Jets as a probe of quark-gluon plasmas

David A. Appel
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 29 August 1985)

We investigate the propagation of jets through a quark-gluon plasma. The transverse-momentum imbalance of a jet pair is shown to be sensitive to multiple scattering off the constituents of the plasma for expected values of the plasma temperature and size. This raises the possibility that such transverse-momentum imbalance could be used as a probe of a quark-gluon plasma produced by partonic interactions in ultrarelativistic nucleus-nucleus collisions.

I. INTRODUCTION

There is an increasing amount of interest in the properties of hadronic matter at extreme temperature and density. Future experiments with ultrarelativistic heavy ions, with energies up to 100 GeV/nucleon in the center-of-mass frame, are expected to produce such matter in the form of a quark-gluon plasma. It is hoped that the study of such a new state of nuclear matter, with its large number of degrees of freedom, could shed light on the nature of the confinement-deconfinement phase transition and the chiral-symmetry phase transition in quantum chromodynamics.[1] Numerical calculations with lattice gauge theories give predictions for the order of these phase transitions, and indicate that their critical temperatures T_c are about 200 MeV, relatively low and accessible to future experiments.

To date, various proposals have been suggested of possible signals of the presence of a quark-gluon plasma,[2] and of probes with which to explore such matter. Studies have been made of dilepton emission rates, strange-particle production, modifications to pion multiplicity distributions, photon production, and other high-energy processes which could play a role in the detection of a quark-gluon plasma.[3] In particular, it has been suggested that studies of jets produced in the same collision as the plasma[4,5] could give information on the mean free paths of quarks and gluons in the plasma, and hence on the basic parameters describing the plasma. Jets, particularly those from hadron-hadron scattering and electron-positron annihilation at high energies, have proven useful as a test of perturbative quantum chromodynamics. Their theory has become increasingly sophisticated, to the point where certain soft-gluon contributions can be calculated to all orders of perturbation theory.[6] Such successes encourage one to consider the production of jets in ultrarelativistic heavy-ion collisions, and to investigate them in an analogous way to that of hadronic collisions.[7] This paper studies the modification of certain cross sections of jets produced in the hard scattering of nucleon constituents due to their interaction with the plasma. The total cross section for jet production will go as A^2 (where A is the mass number of the colliding nuclei), so for very heavy ions the possibility exists that the overall jet cross section could be quite large.

The temperatures that can be expected to be produced in heavy-ion collisions are not particularly high. Recent arguments[8] indicate that at asymptotically large energies quark-gluon thermalization occurs at a temperature that is between 300 and 500 MeV (however, this result depends on the values of several input parameters, and could be off by a factor of 2 or so; in addition, large fluctuations are possible experimentally). In any case, jets produced in hard scatterings will typically have energies much greater than this (on the order of 10's of GeV), so *total* jet cross sections would not be strongly affected and it is reasonable to expect that a single jet would escape the region of the quark-gluon plasma in an identifiable form.[4] In this case, one needs a more sensitive measure of jet behavior, if jets are to prove useful as a plasma probe.

Many different ways of measuring jet properties have been utilized to date. In particular, energy-energy correlations[7,9] and relative transverse-momentum distributions[10] have proven useful as a sensitive measure of gluon effects. These quantities are calculable in QCD and are experimentally accessible.[11,12] In much the same spirit, we propose that the transverse-momentum imbalance of jet pairs produced in the hard scattering of ultrarelativistic-heavy-ion constituents could serve as a sensitive probe of a quark-gluon plasma.

