# Neutrino masses and proton decay modes in SU(3)×SU(3)×SU(3) trinification

K. S. Babu, Xiao-Gang He, and Sandip Pakvasa
*Department of Physics and Astronomy, University of Hawaii at Manoa, Honolulu, Hawaii 96822*
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A detailed study of the Higgs potential and the Higgs-fermion Yukawa couplings in the SU(3)×SU(3)×SU(3) trinification model proposed by de Rújula, Georgi, and Glashow is carried out. Spontaneous symmetry breakdown to SU(3)×SU(2)×U(1) is analyzed, stability of the ground-state vacuum assured, and the Higgs- and gauge-boson mass matrices exhibited. The most economical method of generating all fermion masses and mixings is given. The large corrections to the masses of the neutral leptons are worked out at the one- and two-loop levels and shown to be stable. The new neutral leptons (neutrettos) turn out to be rather heavy ($\sim 100$ TeV). The mass hierarchy of the neutrinos is $\nu_\tau > \nu_\mu > \nu_e$. $\nu_\tau$ can be a few keV, $\nu_\mu$ a few eV, and $\nu_e \sim 10^{-4}$ eV. The $\nu_e$-$\nu_\mu$ mixing angle is about $10^{-2}$. The dominant proton-decay modes are $p \to K^+\bar{\nu}_\mu$ and $p \to \pi^0\mu^+$ with possibly comparable rates. Other modes are suppressed. For example, $\Gamma(p\to\pi^+\bar{\nu}_\mu)/\Gamma(p\to K^+\bar{\nu}_\mu) \sim 10^{-2}$, $\Gamma(p\to e^+\pi^0)/\Gamma(p\to\mu^+\pi^0) \sim \frac{1}{14}$, and $p \to K^0\mu^+$ is absent.

## I. INTRODUCTION

In the late 1950s and early 1960s there was a search for the correct symmetry group of strong interactions. Among the candidates were global symmetry [SU(2)×SU(2)×SU(2)] (Ref. 1) of Gell-Mann and Schwinger, O(7) and other orthogonal groups,[2] $G_2$ of Behrends and Sirlin,[3] and SU(3).[4] We now know that SU(3), in particular the eightfold way,[5] turned out to be the correct choice. Today the search is for the correct choice for a grand unified theory of strong, weak, and electromagnetic interactions.[6–8] Some of the models have a close analogy to the way the old symmetry groups contained isospin. For example, SU(5)[6] is analogous to SU(3), O(10)[7] to O(7), $E_6$ (Ref. 8) to $G_2$, etc. All of these have been thoroughly investigated. The analog of global symmetry, SU(3)×SU(3)×SU(3), as a serious candidate for a grand unified theory was proposed recently by de Rújula, Georgi, and Glashow.[9]

They considered an $SU(3)_C\times SU(3)_L\times SU(3)_R$ gauge group with a discrete $Z_3$ symmetry imposed at the unification scale to guarantee one coupling constant and a desert from $m_W$ to the unification scale. This group has been discussed before either as an electroweak gauge group[10] or as a subgroup of $E_6$.[11] The assignment of fermions and scalars is to the same 27-dimensional representation. They found an interesting pattern of neutrino masses, viz., an inverted hierarchy, scalar-mediated proton decay with $K\mu$ and $K\nu$ as the dominant modes and relatively light ($\sim 10$ GeV) new neutral leptons (neutrettos).

In this paper, we study this model in detail. We confirm most of the results of Ref. 9. However, we find that the additional requirement of reproducing the fermion masses and mixings correctly changes some of their conclusions. In particular, we find that (a) the inverted hierarchy of neutrino masses is possible but not favored, (b) the proton decays dominantly into $K\nu$ but not $K\mu$, and $\pi\mu$ may have a substantial branching ratio, (c) the new neutral leptons (neutrettos) turn out to be rather heavy ($\sim 100$ TeV).

In Sec. II the basic ingredients of the model are outlined. Section III contains the detailed analysis of the Higgs potential, the breaking of symmetry down to SU(3)×SU(2)×U(1) and the stability of the symmetry-breaking vacuum. Yukawa couplings and fermion masses are discussed in detail in Sec. IV. A simple ansatz for obtaining the correct mixing angles is also introduced. The results for neutrino and neutretto masses are derived there. Proton-decay branching ratios are derived in Sec. V and Sec. VI recapitulates the main results.

## II. THE MODEL

In this section we review briefly the essential feature of $[SU(3)]^3\times Z_3$ model.[9] The three SU(3) factor groups are identified as $SU(3)_C$, which is the unbroken color group, $SU(3)_L$ which contains weak SU(2), and $SU(3)_R$ which is the right-handed analog of $SU(3)_L$. To ensure that there is only one gauge coupling constant, an additional discrete symmetry $Z_3$, which is the cyclic group acting on the three factor groups, is imposed. If $(C,L,R)$ is a representation under $[SU(3)]^3$, the effect of $Z_3$ is to symmetrize it in the following way:

$$Z_3(C,L,R)\equiv(C,L,R)+(R,C,L)+(L,R,C)\ . \tag{2.1}$$

In the minimal version, only two irreducible representations are to be considered. The gauge bosons are assigned to the adjoint representation:

$$24=(8,1,1)+(1,8,1)+(1,1,8)\ . \tag{2.2}$$

Just as in SU(5), there are 24 gauge bosons, twelve light (gluons, photon, $W^\pm$, $Z$) and twelve superheavy. However, there is a significant difference: unlike in SU(5), the superheavy gauge bosons here are all integrally charged and do not mediate proton decay.

 

Each family of fermions is assigned to a 27:

$$27=(1,3,3^*)+(3^*,1,3)+(3,3^*,1)\ . \tag{2.3}$$

Under $SU(3)_C\times SU(2)_L\times U(1)_Y$, the 27 decomposes as

$$27=(1,2,\tfrac{1}{2})+2(1,2,-\tfrac{1}{2})+(1,1,1)+2(1,1,0)$$
$$+(3,2,\tfrac{1}{6})+(3,1,-\tfrac{1}{3})+(3^*,1,\tfrac{2}{3})$$
$$+2(3^*,1,-\tfrac{1}{3})\ . \tag{2.4}$$

The transformation properties of the fermions can be understood from the following assignment:[9]

$$(1,3,3^*):\ U_L\begin{bmatrix} E^0 & E^- & e^- \\ E^+ & \bar{E}^0 & \nu \\ e^+ & N_1 & N_2 \end{bmatrix}\bar{V}_R\ ,$$

$$(3^*,1,3):\ V_R\begin{bmatrix} \bar{u}_1 & \bar{u}_2 & \bar{u}_3 \\ \bar{d}_1 & \bar{d}_2 & \bar{d}_3 \\ \bar{B}_1 & \bar{B}_2 & \bar{B}_3 \end{bmatrix}\bar{W}_C\ , \tag{2.5}$$

$$(3,3^*,1):\ W_C\begin{bmatrix} u_1 & d_1 & B_1 \\ u_2 & d_2 & B_2 \\ u_3 & d_3 & B_3 \end{bmatrix}\bar{U}_L\ .$$

$U$, $V$, and $W$ are elements of the three SU(3) factor groups. We have chosen a specific and simple basis, which is always possible with a single family of fermions. If there are several families, one has to consider Cabibbo-Kobayashi-Maskawa mixing as well. This question will be addressed in detail in Sec. IV.

