times\mathbf{j}_{Q\pi}(\mathbf{r})$$

with $j_{Q\pi}$ given in (4.5). $|N\uparrow\rangle$ is the spin-up physical nucleon. To lowest order in the πN coupling there are two terms

$$\mu^{N}_{Q\pi}=Z_N\langle N_0\uparrow|\boldsymbol{\mu}_{Q\pi}\cdot\hat{\mathbf{z}}\tilde{G}_0(m_N)\Lambda_N\tilde{H}_1|N_0\uparrow\rangle+\text{c.c.} \qquad (4.7)$$

which can be represented graphically as in Fig. 2. Inserting a complete set of states we have

$$\mu^{N}_{Q\pi}=2Z_N\sum_{A\mathbf{k}_j}\langle N_0\uparrow|\boldsymbol{\mu}_{Q\pi}\cdot\hat{\mathbf{z}}|A_0\mathbf{k}j\rangle\times\langle A_0\mathbf{k}j\,|\,\tilde{G}_0(m_N)H_I\,|N_0\uparrow\rangle\ . \qquad (4.8)$$

Equation (2.13) gives

$$\langle A_0\mathbf{k}_j\,|\,\tilde{G}_0(m_N)H_I\,|N_0\uparrow\rangle=\frac{-w^{AN}_{\mathbf{k}j}}{\omega_{AN}+\omega_k}$$

with $\omega_{AB}\equiv m_A-m_B$.

To evaluate the matrix element of the magnetic-moment operator, first insert the expression (2.7) for the pion field into (4.5) to obtain

$$\boldsymbol{\mu}_{Q\pi}\cdot\hat{\mathbf{z}}=\frac{1}{2}\left[\frac{-e}{2f_\pi}\right]\epsilon^{il3}\sum_{\mathbf{p}}\int r^i\sum_a\bar{q}_a(\mathbf{r})\gamma^l\gamma^5(\tau_1a_{\mathbf{p}2}-\tau_2a_{\mathbf{p}1})e^{i\mathbf{p}\cdot\mathbf{r}}q_a(\mathbf{r})\theta_V d^3r \qquad (4.9)$$

with the sum over quarks now given explicitly. After substitution of the quark wave functions one finds

$$\langle N_0\uparrow|\boldsymbol{\mu}_{Q\pi}\cdot\hat{\mathbf{z}}|A_0\mathbf{k}_j\rangle=\frac{-e}{4f_\pi}\frac{{N_{1,-1}}^2}{4\pi}\frac{\epsilon^{il3}}{[(2\pi)^3 2\omega_k]^{1/2}}\left\langle N_0\uparrow\left|\sum_a\chi_a^{\dagger}S^{il}(\mathbf{k})(\tau_1\delta_{j,2}-\tau_2\delta_{j,1})\chi_a\right|A_0\right\rangle \qquad (4.10)$$

with

$$S^{il}(\mathbf{k})=\int r^i e^{i\mathbf{k}\cdot\mathbf{r}}\left[j_0{}^2\left[\frac{\omega r}{R}\right]\sigma^l+j_1{}^2\left[\frac{\omega r}{R}\right]\sigma\cdot\mathbf{r}\sigma^l\sigma\cdot\mathbf{r}\right]\theta_V d^e r\ .$$

The angular integration yields

$$(S^{12}-S^{21})(\mathbf{k})=4\pi i(\hat{\mathbf{k}}\times\sigma)\cdot\hat{\mathbf{z}}f(k) \qquad (4.11)$$

with

$$f(k)=\int_0^R r^3(j_0{}^2-j_1{}^2)\left[\frac{\omega r}{R}\right]j_1(kr)dr\ .$$

The main reason why this contribution to the magnetic moments is small is that $j_0^2 - j_1^2 = 0$ at $r = R$, and the integrand goes as r^3.

Inserting (4.10) into (4.8), use the expression (2.14) for the pion creation vertex $w_{kj}^{AN} = v_{kj}^{NA*}$, perform the angular integration, and sum over j. This gives

$$\mu_{Q\pi}^{N} = \sum_A \mu_{Q\pi}^{NA} \tag{4.12}$$

with

$$\mu_{Q\pi}^{NA} = -\left[\frac{eZ_N N_{1,-1}{}^2 f_Q}{54\pi^2 f_\pi m_\pi}\right]\left[\frac{f^{NA}}{f_Q}\right]^2 \times(\delta_{S_A,1/2} - \tfrac{1}{2}\delta_{S_A,3/2})(\delta_{T_A,1/2} - \tfrac{1}{2}\delta_{T_A,3/2}) \times \int_0^\infty \frac{k^3 u(kR) f(k)}{\omega_k(\omega_k + \omega_{AN})} dk , \tag{4.13}$$

the contribution due to the intermediate baryon A. For the nucleon intermediate state $\omega_{AN} = 0$ and the k integration can be done analytically. The result is

$$\mu_{Q\pi}^{NN} = -\left[\frac{mN}{f_\pi^2 R}\right] Z_N \left[\frac{25}{432\pi}\right]\left[\frac{\omega^4}{(\omega-1)^2 \sin^2\omega}\right] \times \left[\frac{1+\alpha}{\alpha^3}\right] e^{-\alpha} I(\alpha) \mu_N , \tag{4.14}$$

where $\mu_N = e/2m_N$ is the nuclear Bohr magneton, $\alpha = m_\pi R$, and

$$I(\alpha) = \int_{-1}^{1} \xi (j_0^2 - j_1^2)(\omega\xi)(1 + \xi\alpha) e^{-\alpha\xi} d\xi$$

must be calculated numerically. This result has the same functional form as the calculation of Bartelski and Szymacha.[8] However, they treat the pion classically and therefore do not handle the intermediate state energy denominators and wave-function renormalization properly.