In the head-on collision of two ultrarelativistic heavy ions, some nucleon constituents can be expected to undergo hard scattering, resulting in the production of two energetic secondary particles ("jets") which then propagate outward from the collision region (we ignore the relatively rare events which result in the production of more than two jets). A true two-body scattering would be confined to a plane; however,[10,12] initial-state bremsstrahlung causes these secondary particles to have a total nonzero momentum normal to any approximate scattering plane. It is this component of the transverse-momentum imbalance which we study. If there were no plasma surrounding the collision region, initial-state bremsstrahlung would be the only contribution which determines the magnitude of the normal momentum. In particular, the fragmentation of partons into hadrons does not produce any net normal momentum. However, scattering off of the particles that comprise the plasma contributes to (in fact, dominates, as we shall see) the normal momentum imbalance, which we call K_η.

Of course, the "true" plane which contains the trajectories of the two scattering ion constituents and the initial trajectories of the outgoing particles is not observable. But since the final particles have energies E much greater than either the energy of the initial-state bremsstrahlung, or of a typical member of the plasma ($\approx T$), the observed plane will deviate from the true plane by an angle of order K_η/E, typically much less than one (as we shall see); and the normal momentum K_η will also be uncertain by only this fraction. This normal momentum can be identified with the transverse momentum along the "bisector" of the two jets in the transverse plane, as in Ref. 12. Thus, with a small error the effective plane of scattering can be found by minimizing over all possible planes the quantity $\sum_i |(K_i)_\eta|$, where i runs over all particles in the final-state jets. Our treatment is similar in spirit to that of the "acoplanarity"[13] sometimes used in treatments of jets produced in e^+e^- annihilation.

The normal momentum K_η is related to the "transverse energy-energy correlation" defined by Ali, Pietarinen, and Stirling.[7] The jets which emerge from the collision have approximately equal and opposite transverse momentum p_T (up to a correction factor of order K_η/p_T). The momentum imbalance described above results in these jets being no longer back-to-back in the transverse plane by the azimuthal angle $\phi \approx K_\eta/p_T$, for small ϕ. The differential cross section $d\sigma/d\phi$ for such scattering is related to the probability (density) for normal momentum production by

$$\frac{1}{\sigma_0(p,p_T)}\frac{1}{p_T}\frac{d\sigma}{d\phi}=\frac{dP}{dK_\eta}, \tag{1}$$

where $\sigma_0(p,p_T)$ is the total cross section for jet production of jets with c.m. momentum p and transverse momentum p_T. The cross section $d\sigma/d\phi$ has been investigated in hadron-hadron collisions in Ref. 7. Here, we calculate the probability density dP/dK_η for scattering to occur with momentum imbalance K_η in the presence of a plasma.

In the parton model there are no bremsstrahlung effects, so we have simply $dP/dK_\eta=\delta(K_\eta)$. With perturbative QCD, multiple gluon emission from the hard scattering can be resummed in perturbation theory,[14] and for the one-dimensional normal momentum density has the form

$$\frac{dP}{dK_\eta}=\frac{1}{\pi}\int_0^\infty db\cos(K_\eta b)\exp[\widetilde{B}(b)]\,. \tag{2}$$

We will show that in the presence of a spherical quark-gluon plasma of radius R and temperature T the above equation is altered, for small K_η, by the simple replacement

$$\widetilde{B}(b)\rightarrow\widetilde{B}(b)+\widetilde{F}(b,R,T)\,. \tag{3}$$

We will see that the magnitude of dP/dK_η is sensitive to reasonable choices for R and T, justifying it as a possible probe of a quark-gluon plasma produced in an ultrarelativistic heavy-ion collision.

In Sec. II we show how multiple scatterings off the plasma constituents are summed to give $\widetilde{F}(b,R,T)$ of Eq. (3) in terms of hard-scattering cross sections and the number densities of the plasma constituents. Section III presents results for dP/dK_η obtained numerically for various values of the relevant parameters (T, R, and jet energy E). Section IV contains a brief discussion of fragmentation effects, which serve only to increase the effect (and possibly even dominate it), and conclusions.

