Photodecay amplitudes in a flux-tube potential model for baryons

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(Received 12 August 1985)

We calculate the photodecay amplitudes of N and Δ resonances within a flux-tube quark model for baryons using a nonrelativistic quark-photon interaction. We find that the configuration mixings previously obtained by diagonalizing the hyperfine interaction give a good qualitative fit to the experimental data. A comparison with other potential models is also given.

I. INTRODUCTION

An important test for the hadron structure is provided by the electromagnetic interactions of baryons. The present status of predictions of the γN decay couplings arising from various quark models[1-4] is summarized by the Particle Data Group.[5]

The purpose of the present work is to calculate the photonic couplings by using the semirelativistic quark model described in Ref. 6 and the quark-photon interaction of Ref. 7. In Ref. 6 the baryon wave functions were obtained by the diagonalization of the hyperfine Hamiltonian in a space spanned by the **56**, **70**, and **20** SU(6) multiplets. The unperturbed wave functions used in the diagonalization were derived variationally by Carlson, Kogut, and Pandharipande[8] in the frame of a flux-tube quark model which yields a linearly confining potential with two- and three-body terms. The unperturbed Hamiltonian (representing the color-electric interaction) and the hyperfine splitting (originating from the color-magnetic interaction) are based on the same quark-gluon dynamics and accordingly give to the model a higher degree of consistency than that of a harmonic-oscillator model.[1,9-12] One important aspect is that the unperturbed spectrum is determined up to a single additive constant.

The outcome of our calculations are the helicity amplitudes $A_{1/2}, A_{3/2}$. Both the ground state of the nucleon and the resonances are described by the mixture of configurations obtained in the diagonalization procedure. We present results for two different sets of values of the quark mass m and the size parameter Λ of the quark used in deriving the hadron spectra. There is no free parameter in the photon-quark interaction.

In Sec. II we briefly describe the quark model of Ref. 6. In Sec. III the derivation of the decay amplitudes is presented. In Sec. IV our numerical results for these amplitudes are shown and compared with the experimental data. Section V is devoted to a discussion and a comparison with other models. Details concerning the wave functions and the analytic derivation of the decay amplitudes are given in the appendixes.

II. THE MODEL

The hadronic states under discussion have been obtained in Ref. 6 by the diagonalization of the hyperfine interaction in a truncated space spanned by the $\mathbf{56}(0^+,2^+)$, $\mathbf{56}'(0^+)$, $\mathbf{70}(0^+,1^-,2^+)$, and $\mathbf{20}(1^+)$ SU(6) multiplets. Each unperturbed wave function has a space part of the form

$$\psi_n(\mathbf{r}_{12},\mathbf{r}_{13},\mathbf{r}_{23})=F_{123}\prod_{i<j} f(r_{ij})\varphi_n(\mathbf{r}_{12},\mathbf{r}_{13},\mathbf{r}_{23}) , \qquad (2.1)$$

where r_{ij} are the interquark distances and f and F_{123} are the two- and three-body parts[8] of the ground-state wave function ($n=0$). The polynomials φ_n with $n\neq 0$ introduce radial or orbital excitations. The wave functions ψ_n are given explicitly in Appendix A in terms of the Jacobi relative coordinates

$$\boldsymbol{\rho}=\frac{1}{\sqrt{2}}(\mathbf{r}_1-\mathbf{r}_2) ,$$
$$\boldsymbol{\lambda}=\frac{1}{\sqrt{6}}(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3) . \qquad (2.2)$$

The functions f and F_{123} have the form

$$\ln f(r)=-W(r)(\lambda_1 r+\lambda_2 r^2)-[1-W(r)]\lambda_{1.5}r^{1.5} , \qquad (2.3)$$

with

$$W(r)=\frac{1+\exp(-r_0/a)}{1+\exp[(r-r_0)/a]} \qquad (2.4)$$

and

$$F_{123}=1-\beta\sqrt{\sigma}\left[\sum_i r_{i4}-\tfrac{1}{2}\sum_{i<j} r_{ij}\right] , \qquad (2.5)$$

TABLE I. Values of parameters of the hyperfine interactions used in the calculations. m is the mass and Λ is the size of the quark.

Set	m (MeV)	Λ (fm)
I	360	0.13
II	324	0.09

TABLE II. Photodecay helicity amplitudes. For each resonance the first and second lines are the results obtained with the parameter sets I and II, respectively. The third line reproduces the experimental data from the Particle Data Group (Ref. 5). Masses are in MeV and amplitudes in 10^{-3} GeV$^{-1/2}$. A factor i has been omitted from the amplitudes of the negative-parity resonances. The theoretical helicity amplitudes are calculated as explained in Appendix C.