The generalization of (4.12) to the other long-lived baryons is simply

$$\mu_{Q\pi}^{B} = \sum_A \mu_{Q\pi}^{BA}$$

with

$$\mu_{Q\pi}^{BA} = -Z_B \left[\frac{m_N \hbar c}{18\pi f_\pi^2}\right]\left[\frac{\omega}{\omega - 1}\right]\left[\frac{f^{BA}}{f_Q}\right]^2 s_B(A) t_B(A) \xi^{BA} \tag{4.15}$$

in nuclear magnetons. Here the isospin factor is[15,19]

$$t_B(A) = [\langle T_A(t_B - 1)1 + 1 \mid T_B t_B \rangle^2 - \langle T_A(t_B + 1)1 - 1 \mid T_B t_B \rangle^2] \tag{4.16}$$

with the spin factor $s_B(A)$ given analogously ($T \to S$, $t \to s$) and

$$\xi^{BA} = \frac{N_{1,-1}{}^2}{4\pi} \int_0^\infty \frac{k^3 f(k) u(kR)}{\omega_k(\omega_k + \omega_{AB})} dk \tag{4.17}$$

in units of fm^{-1}. Note that we use massless quark wave functions in $f(k)$ since the pions do not couple to strange quarks at the photon vertex.

The bare-bag probability is obtained from the normalization condition on the physical baryon, $\langle B \mid B \rangle = 1$, which is

$$Z_B(R) = \left[1 + \sum_A \left[\frac{f^{BA}}{m_\pi}\right]^2 \frac{1}{12\pi^2} \times \int_0^\infty \frac{k^4 u^2(kR)}{\omega_k(\omega_k + \omega_{AB})^2} dk\right]^{-1} .$$

The results for $\mu_{Q\pi}^{B}$ with $B = n, \Sigma^-, \Xi^0$ are plotted as a function of bag radius in Fig. 3. The double integrals ξ^{BA} are done numerically with the standard values for the baryon masses.[3] The other moments are given by

$$\mu_{Q\pi}^{p} = -\mu_{Q\pi}^{n},\ \mu_{Q\pi}^{\Sigma^+} = -\mu_{Q\pi}^{\Sigma^-} ,$$

$$\mu_{Q\pi}^{\Xi^-} = -\mu_{Q\pi}^{\Xi^0},\ \mu_{Q\pi}^{\Lambda} = 0 ,$$

as a consequence of the isovector nature of this term.

The results of these computations are summarized in Table II. But there are terms other than $j_{Q\pi}$ that must be included. These are discussed in Sec. V and included in Table II in the row $\delta\mu$.

As can be seen from the table, the old surface-coupling CBM predictions are quite good, differing from the experimentally measured values by roughly 5–10%. They have actually improved since publication of Ref. 3 because new measurements of μ^{Σ^-} obtained from fine-structure splitting in Σ^- exotic atoms have brought the value much closer to the CBM prediction.[20] The basic result of adding $\mu_{Q\pi}^{B}$ is to lower the total pionic contribution to the magnetic moments. This effect is almost canceled by the terms of Fig. 1(b) (see Sec. V). The new results in the last row of Table II are not distinguishable from those of Ref. 3 and the good agreement with experiment is maintained. Théberge and Thomas include a center-of-mass correction to the quark term due to Donoghue and Johnson[21] but other prescriptions exist and currently there is no consensus on how to make these corrections—see Sec. IV of Ref. 3 for a discussion of the ambiguities. Un-

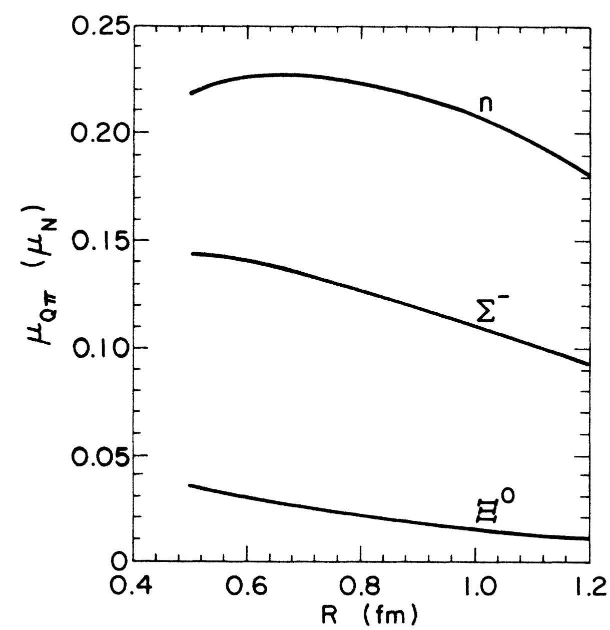

FIG. 3. The baryon-magnetic-moment corrections (μ_{QN}), as associated with the new processes shown in Fig. 2, which arise in the volume coupling CBM, are shown as a function of R.

til better center-of-mass corrections are available and other effects such as recoil and nonstatic corrections are included, one can say that both the surface and volume coupling cloudy-bag models give good predictions for the electromagnetic properties of the baryons.

V. ρ-MESON-LIKE EXCHANGES

The Weinberg-Tomozawa term ($H_{\rm WT}$) allows for the emission of two pions which together carry the quantum numbers of the ρ meson. This virtual "ρ meson" can convert into a photon. There is then another contribution to the expectation value of $j^\mu(x)$, as shown in Fig. 1(b).