With this assignment of fermions, it is clear that the model contains new particles. There is a heavy quark $B$, which is a charge $-\frac{1}{3}$ weak singlet, and a pair of heavy leptons $E^0$ and $E^+$, with charges 0 and 1 and belonging to a weak vector doublet. In addition, there are two neutral chiral states $N_1$ and $N_2$, called neutrettos, which are not necessarily superheavy, as we shall see.

At the unification mass $M_u \sim 10^{14}$ GeV, $[SU(3)]^3\times Z_3$ breaks down spontaneously to $SU(3)_C\times SU(2)_L\times U(1)_Y$. Even though intermediate symmetries are possible, we consider here only the case where the breaking is in one step. The weak mixing angle $\sin^2\theta_W$ is $\frac{3}{8}$ at the unification scale as in SU(5). Hence the model retains the successful SU(5) prediction on $\sin^2\theta_W(m_W)$. Each Higgs multiplet is assigned to a 27 [see Eq. (2.3)]. A single 27-plet of the Higgs multiplet cannot break the symmetry to $SU(3)_C\times SU(2)_L\times U(1)_Y$, but two 27-plets can do the desired job. Each 27 leaves a left-right-symmetric $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)$ stability group, but in general a different one. Together, they leave the $SU(3)_C \times SU(2)_L\times U(1)_Y$ subgroup invariant. Besides symmetry-breaking considerations, there is an independent reason to introduce a second 27: The Cabibbo-Kobayashi-Maskawa matrix will be unity if there is only one 27 that the quarks can couple to. In order to break the symmetry spontaneously, the Higgs multiplets should acquire vacuum expectation values (VEV's) given by

$$\langle 1,3,3^*\rangle_1=\begin{bmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & 0 \\ 0 & 0 & v_1 \end{bmatrix}\ , \tag{2.6}$$

$$\langle 1,3,3^*\rangle_2=\begin{bmatrix} \hat{0} & 0 & 0 \\ 0 & \hat{0} & \hat{0} \\ 0 & v_2 & \hat{0} \end{bmatrix}\ . \tag{2.7}$$

The same Higgs multiplets can acquire VEV's of order $m_W$ and break $SU(2)_L\times U(1)_Y$ to $U(1)_Q$. In (2.6) and (2.7) the entries with a caret correspond to fields which can have VEV's of order $u\sim m_W$.[12] As far as the symmetry breaking is concerned, one such $u$ is enough, but to realize a realistic fermion mass spectrum, at least two such $u$'s are necessary, as will be shown below.

## III. SYMMETRY BREAKING AND THE HIGGS-BOSON STRUCTURE

In this section, we analyze the symmetry breaking of $[SU(3)]^3\times Z_3$ down to $SU(3)_C\times SU(2)_L\times U(1)_Y$. The Higgs potential invariant under $[SU(3)]^3\times Z_3$ is constructed and all the Higgs-boson masses are enumerated. The positivity of these masses is an essential criterion to ensure a consistent symmetry-breaking pattern. The masses of the colored Higgs will turn out to be determining factors on proton lifetime. Mixing among various submultiplets of the Higgs multiplet is necessary to give phenomenologically acceptable masses to the neutral leptons in the theory. This will be addressed in detail in the next section.

As mentioned earlier, two 27-plets of the Higgs multiplet are necessary to break $[SU(3)]^3\times Z_3$ down to $SU(3)_C\times SU(2)_L\times U(1)_Y$. Let $\phi$ and $\chi$ denote these two multiplets. From (2.3) it is clear that $\phi$ and $\chi$ have three pieces. Let us denote them and their charge conjugates by

$$\phi=\phi_C(1,3,3^*)+\phi_L(3^*,1,3)+\phi_R(3,3^*,1)\ , \tag{3.1a}$$

$$\bar{\phi}=\bar{\phi}_C(1,3^*,3)+\bar{\phi}_L(3,1,3^*)+\bar{\phi}_R(3^*,3,1) \tag{3.1b}$$

and similarly for $\chi$. Further, these may be written in a tensor form as $(\phi_C)^i_j, (\phi_L)^i_j$, and $(\phi_R)^i_j$ where the contravariant (covariant) index refers to the $3(3^*)$ of Eq. (3.1). The charge-conjugate fields are then $(\bar{\phi}_C)^i_j=(\phi_C)^j_i$ and so on. Now we are ready to write down the complete Higgs potential. For simplicity, we eliminate terms odd in $\chi$ by imposing a discrete symmetry.[13] Consequently, the potential takes the form

$$V(\phi,\chi)=V_1(\phi)+V_2(\chi)+V_3(\phi,\chi)+V_4(\text{cubic})\ , \tag{3.2}$$

where

$$V_1(\phi)=Z_3\{-\mu_1{}^2\,\mathrm{Tr}(\bar\phi_C\phi_C)+\alpha_1(\mathrm{Tr}\bar\phi_C\phi_C)^2+\alpha_2\,\mathrm{Tr}(\bar\phi_C\phi_C\bar\phi_C\phi_C)+\alpha_3\,\mathrm{Tr}(\bar\phi_C\phi_C)\,\mathrm{Tr}(\bar\phi_L\phi_L)+\alpha_4\,\mathrm{Tr}(\bar\phi_C\phi_L\bar\phi_L\phi_C)$$
$$+[\alpha_5(\phi_C)^i_\alpha(\phi_C)^j_\beta(\bar\phi_L)^m_\gamma(\bar\phi_R)^k_m\epsilon_{ijk}\epsilon^{\alpha\beta\gamma}+\mathrm{H.c.}]\}\ . \qquad (3.3)$$

$V_2(\chi)$ has the same form as above, except that $\mu_1$ is replaced by $\mu_2$ and $\alpha_i$ by $\beta_i$. The $Z_3$ symmetrization is understood as follows:

$$Z_3[\,\mathrm{Tr}(\bar\phi_C\phi_C)\,\mathrm{Tr}(\bar\phi_L\phi_L)]=\mathrm{Tr}(\bar\phi_C\phi_C)\,\mathrm{Tr}(\bar\phi_L\phi_L)+\mathrm{Tr}(\bar\phi_L\phi_L)\mathrm{Tr}(\bar\phi_R\phi_R)+\mathrm{Tr}(\bar\phi_R\phi_R)\,\mathrm{Tr}(\bar\phi_C\phi_C) \qquad (3.4)$$