Resonance	Main component	Mass	$A^p_{3/2}$	$A^p_{1/2}$	$A^n_{3/2}$	$A^n_{1/2}$
$P_{11}(1440)$	$^2N(\mathbf{56}',0^+)\frac{1}{2}^+$	1712		-30		$+19$
		1607		-31		$+19$
		1400–1480		-69 ± 7		$+37\pm19$
$D_{13}(1520)$	$^2N(\mathbf{70},1^-)\frac{3}{2}^-$	1579	$+213$	$+57$	-211	-98
		1496	$+202$	$+45$	-201	-88
		1510–1530	$+167\pm10$	-22 ± 10	-144 ± 14	-65 ± 13
$S_{11}(1535)$	$^2N(\mathbf{70},1^-)\frac{1}{2}^-$	1568		$+210$		-188
		1475		$+203$		-182
		1520–1560		$+73\pm14$		-76 ± 32
$S_{11}(1650)$	$^4N(\mathbf{70},1^-)\frac{1}{2}^-$	1673		$+68$		-41
		1627		$+77$		-48
		1620–1680		$+48\pm16$		-17 ± 37
$D_{15}(1675)$	$^4N(\mathbf{70},1^-)\frac{5}{2}^-$	1690	$+4$	$+3$	-35	-25
		1653	$+7$	$+5$	-39	-27
		1660–1690	$+19\pm12$	$+19\pm12$	-69 ± 19	-47 ± 23
$F_{15}(1680)$	$^2N(\mathbf{56},2^+)\frac{5}{2}^+$	1860	$+96$	$+23$	-28	$+5$
		1754	$+94$	$+22$	-30	$+2$
		1670–1690	$+127\pm12$	-17 ± 10	-30 ± 14	$+31\pm13$
$D_{13}(1700)$	$^4N(\mathbf{70},1^-)\frac{3}{2}^-$	1726	$+17$	$+2$	-58	-14
		1714	$+21$	$+1$	-64	-14
		1670–1730	0 ± 19	-22 ± 13	-2 ± 44	0 ± 56
$P_{11}(1710)$	$^2N(\mathbf{70},0^+)\frac{1}{2}^+$	1868		-40		$+25$
		1795		-48		$+35$
		1680–1740		$+5\pm16$		-5 ± 23
$P_{13}(1720)$	$^2N(\mathbf{56},2^+)\frac{3}{2}^+$	1859	$+47$	-120	-12	$+45$
		1752	$+45$	-118	-12	$+47$
		1690–1800	-35 ± 24	$+52\pm39$	-43 ± 94	-2 ± 26
$F_{17}(1990)$	$^4N(\mathbf{70},2^+)\frac{7}{2}^+$	2028	-3	-2	-10	-8
		1980	-5	-4	-8	-6
		1950–2050	$+31\pm55$	$+24\pm30$	-122 ± 55	-49 ± 45
$P_{33}(1232)$	$^4\Delta(\mathbf{56},0^+)\frac{3}{2}^+$	1308	-159	-90		
		1285	-181	-101		
		1230–1234	-258 ± 11	-141 ± 5		
$P_{33}(1600)$	$^4\Delta(\mathbf{56}',0^+)\frac{3}{2}^+$	1893	-80	-44		
		1904	-106	-38		
		1500–1900	$+1\pm22$	-20 ± 29		

TABLE II. (*Continued*).

Resonance	Main component	Mass	$A^p_{3/2}$	$A^p_{1/2}$	$A^n_{3/2}$	$A^n_{1/2}$
$S_{31}(1620)$	$^2\Delta(\mathbf{70},1^-)\frac{1}{2}^-$	1657		+154		
		1631		+141		
		1600–1650		19±16		
$D_{33}(1700)$	$^2\Delta(\mathbf{70},1^-)\frac{3}{2}^-$	1657	+212	+149		
		1631	+201	+149		
		1630–1740	+77±28	+116±17		
$F_{35}(1905)$	$^2\Delta(\mathbf{70},2^+)\frac{5}{2}^+$	1997	+28	+33		
		1962	+41	+36		
		1890–1920	−47±19	+27±13		
$P_{31}(1910)$	$^2\Delta(\mathbf{70},0^+)\frac{1}{2}^+$	1934		+26		
		1910		+35		
		1850–1950		−12±30		
$P_{33}(1920)$	$^4\Delta(\mathbf{56},2^+)\frac{3}{2}^+$	1986	+13	−45		
		1964	−39	−78		
		1860–2160	+23±?	+40±?		
$F_{37}(1950)$	$^4\Delta(\mathbf{56},2^+)\frac{7}{2}^+$	1985	−22	−17		
		1952	−21	−17		
		1910–1960	−90±13	−73±14		

where r_{i4} is the distance between the quark i and a point $\mathbf{r}_4$ where the flux tubes originating from quarks meet at 120°. For the particular cases where an angle between flux tubes is larger than 120°, the point $\mathbf{r}_4$ coincides with one of the quarks.[8] The parameters λ_1, λ_2, $\lambda_{1.5}$, r_0, a, and β have been determined variationally in Ref. 8. They take the values

$$\begin{aligned} &\lambda_1=0.198\ \mathrm{fm}^{-1}\ ,\\ &\lambda_2=0.637\ \mathrm{fm}^{-2}\ ,\\ &\lambda_{1.5}=1.4\ \mathrm{fm}^{-1.5}\ ,\\ &r_0=0.12\ \mathrm{fm}\ ,\\ &a=0.12\ \mathrm{fm}\ ,\\ &\beta=0.25\ \mathrm{GeV}^{-1}\ , \end{aligned} \tag{2.6}$$

and $\sqrt{\sigma}$ is the string-tension constant

$$\sqrt{\sigma}=1\ \mathrm{GeV\,fm}^{-1}\ .$$

The term containing λ_2 has been overlooked in Eq. (2.2) of Ref. 6 but calculations were in fact performed as indicated here with the above parameter values.

The hyperfine interaction is the sum of a spin-spin term and a tensor term[13] and it has been modified[7] to include a finite-sized quark.

