Transverse-momentum distribution of π^0 and its product gamma rays

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We study the relationship between the transverse-momentum distribution of π^0 and that of its product γ rays. A new relationship between the average transverse momenta is obtained by taking into account the mass of the pion. The correlation between the average transverse momentum and energy density as observed in the Japanese-American Cooperative Emulsion Experiment is discussed.

I. INTRODUCTION

Experimental measurements on the distributions of π^0 often rely on the measurements of the distributions of the product γ rays. Much work[1,2] has been done to relate the moments of these two distributions and the projections of the distributions onto one of the two transverse axes. For the case when the pion mass can be neglected, approximate relations have been obtained[3,4] to relate the energy spectrum of π^0 with that of its product γ rays. The approximate relation

$$\langle p_{\pi T} \rangle = 2\langle p_{\gamma T} \rangle \tag{1.1}$$

was used[5,6] to infer the average transverse momentum $\langle p_{\pi T} \rangle$ of the primary pions from the average transverse momentum $\langle p_{\gamma T} \rangle$ of the detected γ rays.

Recently, there has been much interest in the transverse-momentum distribution of particles produced in nucleus-nucleus collisions at ultrarelativistic energies, as the distribution may reveal much about the dynamics during the collision process and the possibility of producing a quark-gluon plasma.[7–13] Of particular interest is the recent experimental data from the Japanese American Cooperative Emulsion Experiments (JACEE). The JACEE data[14–17] indicate a correlation between the average transverse momentum of the primary π^0 particles $\langle p_{\pi T} \rangle$ and the rapidity density in the central rapidity region indicating the onset of a phenomenon with new characteristics. In analyzing the JACEE data, Eq. (1.1) was used. This is a reasonable assumption as the JACEE measurements involve $\langle p_{\gamma T} \rangle$ greater than or equal to about 0.15 GeV/c. It is nonetheless useful to assess the amount of error by obtaining a more accurate relation between $\langle p_{\pi T} \rangle$ and $\langle p_{\gamma T} \rangle$. A more accurate relation may also help in other types of analysis involving events for which $\langle p_{\gamma T} \rangle$ is not large.

In many other problems, it is desirable to know the detailed shape of one transverse distribution if the other transverse distribution is known, as for example, when the π^0 distribution is a complicated distribution, or when one wants to estimate the photon background in the photon bremsstrahlung of heavy ions.[18–20] In the general case, this can be obtained by a numerical integration. In the special case where the mass of the pion can be neglected, there is a simple relationship between the transverse distribution of π^0 and that of its product γ rays. Such a relationship has been written down[5] and attributed to Sternheimer.[4] However, the result of Sternheimer and an even earlier result of Carlson, Hooper, and King[3] gave a relation between the energy spectrum of π^0 and that of its γ rays, and not between the transverse distributions. Kopylov[1,2] obtained a similar relation between the projections of the distributions onto one of the two transverse axes which are different from the transverse distributions. On the other hand, in experiments such as JACEE, the transverse-momentum distribution is measured. It is desirable to derive the relation between the transverse distributions of $dN_\pi/dp_{\pi T}$ and $dN_\gamma/dp_{\gamma T}$.

In this paper, we study the relationship between the transverse-momentum distribution of π^0 and that of its product γ rays. We discuss in Sec. II a simple analytical relation between $\langle p_{\pi T} \rangle$ and $\langle p_{\gamma T} \rangle$, taking into account the pion rest mass. A simple relation between the transverse distributions of π^0 and that of its product γ rays is derived for the case when the mass of the pion can be neglected. As numerical integration of the pion spectrum to obtain the γ spectrum often runs into the difficulty of false singularities, we would also like to show in Sec. III and the Appendix how one can integrate the pion distribution numerically. In Sec. IV we discuss the correlation between $\langle p_{\pi T} \rangle$ and the energy density as revealed by the JACEE data.