and so on,

$$V_3(\phi,\chi)=Z_3\{\lambda_1\mathrm{Tr}(\bar\phi_C\phi_C)\mathrm{Tr}(\bar\chi_C\chi_C)+\lambda_2\mathrm{Tr}(\bar\phi_C\phi_C)\mathrm{Tr}(\bar\chi_L\chi_L)+\lambda_3\mathrm{Tr}(\bar\phi_C\phi_C)\mathrm{Tr}(\bar\chi_R\chi_R)+\lambda_4\mathrm{Tr}(\bar\phi_C\chi_C)\mathrm{Tr}(\bar\chi_C\phi_C)$$
$$+[\lambda_5(\mathrm{Tr}\bar\phi_C\chi_C)^2+\mathrm{H.c.}]+[\lambda_6\mathrm{Tr}(\bar\phi_C\chi_C)\mathrm{Tr}(\bar\phi_L\chi_L)+\mathrm{H.c.}]+[\lambda_7\mathrm{Tr}(\bar\phi_C\chi_C)\mathrm{Tr}(\bar\chi_L\phi_L)+\mathrm{H.c.}]$$
$$+\lambda_8\mathrm{Tr}(\bar\phi_C\phi_C\bar\chi_C\chi_C)+\lambda_9\mathrm{Tr}(\bar\phi_C\chi_C\bar\chi_C\phi_C)+\lambda_{10}\mathrm{Tr}(\bar\phi_C\phi_C\bar\chi_L\chi_L)+\lambda_{11}\mathrm{Tr}(\bar\phi_C\phi_C\bar\chi_R\chi_R)$$
$$+[\lambda_{12}\mathrm{Tr}(\bar\phi_C\chi_C\bar\phi_C\chi_C)+\mathrm{H.c.}]+[\lambda_{13}\mathrm{Tr}(\bar\phi_C\chi_C\bar\phi_L\chi_L)+\mathrm{H.c.}]+[\lambda_{14}\mathrm{Tr}(\bar\phi_C\chi_C\bar\chi_L\phi_L)+\mathrm{H.c.}]$$
$$+[\lambda_{15}(\phi_C)^i_j(\phi_C)^k_1(\bar\chi_L)^m_\beta(\bar\chi_R)^\alpha_m\epsilon_{ik\alpha}\epsilon^{j1\beta}+\mathrm{H.c.}]+[\lambda_{16}(\chi_C)^i_j(\chi_C)^k_1(\bar\phi_L)^m_\beta(\bar\phi_R)^\alpha_m\epsilon_{ik\alpha}\epsilon^{j1\beta}+\mathrm{H.c.}]\}\ , \qquad (3.5)$$

$$V_4(\text{cubic})=Z_3\{[\gamma_1(\phi_C)^i_\alpha(\phi_C)^j_\beta(\phi_C)^k_\gamma\epsilon_{ijk}\epsilon^{\alpha\beta\gamma}+\mathrm{H.c.}]+[\gamma_2(\phi_C)^i_j(\phi_L)^j_k(\phi_R)^k_i+\mathrm{H.c.}]$$
$$+[\gamma_3(\phi_C)^i_\alpha(\chi_C)^j_\beta(\chi_C)^k_\gamma\epsilon_{ijk}\epsilon^{\alpha\beta\gamma}+\mathrm{H.c.}]+[\gamma_4(\phi_C)^i_j(\chi_L)^j_k(\chi_R)^k_i+\mathrm{H.c.}]\}\ . \qquad (3.6)$$

Here, H.c. stands for Hermitian conjugation. Some of the parameters in (3.3), (3.5), and (3.6) as well as the vacuum expectation values themselves, can be complex in general, but for simplicity we shall assume them to be real by imposing invariance under *CP*.

With the VEV's given by Eqs. (2.6) and (2.7),

$$\langle(\phi_C)^i_j\rangle=v_1\delta^{i3}\delta_{j3},\quad \langle(\chi_C)^i_j\rangle=v_2\delta^{i3}\delta_{j2}\ , \qquad (3.7)$$

it is straightforward to calculate the masses of all the Higgs bosons. The VEV's $v_1$ and $v_2$ are determined by the stationary conditions

$$\mu_1{}^2=2(\alpha_1+\alpha_2)v_1{}^2+(\lambda_1+\lambda_9)v_2{}^2\ , \qquad (3.8)$$

$$\mu_2{}^2=2(\beta_1+\beta_2)v_2{}^2+(\lambda_1+\lambda_9)v_1{}^2\ . \qquad (3.9)$$

In Table I we list the Higgs-boson masses except for a $(1,2,\frac{1}{2})$ and two $(3,1,-\frac{1}{3})$ multiplets from $\phi$ and $\chi$ which obey cubic equations. For the $(1,2,\frac{1}{2})$, the mass matrix in the $(\phi,\phi,\chi)$ basis is

$$\begin{bmatrix} -2\alpha_2v_1{}^2-\lambda_9v_2{}^2 & 6\gamma_1v_1 & -2\gamma_3v_2 \\ 6\gamma_1v_1 & -2\alpha_2v_1{}^2+(\lambda_8-\lambda_9)v_2{}^2 & 2\lambda_{12}v_1v_2 \\ -2\gamma_3v_2 & 2\lambda_{12}v_1v_2 & -2\beta_2v_2{}^2+(\lambda_8-\lambda_9)v_1{}^2 \end{bmatrix}. \qquad (3.10)$$

For one of the $(3,1,-\frac{1}{3})$ mixings, the mass matrix in the $(\phi,\phi,\chi)$ basis is

$$\begin{bmatrix} [-(2\alpha_1+2\alpha_2-\alpha_3-\alpha_4)v_1{}^2 -(\lambda_1-\lambda_3+\lambda_9)v_2{}^2] & \gamma_2v_1 & \lambda_{13}v_1v_2 \\ \gamma_2v_1 & [-(2\alpha_1+2\alpha_2-\alpha_3-\alpha_4)v_1{}^2 -(\lambda_1-\lambda_2+\lambda_9-\lambda_{10})v_2{}^2] & \gamma_4v_2 \\ \lambda_{13}v_1v_2 & \gamma_4v_2 & [-(2\beta_1+2\beta_2-\beta_3-\beta_4)v_2{}^2 -(\lambda_1-\lambda_2+\lambda_9)v_1{}^2] \end{bmatrix}. \qquad (3.11)$$

TABLE I. Masses of the Higgs bosons except for a $(1,2,\frac{1}{2})$ multiplet and the $(3,1,-\frac{1}{3})$ multiplets which obey the cubic equation (see text). The first column indicates the decomposition under $SU(3)_C\times SU(2)_L\times U(1)_Y$. When there is no mixing among $\phi$ and $\chi$ submultiplets, the origin is indicated as a subscript in the first column.

| | |
|---|---|
| $(1,2,\frac{1}{2})$ | $-\frac{1}{2}(\lambda_9(2v_1^2+v_2^2)+2\beta_2v_2^2 \pm\{[\lambda_9(2v_1^2+v_2^2)+2\beta_2v_2^2]^2+4(v_1^2+v_2^2)[4\gamma_3^2-\lambda_9(2\beta_2v_2^2+\lambda_9v_1^2)]\}^{1/2})$ |
| $(1,1,0)$ | $[\lambda_4+\lambda_8\pm2(\lambda_5+\lambda_{12})](v_1^2+v_2^2)$ |
| $(1,1,0)$ | $2((\alpha_1+\alpha_2)v_1^2+(\beta_1+\beta_2)v_2^2 \pm\{[(\alpha_1+\alpha_2)v_1^2+(\beta_1+\beta_2)v_2^2]^2-[4(\alpha_1+\alpha_2)(\beta_1+\beta_2)-(\lambda_1+\lambda_9)^2]v_1^2v_2^2\}^{1/2})$ |
| $(3,2,\frac{1}{6})_\phi$ | $-[(2\alpha_1+2\alpha_2-\alpha_3)v_1^2+(\lambda_1-\lambda_2+\lambda_9)v_2^2]$ |
| $(3,2,\frac{1}{6})_\chi$ | $-[(2\beta_1+2\beta_2-\beta_3)v_2^2+(\lambda_1-\lambda_3+\lambda_9)v_1^2]$ |
| $(3,1,\frac{2}{3})_\phi$ | $-[(2\alpha_1+2\alpha_2-\alpha_3)v_1^2+(\lambda_1-\lambda_3+\lambda_9)v_2^2]$ |
| $(3,1,\frac{2}{3})_\chi$ | $-[(2\beta_1+2\beta_2-\beta_3)v_2^2+(\lambda_1-\lambda_2+\lambda_9)v_1^2]$ |