In the present study we have considered two different sets of values for the mass m and the size Λ of the quark. They are given in Table I. Set I was used in Refs. 6 and 14. Set II was found to give a better overall hadron spectrum and in particular a better nucleon mass. The corresponding resonance masses can be read in Table II. The mixing angles associated with set I can be found in Ref. 6.

III. THE DECAY AMPLITUDES

In order to avoid any confusion we specify below our definitions and conventions.

The quark-photon interaction $\mathscr{H}$ used in these calculations is the same as that used in Refs. 3 and 7. Because of the symmetry of the wave function of three identical quarks, the operator $\mathscr{H}$ can be written as three times the contribution of the third quark. If the charge, momentum, and spin operators of the third quark are denoted by $q^{(3)}$, $\mathbf{p}^{(3)}$, and $\mathbf{S}^{(3)}$, one has

$$\mathscr{H}=-3\frac{eq^{(3)}}{m}(\mathbf{p}^{(3)}\cdot\mathbf{A}+g\mathbf{S}^{(3)}\cdot\mathbf{H})\ , \tag{3.1}$$

where g is the gyromagnetic factor of the quark, $\mathbf{H}$ is the magnetic field, and $\mathbf{A}$ the corresponding vector potential. For the emission of a right-handed polarized photon with momentum $\mathbf{k}$, the potential $\mathbf{A}$ reads

$$\mathbf{A}(\mathbf{r}^{(3)})=\left[\frac{2\pi}{k}\right]^{1/2}\epsilon^{*}a_{\mathbf{k}}^{\dagger}e^{-i\mathbf{k}\cdot\mathbf{r}^{(3)}} \tag{3.2}$$

with

$$\epsilon=-\frac{1}{\sqrt{2}}(1,i,0)\ . \tag{3.3}$$

Taking $\mathbf{k}$ along the quantization axis, $\mathscr{H}$ can be brought to the following convenient form:

$$\mathscr{H}=\mathscr{A}q^{(3)}S_{-}^{(3)}-\mathscr{B}q^{(3)}\ , \tag{3.4}$$

where

$$S_{-}^{(3)}=S_x^{(3)}-iS_y^{(3)}\ , \tag{3.5}$$

$$\mathscr{A}=6\mu\sqrt{\pi k}\,e^{-ikz^{(3)}}\ , \tag{3.6}$$

$$\mathscr{B}=-\frac{6\mu}{g}\sqrt{\pi/k}\,e^{-ikz^{(3)}}(p_x^{(3)}-ip_y^{(3)})\ , \tag{3.7}$$

where μ is the quark magnetic moment. We have taken

$$g=1\ , \tag{3.8}$$

$$\mu=0.13\ \mathrm{GeV}^{-1}\ . \tag{3.9}$$

For photoemission one needs to calculate matrix elements $\langle f\,|\,\mathscr{H}\,|\,i\,\rangle$ where $|\,i\,\rangle$ is the resonance and $|\,f\,\rangle$ is the nucleon ground state plus a photon. After performing the straightforward calculations in the flavor and spin space, the problem reduces to the calculation of the matrix elements of $\mathscr{A}$ and $\mathscr{B}$ in the coordinate space. For this purpose we rewrite $\mathscr{A}$ and $\mathscr{B}$ in terms of the Jacobi relative coordinates (2.2). If the spatial wave functions (2.1) are written as ψ_{LM}^{μ} (Appendix A), where the index μ specifies the symmetry with respect to permutations (ρ, λ, S, and A) of the three-quark wave function, we have to evaluate six-dimensional integrals of the following kind:

$$\langle\psi_{LM}^{\mu}\,|\,\mathscr{A}\,|\,\psi_{L'M'}^{\mu'}\rangle=6\mu\sqrt{\pi k}\int d^3\rho\, d^3\lambda(\psi_{LM}^{\mu})^{*}\times e^{i\sqrt{2/3}\mathbf{k}\cdot\boldsymbol{\lambda}^{(3)}}\psi_{L'M'}^{\mu'} \tag{3.10}$$

and

$$\langle\psi_{LM}^{\mu}\,|\,\mathscr{B}\,|\,\psi_{L'M'}^{\mu'}\rangle=-i2\sqrt{6}\sqrt{\pi/k}\,\mu g^{-1}\times\int d^3\rho\, d^3\lambda(\psi_{LM}^{\mu})^{*}e^{i\sqrt{2/3}\mathbf{k}\cdot\boldsymbol{\lambda}^{(3)}}\times\left[\frac{\partial}{\partial\lambda_x}-i\frac{\partial}{\partial\lambda_y}\right]\psi_{L'M'}^{\mu'}\ . \tag{3.11}$$

It is easy to show that expressions (3.10) and (3.11) reduce to three-dimensional integrals. One example is given in Appendix B. The specific matrix elements we need to calculate are those entering the definitions of the helicity amplitudes $A_{1/2}$ and $A_{3/2}$. By definition the amplitude $A_{1/2}$ is the matrix element $\langle f\,|\,\mathscr{H}\,|\,i\,\rangle$ where the resonance is in a state of angular momentum projection $J_z'=\frac{1}{2}$ and the nucleon has $J_z=-\frac{1}{2}$ in its ground state. The amplitude $A_{3/2}$ corresponds to a matrix element with $J_z'=\frac{3}{2}$ and $J_z=\frac{1}{2}$, respectively.