II. EFFECT OF THE PLASMA

The spacetime evolution of a quark-gluon plasma has been (and continues to be) investigated in considerable theoretical detail. In the high-energy collision of two (Lorentz-contracted) heavy nuclei, most of the phase space divides into two regions: the leading particles which carry the baryon number of the incident nucleus, and the so-called central rapidity region.[15] The latter has a zero net baryon number, and undergoes longitudinal and transverse expansion on a time scale of a few fm.

We imagine in addition a hard-scattering producing jets, which then propagate through the plasma. We simplify the above picture somewhat, and consider this initial hard scattering to take place at the center of a static spherical region of quark-gluon plasma at temperature T, and assume that it produces, besides the soft-gluon bremsstrahlung from the initial state, two energetic particles (quarks or gluons) with a total invariant mass Q. For simplicity, we treat these particles as if they were massless and on-shell. We let $B(\mathbf{k}_T)$ denote the probability density for one of the final hard particles to acquire transverse momentum $\mathbf{k}_T$ due to compensation for the momentum of the initial-state bremsstrahlung, and $F(\mathbf{l}_T)$ denote the probability density for transverse-momentum production $\mathbf{l}_T$ due to scattering off the plasma constituents.[16] Explicit forms are given below. Then the probability density dP/dK_η is obtained by requiring that the net effect of all possible combinations of these two types of scattering add to give K_η:

$$\frac{dP}{dK_\eta}=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{1}{n!}\prod_{i=1}^{n}\int d^2k_{Ti}B(\mathbf{k}_{Ti})\frac{1}{m!}\prod_{j=1}^{m}\int d^2l_{Tj}F(l_{Tj})\delta\left[K_\eta-\sum_{i=1}^{n}(\mathbf{k}_{Ti})_\eta-\sum_{j=1}^{m}(l_{Tj})_\eta\right]\right], \tag{4}$$

where the $n=m=0$ term is simply $\delta(K_\eta)$. Here the factorials compensate for overcounting, and the subscript η on a quantity denotes its projection onto the normal direction. Because of the δ function, the right-hand side does not exponentiate, but its Fourier transform does. That is, we have

$$\int_{-\infty}^{+\infty}dK_\eta\exp(iK_\eta b)\frac{dP}{dK_\eta}=\exp[\widetilde{B}(b)+\widetilde{F}(b)]\,, \tag{5}$$

where

$$\widetilde{F}(b)=\int d^2 l_T \exp(i l_\eta b) F(\boldsymbol{l}_T) , \qquad (6)$$

and similarly for $\widetilde{B}(b)$. In Eq. (6), $l_\eta \equiv (\boldsymbol{l}_T)_\eta$.

The function $\widetilde{B}(b)$ has been calculated to various degrees of approximation, and the form which we use is that given by Davies, Webber, and Stirling[17] (the Q dependence of $\widetilde{B}$ is not indicated explicitly):

$$\widetilde{B}(b)=-\int_{(b_0/b)^2}^{Q^2} \frac{dq^2}{q^2}\left[\ln\left[\frac{Q^2}{q^2}\right] A'(\alpha_s(q))+B'(\alpha_s(q))\right] , \qquad (7)$$

where to lowest order in α_s

$$A'(\alpha_s)=(\alpha_s/2\pi)2C_F , \qquad (8a)$$

$$B'(\alpha_s)=(\alpha_s/2\pi)(-3C_F) , \qquad (8b)$$

and $b_0=2\exp(-\gamma_E)=1.123$ and $C_F=\frac{4}{3}$. Carrying out the integration, using the running coupling constant

$$\alpha_s(q)=\frac{1}{\beta\ln(q^2/\Lambda^2)} , \qquad (9)$$

where $\beta=(33-2N_F)/12\pi$ for N_F active flavor degrees of freedom, and Λ is the QCD scale parameter, gives

$$\widetilde{B}(b)=-\frac{C_F}{\pi\beta}\left[\left[\ln\ln\frac{Q^2}{\Lambda^2}-\ln\ln\frac{(b_0/b)^2}{\Lambda^2}\right]\left[\ln\frac{Q^2}{\Lambda^2}-\frac{3}{2}\right] -\ln\left[\frac{Q^2}{(b_0/b)^2}\right]\right] . \qquad (10)$$

