Total cross section and extraction of low-energy parameters of Λp scattering

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Since the lowest momentum at which σ_T data is available for Λp scattering is 120 MeV/c, extraction of low-energy parameters amounts to extrapolation of the data to low energies. Using the analytic structure of the forward scattering amplitude to advantage, a parametrization of σ_T is presented which it is hoped is more reliable for the purpose of deriving results through extrapolation. The scattering lengths and effective ranges of Λp scattering are then estimated.

I. INTRODUCTION

In spite of the fact that experimental investigation of the hyperon-nucleon interaction is severely handicapped by the short hyperon mean lifetime (few times 10^{-10} sec), some amount of consistency in the total-cross-section data of Λp scattering has been achieved over the last 15 years. The first data for this cross section obtained in the 81-cm Saclay hydrogen bubble chamber at CERN were analyzed by Alexander *et al.*[1] and Sechi-Zorn, Kehoe, Twithy, and Burnstein.[2] This was followed by the compilation of data by the Particle Data Group[3] at Lawrence Berkeley Laboratory. And, in the last decade, total-cross-section data have been obtained[4] up to a momentum of 20 GeV/c. With the availability of these data there have been, of course, efforts to check the hypothesis[5] that quark-quark amplitudes are additive, i.e., for the Λp scattering the difference between the Λp and pp total cross sections is the difference due to the single strangle quark which also appears in the difference between K^-n and π^+p total cross sections. Using this hypothesis, the quark-model result is $\sigma_T^{\Lambda p}=35.2\pm0.6$ mb; through data analysis, Gjesdal *et al.*[4] reported the value $\sigma_T^{\Lambda p}=34.6\pm0.41$ mb at momentum ≥ 6 GeV/c. However, in the analysis of total-cross-section data of Λp scattering the major efforts have been to extract estimates of the low-energy parameters, i.e., the scattering lengths and effective ranges. This one does by writing[6,7]

$$k\cot\delta=-\frac{1}{a}+\frac{rk^2}{2}\ , \tag{1}$$

where k is the center-of-mass momentum, a the scattering length, r the effective range, and δ the phase shift. Because the low-energy scattering receives significant contribution only from S states, one uses the parameters of the singlet and triplet S states to obtain[8] an expression for the total cross section:

$$\sigma_T=\frac{3\pi}{|\,k\cot\delta_t-ik\,|^2}+\frac{\pi}{|\,k\cot\delta_s-ik\,|^2}\ . \tag{2}$$

One then tries to obtain estimates of the four parameters a_s, a_t, r_s, r_t by making a least-χ^2 fit to the data on σ_T.

In the absence of reliable Λp scattering data de Swart and Dullemond[9] were the first to obtain values for a_s and a_t by an analysis[10] of the light hyperfragments using a shape-independent relation connecting the volume integral of different potentials[11] and the scattering lengths. With the availability of fairly accurate data there have been efforts[1,2] to exploit the relation of de Swart and Dullemond,[9]

$$r=b\left(1-\frac{b}{2a}\right), \tag{3}$$

between effective range r and the intrinsic range b, and obtain values of these parameters. However, these values were potential dependent. On the other hand, Fast and de Swart[12] reduced the number of free parameters to two by correlating the scattering length and effective range in a suitable potential model and then obtained values for the remaining two unknown parameters, but these values were again potential dependent. Fast, Helder, and de Swart[13] used as an input a Λp resonance in the $^3S_1$ state and making a fit to all the existing hyperon-nucleon data arrived at the values

$$\begin{aligned} a_s&=-1.7\pm0.5\ \text{fm},\quad r_s=2.5^{+1.0}_{-0.5}\ \text{fm}\ ,\\ a_t&=-1.5\pm0.05\ \text{fm},\quad r_t=2.0\pm0.05\ \text{fm}\ . \end{aligned} \tag{4}$$