The main motivation of this study was to investigate the role of the configuration mixings produced by the model described in Ref. 6. Both the initial and final state $|\,f\,\rangle$ are a mixture of SU(6) basis states with specific angular momentum and parity. For the nucleon ground state we consider two versions of configuration mixings due to parameter sets I and II of Table I. These are

$$N\simeq 0.988\,|\,^2N(\mathbf{56},0^+)\tfrac{1}{2}^+\rangle+0.106\,|\,^2N(\mathbf{56'},0^+)\tfrac{1}{2}^+\rangle-0.110\,|\,^2N(\mathbf{70},0^+)\tfrac{1}{2}^+\rangle \tag{3.12}$$

for set I,

$$N\simeq 0.969\,|\,^2N(\mathbf{56},0^+)\tfrac{1}{2}^+\rangle+0.174\,|\,^2N(\mathbf{56'},0^+)\tfrac{1}{2}^+\rangle-0.172\,|\,^2N(\mathbf{70},0^+)\tfrac{1}{2}^+\rangle\ , \tag{3.13}$$

for set II. There are also other components but their amplitudes are smaller than 0.04 and have been neglected. The configuration mixings associated with the resonances obtained with set I are taken from Table I of Ref. 6. The matrix elements $\langle f\,|\,\mathscr{H}\,|\,i\,\rangle$ for $|\,f\,\rangle=|\,^2N(\mathbf{56},0^+)\frac{1}{2}^+\rangle$ and $|\,i\,\rangle$ equal to all SU(6) configurations entering the present calculations expressed as linear combinations of matrix elements of $\mathscr{A}$ and $\mathscr{B}$ are given in Appendix C.

IV. RESULTS

The photon momentum k is evaluated in the resonance center-of-mass frame where the partial-wave analysis is performed. It reads

$$k=\frac{m_R{}^2-m_N{}^2}{2m_R}\ , \tag{4.1}$$

where m_N and m_R are the calculated nucleon and resonance masses, respectively. They are given in column 3 of Table II for the resonances available from the Particle Data Group.[5] In this table each resonance is represented by three lines: the first and the second reproduce our numerical results corresponding to sets I and II, respectively, and the third line represents experimental data. The columns four to seven give the helicity amplitudes in the sequence $A_{3/2}^{p}$, $A_{1/2}^{p}$, $A_{3/2}^{n}$, and $A_{1/2}^{n}$. As mentioned, set II gives a better overall description of the N and Δ spectra than set I but for the decay amplitudes it does not always make a substantial change. This happens in cases where although m_N and m_R are changed, the photon momentum (4.1) varies only little. A change of sign takes place when set I is replaced by set II only in the case of the $A_{1/2}^{p}$ amplitude for the $P_{33}(1920)$ resonance.

For both sets the comparison with experimental data shows that the sign of the decay amplitudes is well reproduced in most cases. The few exceptions can be read in Table II. The results for the magnitude of the amplitudes are quite satisfactory. In particular, the absolute value of $A_{1/2}^{p}$ for $F_{15}(1680)$ turns out to be small (backward "missing" resonance) without imposing any constraint, as in Refs. 3 and 7, where the harmonic-oscillator parameter was fitted to cancel the amplitude $A_{1/2}^{p}$ of this resonance. As already established in Ref. 3, configuration mixings play an important role. For instance, the nonzero values of $A^p_{1/2}$ and $A^p_{3/2}$ of the F_{17} and D_{15} resonances arise entirely from the $|{}^2N(\mathbf{70},0^+)\frac{1}{2}^+\rangle$ component of the nucleon ground state, as it can easily be understood with the help of Appendix C. The results of Table II contain both the resonance and ground-state mixings. We have made a detailed analysis of each of these contributions separately. We found that the resonance mixing alone produces a sign change in the following cases: $A^{p,n}_{3/2}$ and $A^{p,n}_{1/2}$ of F_{35}, $A^n_{1/2}$ of $S_{11}(1650)$ and $A^n_{1/2}$ of P_{13} for both sets, $A^{p,n}_{1/2}$ of $P_{11}(1440)$ and $A^{p,n}_{3/2}$ of $P_{33}(1920)$ for set II. By the addition of the ground-state mixing to the resonance mixing, some amplitudes vary by a large factor. For instance, the $A^p_{1/2}$ amplitude of the Roper resonance changes from -2 to -30 (10^{-3} GeV$^{-1/2}$) for the set I case.

V. DISCUSSION

We think it useful to discuss the merits and failures of the present calculations in comparison with the results of other quark models in order to understand the kinds of improvements which can be done.

Comparison will be made with the work of Barbour and Ponting[2] and that of Koniuk and Isgur[3] because they are the most closely related to our study. An essential common feature with Ref. 2 is the linear confinement. An important difference is our inclusion of the $(\mathbf{70},0^+)$ and $(\mathbf{70},2^+)$ configuration in the baryon wave functions. The similarities between Ref. 3 and our work consist in considering entirely the same SU(6) subspace and neglecting the spin-orbit coupling. The main difference resides in the choice of the unperturbed Hamiltonian.

Because of the chosen SU(6) subspace we have common selection rules and common effects with Ref. 3, in particular, the role of the $|{}^2N(\mathbf{70},0^+)\frac{1}{2}^+\rangle$ configuration in the ground state and the importance of resonance mixing in changing the sign of some amplitudes.

Despite the differences in the nonelectromagnetic part of the Hamiltonian (type of confinement, quark-size effects), it turns out that the present helicity amplitudes are qualitatively comparable to those of Ref. 3.