This form for $\widetilde{B}(b)$ will be valid provided that $b \lesssim \Lambda^{-1}$. For $F(\boldsymbol{l}_T)$, the probability density for scattering elastically off the plasma constituents with transverse-momentum transfer $\boldsymbol{l}_T$, we propose the following form:

$$F(\boldsymbol{l}_T)=\sum_x n_x R \frac{d^2\sigma_x}{d^2 l_T} , \qquad (11)$$

where x runs over the different particle types comprising the plasma ($x=g,q_i,\bar{q}_i$), with n_x their number density. This equation essentially relates the plasma mean free path to the available distance for scattering (R) for each particular $\boldsymbol{l}_T$.

dP/dK_η can now be obtained by taking the inverse Fourier transform of Eq. (5); this gives Eq. (2), but with the replacement indicated in Eq. (3). To assure that this is properly normalized, it is necessary that $\widetilde{B}(0)=\widetilde{F}(0)=0$. It is consistent with the leading-logarithm approximation used to obtain $\widetilde{B}(b)$ to take $\widetilde{B}(b\leq b_0/Q)=0$.[18] This condition is obtained for $\widetilde{F}$ by amending Eq. (6) to read

$$\widetilde{F}(b)=\int d^2 l_T[\exp(i l_\eta b)-1]F(\boldsymbol{l}_T) \qquad (12)$$

(formally, this is equivalent to the usual "+ prescription" for distributions). Choosing appropriate coordinates for the $\boldsymbol{l}_T$ integration, so that its polar angle is measured from the normal direction, gives

$$\widetilde{F}(b)=2\pi\int dl_T l_T[J_0(b l_T)-1]F(l_T) , \qquad (13)$$

where $l_T=|\boldsymbol{l}_T|$.

The motivation for considering the momentum imbalance in the η direction now becomes apparent. With this choice $\widetilde{F}(b)$ is independent of the angle between the incoming nuclei and the outgoing jet, and as a consequence one needs only one Fourier transform to obtain dP/dK_η [cf. Eq. (2)]. Equally significant from an experimental point of view is that a detailed event-by-event analysis based on the jet angle is not necessary; all events of this class can be treated (and can contribute) equally.

To further develop the form of $\widetilde{F}(b)$, we proceed as follows. First, the right-hand side of Eq. (13) is multiplied by a factor of 2 because there are two hard particles in the final state which contribute equally and independently to K_η. In terms of the usual Mandelstam variable t we have

$$\frac{d^2\sigma}{d^2 l}=\frac{1}{\pi}\left.\frac{d\sigma}{dt}\right|_{t=-l^2} . \qquad (14)$$

The expressions for $d\sigma/dt$ were originally given by Horgan and Jacob;[19] the dominant contributions come from the terms $\sim 1/t^2$ due to gluon exchange ($|t| \ll s, |u|=s+t\approx s$). We consider only this singular dependence. $d\sigma/dt$ is then independent of Q, and hence of the hard-scattering angle. Since the cross sections for the processes $gg\to gg$, $gq_i\to gq_i$, and $g\bar{q}_i\to g\bar{q}_i$ are approximately an order of magnitude greater than those of quarks (or antiquarks) scattering off one another (an equivalent statement is that the mean free paths of gluons propagating through the plasma are less than that of quarks or antiquarks), we restrict our considerations to gluon jets. The relevant cross sections are

$$\frac{d\sigma}{dt}=\begin{cases}\frac{9}{2}\pi\alpha_s{}^2/t^2 & \text{for } gg\to gg ,\\ 2\pi\alpha_s{}^2/t^2 & \text{for } gq_i\to gq_i \text{ and } g\bar{q}_i\to g\bar{q}_i .\end{cases} \qquad (15)$$