The small errors in a_t and r_t reflected[8] only the uncertainty in the position of the resonance, set by hand, and not on any theoretical uncertainties. In continuing the analysis of the low-energy hyperon-nucleon scattering Nagels, Rijken, and de Swart[14] used a meson-theoretic potential in a multichannel Schrödinger equation. They employed meson-exchange potentials for the pseudoscalar and vector nonets and the uncorrelated two-pion exchange. Further meson-baryon coupling constants were taken from previous analyses together with SU(3) and SU(6) invariance. Making a simultaneous description of the nucleon-nucleon and hyperon-nucleon scattering they obtained the Λp low-energy parameters as

$$a_s = -2.16 \pm 0.26 \text{ fm}, \quad r_s = 2.03 \pm 0.10 \text{ fm} ,$$
$$a_t = -1.32 \pm 0.07 \text{ fm}, \quad r_t = 2.31 \pm 0.08 \text{ fm} . \tag{5}$$

A comparison of the values in Eqs. (4) and (5) shows that there is a difference of 15–20% in the values of the parameters. In fact, although all the earlier workers agree on the sign of these values, still there are similar differences between the magnitudes of these values and so one gets a feeling only about the range of values of these parameters from the efforts so far. These uncertainties regarding the values of these parameters could possibly be due to the fact that the effective-range calculations are expected to yield reliable results when one uses data only[15] at very low energies. And the lowest momentum at which the total-cross-section data of Λp scattering is available is around 120 MeV/c. Thus, for all practical purposes, when one is trying to obtain estimates of these parameters one is performing an extrapolation through the analytic continuation of the data to regions where experimental information is not available. In attempting such an extrapolation, one has to then choose a procedure which has a greater information storage capacity and thus is likely to lead to more stable and reliable results through extrapolation. However, in the absence of a universal algorithm for such an analytic extrapolation, one hopes that optimal exploitation of the analytic structure could perhaps be a better tool for this due to its information conveying ability.[16] This faith in the analyticity of the amplitude in the energy plane stems from certain rigorous proofs[17–19] connecting it with the validity of causality down to very short distances. In recent time there have been prescriptions by Cutkosky and Deo[20] and Ciulli[20] for providing a relatively stable extrapolation procedure, by optimally exploiting the analytic structure of the scattering amplitude in the energy plane. We, in this paper, have tried to store the available physical information in the coefficients of an accelerated convergent expansion of σ_T. Then we extrapolate the function σ_T, thus constructed, to energies lower than 100 MeV and use these values as our data to obtain the low-energy parameters of Λp scattering.

The paper is planned as follows: In Sec. II the scheme of parametrization of σ_T is presented. Section III contains our results and concluding remarks.

II. SCHEME OF PARAMETRIZATION OF σ_T

Writing the forward-scattering amplitude as

$$f(s,0) = f_{\text{Re}}(s,0) + i f_{\text{Im}}(s,0) \tag{6}$$

and using the optical theorem we obtain

$$\sigma_T = \frac{4\pi}{k} f_{\text{Im}}(s,0) , \tag{7}$$

where s is the square of the center-of-mass energy. To parametrize $f(s,0)$ we note that it is holomorphic in the S plane except for the cuts $S_R \le S \le \infty$ and $-\infty \le S \le S_L$, Fig. 1(a), where

$$S_R = (M_\Lambda + M_p)^2 , \tag{8}$$

$$S_L = 2(M_\Lambda{}^2 - M_p{}^2) \tag{9}$$

which correspond to the opening up of two-particle thresholds in the direct and crossed channels, respectively.

Now our aim is to bring the whole analytic domain of $f(s,0)$ inside a regular region by a suitable conformal mapping such that the cuts form the boundaries of this region. Then we will choose a polynomial whose figure of convergence tallies with this regular region and expand $f(s,0)$ as a series in this polynomial. This will accelerate the degree of convergence of the series and thus a judiciously truncated series will still be a faithful representation of the actual $f(s,0)$ insofar as a fit to the available data is concerned.