There is an astonishing result in our calculations. Although the Roper resonance is too high by about 150 MeV with respect to the nucleon ground state, its decay amplitudes are quite well reproduced. Also the negative-parity spectrum seems to be a success but some of the corresponding decay amplitudes are too large.

In conclusion it seems to us that the decay amplitudes are satisfactorily well reproduced by the present calculations where there is no free parameter for a chosen calculated spectrum.[6] The model used in deriving the spectrum is based on QCD-inspired ideas and therefore has a certain degree of consistency due to the common origin of the spin-independent (linear confinement) and spin-dependent (hyperfine interaction) parts of the effective Hamiltonian.

ACKNOWLEDGMENTS

Useful discussions with N. Isgur, J. Paton, and R. Cashmore are gratefully acknowledged.

APPENDIX A

In this appendix we provide the explicit expressions of the wave functions ψ^{μ}_{LM} considered in the text. Only the wave functions for $M=0$ will be given here; the others can be obtained from those given below by using standard methods.

By writing

$$F=F_{123}\prod_{i<j} f(r_{ij})\ , \tag{A1}$$

one has, in terms of the Jacobi coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ [see Eqs. (2.2)],

$$\psi^S_{00}=N^S_{00}F\ , \tag{A2}$$

$$\psi^{S'}_{00}=N^{S'}_{00}[1-\alpha(\rho^2+\lambda^2)]F\ , \tag{A3}$$

$$\psi^{\rho}_{00}=N^{\rho}_{00}\boldsymbol{\rho}\cdot\boldsymbol{\lambda}F\ , \tag{A4}$$

$$\psi^{\lambda}_{00}=N^{\rho}_{00}\tfrac{1}{2}(\rho^2-\lambda^2)F\ , \tag{A5}$$

$$\psi^{\rho}_{10}=N^{\rho}_{10}\rho_0F\ , \tag{A6}$$

$$\psi^{\lambda}_{10}=N^{\rho}_{10}\lambda_0F\ , \tag{A7}$$

$$\psi^{A}_{10}=N^{A}_{10}(\rho_-\lambda_+-\rho_+\lambda_-)F\ , \tag{A8}$$

$$\psi^S_{20}=N^S_{20}[3(\rho_0{}^2+\lambda_0{}^2)-(\rho^2+\lambda^2)]F\ , \tag{A9}$$

$$\psi^{\rho}_{20}=N^{\rho}_{20}(3\rho_0\lambda_0-\boldsymbol{\rho}\cdot\boldsymbol{\lambda})F\ , \tag{A10}$$

$$\psi^{\lambda}_{20}=\tfrac{1}{2}N^{\rho}_{20}[3(\rho_0{}^2-\lambda_0{}^2)-(\rho^2-\lambda^2)]F\ . \tag{A11}$$

In the above equations, we have used the notations

$$\rho_{\pm}=\rho_x\pm i\rho_y,\quad \lambda_{\pm}=\lambda_x\pm i\lambda_y \tag{A12}$$

and N^{μ}_{L0} to designate the normalization factors given in Ref. 6.

APPENDIX B

As an example of the reduction of the $\mathscr{A}$ and $\mathscr{B}$ matrix elements to three-dimensional integrals, we consider here the $\langle\psi^S_{00}|\mathscr{A}|\psi^{\lambda}_{20}\rangle$ matrix element. In this case, Eq. (3.10) becomes

$$\langle\psi^S_{00}|\mathscr{A}|\psi^{\lambda}_{20}\rangle=6\mu\sqrt{\pi k}\times\int d^3\rho\, d^3\lambda\,\psi^S_{00}\cos(\sqrt{2/3}k\lambda_0)\psi^{\lambda}_{20}\ , \tag{B1}$$

or by using expressions (A2) and (A9),

$$\langle\psi^S_{00}|\mathscr{A}|\psi^{\lambda}_{20}\rangle=3\mu\sqrt{\pi k}N^S_{00}N^{\rho}_{20}\times\int d^3\rho\, d^3\lambda[3(\rho_0{}^2-\lambda_0{}^2)-(\rho^2-\lambda^2)]\times\cos(\sqrt{2/3}k\lambda_0)F^2\ . \tag{B2}$$

Since F depends only on ρ, λ, and $\boldsymbol{\rho}\cdot\boldsymbol{\lambda}$, one can expand F^2 in terms of the Legendre polynomials:

$$F^2=\sum_L f_L(\rho,\lambda)P_L(x)\ , \tag{B3}$$

with

$$x=\frac{\boldsymbol{\rho}\cdot\boldsymbol{\lambda}}{\rho\lambda} \tag{B4}$$

and

$$f_L(\rho,\lambda)=\frac{2L+1}{2}\int_{-1}^{+1}F^2(\rho,\lambda,x)P_L(x)dx\ . \tag{B5}$$

By introducing the expansion (B3) in Eq. (B2) and by using the addition theorem for the spherical harmonics, the integrations over $\hat{\boldsymbol{\rho}}$ and $\hat{\boldsymbol{\lambda}}$ can be performed in a straightforward manner. One is then left with the ρ, λ, and x integration variables. Explicitly, one obtains

$$\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^\lambda\rangle=3\mu\sqrt{\pi/k}\,N_{00}^S N_{20}^\rho\left[\sqrt{6}[S_2(4,1)-S_0(2,3)]+9/k[C_2(4,0)-C_0(2,2)]+\frac{9\sqrt{6}}{2k^2}[S_0(2,1)-S_2(4,-1)]\right]\ , \tag{B6}$$

where we use the notations

$$S_L(n,m)=16\pi^2\int_0^\infty\rho^n d\rho\int_0^\infty\lambda^m\sin(\sqrt{2/3}k\lambda)d\lambda\times\int_0^1 F^2P_L(x)dx\ , \tag{B7}$$

$$C_L(n,m)=16\pi^2\int_0^\infty\rho^n d\rho\int_0^\infty\lambda^m\cos(\sqrt{2/3}k\lambda)d\lambda\times\int_0^1 F^2P_L(x)dx\ . \tag{B8}$$