Finally, treating the plasma as an ideal relativistic quantum gas with zero chemical potential, the number density of bosonic (fermionic) constituents of the plasma with g_B (g_F) degrees of freedom is given by

$$n(T)=\begin{cases}\frac{\zeta(3)}{\pi^2}g_B T^3 & \text{for bosons} ,\\ \frac{\zeta(3)}{\pi^2}\frac{3}{4}g_F T^3 & \text{for fermions} .\end{cases} \qquad (16)$$

Substitution into Eq. (11) gives finally

$$F(l_T)=9aRT^3\left[1+\frac{N_F}{4}\right]\frac{\alpha_s{}^2(l_T)}{l_T{}^4} , \qquad (17)$$

where $a=16\zeta(3)/\pi^2=1.949$. This is then substituted into Eq. (13), and it remains only to specify the integration limits for l_T.

The upper limit is given by phase-space limitations and, ignoring detailed kinematics and constants of order unity, is given by

$$l_{T,\max}=\sqrt{2ET} \ , \tag{18}$$

where E is the energy of the hard outgoing gluons ($\approx Q/\sqrt{2}$ for on-shell massless gluons). The integral is not strongly dependent on $l_{T,\max}$, justifying our lack of detailed kinematics. The integral is more sensitive to the choice for the lower integration limit. An ideal quark-gluon plasma screens color charge[20] with a Debye screening length $r_D \approx n_{\text{tot}}^{-1/3}$, where n_{tot} is the total number density of the plasma. A more precise specification of this relationship would require a complete analysis of the two-body effective potential; we therefore ignore possible numerical factors here (which we expect to be ~ 1), and take

$$l_{T\,\min}=r_D^{-1}=n_{\text{tot}}^{1/3}=[a(1+\tfrac{9}{16}N_F)]^{1/3}T \ , \tag{19}$$

with the understanding that the integral in Eq. (13) could have a significant dependence on the exact choice for $l_{T,\min}$.

For reasonable values of the plasma temperature T, a significant portion of the gluon-plasma scattering takes place in the nonperturbative region $l_T \lesssim \Lambda$. Various schemes have been proposed that enable one to deal with this region by altering the form of the coupling constant $\alpha_s(l)$.[21] For timelike processes, Pennington and Ross[22] argue that a more natural expansion parameter is $\bar{\alpha}_s(l)$, where

$$\bar{\alpha}_s(l)^{-1}=\beta\left[\ln^2\left[\frac{l^2}{\Lambda^2}\right]+\pi^2\right]^{1/2} . \tag{20}$$

Unless otherwise stated, this is the form for α_s which we employ in Eq. (17).

III. NUMERICAL RESULTS

Equations (10), (13), and (17)–(19) form the basis for the results which follow. Numerical integrations were performed to obtain the probability density dP/dK_η for reasonable ranges of the independent parameters T, Q, and R. These results are discussed below. We also discuss briefly what effect our choice for the effective coupling constant has on the final results.

We begin by multiplying the right-hand side of Eq. (2) by 2, so that dP/dK_η will be normalized to unity if one considers only positive K_η:

$$\int_0^\infty dK_\eta \frac{dP}{dK_\eta}=1 \ . \tag{21}$$

In addition, the upper limit on the b integral is replaced by Λ^{-1}; for $b>\Lambda^{-1}$ the integrand is negligible, so the proper normalization is still obtained. Thus our final expression for the transverse-momentum imbalance is

$$\frac{dP}{dK_\eta}=\frac{2}{\pi}\int_0^{\Lambda^{-1}} db\cos(K_\eta b)\exp[\tilde{B}(b)+\tilde{F}(b)] \ . \tag{22}$$

We take $N_F=3$ and $\Lambda=200$ MeV. For the independent parameters we choose as reasonable values $T=200$ MeV, $R=5$ fm (the approximate size of a heavy ion with $A\sim 100$), and $Q=10$ GeV, and consider the effect of varying each separately.