The appearance of a closed pion loop indicates that the virtual pions can have quite high momentum. Such terms are beyond the scope of the cloudy-bag model which uses a long-wavelength approximation. Here such a treatment is not justified, since that approximation leads to infinite values. However, we shall show that including the effects of the pions' small but finite extent leads to finite (and small) values for the terms of Fig. 1(b).

Turn now to the evaluation of the nucleonic expectation value of the electromagnetic current $\hat{j^\mu}$ for the terms of Fig. 1(b). This is $\langle \delta j^\mu \rangle$, with

$$\langle \delta j^\mu(\mathbf{r}) \rangle = \sum_{lm} 2 \int d^3p\, d^3k \frac{\langle N \mid H_{\rm WT} \mid N, \mathbf{p}(m)\mathbf{k}(l) \rangle}{-(\omega_p+\omega_k)} \times \langle N, \mathbf{p}(m), \mathbf{k}(l) \mid j^\mu_\pi(\mathbf{r}) \mid N \rangle , \tag{5.1}$$

where $\mathbf{p}(m), \mathbf{k}(l)$ are the quantum numbers of the intermediate pions. The factor of 2 is inserted to include the term in which the order of $H_{\rm WT}$ and j^μ is reversed.

The expression for j^μ_π in terms of pion creation and destruction operators has been given in many places (e.g., Refs. 1, 3, and 5). Here we relate the term relevant for the present computation:

$$j^\mu_\pi(\mathbf{r}) \Longrightarrow \frac{ie}{2} \sum_{jj'} \epsilon_{jj'3} \int \frac{d^3Q\, d^3k\, e^{i(\mathbf{k}-\mathbf{Q})\cdot\mathbf{r}}}{(2\pi)^3(\omega_Q\omega_k)^{1/2}} \times k^\mu a_{j'}(-\mathbf{Q}) a_j(\mathbf{k}) \tag{5.2}$$

with $g^{00}=1$, $g^{ii}=-1$. The relevant piece of the Weinberg-Tomozawa term

$$\mathscr{H}^{(x)}_{\rm WT} = \frac{\bar{q}(x)}{4f^2} \tau\gamma_\mu q(x)\cdot(\boldsymbol{\phi}\times\partial^\mu\boldsymbol{\phi})(x) , \tag{5.3}$$

is obtained by using the π mode sum (2.7) along with Eq. (2.10) to describe the quarks. That procedure gives

$$\mathscr{H}^{++}_{\rm WT} \Longrightarrow \frac{+1}{4f^2} \int \frac{d^3p\, d^3k\, \epsilon_{lmk}}{\sqrt{2\omega_k}\sqrt{2\omega_p}(2\pi)^3} a^\dagger_l(\mathbf{k}) a^\dagger_m(\mathbf{p}) e^{-i(\mathbf{p}+\mathbf{k})\cdot\mathbf{x}} \tau_k [i\omega_p \rho(x) - i\mathbf{p}\cdot\mathbf{m}(\mathbf{x})] . \tag{5.4}$$

The term $\mathscr{H}^{++}_{\rm WT}$ is the one in which two pions are created. Its adjoint destroys two pions. The functions $\rho(x)$ and $\mathbf{m}(\mathbf{x})$ are given by

$$\rho(x) = \frac{{N_{1,-1}}^2}{4\pi} \left[{j_0}^2 \left[\frac{\omega x}{R} \right] + {j_1}^2 \left[\frac{\omega x}{R} \right] \right], \quad \mathbf{m}(\mathbf{x}) = \frac{{N_{1,-1}}^2}{4\pi} 2(\boldsymbol{\sigma}\times\hat{\mathbf{x}}) j_0 \left[\frac{\omega x}{R} \right] j_1 \left[\frac{\omega x}{R} \right] \equiv \mu(x)(\boldsymbol{\sigma}\times\hat{\mathbf{x}}) . \tag{5.5}$$

It is worthwhile to compute the corrections to charge radii and magnetic moments separately. Thus first choose $\mu=0$. The use of (5.2) and (5.4) in (5.1) gives

$$\left\langle N \middle| \int d^3r\, \delta j^0(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} \middle| N \right\rangle = \frac{-e}{8(2\pi)^3 f^2} \left\langle N \middle| \sum_{a=1}^{3} \tau_3^{(a)} \middle| N \right\rangle \int d^3x\, \rho(x) e^{-i\mathbf{q}\cdot\mathbf{x}} \int \frac{d^3p}{\omega_p} \frac{(\omega(|\mathbf{p}+\mathbf{q}|)-\omega(p))^2}{\omega(|\mathbf{p}+\mathbf{q}|)(\omega(|\mathbf{p}+\mathbf{q}|)+\omega(p))} . \tag{5.6}$$

The evaluation of the change in the mean square charge radii $\langle \delta r^2 \rangle$ is our concern here. Thus we take the small q^2 limit and obtain the coefficient of the leading term. It is

$$\langle N \mid \delta r^2 \mid N \rangle = \frac{e}{8(2\pi)^3 f^2} \left\langle N \middle| \sum_a \tau_3^{(a)} \middle| N \right\rangle \times \int d^3p \frac{p^2}{{\omega_p}^5} . \tag{5.7}$$

As expected, loop integrals lead to divergent terms; in this case there is a logarithmic divergence. To evaluate the integral, the influence of finite size of the π in the quadratic pion interaction terms must be included. This is a difficult problem, since the two pions are far from the mass shell. All we can do is make a rough guess at how the finite size enters. One way is to assume a nonlocal pion quark coupling.[22] That is, make the replacement

$$\boldsymbol{\pi}(\mathbf{x}) \rightarrow \int d^3x' \boldsymbol{\pi}(\mathbf{x}') f(|\mathbf{x}-\mathbf{x}'|) . \tag{5.8a}$$

in *the quadratic pion interaction terms.* For those terms the pion destruction operator $a_j(\mathbf{p})$ is replaced by $a_j(\mathbf{p})\tilde{f}(|\mathbf{p}|)$ where $\tilde{f}$ is the Fourier transform of f. The substitution (5.8a) renders loop integrals finite, but does not affect (by definition) the simple pion quark interaction.