All other matrix elements can be treated in the same manner. The labor involved has been lightened by performing most of the algebraic calculations with the REDUCE symbolic-manipulation program.[15]

APPENDIX C

Here we explicitly provide the expressions of the matrix elements $\langle f|\mathscr{H}|i\rangle$ with $|i\rangle$ running over the 30 basis states considered in Ref. 6 and $|f\rangle=|^2N(\mathbf{56},0^+)\frac{1}{2}^+\rangle$ corresponding to the main component of the nucleon ground state (3.12) or (3.13). We shall use the notation

$$A_{1/2}^{p,n}=\langle\text{proton, neutron};\ J_z=-\tfrac{1}{2}\,|\mathscr{H}|\ \text{resonance};\ J_z'=\tfrac{1}{2}\rangle\ , \tag{C1}$$

$$A_{3/2}^{p,n}=\langle\text{proton, neutron};\ J_z=\tfrac{1}{2}\,|\mathscr{H}|\ \text{resonance};\ J_z'=\tfrac{3}{2}\rangle\ . \tag{C2}$$

We list only the amplitudes $A_{1/2,3/2}^{p,n}$ which do not vanish trivially due to total angular momentum conservation. For Δ resonances, we provide only the $A_{1/2}^p$ and $A_{3/2}^p$ amplitudes since one has

$$A_{1/2}^n=A_{1/2}^p\ , \tag{C3}$$

$$A_{3/2}^n=A_{3/2}^p\ . \tag{C4}$$

1. $|i\rangle=|^4N(\mathbf{70},2^+)\frac{7}{2}^+\rangle$

$$A_{1/2}^p=0\ , \tag{C5}$$

$$A_{1/2}^n=\frac{\sqrt{35}}{105}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^\lambda\rangle\ , \tag{C6}$$

$$A_{3/2}^p=0\ , \tag{C7}$$

$$A_{3/2}^n=\frac{\sqrt{21}}{63}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^\lambda\rangle\ . \tag{C8}$$

2. $|i\rangle=|^4\Delta(\mathbf{56},2^+)\frac{7}{2}^+\rangle$

$$A_{1/2}^p=-\frac{2\sqrt{35}}{105}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^S\rangle\ , \tag{C9}$$

$$A_{3/2}^p=-\frac{2\sqrt{21}}{63}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^S\rangle\ . \tag{C10}$$

3. $|i\rangle=|^2N(\mathbf{56},2^+)\frac{5}{2}^+\rangle$

$$A_{1/2}^p=\frac{\sqrt{15}}{15}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^S\rangle-\frac{\sqrt{10}}{15}\langle\psi_{00}^S|\mathscr{B}|\psi_{21}^S\rangle\ , \tag{C11}$$

$$A_{1/2}^n=-\frac{2\sqrt{15}}{45}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^S\rangle\ , \tag{C12}$$

$$A_{3/2}^p=-\frac{2\sqrt{5}}{15}\langle\psi_{00}^S|\mathscr{B}|\psi_{21}^S\rangle\ , \tag{C13}$$

$$A_{3/2}^n=0\ . \tag{C14}$$

4. $|i\rangle=|^2N(\mathbf{70},2^+)\frac{5}{2}^+\rangle$

$$A_{1/2}^p=\frac{\sqrt{30}}{30}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^\lambda\rangle-\frac{\sqrt{5}}{15}\langle\psi_{00}^S|\mathscr{B}|\psi_{21}^\lambda\rangle\ , \tag{C15}$$

$$A_{1/2}^n=-\frac{\sqrt{30}}{90}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^\lambda\rangle+\frac{\sqrt{5}}{15}\langle\psi_{00}^S|\mathscr{B}|\psi_{21}^\lambda\rangle\ , \tag{C16}$$

$$A_{3/2}^p=-\frac{\sqrt{10}}{15}\langle\psi_{00}^S|\mathscr{B}|\psi_{21}^\lambda\rangle\ , \tag{C17}$$

$$A_{3/2}^n=\frac{\sqrt{10}}{15}\langle\psi_{00}^S|\mathscr{B}|\psi_{21}^\lambda\rangle\ . \tag{C18}$$

5. $|i\rangle=|^4N(\mathbf{70},2^+)\frac{5}{2}^+\rangle$

$$A_{1/2}^p=0\ , \tag{C19}$$

$$A_{1/2}^n=-\frac{\sqrt{210}}{630}\langle\psi_{00}^S|\mathscr{A}|\psi_{20}^\lambda\rangle\ , \tag{C20}$$

$$A^{p}_{3/2}=0\ , \tag{C21}$$

$$A^{n}_{3/2}=-\frac{\sqrt{105}}{105}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle\ . \tag{C22}$$

6. $|i\rangle=|{}^{4}\Delta(\mathbf{56},2^{+})\tfrac{5}{2}^{+}\rangle$

$$A^{p}_{1/2}=\frac{\sqrt{210}}{315}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{20}\rangle\ , \tag{C23}$$

$$A^{p}_{3/2}=\frac{2\sqrt{105}}{105}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{20}\rangle\ . \tag{C24}$$