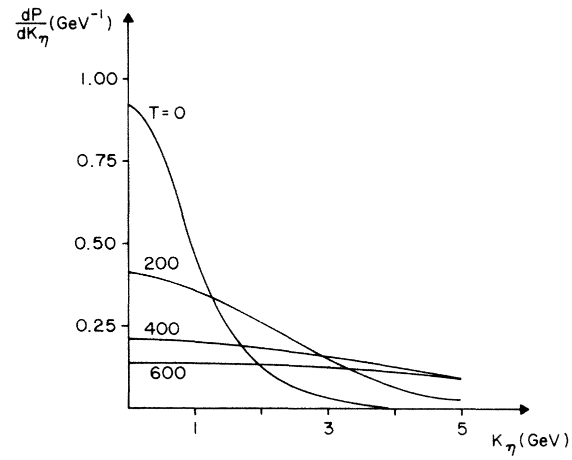

FIG. 1. The probability density dP/dK_η vs K_η at various temperatures T (in MeV), for plasma radius $R=5$ fm and jet-pair invariant mass $Q=10$ GeV.

In Fig. 1 is shown the variation of the scattering probability density with plasma temperature. As T increases, the interaction of the jet with the plasma constituents become more energetic (and more frequent), and as is seen, the probability density is significantly altered from the $T=0$ (no plasma) case. This is particularly true at high temperatures, but also even at the relatively low temperature of $T=200$ MeV. Based on this, one is encouraged to conjecture that someday jet behavior could be used as an effective thermometer of a QCD plasma.

Figure 2 shows what effect varying the jet energy (essentially Q) has on the probability distribution by repeating the calculation that leads to Fig. 1, but with $Q=50$ GeV. The initial-state bremsstrahlung leads in this case to a broader K_η distribution, but the effect of plasma scattering is still to decrease the small K_η probability density by a factor of over 2 at $T=400$ MeV. The effect of varying the plasma radius R is shown in Fig. 3. This dependence is also smooth. QCD plasmas are expected to be considerably greater than 1 fm in transverse dimension, so that deviations from the case of no plasma are still considerable for a wide range of plasma dimensions.

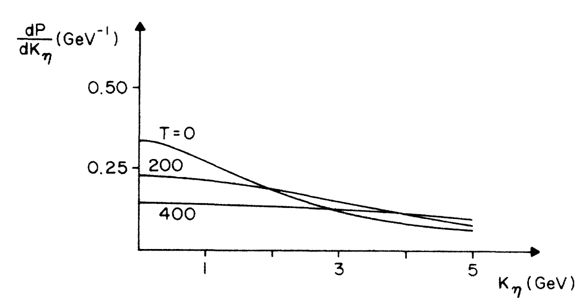

FIG. 2. dP/dK_η at various temperatures for $R=5$ fm and $Q=50$ GeV.

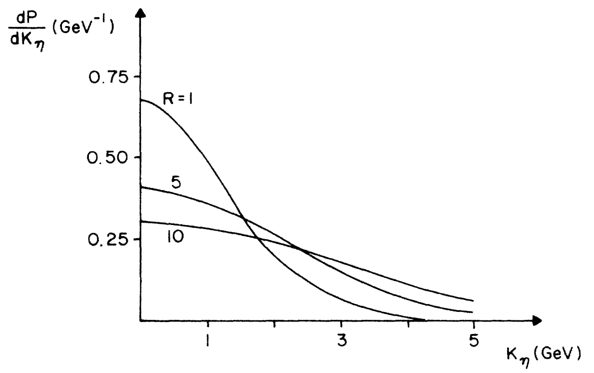

FIG. 3. dP/dK_η for various plasma radii R (in fm) at $T=200$ MeV and $Q=10$ GeV.