To proceed one must specify f. From (5.8) we see that $\tilde{f}$ is roughly related to the pion's internal wave function. We use the simple form

$$\tilde{f}(p)=e^{-p^2R_\pi{}^2/12} \tag{5.8b}$$

with the expectation that R_π is the relevant mean square radius of the pion. [We use a $-p^2R_\pi{}^2/12$ argument since it is $\tilde{f}^2(p)$ that would enter into π-electron scattering.]

The pion's rms charge radius is 0.66 fm, but the volume occupied by the $q\bar{q}$ valence pair is probably much smaller. For example, Brodsky and Lapage[23] and others[24] have shown $R_\pi \lesssim 0.4$ fm. Furthermore that small value of R_π is consistent[25] with the notion of using dynamical rescaling to relate the nucleon and pion's structure functions.

The volume occupied by the $q\bar{q}$ pair is more relevant for computing the terms of Fig. 1(b), since that component of the pion's Fock-space wave function would dominate the calculations of the necessary couplings.

There are two places where quadratic π terms enter in the evaluation of Fig. 1(b). Thus the expression (5.7) is replaced by

$$\langle N\,|\,\delta r^2\,|\,N\rangle=\frac{e}{16\pi^2f^2}\int\frac{p^4dp}{(p^2+m_\pi{}^2)^{5/2}}e^{-p^2R_\pi{}^2/3}\times\left\langle N\left|\sum_{a=1}^{3}\tau_3^{(a)}\right|N\right\rangle . \tag{5.9}$$

TABLE III. Change δr^2 in proton mean charge squared radius caused by the terms of Fig. 1(b). The value $m_\pi R_\pi=0.28$ corresponds to the favored value of $R_\pi=0.4$ fm.

$m_\pi R_\pi$	δr^2 (fm^2)
0.24	0.032
0.28	0.029
0.32	0.026
0.36	0.023
0.40	0.021
0.44	0.019
0.48	0.017

A final manipulation is the replacement of $f=93$ MeV by the value necessary to reproduce the physical pion nucleon coupling constant. This multiplies the right-hand side of (5.9) by 1.48. See Ref. 3.

The results of the numerical evaluation of Eq. (5.9) are given in Table III. This effect is very small. The two terms of (5.9) have opposite signs. Even so, a vanishing value of R_π would lead to an infinite result.

Now turn to the magnetic-moment contribution, $\langle\delta\boldsymbol{\mu}\rangle$,

$$\langle\delta\boldsymbol{\mu}\rangle=\tfrac{1}{2}\int d^3r\,\mathbf{r}\times\langle\mathbf{j}(\mathbf{r})\rangle \ . \tag{5.10}$$

Using (5.2), (5.4), and (5.5) in (5.10) gives

$$\langle\delta\boldsymbol{\mu}\rangle=\frac{-(1.48)}{(2\pi)^6}\frac{e}{f^2 16}\int\frac{d^3x\,d^3r\,d^3k\,d^3p}{\omega_k\omega_p}e^{i(\mathbf{k}+\mathbf{p})\cdot(\mathbf{r}-\mathbf{x})}\sum_{a=1}^{3}\langle\tau_3^{(a)}\mu(x)\sigma^{(a)}\rangle\frac{\mathbf{r}\times(\mathbf{k}-\mathbf{p})}{\omega_k+\omega_p} , \tag{5.11}$$

with the "renormalization" factor of 1.48 included.

The integrals of (5.11) are evaluated by first replacing the variable $\mathbf{k}$ by $\mathbf{P}=\mathbf{p}+\mathbf{k}$ and integrating over $\mathbf{r}$. This gives

$$\langle\delta\boldsymbol{\mu}\rangle=\frac{e}{(2\pi)^3}\frac{1.48}{16f^2}\int d^3x\,\mu(x)\int d^3P\,d^3p\frac{\sum_a\langle\tau_3^{(a)}\sigma^{(a)}\cdot[\hat{\mathbf{x}}\times(2\mathbf{p}-\mathbf{P})]\rangle}{\omega_p\omega_{\mathbf{P}-\mathbf{p}}(\omega_{\mathbf{P}-\mathbf{p}}+\omega_p)}i\nabla_p\delta^{(3)}(\mathbf{p})\times(\mathbf{P}-2\mathbf{p}) \ . \tag{5.12}$$

Proceed by doing the integration of ∇_P by parts. Only the term $\nabla_P e^{-iP\cdot x}$ can contribute to the integral on $\mathbf{x}$. Then one has

$$\langle\delta\mu_z\rangle=\frac{1.48e}{8f^2(2\pi)^3}\int d^3x\,\mu(x)\int\frac{d^3p}{\omega_p{}^3}\mathbf{p}\cdot\sum_a\langle\tau_3^{(a)}(\sigma^{(a)}\times\hat{\mathbf{x}})\rangle(\mathbf{x}\times\mathbf{p})^3 \ . \tag{5.13}$$

This yields the result

$$\langle\delta\mu_z\rangle=\frac{5(1.48)}{216\pi^2f^2}\mu_0\int\frac{p^4dp}{\omega_p{}^3} \tag{5.14}$$

for the proton. Here

$$\mu_0=e\int d^3x\,x\mu(x)=1.21m_NR\mu_N \ .$$

Once again there is a divergent integral. This is remedied by the insertion of $u^4(Q)$, see (5.8) and (5.9). Thus, e.g.,

$$\delta\mu_z\equiv\langle P\uparrow\,|\,\delta\mu_z\,|\,P\uparrow\rangle$$

$$=\frac{5(1.48)}{216\pi^2}\frac{\mu_0}{f^2}\int_0^\infty\frac{p^4}{(p^2+m_\pi{}^2)^{3/2}}e^{-p^2R^2/3}dp \tag{5.15}$$

for the proton matrix element. The results of a numerical evaluation are shown in Table IV. For $m_\pi R_\pi=0.28$, and $R=1.0$ fm, $\delta\mu_p=0.12\mu_N$. There is not much sensitivity to the value of R_π.