II. RELATION BETWEEN $\langle p_{\pi T} \rangle$ AND $\langle p_{\gamma T} \rangle$

We consider the distribution of the pions in the transverse direction $dN_\pi/dp_{\pi T}$ and the same distribution projected onto one of the two transverse axes $dN_\pi/dp_{\pi x}$. They are normalized according to

$$\int_0^\infty (dN_\pi/dp_{\pi T})dp_{\pi T} = 2\int_0^\infty (dN_\pi/dp_{\pi x})dp_{\pi x} = N_\pi \,, \tag{2.1}$$

where N_π is the π^0 multiplicity. The two distributions are related by[1,2]

$$\frac{dN_\pi}{dp_{\pi x}} = \frac{1}{\pi}\int_{|p_{\pi x}|}^\infty \frac{dN_\pi}{dp_{\pi T}} \frac{dp_{\pi T}}{(p_{\pi T}{}^2 - p_{\pi x}{}^2)^{1/2}} \tag{2.2}$$

and by the inverse transformation

$$\frac{dN_\pi}{dp_{\pi T}} = -2\frac{d}{dp_{\pi T}}\int_{p_{\pi T}}^{\infty}\frac{dN_\pi}{dp_{\pi x}}\frac{p_{\pi x}dp_{\pi x}}{(p_{\pi x}{}^2-p_{\pi T}{}^2)^{1/2}} . \quad (2.3)$$

Thus, the expectation values of $p_{\pi T}$ and $p_{\pi x}$ are related as follows:

$$\langle p_{\pi T}\rangle = \pi\int_0^\infty dp_{\pi x}\frac{dN_\pi}{dp_{\pi x}}p_{\pi x} = \pi\langle p_{\pi x}\rangle . \quad (2.4)$$

There are similar relations for the distributions of the γ rays in the transverse direction $p_{\gamma T}$ and in its projection onto one of the transverse axes $p_{\gamma x}$. A relation between the π^0 distribution $dN_\pi/dp_{\pi x}$ and the γ distribution $dN_\gamma/dp_{\gamma x}$ has been given by Kopylov[1,2] [Eq. (18) of Ref. 2]. From this relation, it is easy to show that

$$\langle p_{\gamma T}\rangle = \frac{\pi}{8}\int_0^\infty dp_{\pi x}\frac{dN_\pi(p_{\pi x}+E_{\pi x})^2}{dp_{\pi x}E_{\pi x}}$$

$$= \frac{\pi}{8}\langle (p_{\pi x}+E_{\pi x})^2/E_{\pi x}\rangle , \quad (2.5)$$

where $E_{\pi x}$ is defined as

$$E_{\pi x} = (m_\pi{}^2+p_{\pi x}{}^2)^{1/2} . \quad (2.6)$$

To proceed further, we need an explicit form of the π^0 distribution $dN_\pi/dp_{\pi x}$. We note that the low-p_T distribution of pions in nucleon-nucleon or nucleus-nucleus collisions follows an exponential behavior as[21,22]

$$dN_\pi/d\mathbf{p}_{\pi T} \propto \exp(-p_{\pi T}/\alpha_\pi) . \quad (2.7)$$

Thus, the projected distribution of π^0 along one of the two transverse axes is given by

$$dN_\pi/dp_{\pi x} \propto p_{\pi x}K_1(p_{\pi x}/\alpha_\pi)/\pi\alpha_\pi , \quad (2.8)$$

where K_1 is a modified Bessel function.[23] The distribution $dN_\pi/dp_{\pi x}$ is finite at $p_{\pi x}=0$ and has an exponential decay. We approximate the distribution (2.8) of $dN_\pi/dp_{\pi x}$ by

$$dN_\pi/dp_{\pi x} \sim \pi\exp(-\pi\,|p_{\pi x}|\,/\langle p_{\pi T}\rangle)/2\langle p_{\pi T}\rangle . \quad (2.9)$$

With this approximation, Eq. (2.5) gives a relation between the average transverse momenta of π^0 and the produced photons as follows:

$$\frac{\langle p_{\gamma T}\rangle}{\langle p_{\pi T}\rangle} = \frac{1}{4}\left[1+\frac{\pi}{2}z[\mathbf{H}_1(z)-Y_1(z)] - \frac{\pi}{4}z^2[\mathbf{H}_0(z)-Y_0(z)]\right] , \quad (2.10)$$

where z is

$$z = \pi m_\pi/\langle p_{\pi T}\rangle , \quad (2.11)$$

$\mathbf{H}$ is the Sturve function, and Y is a Bessel function.[23] One can easily show that Eq. (2.10) has the proper limits. As $\langle p_{\pi T}\rangle$ approaches infinity, it gives Eq. (1.1). As $\langle p_{\pi T}\rangle$ approaches zero, it gives $\langle p_{\gamma T}\rangle = \pi m_\pi/8$, as it should.