As previously mentioned, there exist coupling-constant schemes other than our choice in Eq. (20). We have checked that such choices do not significantly affect our results. The variation from our results is at most 20%, and then only at very small K_η (<1 GeV). On the other hand, the pure one-loop coupling of Eq. (9) gives results which vary by as much as 50% for $K_\eta < 3$ GeV.

IV. DISCUSSION

As our numerical results show, jet pairs can acquire considerable transverse-momentum imbalance as they propagate through a quark-gluon plasma. This leads us to suggest the possibility that such jet pairs could serve as a probe of the plasma itself. While admittedly our results could be sensitive to nonperturbative effects, we feel that they indicate the potential of jet physics as an important tool of investigation for ultrarelativistic heavy-ion collisions.

Our treatment of the quark-gluon plasma in this paper has admittedly been somewhat simplistic. We have not considered the expansion of the plasma, which would tend to smear the distribution over a range of temperatures. In addition, jets would be produced upon scattering from nonthermalized matter that would be produced in nuclear collisions, leading to a background jet distribution; jet scattering from the thermalized plasma, if not dominant, would need to be extracted from this background if jets are to be useful as a probe of quark-gluon plasmas.

However, because of fragmentation effects, we believe that our results are only a lower bound on the actual momentum imbalance that would occur. As a hard gluon propagates outward from the collision region, it will eventually undergo fragmentation into additional gluons and quark-antiquark pairs. Each of these particles will then scatter off the plasma, and in doing so obtain a nonzero normal momentum exactly as discussed above. They themselves fragment, and so on, until finally this aggregate of particles (all in the cone of the jet, more or less) escapes the plasma and is detected as hadrons. The crucial point is that such fragmentation is independent of the initial state, and hence $\widetilde{B}(b)$ remains entirely unaffected. However, $\widetilde{F}(b,R,T)$ will increase significantly, in rough proportion to the total plasma distance traversed by the initial gluon together with all of the particles, particularly gluons, created by all of the fragmentations. This geometrical increase of particles could result in an effective exponential dependence on R in $\widetilde{F}$, which could easily dominate the probability density dP/dK_η. We have not attempted to incorporate this potentially important effect into our analysis. Nonetheless, it can only add to the significance of jet transverse-momentum imbalance as a probe of a QCD plasma.

ACKNOWLEDGMENTS

The author is very grateful to George Sterman for suggesting the problem, and for encouragement. This work was supported in part by the National Science Foundation under Contract No. PHY-81-09110.

[1]For reviews, see M. Jacob and J. Tran Thanh Van, Phys. Rep. **88**, 321 (1982); T. W. Ludlam and H. E. Wegner, Nucl. Phys. **A418**, 1 (1984); L. McLerran, Fermilab Report No. Fermilab-Conf-84/101-T, 1984 (unpublished); J. Cleymans, R. V. Gavai, and E. Suhonen, Brookhaven Report No. BI-TP 85/05 (unpublished).

[2]For example, see B. Sinha, Phys. Lett. **157B**, 221 (1985); F. Halzen and H. C. Liu, Phys. Rev. D **25**, 1842 (1982).

[3]For example, see L. D. McLerran and T. Toimela, Phys. Rev. D **31**, 545 (1985); R. C. Hwa and K. Kajantie, *ibid.* **32**, 1109 (1985); E. L. Feinberg, Nuovo Cimento **34A**, 391 (1976); K. Kajantie and M. I. Miettinen, Z. Phys. C **9**, 341 (1981); **14**, 357 (1982); G. Domokos and J. I. Goldman, Phys. Rev. D **23**, 203 (1981); G. Domokos, *ibid.* **28**, 123 (1983); E. V. Shuryak, Phys. Lett. **78B**, 150 (1978); J. Rafelski and B. Miller, Phys. Rev. Lett. **48**, 1066 (1982).

[4]J. D. Bjorken, Fermilab Report No. Fermilab-Pub-82/59-THY, 1982 (unpublished), and erratum.