It is noteworthy that this term $\delta\mu_p$ has about the same magnitude and opposite sign as $\mu_{Q\pi}^B$. The sum $\mu_{e\pi}^B+\delta\mu_p$ is very small, as shown in Table II.

One can also compute $\delta\mu_z$ and $\delta\mu_\Xi$. To do this use

$$\left\langle\Sigma^+\left|\sum_{a=1}^{3}\sigma_3^{(a)}\tau_3^{(a)}\right|\Sigma^+\right\rangle=\tfrac{4}{3}$$

and

$$\left\langle\Xi^0\left|\sum_{a=1}^{3}(\sigma_3\tau_3)_a\right|\Xi^0\right\rangle=-\tfrac{1}{3} \ .$$

TABLE IV. Change in proton magnetic moment (in nuclear magnetons) caused by the terms of Fig. 1(b).

$m_\pi R_\pi$	$\delta\mu$
0.24	0.123
0.28	0.120
0.32	0.117
0.36	0.114
0.40	0.110
0.44	0.107
0.48	0.104

The specific values of Table II are obtained with $R=1.0$ fm and $R_\pi=0.4$ fm.

Another question concerns the influence of the substitution, (5.8b), on the previously calculated pion cloud terms (μ_π^B) of the CBM. This effect is zero, since (5.8b) is to be made only on the quadratic terms. The numerical result is a 5% reduction of μ_π^B. This can be compensated by a slight change in the bag radius.

In our treatment of the electromagnetic properties the anomaly terms, generated by the Wess-Zumino term[26] in the usual nonlinear σ model, are not included. Such terms should have only a small influence here. For example, the pion—two-photon term involves an extra photon and must be small. The photon—three-pion term is of order $1/f^6$, a much higher order than any of the included terms. We expect that it is small. The anomaly terms generated by the Wess-Zumino term play a significant role in other theories, e.g., Ref. 15.

The net result of all of this is that the original CBM computations of the baryon magnetic moments are virtually unchanged.

VI. CORRECTION TO g_A

The Weinberg-Tomozawa term for pion-nucleon scattering is expected to play a role in the computation of g_A. As shown by Adler and Weisberger

$$1=g_A{}^2+\frac{2}{\pi}f^2\int_{m_\pi}^{\infty}|\mathbf{q}|\frac{d\nu}{\nu}[\sigma_{\rm tot}^{(-)}(\nu)-\sigma_{\rm tot}^{(+)}(\nu)]\ , \tag{6.1}$$

where $|\mathbf{q}|=(\nu^2-m_\pi{}^2)^{1/2}$ and $\sigma_{\rm tot}^{(\pm)}(\nu)$ are total cross sections for $\pi^\pm p$ scattering. Any term contributing to the difference $\sigma_{\rm tot}^{(-)}(\nu)-\sigma_{\rm tot}^{(+)}(\nu)$ will change the integral of (6.1). Since the Weinberg-Tomozawa term is of an isovector form, g_A will be influenced.

To find the modification to g_A it is necessary to construct the conserved axial-vector current A_μ. Consider the Lagrangian density of (3.5). It is easy to show that $\delta\mathcal{L}'_{\rm CBM}=0$ (with $m_\pi=0$) under the infinitesimal local gauge transformation:

$$q\to q-\frac{i\tau}{4f}\cdot(\epsilon\times\phi)q,\ \ \phi\to\phi+\epsilon f\ . \tag{6.2}$$

The corresponding conserved axial-vector current $\mathbf{A}_\mu$ is given to first order in ϕ by the relation

$$\mathbf{A}_\mu=\bar q\tau\gamma_\mu\gamma_5 q+f\partial_\mu\phi+\frac{-\bar q\gamma_\mu\tau q}{2f}\times\phi\ . \tag{6.3}$$

The third term of (6.3) is a new one, define it to be $\delta\mathbf{A}_\mu$. The computation of the corresponding change δg_A in g_A requires evaluating the nucleonic expectation value of

$$\delta\mathbf{A}=\frac{-\bar q\gamma\tau q}{2f}\times\phi\ . \tag{6.4}$$

This term is depicted in Fig. 1(c). One then gets an additional contribution to the axial-vector coupling constant:

$$\delta g_A=\int d^3x\langle\tilde p\uparrow|2\delta A_{z3}|\tilde p\uparrow\rangle\ , \tag{6.5}$$

where $|\tilde p\uparrow\rangle$ is the physical proton wave function (with spin up). The influence of virtual pions is included in $|\tilde p\uparrow\rangle$ so that the expectation value of $\delta\mathbf{A}$ does not vanish.