From the tabulated values of the Sturve and Bessel functions, we can obtain the relation between the two average transverse momenta as shown in Fig. 1. The result by neglecting the mass of the pion, as given by Eq. (1.1), is also exhibited as the dashed curve. One finds that there is a difference of about 15% for $\langle p_{\gamma T}\rangle \sim 0.15$ GeV/c, and the difference decreases as $\langle p_{\gamma T}\rangle$ increases.

For the case when the mass of the pion can be neglected, as in the case when $\langle p_{\pi T}\rangle$ is much greater than m_π, we can prove that

$$\frac{dN_\gamma}{dp_{\gamma T}} = -2\frac{d}{dp_{\gamma T}}\int_{p_{\gamma T}}^{\infty}dp_{\pi x}\frac{dN_\pi}{dp_{\pi x}} \times (p_{\pi x}{}^2-p_{\gamma T}{}^2)^{1/2}\frac{1}{p_{\pi x}} . \quad (2.12)$$

We can rewrite the π^0 transverse-momentum distribution $dN_\pi/dp_{\pi T}$ of Eq. (2.3) in the following form:

$$\frac{dN_\pi}{dp_{\pi T}} = 2p_{\pi T}\left[\frac{d}{dp_{\pi T}}\right]^2 \times \int_{p_{\pi T}}^{\infty}dp_{\pi x}\frac{dN_\pi}{dp_{\pi x}}(p_{\pi x}{}^2-p_{\pi T}{}^2)^{1/2}\frac{1}{p_{\pi x}} . \quad (2.13)$$

By comparing Eqs. (2.12) and (2.13), we obtain

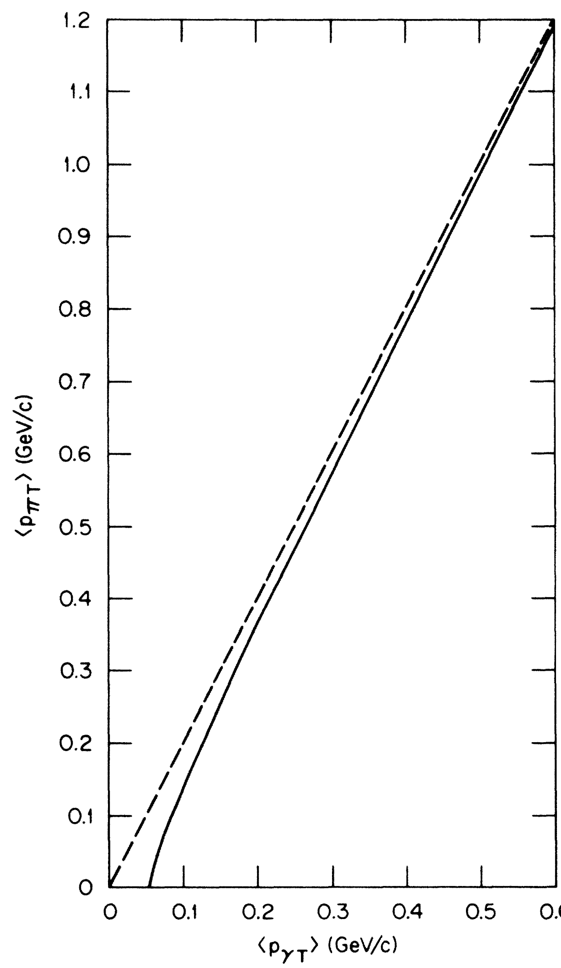

FIG. 1. The relation between $\langle p_{\pi T}\rangle$ and $\langle p_{\gamma T}\rangle$ as evaluated from the new relation (2.10) is shown as the solid curve. The dashed curve is the approximate relation $\langle p_{\pi T}\rangle = 2\langle p_{\gamma T}\rangle$ of Eq. (1.1).

$$\frac{dN_\pi}{dp_{\pi T}} = -p_{\pi T}\left[\frac{d}{dp_{\gamma T}}\frac{dN_\gamma}{dp_{\gamma T}}\right]_{p_{\gamma T}=p_{\pi T}} , \qquad (2.14)$$

which is the desired formula for transverse-momentum distributions. This equation is similar in form to those of Carlson, Hooper, and King,[3] Sternheimer,[4] and Kopylov[1,2] but differs in substance. The relation obtained by Carlson, Hooper, and King and Sternheimer relates energy spectra while the relation obtained by Kopylov relates the projection of the transverse-momentum distribution onto one of the transverse axes.