For the other $(3,1,-\frac{1}{3})$, the mass matrix in the $(\phi,\chi,\chi)$ basis is

$$\begin{bmatrix} [-(2\alpha_1+2\alpha_2-\alpha_3)v_1^2 -(\lambda_1-\lambda_3+\lambda_9-\lambda_{11})v_2^2] & \lambda_{14}v_1v_2 & \gamma_4v_2 \\ \lambda_{14}v_1v_2 & [-(2\beta_1+2\beta_2-\beta_3)v_2^2 -(\lambda_1-\lambda_2+\lambda_9-\lambda_{10})v_1^2] & \gamma_4v_1 \\ \gamma_4v_2 & \gamma_4v_1 & [-(2\beta_1+2\beta_2-\beta_3-\beta_4)v_2^2 -(\lambda_1-\lambda_3+\lambda_9-\lambda_{11})v_1^2] \end{bmatrix} . \quad (3.12)$$

The (1,1,1) multiplet and two of the (1,1,0) multiplets from both $\phi$ and $\chi$, together with a mixed $(1,2,\frac{1}{2})$, account for the twelve would-be Goldstone bosons which are eaten up by the 12 gauge bosons which have become superheavy, thereby breaking the symmetry to $SU(3)_C\times SU(2)_L\times U(1)_Y$. In Table II, we list the masses of these gauge bosons. $W^\pm,Z^0$ and the photon remain massless at this stage of the symmetry breaking. There is a range of parameters in the Higgs potential for which all the Higgs-boson masses are positive, thereby ensuring the stability of the $SU(3)_C\times SU(2)_L\times U(1)_Y$-symmetric vacuum.

The same $\phi$ and $\chi$ can have VEV's of order $u\sim m_W$ and thereby break $SU(2)_L\times U(1)_Y$ to ordinary $U(1)_Q$.

TABLE II. Masses of the superheavy gauge bosons. The first column indicates their classification under $SU(3)_C\times SU(2)_L\times U(1)_Y$.

| | |
|---|---|
| $(1,2,\frac{1}{2})$ | $\frac{1}{2}g^2(v_1^2+v_2^2)$ |
| $(1,1,1)$ | $\frac{1}{2}g^2v_1^2$ |
| $(1,1,1)$ | $\frac{1}{2}g^2v_2^2$ |
| $(1,1,0)$ | $\frac{1}{2}g^2(v_1^2+v_2^2)$ |
| $(1,1,0)$ | $\frac{2}{3}g^2[v_1^2+v_2^2\pm(v_1^4+v_2^4-\frac{7}{4}v_1^2v_2^2)^{1/2}]$ |

That no new representations need be introduced to break the electroweak symmetry is an attractive feature of the model. As far as the symmetry breaking is concerned, one VEV $u$ of order $m_W$ is sufficient, but in order to obtain a realistic fermion mass spectrum at least two $u$'s are necessary (see below). The nonvanishing VEV's of the Weinberg-Salam doublets will induce shifts in the VEV's $v_1$ and $v_2$ of (3.7). These shifts must be much smaller than $v_1$ and $v_2$ themselves, for consistency. Let $v_1\to v_1(1+\epsilon_1)$ and $v_2\to v_2(1+\epsilon_2)$. We shall see that there is indeed a solution corresponding to the minimum of the potential, where $\epsilon_1$ and $\epsilon_2$ are much smaller than 1. A minimal choice of the VEV's is

$$\langle(\phi_C)^i_j\rangle=u_1\delta^{i1}\delta_{j1}+u_2\delta^{i2}\delta_{j2} +v_1(1+\epsilon_1)\delta^{i3}\delta_{j3} , \quad (3.13a)$$

$$\langle(\chi_C)^i_j\rangle=v_2(1+\epsilon_2)\delta^{i3}\delta_{j2} . \quad (3.13b)$$

Here $v_1$ and $v_2$ are determined by (3.8) and (3.9). The stationary conditions which determine $u_1$, $u_2$, $\epsilon_1$, and $\epsilon_2$ are[14]

$$u_1[-\mu_1^2+2\alpha_1(u_1^2+u_2^2+v_1^2)+2\alpha_2 u_1^2+\lambda_1 v_2^2]+6\gamma_1 u_2 v_1=0\ , \tag{3.14a}$$

$$u_2[-\mu_1^2+2\alpha_1(u_1^2+u_2^2+v_1^2)+2\alpha_2 u_2^2+\lambda_1 v_2^2+\lambda_8 v_2^2]+6\gamma_1 u_1 v_1=0\ , \tag{3.14b}$$

$$\epsilon_1=\frac{[4\alpha_1(\beta_1+\beta_2)-\lambda_1(\lambda_1+\lambda_9)](u_1^2+u_2^2)-\lambda_8(\lambda_1+\lambda_9)u_2^2+\dfrac{16\gamma_1}{v_1}(\beta_1+\beta_2)u_1u_2}{2v_1^2[(\lambda_1+\lambda_9)^2-4(\alpha_1+\alpha_2)(\beta_1+\beta_2)]}\ , \tag{3.14c}$$

$$\epsilon_2=-\frac{\alpha_1(u_1^2+u_2^2)+\dfrac{3\gamma_1}{v_1}u_1u_2+2(\alpha_1+\alpha_2)\epsilon_1 v_1^2}{(\lambda_1+\lambda_9)v_2^2}\ . \tag{3.14d}$$

$\epsilon_1$ and $\epsilon_2$ are indeed very small, of order $u^2/v^2$, and hence the hierarchical symmetry breaking is consistent. It may also be mentioned that there is another solution where $\epsilon_1$ and $\epsilon_2$ are of order $u/v$ which is also small, but in this case the mass terms $\mu_1^2$ and $\mu_2^2$ are of order $u^2$ rather than of order $v^2$. Because of the various possible mixings, once the electroweak symmetry is broken, the corrections to the Higgs-boson masses become too involved, and we do not attempt to derive them here. All the physical Higgs bosons, except one neutral, have masses of order $v$, in agreement with general principles of naturalness.[15]

Problems of gauge hierarchy and fine-tuning usually associated with grand unified theories are present here as well. The mystery why $m_W \ll m_x$ persists. At each order of perturbation theory, certain parameters in the Higgs potential are to be fine-tuned in order to keep the light scalar light.

## IV. FERMION MASSES

With the assignment of each family of fermions to a 27 as in (2.5), there are two types of trilinear invariants that can be constructed [see Eq. (3.6)]:

$$\begin{aligned}[\psi(1,3,3^*)\psi(3,3^*,1)\phi_1(3^*,1,3)\\ +\psi(3^*,1,3)\psi(1,3,3^*)\phi_1(3,3^*,1)\\ +\psi(3,3^*,1)\psi(3^*,1,3)\phi_1(1,3,3^*)+\text{H.c.}]\end{aligned} \tag{4.1a}$$

and

$$\begin{aligned}[\psi(1,3,3^*)\psi(1,3,3^*)\phi_2(1,3,3^*)\\ +\psi(3,3^*,1)\psi(3,3^*,1)\phi_2(3,3^*,1)\\ +\psi(3^*,1,3)\psi(3^*,1,3)\phi_2(3^*,1,3)+\text{H.c.}]\ .\end{aligned} \tag{4.1b}$$