[5]L. McLerran, in *Quark Matter '84*, proceedings of the Fourth International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions, Helsinki, Finland, edited by K. Kajantie (Springer, Berlin, 1985), pp. 1–16.

[6]G. Parisi and R. Petronzio, Nucl. Phys. **B154**, 427 (1979); G. Curci, M. Greco, and Y. Srivastave, Phys. Rev. Lett. **43**, 834 (1979); J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981); **B197**, 446 (1982); **B213**, 545(E) (1983).

[7]A. Ali, E. Pietarinen, and W. J. Stirling, Phys. Lett. **141B**, 447 (1984).

[8]R. C. Hwa and K. Kajantie, University of Oregon Report Nos. OITS-289, 1985 (unpublished); and OITS-293, 1985 (unpublished).

[9]C. L. Basham, L. S. Brown, S. D. Ellis, and S. T. Love, Phys. Rev. Lett. **41**, 1585 (1978); Phys. Rev. D **19**, 2018 (1979); Y. Dokshitzer, D. I. D'Yakanov, and S. I. Troyan, Phys. Lett. **78B**, 290 (1978); for a review, see J. C. Collins and D. E. Soper, University of Oregon Report No. OITS-273, 1985 (unpublished).

[10]M. Greco, Z. Phys. C **26**, 567 (1985).

[11]PLUTO Collaboration, Ch. Berger *et al.*, Phys. Lett. **99B**, 292 (1981); Mark II Collaboration, D. Schlatter *et al.*, Phys. Rev. Lett. **49**, 251 (1982); Mark J Collaboration, B. Adeva *et al.*, *ibid.* **50**, 2051 (1983); CELLO Collaboration, H. J. Behrend *et al.*, DESY Report No. 82-022, 1982 (unpublished); JADE Collaboration, W. Bartel *et al.*, Z. Phys. C **25**, 231 (1984).

[12]P. Bagnaia *et al.*, Phys. Lett. **144B**, 283 (1984).

[13]A. De Rújula, J. Ellis, E. G. Floratos, and M. K. Gaillard, Nucl. Phys. **B138**, 387 (1978).

[14]M. Greco, Nucl. Phys. **B250**, 450 (1985).

[15]J. D. Bjorken, Phys. Rev. D **27**, 140 (1983); also in *Current Induced Reactions,* lectures at the International Summer Institute on Theoretical Particle Physics, Hamburg, 1975, edited by J. G. Korner, G. Kramer, and D. Schildknecht (Springer, Berlin, 1976); R. Anishetty, P. Koehler, and L. McLerran, Phys. Rev. D **22**, 2793 (1980); K. Kajantie, R. Raitio, and P. V. Ruuskanen, Nucl. Phys. **B222**, 152 (1983).

[16]F also depends on R and T, but usually we will suppress indication of this dependence.

[17]C. T. H. Davies, B. R. Webber, and W. J. Stirling, Nucl. Phys. **B256**, 413 (1985).

[18]In addition, it is necessary that $b < b_0/\Lambda$ in order for the double logarithm in Eq. (10) to be defined. It turns out that we only need $\widetilde{B}$ for $b \le 1/\Lambda$.

[19]R. Horgan and M. Jacob, Nucl. Phys. **B179**, 441 (1981).

[20]E. V. Shuryak, Zh. Eksp. Teor. Fiz. **74**, 408 (1978) [Sov. Phys. JETP **47**, 212 (1978)]; see also Phys. Rep. **61**, 71 (1980).

[21]For example, see G. Parisi and R. Petronzio, Nucl. Phys. **B154**, 427 (1979); P. Chiapetta and M. Greco, *ibid.* **B221**, 269 (1983); M. P. Tuite, *ibid.* **B236**, 48 (1984).

[22]M. R. Pennington and G. G. Ross, Phys. Lett. **102B**, 167 (1981).