III. NUMERICAL INTEGRATION TO OBTAIN THE PHOTON DISTRIBUTION

In the general case when the mass of the pion cannot be neglected or when the pion distribution is a complicated function, the distribution of the product γ rays can be obtained only by numerical methods. The momentum distribution of the photons from the decay of the π^0 particles $E_\gamma dN_\gamma/d\mathbf{p}_\gamma$ can be obtained in terms of the π^0 momentum distribution $E_\pi dN_\pi/d\mathbf{p}_\pi$ by

$$\frac{E_\gamma dN_\gamma}{d\mathbf{p}_\gamma}(\mathbf{p}_\gamma) = M\int\frac{d\mathbf{p}_\pi}{E_\pi}\frac{E_\pi dN_\pi}{d\mathbf{p}_\pi}(\mathbf{p}_\pi)\int\frac{d\mathbf{p}}{E}\frac{\delta(E-m_\pi/2)}{2\pi m_\pi}\delta(\mathbf{p}_\gamma - L(\mathbf{p},\mathbf{p}_\pi))E_\gamma . \qquad (3.1)$$

Here, $M=2$ is the photon multiplicity for each π^0 decay, $(E,\mathbf{p})$ is the four-momentum of the detected photon in the π^0 rest frame, and L is the Lorentz transformation of $\mathbf{p}$ from the π^0 rest frame to the frame in which $\mathbf{p}_\gamma$ is studied (which we choose to be the laboratory frame). After making use of the identity

$$\delta(\mathbf{p}_\gamma - L(\mathbf{p},\mathbf{p}_\pi))E_\gamma = \delta(\mathbf{p} - L^{-1}(\mathbf{p}_\gamma,\mathbf{p}_\pi))E , \qquad (3.2)$$

where L^{-1} is the inverse Lorentz transformation, we obtain the result

$$\frac{E_\gamma dN_\gamma}{d\mathbf{p}_\gamma} = M\int\frac{d\mathbf{p}_\pi}{E_\pi}\frac{E_\pi dN_\pi}{d\mathbf{p}_\pi}(\mathbf{p}_\pi)\frac{\delta(E-m_\pi/2)}{2\pi m_\pi} . \qquad (3.3)$$

Many ways of numerical integration of the above would lead to a false singularity near the origin and a numerically inaccurate result. To bypass such a difficulty, it is best to work in a rotated coordinate system in which the momentum vector $\mathbf{p}_\gamma$ lies in the z' axis and the laboratory z axis lies in the azimuthal plane of $\phi'_z=0$. We give in the Appendix the details of the procedure.

For numerical purposes, we take a primary π^0 momentum distribution of the form[21,22,24]

$$\frac{E_\pi dN_\pi}{d\mathbf{p}_\pi}(\mathbf{p}_\pi) = C[(1-x_+)(1-x_-)]^4\exp(-p_{\pi T}/\alpha_\pi) . \qquad (3.4)$$

The light-cone variables x_+ and x_- are given by

$$x_+ = \frac{m_{\pi T}}{m_N}\exp(y-y_B) \qquad (3.5)$$

and

$$x_- = \frac{m_{\pi T}}{m_N}\exp(y_T-y) , \qquad (3.6)$$

where $m_{\pi T}$ is the π^0 transverse mass, m_N is the nucleon rest mass, and y_B and y_T are the average beam rapidity and target rapidity, respectively. For $C=C_{NN}=4.14$ GeV$^{-2}c^3$ and $\alpha_\pi=0.182$ GeV/c, such a parametrization gives the correct distribution of π^+ particles for pp collisions at $\sqrt{s}=30.6$ GeV (Refs. 21 and 22). For nucleus-nucleus collisions, the constant C needs to be chosen to give the plateau rapidity density dN/dy and the quantities y_B and y_T need to take into account the slowing down of the nucleons as they traverse the other nuclei. The relationship between $\langle p_{\gamma T}\rangle$ and $\langle p_{\pi T}\rangle$ is independent of C and is not sensitive to y_B and y_T.