If $\phi_1$ and $\phi_2$ are distinct Higgs multiplets in (4.1), then definite quark numbers may be assigned to them. From (4.1a), $\phi_1(1,3,3^*)$ takes zero quark number, $\phi_1(3,3^*,1)$ takes $+1$, and $\phi_1(3^*,1,3)$ takes $-1$. From (4.1b), $\phi_2(1,3,3^*)$ takes zero quark number, $\phi_2(3,3^*,1)$ minus 2, and $\phi_2(3^*,1,3)$ plus 2. If the Higgs potential has no quark-number-violating terms, then the baryon number is exactly conserved, and there is no proton decay. However, if the quarks couple only to one Higgs multiplet, the up- and down-quark matrices are proportional resulting in

$$m_u/m_d=m_c/m_s=m_t/m_b\ . \tag{4.2}$$

The second half of Eq. (4.2) predicts a top mass of $\sim 35$ GeV, but the first half is clearly unacceptable. Furthermore, the mixing angles are all zero. Therefore the quarks must couple to a second Higgs multiplet. If the Higgs sector is constrained to have only two 27's, the quarks couple to both, and quark number is no longer well defined for the Higgs multiplet, naturally resulting in baryon-number-violating processes. It should be also mentioned that proton decays inevitably only in the minimal version of the model, where only two 27-plets of the Higgs multiplet are introduced. A third 27, for example, can stabilize the proton, while correcting the mass relation (4.2) and the Cabibbo angle, at the same time. With the VEV's of $\phi$ and $\chi$ given by (3.7), the leptons can couple only to the $\phi$ field, lest the electron be superheavy.

As it turns out, the most economic way of obtaining a realistic fermion mass spectrum requires two VEV's of $u\sim 10^2$ GeV. There are two generically different ways of arranging these:

$$\text{(A)}\quad \langle\phi(1,3,3^*)\rangle=\begin{bmatrix}u_1&0&0\\0&u_2&0\\0&0&v_1\end{bmatrix}, \tag{4.3a}$$

$$\langle\chi(1,3,3^*)\rangle=\begin{bmatrix}0&0&0\\0&0&0\\0&v_2&0\end{bmatrix}; \tag{4.3b}$$

$$\text{(B)}\quad \langle\phi(1,3,3^*)\rangle=\begin{bmatrix}0&0&0\\0&u_2&0\\0&0&v_1\end{bmatrix}, \tag{4.4a}$$

$$\langle\chi(1,3,3^*)\rangle=\begin{bmatrix}u_1&0&0\\0&0&0\\0&v_2&0\end{bmatrix}. \tag{4.4b}$$

Any other arrangement will necessarily result in either the quark mass or the electron mass being zero at the tree level. Again, we choose all the VEV's to be real, which is not obligatory.

Let us suppose for a moment, that there is no flavor mixing in the quark sector. The general case, where there is such mixing, is deferred to the end of this section. The Yukawa coupling responsible for the quark masses is then

$$L_q=(\psi_L)^j_i(\psi_R)^i_k[g(\phi_C)^k_j+g'(\chi_C)^k_j]+\text{H.c.} \tag{4.5}$$

The term which gives rise to lepton masses is

$$L_1 = h(\psi_C)^i_\alpha(\psi_C)^j_\beta(\phi_C)^k_\gamma \epsilon_{ijk}\epsilon^{\alpha\beta\gamma} + \text{H.c.} \tag{4.6}$$

The up-quark mass is then

$$m_u = g u_1 \ [\text{case (A)}]$$
$$= g' u_1 \ [\text{case (B)}] . \tag{4.7}$$

The mass matrix for $d$ and $B$ quarks is

$$\begin{bmatrix} g u_2 & 0 \\ g' v_2 & g v_1 \end{bmatrix} . \tag{4.8}$$

This non-Hermitian mass matrix can be easily diagonalized yielding

$$m_d \simeq g^2 u_2 v_1 / (g^2 v_1{}^2 + g'^2 v_2{}^2)^{1/2} , \tag{4.9a}$$

$$m_B \simeq (g^2 v_1{}^2 + g'^2 v_2{}^2)^{1/2} . \tag{4.9b}$$

These results are valid for both cases (A) and (B). From (4.7) and (4.8), it is clear that if $g'=0$, the unacceptable mass ratios of Eq. (4.2) are obtained. The situation is unaltered even when different families mix, since one can always choose a basis where this mixing goes away. Hence the need to couple the quarks to both $\phi$ and $\chi$.

From (4.6) electron and $E^\pm$ get masses, for both cases (A) and (B),

$$m_e = -4hu_2 , \tag{4.10a}$$

$$m_{E^\pm} = -4hv_1 . \tag{4.10b}$$

The neutretto $N_1$ and the neutrino mix with a tree-level mass matrix, for case A,

$$\begin{bmatrix} 0 & -2hu_1 \\ -2hu_1 & 0 \end{bmatrix} \tag{4.11}$$

resulting in

$$m_{N_1} = m_\nu = -2hu_1 . \tag{4.12}$$

For case (B), both $N_1$ and $\nu$ remain massless at the tree level. $E^0$ and $N_2$ have a mass matrix, in the $(E^0, N_2)$ basis,

$$\begin{bmatrix} 4hv_1 & 2h(u_1+u_2) \\ 2h(u_1+u_2) & 0 \end{bmatrix} . \tag{4.13}$$

[For case (B), set $u_1 = 0$.] Then

$$m_{E^0} \simeq 4hv_1 , \tag{4.14a}$$

$$m_{N_2} \simeq -h(u_1+u_2)^2/v_1 . \tag{4.14b}$$

The masses of the neutrino and $N_1$ being of the same order as the electron mass is clearly a disastrous result. So is $N_2$ being superlight. Fortunately, these results are automatically corrected by one-loop corrections, which we proceed to discuss now.

The presence of cubic self-couplings in the Higgs potential ensures mixing among the colored multiplets $(3^*,1,3)$ and $(3,3^*,1)$. This results in large corrections to the neutrino and neutretto masses at the one-loop level. A typical diagram is shown in Fig. 1. Such diagrams connect $N_1$ with itself and with $N_2$, and $N_2$ with itself, with contributions much larger than the tree-level contributions. We have tacitly assumed here that the cubic couplings in the Higgs potential have coefficients of order $v$. However, $\gamma_2/v$ and $\gamma_4/v$ should be chosen somewhat smaller than the other couplings in the potential, to guarantee convergence of the perturbation series. The same $\gamma_2$ and $\gamma_4$ appear in proton-decay diagrams (see below) and the lowest-order diagrams dominate only if $\gamma_{2,4}v/m_H{}^2$ are small compared to 1. Since the couplings $g$ and $g'$ are larger than $h$, by roughly an order of magnitude or so, it is enough to consider diagrams of order $g^3 v$ and $g'^3 v$.