To evaluate (6.5) use the pion mode expansion (2.8) and the $\kappa=-1$ MIT quark wave function. The relation

$$a_m(\mathbf{k})|\tilde p\uparrow\rangle=-\frac{1}{\omega_k+H}v_{\mathbf{k}m}^\dagger|\tilde p\uparrow\rangle\ , \tag{6.6}$$

where

$$v_{km}^\dagger=-[H_I,a_m(\mathbf{k})]\ , \tag{6.7}$$

is also employed [see (2.13) and (2.14)]. To obtain a result in the lowest nonvanishing order include only the linear pion-quark term in H_I. A standard calculation gives

$$\delta g_A=\frac{1}{f^2}\frac{3}{5}g_A(Z_NZ_1)^2\sum_{nm}\epsilon_{nm3}\int\frac{d^3k\,u(kR)}{2\omega_k(2\pi)^3}kF(k)\times\sum_\alpha\sum_{i,j=1}^{3}\left[\left\langle p\uparrow\left|(\sigma_i\times\hat{\mathbf{k}})_z\tau_{in}\frac{|\alpha\rangle\langle\alpha|\sigma_j\cdot\mathbf{k}}{\omega_k+(E_\alpha-M_N)}\tau_{jm}\right|p\uparrow\right\rangle+\frac{\langle p\uparrow|\tau_{jm}\sigma_j\cdot\mathbf{k}|\alpha\rangle\langle\alpha|(\sigma_i\times\hat{\mathbf{k}})_z\tau_{in}|p\uparrow\rangle}{\omega_k+(E_\alpha-M_N)}\right]\ . \tag{6.8}$$

Here g_A is the MIT-bag-model value of $g_A=\frac{5}{9}\omega/(\omega-1)=1.09$ and $|\alpha\rangle$ is the intermediate baryon state, a nucleon or Δ, of energy M_N or $M_N+\omega_0$ ($\omega_0=297$ MeV). Both $|p\rangle$ and $|\alpha\rangle$ are bare states: the wave function Z_N and vertex Z_1 renormalization are then required.

The function $F(k)$ is

$$F(k)=N^2\int_0^R dr\,r^2j_0\left[\frac{\omega r}{R}\right]j_1\left[\frac{\omega r}{R}\right]j_1(kr)\ . \tag{6.9}$$

As usual $u(R)=3j_1(kR)/kR$.

To simplify the algebra we use a closure approximation and replace $E_\alpha-M_N$ by 0. This is a reasonable approximation. Then one can find that

$$\delta g_A = \frac{-g_A\langle\ \rangle}{10\pi^2 f^2}\int_0^\infty dk \frac{k^3 F(k) u(k)}{\omega_k \omega_k}, \quad (6.10)$$

where

$$\langle\ \rangle = \sum_{i,j=1}^{3} \langle p\uparrow | (\sigma_i \times \sigma_j)_z (\tau_i \times \tau_j)_z | p\uparrow\rangle = -\tfrac{44}{3}. \quad (6.11)$$

The net result is

$$\frac{\delta g_A}{g_A} = \frac{22}{15}\frac{(Z_N Z_1)^2}{\pi^2 f^2}\int_0^\infty dk \frac{k^3 F(k) u(k)}{\omega_k \omega_k}. \quad (6.12)$$

The $(Z_N Z_1)^2$ factor is taken from Refs. 3 or 18. The values of $Z_N Z_1$ range smoothly from 0.86 for $R=0.7$ fm to 0.93 for $R=1.1$ fm.

The numerical results for $\delta g_A/g_A$ are shown in Table V. The bag contribution of 1.09 must be multiplied by $Z_N Z_1$ to account for that fraction of the time when there is no pion Z_N or one pion $(Z_1-1)Z_N$. Thus 1.09 is multiplied by $Z_N Z_1$ and the result is shown in the second column of Table V. The agreement with the experimental value of 1.26 is rather good for bag radii between 0.9 and 1.1 fm.

The computed pion nucleon coupling constant increases from 0.23 to 0.27, which is quite close to the experimental value of 0.28, as a result of including g_A. Of course, center-of-mass corrections to $g_A Z_N Z_1$ could add another 0.1 or so to g_A (Ref. 27).

VII. REMARKS

The newer, volume-coupling version of the CBM has several formal and practical advantages. The linear boundary condition on the quark wave functions is independent of the pion field operators. The Weinberg-Tomozawa term is included automatically; there is no need to sum over antiquark states. The Lagrangian contains new electromagnetic interactions, Figs. 1(a) and 1(b), but the influence of these on computed charge radii and magnetic moments is very small. The inclusion of the Weinberg-Tomozawa term leads to a new axial-vector interaction, Fig. 1(c). For a nucleon bag of radius 1 fm, the computed value of g_A is in excellent agreement with the experimental one.

Finally we note that this model has also been applied with considerable success to pion photoproduction[28] as well as to KN (Ref. 29) and $\bar{K}N$ (Ref. 30) scattering.

TABLE V. Contributions to g_A.

R (fm)	g_A [Eq. (6.12)]	$g_A^B Z_N Z_1$	g_A
0.7	0.46	0.94	1.40
0.8	0.38	0.96	1.34
0.9	0.30	0.98	1.28
1.0	0.25	1.00	1.25
1.1	0.21	1.01	1.22