As an illustration, we plot in Fig. 2 the distributions of

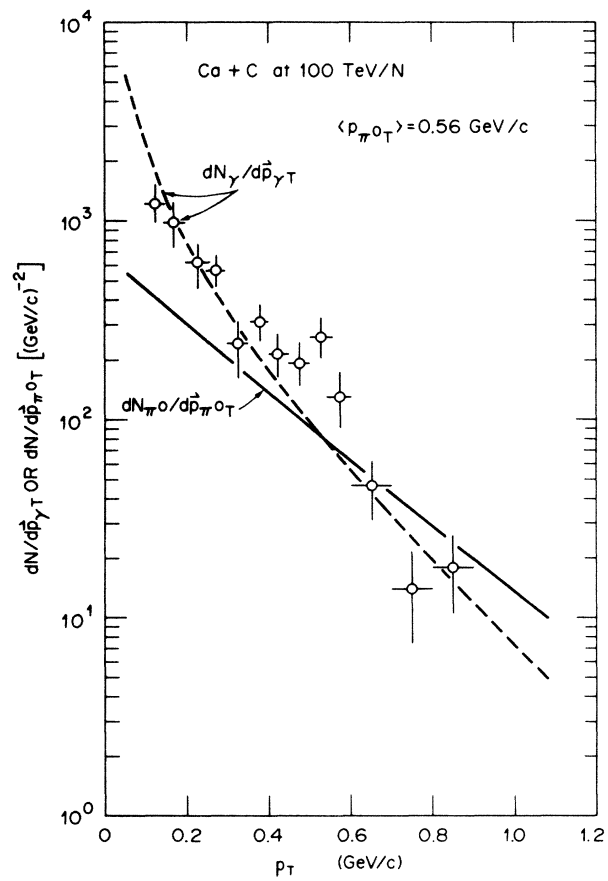

FIG. 2. Transverse-momentum distribution of π^0 and the corresponding product photons. The solid curve is the transverse-momentum distribution of π^0 as obtained from Eq. (3.4) with $\alpha_\pi=0.286$ GeV/c. The dashed curve gives the corresponding transverse-momentum distribution of the product photons.

$dN_\gamma/d\mathbf{p}_{\gamma T}$ and $dN_\pi/d\mathbf{p}_{\pi T}$ as a function of p_T ($p_{\gamma T}$ or $p_{\pi T}$), for the case $\alpha_\pi=0.286$ GeV/c. In this case, $\langle p_{\pi T}\rangle$ is 0.561 GeV/c. The data points are those of Ref. 14 for the collision of Si on Ag at an energy of 100 TeV per projectile nucleon and the constant C is normalized to the experimental data at small $p_{\pi T}$. The experimental data has an unusual bump at $p_{\pi T}\sim 0.5$ GeV/c which is difficult to be accounted for. Depending on the weight one assigns to this bump, the data give different values of $\langle p_{\pi T}\rangle$. For example, Ref. 14 gives $\langle p_{\pi T}\rangle=0.7$ GeV/c. Clearly, there is a high degree of uncertainty in $p_{\pi T}$ for this event because of the unusual shape of the distribution.

The logarithm of the theoretical π^0 transverse-momentum distribution has a linear slope, as is expected from the input distribution (3.4). One observes that the logarithm of the theoretical photon transverse-momentum distribution changes its slope as p_T increases. The distribution rises for small values of p_T because photons with small values of $p_{\gamma T}$ can come from the same side of the azimuthal angle and also from the other side. In fact, from Eqs. (2.14), if $dN_\pi/dp_{\pi T}\propto \exp(-p_{\pi T}/\alpha_\pi)$, then $dN_\gamma/d\mathbf{p}_{\gamma T}\propto \exp(-p_{\gamma T}/\alpha_\gamma)/p_{\gamma T}$. For the case shown in Figure 3, the theoretical distributions satisfy such a relationship.