To this end we first symmetrize the cuts on the real axis of the X plane by the mapping Fig. 1(b)

$$X = s - \frac{S_R + S_L}{2} . \tag{10}$$

Taking note of the fact that mapping of the analytic domain of the scattering amplitude into a strip along the real axis and use of this mapped variable to construct the scattering amplitude have been recently found[21] to be use-

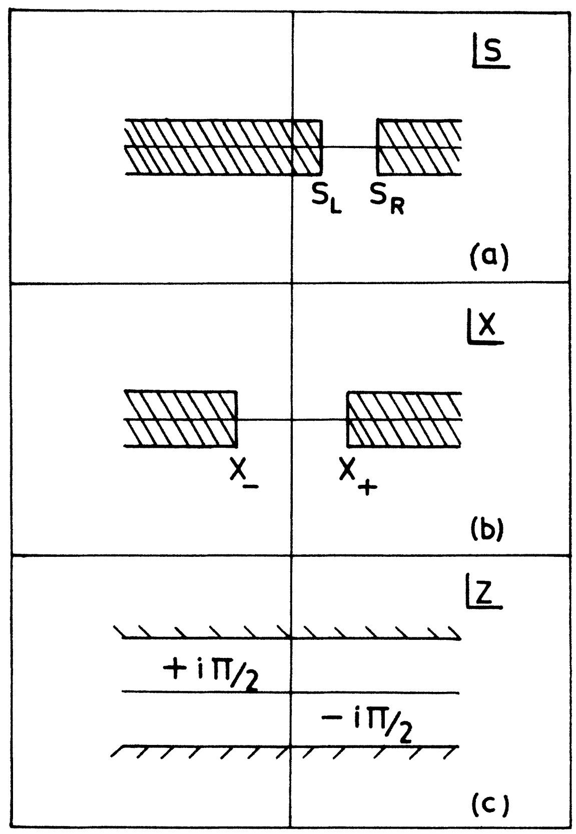

FIG. 1. (a) Analytic structure of Im $f(s,0)$ in the S plane. S_R and S_L are the position of the right-hand and left-hand cuts, respectively. (b) X_+, X_- are the position of the cuts in the symmetrized X plane being symmetrized. (c) Mapping of the X plane into the interior of a strip along the real axis of the Z plane with the cuts forming the boundaries at $\pm i\pi/2$.

ful for an analysis of the total cross section of pp, $\bar{p}p$, $\pi^{\pm}p$, and $K^{\pm}p$ scatterings, we map the whole of the analytic X plane into the interior of a strip along the real axis of the Z plane with the cuts forming the boundaries at $\pm i\pi/2$, Fig. 1(c):

$$Z = -i \arcsin X . \tag{11}$$

We note that for

$$s \geq (S_R - S_L)/2 , \tag{12}$$

$$Z = \ln[X + (X^2-1)^{1/2}] - i\frac{\pi}{2} . \tag{13}$$

Now since the domain of convergence of the Hermite polynomial is a strip along the real axis and σ_T is related to the imaginary part of $f(s,0)$, we expand $f(s,0)$ as,

$$f(s,0) = i \sum_{n=0}^{\infty} a_n H_n(Z) , \tag{14}$$

where a_n are real. Further for larger s

$$Z = \ln(2s) - i\frac{\pi}{2}$$
$$= \ln\left[2s \exp\left[-i\frac{\pi}{2}\right]\right] \tag{15}$$

and so, the Froissart bound requires that the series in (14) must be terminated at $n=2$. Thus we have

$$f(s,0) = i(b_0 + b_1 Z + b_2 Z^2) , \tag{16}$$

where

$$b_0 = a_0 - 2a_2 ,$$
$$b_1 = 2a_1 , \tag{17}$$
$$b_2 = 4a_2 .$$

Then we have

$$\sigma_T = \frac{4\pi}{k} f_{\rm Im}(s,0)$$
$$= \frac{4\pi}{k}\left[C_0 + C_1 y + C_2\left[y^2 - \frac{\pi}{4}\right]\right], \quad s \geq S_R , \tag{18}$$

where

$$y = \ln[X + (X^2-1)^{1/2}] . \tag{19}$$

The coefficients C_0, C_1, and C_2 can be estimated by a direct fit to σ_T experimental data.