7. $|i\rangle=|{}^{2}\Delta(\mathbf{70},2^{+})\tfrac{5}{2}^{+}\rangle$

$$A^{p}_{1/2}=\frac{\sqrt{30}}{90}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle+\frac{\sqrt{5}}{15}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ , \tag{C25}$$

$$A^{p}_{3/2}=\frac{\sqrt{10}}{15}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ . \tag{C26}$$

8. $|i\rangle=|{}^{4}N(\mathbf{70},0^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=0\ , \tag{C27}$$

$$A^{n}_{1/2}=\frac{\sqrt{2}}{18}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{00}\rangle\ , \tag{C28}$$

$$A^{p}_{3/2}=0\ , \tag{C29}$$

$$A^{n}_{3/2}=\frac{\sqrt{6}}{18}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{00}\rangle\ . \tag{C30}$$

9. $|i\rangle=|{}^{2}N(\mathbf{56},2^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=-\frac{\sqrt{10}}{15}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{20}\rangle-\frac{\sqrt{15}}{15}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{S}_{21}\rangle\ , \tag{C31}$$

$$A^{n}_{1/2}=\frac{2\sqrt{10}}{45}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{20}\rangle\ , \tag{C32}$$

$$A^{p}_{3/2}=\frac{\sqrt{5}}{15}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{S}_{21}\rangle\ , \tag{C33}$$

$$A^{n}_{3/2}=0\ . \tag{C34}$$

10. $|i\rangle=|{}^{2}N(\mathbf{70},2^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=-\frac{\sqrt{5}}{15}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle-\frac{\sqrt{30}}{30}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle, \tag{C35}$$

$$A^{n}_{1/2}=\frac{\sqrt{5}}{45}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle+\frac{\sqrt{30}}{30}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ , \tag{C36}$$

$$A^{p}_{3/2}=\frac{\sqrt{10}}{30}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ , \tag{C37}$$

$$A^{n}_{3/2}=-\frac{\sqrt{10}}{30}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ . \tag{C38}$$

11. $|i\rangle=|{}^{4}N(\mathbf{70},2^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=0\ , \tag{C39}$$

$$A^{n}_{1/2}=-\frac{\sqrt{10}}{90}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle\ , \tag{C40}$$

$$A^{p}_{3/2}=0\ , \tag{C41}$$

$$A^{n}_{3/2}=\frac{\sqrt{30}}{90}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle\ . \tag{C42}$$

12. $|i\rangle=|{}^{2}N(\mathbf{20},1^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=0\ , \tag{C43}$$

$$A^{n}_{1/2}=0\ , \tag{C44}$$

$$A^{p}_{3/2}=0\ , \tag{C45}$$

$$A^{n}_{3/2}=0\ . \tag{C46}$$

13. $|i\rangle=|{}^{4}\Delta(\mathbf{56},0^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=-\frac{\sqrt{2}}{9}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{00}\rangle\ , \tag{C47}$$

$$A^{p}_{3/2}=-\frac{\sqrt{6}}{9}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{00}\rangle\ . \tag{C48}$$

14. $|i\rangle=|{}^{4}\Delta(\mathbf{56}',0^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=-\frac{\sqrt{2}}{9}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S'}_{00}\rangle\ , \tag{C49}$$

$$A^{p}_{3/2}=-\frac{\sqrt{6}}{9}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S'}_{00}\rangle\ . \tag{C50}$$

15. $|i\rangle=|{}^{4}\Delta(\mathbf{56},2^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=\frac{\sqrt{10}}{45}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{20}\rangle\ , \tag{C51}$$

$$A^{p}_{3/2}=-\frac{\sqrt{30}}{45}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{S}_{20}\rangle\ . \tag{C52}$$

16. $|i\rangle=|{}^{2}\Delta(\mathbf{70},2^{+})\tfrac{3}{2}^{+}\rangle$

$$A^{p}_{1/2}=-\frac{\sqrt{5}}{45}\langle\psi^{S}_{00}|\mathcal{A}|\psi^{\lambda}_{20}\rangle+\frac{\sqrt{30}}{30}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ , \tag{C53}$$

$$A^{p}_{3/2}=-\frac{\sqrt{10}}{30}\langle\psi^{S}_{00}|\mathcal{B}|\psi^{\lambda}_{21}\rangle\ . \tag{C54}$$

17. $|i\rangle = |{}^2N(\mathbf{56},0^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = \tfrac{1}{3}\langle \psi^S_{00} | \mathscr{A} | \psi^S_{00}\rangle \; , \tag{C55}$$

$$A^n_{1/2} = -\tfrac{2}{9}\langle \psi^S_{00} | \mathscr{A} | \psi^S_{00}\rangle \; . \tag{C56}$$

18. $|i\rangle = |{}^2N(\mathbf{56'},0^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = \tfrac{1}{3}\langle \psi^S_{00} | \mathscr{A} | \psi^{S'}_{00}\rangle \; , \tag{C57}$$

$$A^n_{1/2} = -\tfrac{2}{9}\langle \psi^S_{00} | \mathscr{A} | \psi^{S'}_{00}\rangle \; . \tag{C58}$$

19. $|i\rangle = |{}^2N(\mathbf{70},0^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = \frac{\sqrt{2}}{6}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{00}\rangle \; , \tag{C59}$$

$$A^n_{1/2} = -\frac{\sqrt{2}}{18}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{00}\rangle \; . \tag{C60}$$