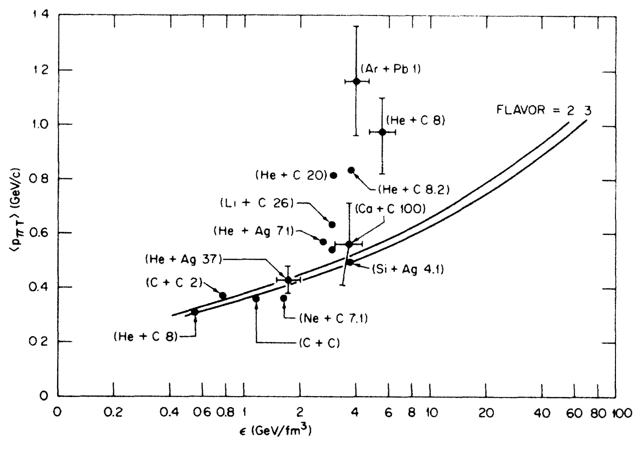

FIG. 3. Correlation of the average transverse momentum of the produced π^0 and the energy density ϵ for JACEE events. The label next to each data point specifies the colliding projectile and target and the energy in TeV per projectile nucleon. The solid curves give the theoretical correlation based on the equation of state of a quark-gluon plasma with two or three flavors.

IV. DISCUSSIONS

The results of Sec. II show that the approximate expression (1.1) gives rise to an error of about 15% for $\langle p_{\gamma T}\rangle\sim 0.15$ GeV/c and the error decreases with $\langle p_{\gamma T}\rangle$. As the JACEE data begin with $\langle p_{\gamma T}\rangle$ about 0.15 GeV/c, we can assess the average transverse momenta in the JACEE analysis to have an error of about 15% for the low end of the average transverse momenta but a much smaller error for the high end. However, Sec. III shows that for some of the high-transverse-momentum events as in the collision of Ca on C at 100 TeV per nucleon, the experimental transverse-momentum spectra have peculiar shapes which lead to a large degree of uncertainty of about 15% in the average transverse momentum. Therefore, one can assess the average transverse momenta in the JACEE analysis to have an error of about 15%.

The JACEE group also gives the energy density in the central rapidity region. As the average transverse momentum can be considered as a measure of the temperature,[25] it is of interest to compare the JACEE correlation with the quark-gluon equation of state. For this purpose, we use the results of Fig. 1, and calculate the initial energy density ϵ by using the expression[5]

$$\epsilon=\frac{3}{2}\frac{dN}{dy}\frac{(m_\pi{}^2+\langle p_{\pi T}\rangle^2)^{1/2}}{t_0\pi r_0{}^2A^{2/3}}\ , \tag{4.1}$$

where dN/dy is evaluated at the central rapidity region. We set $t_0=1$ fm/c and $r_0=1.2$ fm (Ref. 26). The results are shown in Fig. 3 which has only minor differences from those of Refs. 14—17.

It is difficult to pinpoint the origin of the correlation. Multiple collisions of nucleons give $\langle p_{\pi T}\rangle$ nearly independent of projectile, target, and incident momentum combinations.[21] Experimental measurements at low energies give $\langle p_{\pi T}\rangle$ nearly independent of projectile and target.[27] We shall attempt a model comparison as a way to initiate a discussion. One assumes that a quark-gluon plasma is produced and each parton of the plasma (a quark or a gluon) hadronizes into a detected particle by either picking up a sea quark in the case of a quark-parton or converting into a $q\bar{q}$ pair in the case of a gluon parton. In the idealized case with no transverse flow and no momentum loss in picking up a sea quark, the detected particle carries the momentum of the parton and $\langle p_{\pi T}\rangle$ is related to the temperature kT by[25] $\langle p_{\pi T}\rangle=2.35kT$. We obtain a correlation between the energy density and $\langle p_{\pi T}\rangle$ from the equation of state for an ideal relativistic quark-gluon gas shown in Fig. 3 for the case of two flavors and three flavors. We find that the data points at high energy densities deviate significantly from the equations of state of a quark-gluon gas. If the systematics of these high-$\langle p_{\pi T}\rangle$ events are real, they are unlikely to arise from the temperature of the plasma alone as the temperature of the partons would be about 450 MeV and the energy density would be much greater than that inferred from dN/dy. In order to have $\langle p_{\pi T}\rangle$ as high as 1 GeV/c, it is necessary to have some type of collective transverse flow which may come from the hydrodynamical expansion of the quark-gluon plasma[28] or from the collective hydrodynamics of the shock waves.[29] The large-transverse-momentum events in $p\bar{p}$ experiments have also been interpreted as due to the occurrence of minijets[30—32] in which many low-x partons have now enough energy to undergo hard scattering and produce many minijets of a few GeV each, which fragment independently from one another. On the other hand, if there is a phase transition of a first order, we expect that during the phase transition the plasma temperature would remain at the critical temperature even for events with higher-energy densities. This means that $\langle p_{\pi T}\rangle$ should flatten out as energy density increases beyond a few GeV/fm^3. Such a behavior may constitute a signal for the formation of a quark-gluon plasma with a sharp first-order phase transition. It is important to accumulate more data points to distinguish the different possibilities.