By demanding $\gamma_2/v$ and $\gamma_4/v$ be somewhat smaller than the other Higgs self-couplings, we have guaranteed stability of the neutretto masses while going from one loop to two loops. However, for the neutrino mass, this is not the case and one has to go to two-loop diagrams for its stability. At the one-loop level, there is no $\nu$-$\nu$ diagram, and the large tree-level mixing term of $\nu$ with $N_1$ results only in a very small neutrino mass of order $g^3u^2/v$ since the Yanagida–Gell-Mann–Ramond–Slansky seesaw mechanism[16] is operative in this case. The contribution for $\nu$-$\nu$ mixing from two-loop diagrams is comparable to this mass. Since $E^0$ effectively decouples, we have the mass matrix for $(N_1, \nu, N_2)$ mixing,

$$\begin{bmatrix} g^3 v_2 f_1 & -2hu_1 + g^3 u_2 f_3 & g^3 v_2 f_2 \\ -2hu_1 + g^3 u_2 f_3 & \epsilon & 0 \\ g^3 v_2 f_2 & 0 & g^3 v_2 f_3 \end{bmatrix} , \tag{4.15}$$

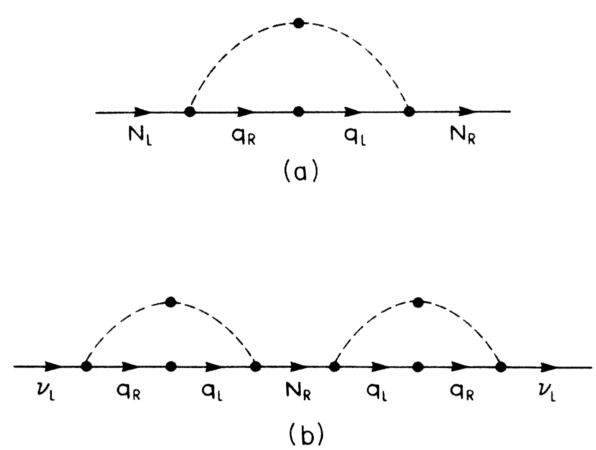


FIG. 1. (a) One-loop diagram for the neutrino and neutretto masses. The dashed lines correspond to the Higgs multiplets $(3^*,1,3)$ and $(3,3^*,1)$ which mix via the cubic terms in the Higgs potential. The solid internal lines represent quarks. (b) Diagram responsible for two-loop correction to the neutrino masses and mixings.

where $f_{1,2,3}$ are functions of the ratios $g'/g$ and $v_1/v_2$ and include the contributions from the loop integrals. These integrals are convergent, and have particularly simple forms if we neglect the mass of the internal fermions in comparison with the Higgs-boson mass. Here, we have kept the two-loop contribution $\epsilon$, for the $\nu$-$\nu$ entry [Fig. 1(b)] (all other two-loop terms contribute to only higher-order corrections of the neutrino masses):

$$\epsilon=(g^3u_2^2/v_2)f_4 \; . \tag{4.16}$$

The functions $f_{1,2,3,4}$ are given in terms of the loop integrals $K_{1,2,3,4}$,

$$f_1=(K_3+K_4)g'^2/g^2 \; , \tag{4.17a}$$

$$f_2=(K_3+K_4)(v_2/v_1)(g'/g)+K_1(v_1/v_2)(g'/g)+K_2(g'/g)^3 \; , \tag{4.17b}$$

$$f_3=K_1(v_1/v_2)+K_2(v_1/v_2)(g'/g)^2 \; , \tag{4.17c}$$

$$f_4=2(K_1+K_2)^2/((f_1+f_2)\{1+[(f_1-f_3)/(f_1+f_3)][1+4f_2^2/(f_1-f_3)^2]^{1/2}\}) \; , \tag{4.17d}$$

where

$$\begin{aligned}
K_1&=\frac{3}{8\pi^2}\frac{\gamma_2 v_1}{(m_{L3}^2-m_{R3}^2)}\ln\left[\frac{m_{L3}^2}{m_{R3}^2}\right] ,\\
K_2&=\frac{3}{8\pi^2}\frac{\gamma_4 v_1}{(\mu_{L3}^2-\mu_{R3}^2)}\ln\left[\frac{\mu_{L3}^2}{\mu_{R3}^2}\right] ,\\
K_3&=\frac{3}{8\pi^2}\frac{\gamma_4 v_2}{(m_{L2}^2-\mu_{R3}^2)}\ln\left[\frac{m_{L2}^2}{\mu_{R3}^2}\right] ,\\
K_4&=\frac{3}{8\pi^2}\frac{\gamma_4 v_2}{(\mu_{L3}^2-m_{R3}^2)}\ln\left[\frac{\mu_{L3}^2}{m_{R3}^2}\right] .
\end{aligned} \tag{4.18}$$

Here the $m$'s refer to the mass of the colored Higgs from $\phi$ and $\mu$ from $\chi$.

From (4.14), the masses of the neutrino and neutrettos follow:

$$m_{N_1,N_2}\simeq\tfrac{1}{2}g^3v_2(f_1+f_3)\times\left[1\pm\left[\frac{f_1-f_3}{f_1+f_3}\right]\left[1+\frac{4f_2^2}{(f_1-f_3)^2}\right]^{1/2}\right] , \tag{4.19}$$

$$m_\nu\simeq\frac{4h^2}{g^3}\frac{u_1^2}{v_2}\frac{f_3}{(f_2^2-f_1f_3)}+g^3\frac{u_2^2}{v_2}\frac{f_3^3}{(f_2^2-f_1f_3)}+\epsilon \; . \tag{4.20}$$

If the neutrino mass at the tree level is zero [i.e., case (B)], the $\nu-N_1$ entry in (4.15) is merely the one-loop contribution, which is $g^3u_2f_3$. Hence the neutrino mass for case (B) is

$$m_\nu\simeq g^3\frac{u_2^2}{v_2}\frac{f_3^3}{(f_2^2-f_1f_3)}+\epsilon \; . \tag{4.21}$$

A few comments are in order on these masses. Neutrettos have become heavy, but their masses may be as low as a few GeV. Neutrino mass can be a few eV in case (A), but is essentially zero in case B. We have

$$m_{\nu e}/m_{\nu\mu}\sim(m_e/m_\mu)^2(m_c/m_u)^3$$

in case (A), so that the electron neutrino is the heavier neutrino. As we show below, however, the requirement of reproducing correct masses and mixing for quarks reverses this mass pattern. In case (B), $m_{\nu e}/m_{\nu\mu}\sim(m_u/m_c)^3$ so that neutrino mass increases with generation, as in most other models. For the $\tau$ neutrino (or any other heavy flavor neutrino) the contribution from (4.21) will dominate. $\nu_\tau$ can be as heavy as a few keV and is heavier than the first two generation counterparts, for both choices of VEV's, (A) and (B).

We now proceed to construct a simple phenomenological model for quark mixing. We shall consider the choice of VEV's given by (4.3) only [i.e., case (A)]. Since the mixing of the third generation (or higher generations) into the first two is known to be very small, we consider here only the Cabibbo mixing between the first two generations of quarks. Flavor mixing in the leptonic sector, even if it exists, is very small and is neglected here (except for the neutral leptons, which can mix at the one-loop level through quark mixing).