20. $|i\rangle = |{}^4N(\mathbf{70},2^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = 0 \; , \tag{C61}$$

$$A^n_{1/2} = \frac{\sqrt{10}}{90}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{20}\rangle \; . \tag{C62}$$

21. $|i\rangle = |{}^2N(\mathbf{20},1^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = 0 \; , \tag{C63}$$

$$A^n_{1/2} = 0 \; . \tag{C64}$$

22. $|i\rangle = |{}^2\Delta(\mathbf{70},0^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = \frac{\sqrt{2}}{18}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{00}\rangle \; . \tag{C65}$$

23. $|i\rangle = |{}^4\Delta(\mathbf{56},2^+)\frac{1}{2}^+\rangle$

$$A^p_{1/2} = -\frac{\sqrt{10}}{45}\langle \psi^S_{00} | \mathscr{A} | \psi^S_{20}\rangle \; . \tag{C66}$$

24. $|i\rangle = |{}^4N(\mathbf{70},1^-)\frac{5}{2}^-\rangle$

$$A^p_{1/2} = 0 \; , \tag{C67}$$

$$A^n_{1/2} = \frac{\sqrt{30}}{90}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle \; , \tag{C68}$$

$$A^p_{3/2} = 0 \; , \tag{C69}$$

$$A^n_{3/2} = \frac{\sqrt{15}}{45}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle \; . \tag{C70}$$

25. $|i\rangle = |{}^2N(\mathbf{70},1^-)\frac{3}{2}^-\rangle$

$$A^p_{1/2} = \frac{\sqrt{3}}{9}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle - \frac{\sqrt{6}}{18}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; , \tag{C71}$$

$$A^n_{1/2} = -\frac{\sqrt{3}}{27}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle + \frac{\sqrt{6}}{18}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; , \tag{C72}$$

$$A^p_{3/2} = -\frac{\sqrt{2}}{6}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; , \tag{C73}$$

$$A^n_{3/2} = \frac{\sqrt{2}}{6}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; . \tag{C74}$$

26. $|i\rangle = |{}^4N(\mathbf{70},1^-)\frac{3}{2}^-\rangle$

$$A^p_{1/2} = 0 \; , \tag{C75}$$

$$A^n_{1/2} = -\frac{\sqrt{30}}{270}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle \; , \tag{C76}$$

$$A^p_{3/2} = 0 \; , \tag{C77}$$

$$A^n_{3/2} = -\frac{\sqrt{10}}{30}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle \; . \tag{C78}$$

27. $|i\rangle = |{}^2\Delta(\mathbf{70},1^-)\frac{3}{2}^-\rangle$

$$A^p_{1/2} = \frac{\sqrt{3}}{27}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle + \frac{\sqrt{6}}{18}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; , \tag{C79}$$

$$A^p_{3/2} = \frac{\sqrt{2}}{6}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; . \tag{C80}$$

28. $|i\rangle = |{}^2N(\mathbf{70},1^-)\frac{1}{2}^-\rangle$

$$A^p_{1/2} = -\frac{\sqrt{6}}{18}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle - \frac{\sqrt{3}}{9}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; , \tag{C81}$$

$$A^n_{1/2} = \frac{\sqrt{6}}{54}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle + \frac{\sqrt{3}}{9}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; . \tag{C82}$$

29. $|i\rangle = |{}^4N(\mathbf{70},1^-)\frac{1}{2}^-\rangle$

$$A^p_{1/2} = 0 \; , \tag{C83}$$

$$A^n_{1/2} = -\frac{\sqrt{6}}{54}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle \; . \tag{C84}$$

30. $|i\rangle = |{}^2\Delta(\mathbf{70},1^-)\frac{1}{2}^-\rangle$

$$A^p_{1/2} = -\frac{\sqrt{6}}{54}\langle \psi^S_{00} | \mathscr{A} | \psi^\lambda_{10}\rangle + \frac{\sqrt{3}}{9}\langle \psi^S_{00} | \mathscr{B} | \psi^\lambda_{11}\rangle \; . \tag{C85}$$

The amplitudes corresponding to $|f\rangle = |{}^2N(\mathbf{56'},0^+)\frac{1}{2}^+\rangle$ can be obtained from the amplitudes (C5) to (C85) by replacing $\langle \psi^S_{00}|$ by $\langle \psi^{S'}_{00}|$. Those corresponding to $|f\rangle = |{}^2N(\mathbf{70},0^+)\frac{1}{2}^+\rangle$ are not given explicitly here but are available on request.

We note that our expressions do not depend on the model which is used to obtain the spatial wave functions ψ^{μ}_{LM}. In particular, they also apply to the harmonic-oscillator model used, for instance, in Refs. 1, 3, and 7.

Before comparing a calculated helicity amplitude to the experimental one, we have to multiply it by the sign of the helicity amplitude $A_{1/2\pi N}$ for the decay into $N+\pi$. This is because the photoproduction experiment and analysis determine the quantity[16] $A^{p,n}_{1/2,3/2}\mathrm{sgn}\,(A_{1/2\pi N})$. In Table II we have assumed that $\mathrm{sgn}(A_{1/2\pi N})$ can be obtained from Ref. 3 adapted to our phase conventions for the spatial wave functions (Appendix A). An extra minus sign is moreover required for the decay of N^* resonances.[3]

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