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APPENDIX

We work in a rotated coordinate system in which the momentum vector $\mathbf{p}_\gamma$ lies in the z' axis and the laboratory z axis lies in the azimuthal plane of $\phi_z'=0$ (Fig. 4). In this rotated coordinate system, the momentum vector $\mathbf{p}_\pi$ expressed in polar coordinate is

$$\mathbf{p}_\pi=(p_\pi'=p_\pi,\theta_\pi',\phi_\pi') \tag{A1}$$

while the unit vector along the laboratory z axis becomes in the rotated frame

$$\hat{\mathbf{z}}=(1,\theta_z',\phi_z')\ . \tag{A2}$$

But θ_z' is equal to the polar angle θ_γ in the laboratory frame, and $\phi_z'=0$ by the choice of the rotated frame.

Integrating the δ function with respect to the polar coordinates θ_π', we obtain

$$\frac{E_\gamma dN_\gamma}{d\mathbf{p}_\gamma}=\frac{M}{2\pi|\mathbf{p}_\gamma|}\times\int_{p_\pi'(\min)}^{\infty}\frac{p_\pi' dp_\pi' d\phi_\pi'}{[(p_\pi')^2+m_\pi{}^2]^{1/2}}\times\frac{E_\pi dN_\pi}{d\mathbf{p}_\pi}[\mathbf{p}_\pi(\mathbf{p}_\pi',\mathbf{p}_\gamma)]\ . \tag{A3}$$

In the above equation, the distribution in $\mathbf{p}_\pi$ is expressed in terms of the other quantities of $\mathbf{p}_\pi'$ and $\mathbf{p}_\gamma$ by means of the following:

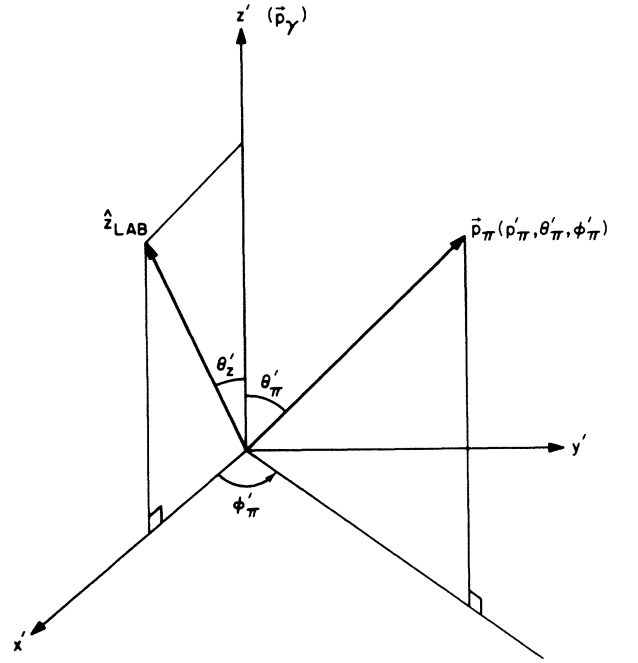

FIG. 4. Coordinate system used in the numerical integration of the pion distribution. The vector $\mathbf{p}_\pi$ is the three-momentum of π^0, and $\mathbf{p}_\gamma$ is the three-momentum of a detected photon which is chosen to lie along the z' axis. The unit vector along the z axis in the laboratory frame is chosen to lie in the x'-z' plane.

$$p_{\pi z}=p_\pi(\cos\theta_\pi'\cos\theta_\gamma-\sin\theta_\pi'\sin\theta_\gamma\cos\phi_\pi') \tag{A4}$$

and

$$p_{\pi T}=(p_\pi{}^2-p_{\pi z}{}^2)^{1/2}\ . \tag{A5}$$

The lower limit of integration over p_π' is

$$p_\pi'(\min)=|p_\gamma-m_\pi{}^2/4p_\gamma|\ . \tag{A6}$$

In Eq. (A3), the integrand is finite in the entire region of integration and can be integrated numerically without difficulty.

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