As pointed out earlier, one can always choose a basis where the quark mixing angles vanish if there is only one Higgs multiplet. When two Higgs multiplets couple to the quarks, only one such mixing can be rotated away. We choose to work in a basis where $\chi$ does not connect different families. Imposing an explicit left-right symmetry and assuming for simplicity that the coupling of $\chi$ to the first generation is negligible, the Lagrangian which generates the quark masses and mixing is

$$\begin{aligned}L_q=Z_3\{&g_1\psi_1(3,3^*,1)\psi_1(3^*,1,3)\phi(1,3,3^*)+g_{12}\psi_1(3,3^*,1)\psi_2(3^*,1,3)\phi(1,3,3^*)\\&+g_{12}\psi_2(3,3^*,1)\psi_1(3^*,1,3)\phi(1,3,3^*)+[g_2\phi(1,3,3^*)+g_2'\chi(1,3,3^*)]\psi_2(3,3^*,1)\psi_2(3^*,1,3)+\text{H.c.}\} \; .\end{aligned} \tag{4.22}$$

Consequently, the up-quark matrix for $(u,c)$ mixing is

$$\begin{bmatrix} g_1u_1 & g_{12}u_1\\ g_{12}u_1 & g_2u_1\end{bmatrix} \tag{4.23}$$

and the down-quark matrix for $(d,s,B,B_s)$ is ($B_s$ is the second-generation analog of $B$)

$$\begin{bmatrix} g_1u_2 & g_{12}u_2 & 0 & 0 \\ g_{12}u_2 & g_2u_2 & 0 & g_2'v_2 \\ 0 & 0 & g_1v_1 & g_{12}v_1 \\ 0 & 0 & g_{12}v_1 & g_2v_1 \end{bmatrix} . \tag{4.24}$$

The quark masses and mixing follow:

$$m_u+m_c=(g_1+g_2)u_1 , \tag{4.25a}$$

$$m_um_c=|g_1g_2-g_{12}^2|u_1^2 , \tag{4.25b}$$

$$m_d^2+m_s^2=\frac{(g_1^2+g_{12}^2)^2g_2'^2\frac{v_2^2}{v_1^2}+(g_1^2+g_2^2+2g_{12}^2)(g_1g_2-g_{12}^2)^2}{(g_1g_2-g_{12}^2)^2+(g_1^2+g_{12}^2)g_2'^2\frac{v_2^2}{v_1^2}}u_2^2 , \tag{4.25c}$$

$$m_d^2m_s^2=\frac{(g_1g_2-g_{12}^2)^4u_2^4}{(g_1g_2-g_{12}^2)^2+(g_1^2+g_{12}^2)g_2'^2\frac{v_2^2}{v_1^2}} , \tag{4.25d}$$

and

$$\tan2\theta_u=\frac{2g_{12}}{g_2-g_1} , \tag{4.26a}$$

$$\tan\theta_d=\frac{g_1(g_1g_2-g_{12}^2)u_2^2-g_2m_d^2}{g_{12}(g_1g_2-g_{12}^2)u_2^2+g_{12}m_d^2} . \tag{4.26b}$$

Equations (25) and (26) can be solved in the approximation $g_2 \gg g_1$, $g_{12}$. With $m_u \simeq 5$ MeV, $m_d \simeq 10$ MeV, $m_c \simeq 1500$ MeV, $m_s \simeq 150$ MeV, and $\theta_c \simeq 0.23$ as inputs, we obtain

$$g_1:g_{12}:g_2 \simeq 1:4:280 , \tag{4.27a}$$

$$\frac{g_2'}{g_2}\frac{v_2}{v_1}\simeq 45 , \tag{4.27b}$$

$$\frac{u_1}{u_2}\simeq\frac{1}{6} . \tag{4.27c}$$

Equation (4.27c) also implies an upper bound of about 45 GeV on the mass of the top quark ($m_t=g_3u_1$), necessary for perturbative unification. Equation (4.27) will turn out to be significant in the context of proton-decay branching fractions, which is discussed in the next section. It may be mentioned that Eqs. (4.27) are a particular solution to Eqs. (4.25) and (4.26), and our choice is based only on the simplicity.

Mixing among different quark flavors induces mixing among different generations of neutral leptons at the one-loop level. Note that the second-generation neutrettos, even in the absence of family mixing among quarks, are much heavier than their first-generation counterparts, by roughly a factor of $(m_c/m_u)^3 \sim 10^7$. The effect of mixing is to increase the first-generation neutretto masses by a few orders of magnitude, and at the same time decrease the neutrino mass arising from its tree-level and one-loop level mixings with $N_1$, by several orders of magnitude. The source of the latter is again the see-saw mechanism. The two-loop contribution for the $\nu$-$\nu$ entry will survive and eventually dictate the neutrino mass hierarchy. To get some quantitative idea, let us assume that all the loop integrals give equal contribution (i.e., $K_1=K_2=K_3=K_4=K$). Then, if $g_{12}g_2^2v_1K \simeq 10^6$ GeV, which is a natural choice, the neutrettos get masses of order $10^6$ and $10^{13}$ GeV. Without the two-loop contribution, the neutrino masses are less than $10^{-8}$ eV. The two-loop terms, which are of the form (4.16), are clearly much larger and will correspond to the corrected neutrino masses. The mass hierarchy of the neutrinos is then $\nu_e < \nu_H < \nu_\tau$, for both case (A) and case (B), as in most other models. Recall that for case (A) in the absence of quark mixing $\nu_\mu$ was lighter than $\nu_e$, which is no longer true. $\nu_\mu$ can be a few eV and $\nu_e \sim 10$ eV. $\nu_\tau$ is in the keV range as before. Neutrino oscillation between $\nu_e$ and $\nu_\mu$ occurs, but with a small mixing angle $\sim 0.01$.

## V. PROTON DECAY

The Higgs field $\phi$ has no well-defined quark number, and couples to both leptons and quarks, without conserving baryon number, and can mediate proton decay. The Higgs potential is baryon-number nonconserving, and can, in principle, result in further contribution to proton decay, but this contribution is a factor $u/v$ smaller than the other mechanism. As one expects in any Higgs-boson-mediated proton-decay models, the heavy-flavor modes dominate. In our case, the decay modes expected are $\pi^0e^+$, $\pi^0\mu^+$, $\pi^+\bar{\nu}$, and $K^+\bar{\nu}$, with dominant ones being $K^+\bar{\nu}_\mu$ and $\pi^0\mu^+$. Here, $K^+\bar{\nu}_\mu$, for example, stands for all final states with the quantum numbers of $K^+\bar{\nu}_\mu$ (i.e., $\pi^0K^+\bar{\nu}_\mu$, $\pi^+\pi^-K^+\bar{\nu}_\mu$, etc.).[17] $p \to K^0\mu^+$ is not present in the theory. This can be understood as follows: Two processes can, in principle, lead to $p \to K^0\mu^+$. (i) $uu \to \bar{s}\mu^+$. This does not occur since there is no charge $\frac{4}{3}$ Higgs field.

(ii) $us \rightarrow u\mu^+$. This also does not occur since the Yukawa coupling matrix for the leptons is diagonal. It is so because Yukawa couplings of the form Eq. (4.6) takes place only with the Higgs field $\phi$. (Otherwise, the electron will become superheavy.) Though no prediction can be made of the absolute decay rate, due to our lack of knowledge of the Higgs-boson masses, certain branching ratios can be calculated. We emphasize again that proton decay is a consequence of the minimality of the model. If the 27-plet of the Higgs multiplet that couples to the leptons is a third one, then by imposing on the Higgs potential, an additional global symmetry, proton can be made absolutely stable.