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APPENDIX A: PROOF THAT THE CBM SATISFIES CURRENT ALGEBRA

In this appendix we show that the components of the vector and axial-vector currents given in Eqs. (2.3) and (2.4) satisfy the SU(2)×SU(2) current-algebra equal-time commutation relations

$$[V_i^0, V_j^0] = i\delta^3(\mathbf{x}-\mathbf{y})\epsilon^{ijk} V_k^0(\mathbf{x}), \quad (A1)$$

$$[A_i^0, A_j^0] = i\delta^3(\mathbf{x}-\mathbf{y})\epsilon^{ijk} V_k^0(\mathbf{x}), \quad (A2)$$

$$[V_i^0, A_j^0] = i\delta^3(\mathbf{x}-\mathbf{y})\epsilon^{ijk} A_k^0(\mathbf{x}), \quad (A3)$$

to the first nontrivial order in ${f_\pi}^{-1}$. The notation employed throughout this appendix for commutators is that the first argument occurs at $\mathbf{x}$ and the second at $\mathbf{y}$. The quark contributions to the currents satisfy (A1)–(A3). For $\mathbf{x}$ inside the bag one may expand the quark field operators in a complete set of eigenmodes. The completeness of the wave functions, along with the usual fermion anticommutation relations gives immediately (A1)–(A3). For $\mathbf{x}$ outside the bag one gets 0=0. Next we consider the pion contributions to the currents. *In this appendix we use a different notation* in which the pion field is denoted by $\boldsymbol{\phi}(\mathbf{x})$ to distinguish it from the conjugate field $\boldsymbol{\pi}(\mathbf{x})$. The conjugate field in terms of $\dot{\boldsymbol{\phi}} \equiv \partial_t \boldsymbol{\phi}$ and $\boldsymbol{\phi}$ is

$$\boldsymbol{\pi} = \frac{\partial \mathscr{L}}{\partial \dot{\boldsymbol{\phi}}} = \dot{\boldsymbol{\phi}} - \frac{\phi^2}{3{f_\pi}^2}[\dot{\boldsymbol{\phi}} - (\hat{\boldsymbol{\phi}}\cdot\dot{\boldsymbol{\phi}})\hat{\boldsymbol{\phi}}] + O({f_\pi}^{-4}) \quad (A4)$$

with the higher-order terms due to the presence of the covariant derivative in (2.1). Postulating the usual equal-time commutation relations

$$[\phi_i, \phi_j] = 0, \quad (A5)$$

$$[\pi_i, \pi_j] = 0, \quad (A6)$$

$$[\pi_i, \phi_j] = i\delta_{ij}\delta^3(\mathbf{x}-\mathbf{y}), \quad (A7)$$

we now show that the commutation relations (A1)–(A3) follow to low order in ${f_\pi}^{-1}$.

In order to evaluate (A1)–(A3) we first need the commutation relations (A6) and (A7) in terms of $\boldsymbol{\phi}$ and $\dot{\boldsymbol{\phi}}$. This is done by plugging (A4) into (A6) and (A7) and solving self-consistently to a given order. To $O({f_\pi}^{-2})$ we find

$$[\dot{\phi}_i, \dot{\phi}_j] = \frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3{f_\pi}^2}(2\phi_j\dot{\phi}_i - 2\phi_i\dot{\phi}_j - \dot{\phi}_j\phi_i + \dot{\phi}_i\phi_j), \quad (A8)$$

$$[\dot{\phi}_i, \phi_j] = -i\delta_{ij}\delta^3(\mathbf{x}-\mathbf{y}) + \frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3{f_\pi}^2}(\phi_j\phi_i - \delta_{ij}\phi^2). \quad (A9)$$

Expanding expressions (2.3) and (2.4) for the currents in powers of ${f_\pi}^{-1}$

$$\mathbf{V}^0 = -\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi} + \frac{1}{3{f_\pi}^2}\phi^2(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi}) + O({f_\pi}^{-4}), \quad (A10)$$

$$\mathbf{A}^0=f_\pi\dot{\boldsymbol{\phi}}-\frac{2}{3f_\pi}\boldsymbol{\phi}\times(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi})+O(f_\pi{}^{-3})\ , \tag{A11}$$

and using the commutation relations (CR's) (A5), (A8), and (A9) we can now check the validity of the current-algebra relations (A1)—(A3).

First let us consider (A1) which to lowest order is

$$[(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi})_i,(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi})_j]=-i\delta^3(\mathbf{x}-\mathbf{y})\epsilon^{ijk}(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi})_k$$

and can be easily confirmed by using the CR's to lowest order. The next nonvanishing terms on both sides of (A1) are of order $f_\pi{}^{-2}$. There are two contributions to the left-hand side (LHS). The first comes from the lowest-order terms of $\mathbf{V}^0$

$$L^a_{(\mathrm{A1})}=\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi{}^2}\epsilon^{abi}\epsilon^{cdj}(\dot{\phi}_a\phi^2\phi_d\delta_{cb}-\dot{\phi}_c\phi^2\phi_b\delta_{ad}+2\phi_c\dot{\phi}_a\phi_d\phi_b-2\phi_a\dot{\phi}_c\phi_d\phi_b)\ ,$$

where we have kept only the $O(f_\pi{}^{-2})$ piece of the CR's. The second contribution to the LHS of (A1) comes from the second-order terms in V^0

$$L^b_{(\mathrm{A1})}=\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi{}^2}\epsilon^{abi}\epsilon^{cdj}(-\dot{\phi}_a\phi^2\phi_d\delta_{bc}+\phi^2\dot{\phi}_c\phi_b\delta_{ad}+\dot{\phi}_c\phi^2\phi_b\delta_{da}-\phi^2\dot{\phi}_a\phi_d\delta_{cb}+2\phi_a\dot{\phi}_c\phi_d\phi_b-2\phi_c\dot{\phi}_a\phi_b\phi_d)$$

having used the CR's to lowest order. The full expression to $O(f_\pi{}^{-2})$ for the LHS of (A1) becomes

$$L^a_{(\mathrm{A1})}+L^b_{(\mathrm{A1})}=\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi{}^2}\phi^2(\dot{\phi}_i\phi_j-\dot{\phi}_j\phi_i)$$

which is identical to the right-hand side (RHS) of (A1) to this order.

Next we consider (A2). To $O(f_\pi{}^2)$ the LHS involves $[\dot{\phi}_i,\dot{\phi}_i]$, which is zero by (A8) to this order. The next nontrivial order is $O(f_\pi^0)$ for which again we have two contributions to the LHS. First there is the contribution from the lowest-order terms of $\mathbf{A}^0$ keeping only the $O(f_\pi{}^{-2})$ terms from the CR's:

$$L^a_{(\mathrm{A2})}=\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3}(2\phi_j\dot{\phi}_i-2\phi_i\dot{\phi}_j-\dot{\phi}_j\phi_i+\dot{\phi}_i\phi_j)\ .$$

Second, there is the contribution from the next-order terms in $\mathbf{A}^0$ keeping only the lowest-order terms from the CR's:

$$L^b_{(\mathrm{A2})}=\frac{2i\delta^3(\mathbf{x}-\mathbf{y})}{3}(\phi_i\dot{\phi}_j-\phi_j\dot{\phi}_i-2\dot{\phi}_i\phi_j+2\dot{\phi}_j\phi_i)\ .$$

The sum

$$L^a_{(\mathrm{A2})}+L^b_{(\mathrm{A2})}=i\delta^3(\mathbf{x}-\mathbf{y})(\dot{\phi}_j\phi_i-\dot{\phi}_i\phi_j)$$

is identical to the RHS of (A2) to lowest order.

Finally, to lowest order (A3) is

$$[-(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi})_i,f_\pi\dot{\phi}_j]=i\delta^3(\mathbf{x}-\mathbf{y})\epsilon^{ijk}f_\pi\dot{\phi}_k\ ,$$

in agreement with the lowest-order CR's. The next nontrivial order is $O(f^{-1})$ for which the LHS of (A3) has three contributions. First is the contribution from V^0 and A^0 to lowest order keeping only $O(f_\pi{}^{-2})$ in the CR's:

$$L^a_{(\mathrm{A3})}=\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi}\times\epsilon^{abi}[\dot{\phi}_a(\phi_b\phi_j-\phi^2\delta_{bj})-(2\phi_j\dot{\phi}_a-2\phi_a\dot{\phi}_j-\dot{\phi}_j\phi_a+\dot{\phi}_a\phi_j)\phi_b]\ .$$

Then there are two terms involving the CR's to lowest order:

$$L^b_{(\mathrm{A3})}=\frac{2i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi}\epsilon^{abi}(\dot{\phi}_a\phi^2\delta_{bj}+\boldsymbol{\phi}\cdot\dot{\boldsymbol{\phi}}\phi_b\delta_{aj}-\phi_a\dot{\phi}_j\phi_b)\ ,$$

$$L^c_{(\mathrm{A3})}=\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi}\epsilon^{abi}(\phi^2\dot{\phi}_a\delta_{bj}+2\phi_j\dot{\phi}_a\phi_b)\ .$$

Adding these terms together we find

$$L^a_{(\mathrm{A3})}+L^b_{(\mathrm{A3})}+L^c_{(\mathrm{A3})}=-\frac{i\delta^3(\mathbf{x}-\mathbf{y})}{3f_\pi}\epsilon^{ijk}[(\dot{\phi}_k\phi^2+\phi^2\dot{\phi}_k)-2\boldsymbol{\phi}\cdot\dot{\boldsymbol{\phi}}\phi_a]=-i\delta^3(\mathbf{x}-\mathbf{y})\epsilon^{ijk}\frac{2}{3f_\pi}\boldsymbol{\phi}\times(\dot{\boldsymbol{\phi}}\times\boldsymbol{\phi})_k\ ,$$

proving the validity of (A3) to this order.

APPENDIX B: PROOF THAT THE AB VERTEX IS IDENTICAL IN BOTH VERSIONS OF THE CBM

From Eq. (3.5) we find the lowest-order interaction Hamiltonian for the volume-coupling CBM is

$$\begin{aligned}H_I'&=-\frac{1}{2f_\pi}\int d^3x\,\bar{q}(x)\mathbf{q}(x)\boldsymbol{\gamma}\gamma^5\boldsymbol{\tau}q(x)\cdot\nabla\boldsymbol{\pi}(x)\theta_V\\&=\frac{1}{2f_\pi}\int d^3x\{\bar{q}\boldsymbol{\gamma}\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\pi})q\cdot\nabla\theta_V+[(\nabla\bar{q})\cdot\boldsymbol{\gamma}\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\pi})q+\bar{q}\boldsymbol{\gamma}\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\cdot(\nabla q)]\theta_V\}\ ,\end{aligned}\tag{B1}$$

where we have integrated by parts to obtain the last expression. Restricting the quarks to the ground state the Dirac equation gives

$$\boldsymbol{\gamma}\cdot\nabla q=i\frac{\omega}{R}\gamma^0q$$

and

$$\nabla\bar{q}\cdot\boldsymbol{\gamma}=-i\frac{\omega}{R}\bar{q}\gamma^0$$

which upon substitution into (B1) leaves only the first term. Using $\nabla\theta_V=-\hat{\mathbf{r}}\Delta_S$, we find

$$H_I'=\frac{1}{2f_\pi}\int d^3x\,\bar{q}\gamma^5\boldsymbol{\tau}\cdot\boldsymbol{\pi}\boldsymbol{\gamma}q\cdot\hat{\mathbf{r}}\Delta_S$$

and finally, the boundary condition (2.2b) $-i\hat{\mathbf{r}}\cdot\gamma q = q$ gives

$$H_1' = \frac{1}{2f_\pi}\int d^3x\, \bar{q}\gamma^5 \boldsymbol{\tau}\cdot\boldsymbol{\pi} q \Delta_S = H_I$$

the same interaction Hamiltonian as in the surface-coupling CBM. Since the interaction Hamiltonians are identical with quarks restricted to the ground state, the πAB vertices will also be identical.

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