We consider both the exchange diagrams and the fusion diagrams contributing to proton decay.[18] For a decay process, $a \rightarrow bc\bar{d}$ with an effective four-fermion interaction Lagrangian

$$\mathscr{L} = \frac{f}{m_H^2}\bar{b}\left[\frac{1-\gamma_5}{2}\right]a\bar{c}\left[\frac{1-\gamma_5}{2}\right]d \tag{5.1}$$

the decay rate can be calculated, in the approximation that $m_b$ is zero

$$\Gamma = \frac{f^2}{512\pi^3}\frac{m_a^5}{m_H^4}P , \tag{5.2}$$

where $P$ is the phase-space factor

$$P = \frac{s}{2}[1-3(x_1^2+x_2^2)+x_1^2x_2^2(1-x_1^2-x_2^2)] - \frac{s^3}{12}(5-x_1^2-x_2^2)+(x_1^4+x_2^4-2x_1^4x_2^4)\ln\left[\frac{1+s-x_1^2-x_2^2}{2x_1x_2}\right] + |x_1^4-x_2^4|\ln\left|\frac{x_1^2+x_2^2-|x_1^2-x_2^2|s-(x_1^2-x_2^2)^2}{2x_1x_2}\right| , \tag{5.3}$$

where

$$x_1 = \frac{m_c}{m_a}, \quad x_2 = \frac{m_d}{m_a} \tag{5.4}$$

and

$$s = [1-2(x_1^2+x_2^2)+(x_1^2-x_2^2)^2]^{1/2} . \tag{5.5}$$

To estimate proton lifetime and various branching fractions, we assume that the decay process is roughly described by Eqs. (5.1)–(5.5), with $m_H$ corresponding to the mass of the Higgs boson which mediates the decay. Then from (5.2), the part that depends on the specific model is $f^2/m_H^4$, which contains the Higgs-boson mass and the Yukawa couplings to the fermions. Since the rate is very sensitive to the Higgs-boson masses, the absolute decay rate cannot be reliably estimated. However, certain branching fractions can be calculated: Using (5.3)–(5.5),

$$\frac{\Gamma(p \rightarrow \pi^0 e^+)}{\Gamma(p \rightarrow \pi^0 \mu^+)} = \frac{g_1^2}{g_{12}^2}\frac{P_{\pi^0 e^+}}{P_{\pi^0 \mu^+}} \simeq \frac{1}{14} , \tag{5.6}$$

$$\frac{\Gamma(p \rightarrow \pi^+ \bar{\nu}_\mu)}{\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)} = 4\frac{g_{12}^2}{g_2^2}\frac{P_{\pi^+\nu_\mu}}{P_{K^+\nu_\mu}} \simeq \frac{1}{150} . \tag{5.7}$$

To arrive at the numbers in (5.6) and (5.7), we made use of (4.26) and assumed $m_a = m_p$, $m_c =$ the lepton mass, and $m_d =$ the final-state hadron mass, which seems reasonable.

The branching ratio $\Gamma(p \rightarrow \pi^0\mu^+)/\Gamma(p \rightarrow K^+\bar{\nu}_\mu)$ is given by

$$\frac{\Gamma(p \rightarrow \pi^0 \mu^+)}{\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)} \simeq 9\frac{g_{12}^2}{g_2^2}\left[1+\frac{m_{L3}^2}{m_{R3}^2}\right]^2\frac{P_{\pi^0\mu^+}}{P_{K^+\nu_\mu}} , \tag{5.8}$$

where we have neglected $\gamma_2$ and $\gamma_4$ contributions as explained previously. From this we find

$$2\% \lesssim \frac{\Gamma(p \rightarrow \pi^0 \mu^+)}{\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)} \lesssim 1 \tag{5.9}$$

as $m_{L3}/m_{R3}$ varies from $\frac{1}{2}-\frac{1}{3}$ to 2–3. Hence for $m_{L3}/m_{R3}$ of the order one $B(\pi^0\mu^+)$ and $B(K^+\bar{\nu}_\mu)$ are comparable with $B(K^+\bar{\nu}_\mu)$ somewhat larger.

Bound neutrons in the nuclei will also decay, violating baryon number. The decay modes expected are $\pi^- e^+$, $\pi^-\mu^+$, $\pi^0\bar{\nu}_e$, $\pi^0\bar{\nu}_\mu$, $K^0\bar{\nu}_e$, and $K^0\bar{\nu}_\mu$. Again, $n \rightarrow K^-\mu^+$ mode is absent. The branching fractions can be expressed in terms of the proton-decay branching fractions as follows:

$$\frac{\Gamma(n \rightarrow \pi^- e^+)}{\Gamma(p \rightarrow \pi^0 e^+)} \simeq \frac{\Gamma(n \rightarrow \pi^- \mu^+)}{\pi(p \rightarrow \pi^0 \mu^+)} \simeq \frac{4}{9} , \tag{5.10}$$

$$\frac{\Gamma(n \rightarrow \pi^0 \bar{\nu}_\mu)}{\Gamma(p \rightarrow \pi^+ \bar{\nu}_\nu)} \simeq \frac{9}{4}, \quad \frac{\Gamma(n \rightarrow K^0 \bar{\nu}_\mu)}{\Gamma(p \rightarrow K^+ \bar{\nu}_\mu)} \simeq 1 . \tag{5.11}$$

## VI. CONCLUSION

We have studied in detail, various aspects of the $[SU(3)]^3 \times Z_3$ grand-unification model. Only two energy scales, $m_u \sim 10^{14}$ GeV and $m_W \sim 10^2$ GeV are considered here. It is also possible that $[SU(3)]^3 \times Z_3$ first breaks down to one or more intermediate symmetric groups, which then breaks down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. This alternative will be reported elsewhere.

The $SU(3)_3 \times SU(2)_L \times U(1)_Y$-symmetric vacuum is shown to be stable. We have summarized the Higgs- and gauge-boson masses in Tables I and II. The quark and lepton masses have been worked out. All the new particles in the theory are much heavier than currently accessible energies, thus posing no phenomenological problem. We have constructed a simple and explicit model for quark mixing which correctly reproduces all the known fermion masses and the Cabibbo angle. The masses of the neutral leptons, i.e., the neutrinos and neutrettos, are evaluated at the one- and two-loop levels and shown to be stable. Once family mixing among quarks takes place, the lightest neutretto is about $10^5$ GeV. The neutrino mass hierarchy is $\nu_e < \nu_\mu < \nu_\tau$. $\nu_\tau$ can be a few keV, $\nu_\mu$ a few eV and $\nu_e \sim 10^{-4}$ eV.

Proton decay in the model is Higgs-boson mediated, and has to take place once we allow the same Higgs multiplet to couple to both quarks and leptons. If the Higgs sector is constrained to have only two 27-plets, quarks should couple to both, or else the Cabibbo angle will vanish, and $m_u/m_d = m_c/m_s = m_t/m_b$ will result. The proton can be made absolutely stable if there are at least three 27 of the Higgs multiplet. Because of the many uncertainties in the Higgs-boson masses, we cannot reliably estimate proton lifetime, but certain branching ratios can be predicted. $p \to K^+\bar{\nu}$ and $p \to \pi^0\mu^+$ are the dominant decay modes and may have comparable rates. The branching ratios

$$\Gamma(p \to \pi^+\bar{\nu}_\mu)/\Gamma(p \to K^+\bar{\nu}_\mu) \simeq 10^{-2}$$

and

$$\Gamma(p \to e^+\pi^0)/\Gamma(p \to \mu^+\pi^0) \simeq \tfrac{1}{14}$$

in our model and the $p \to K^0\mu^+$ mode is absent. Grand unified monopoles are present in the theory, but they do not induce proton decay. The ongoing proton-decay experiments will tell us if the proton decays dominantly to heavy flavors or not (if it decides to decay at all). That will establish the validity or otherwise of the model in its minimal version. We shall wait and see.

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[13]Cubic terms in $\phi$ (or $\chi$) are necessary to realize phenomenologically acceptable masses for the neutral leptons.

[14]A subsidiary condition, viz., $\lambda_{12}u_2v_1 - 2\gamma_3u_1 = 0$ has to be satisfied for this choice of VEV's. A nonzero $\langle(\chi_c)_3{}^2\rangle$ will eliminate this extra constraint.

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