III. RESULTS AND CONCLUSION

By using Eq. (18) we tried to fit all the Λp scattering σ_T data[22] up to 20 GeV/c. The best fit, Fig. 2, is obtained with values

$$C_0 = 9.1^{+0.001}_{-0.006} ,$$
$$C_1 = -4.3^{+0.001}_{-0.008} , \tag{20}$$
$$C_2 = 1.8^{+0.006}_{-0.003}$$

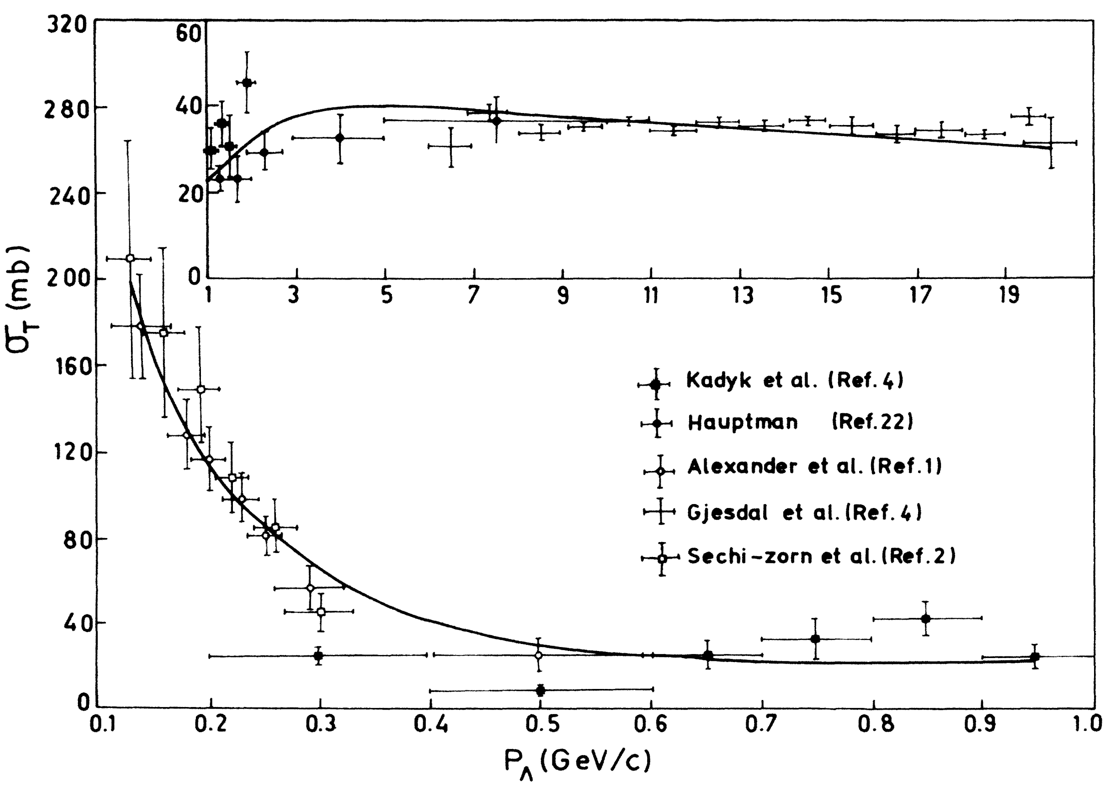

FIG. 2. Fit for the total-cross-section curve for the data in the energy range (i) up to 0.95 GeV/c and (ii) from 1 GeV/c to 20 GeV/c (inset).

TABLE I. Values of scattering lengths and effective ranges in fm from this and also earlier analysis (Refs. 1, 2, 12, 14, 23, and 24).

Parameters	This analysis	Nagels *et al.* (Ref. 24) 1979	Nagels *et al.* (Ref. 23) 1977	Nagels *et al.* (Ref. 14) 1973	Fast *et al.* (Ref. 12) 1969	Sechi-Zorn *et al.* (Ref. 2) 1968	Alexander *et al.* (Ref. 1) 1968
a_s	-2.2 ± 0.1	-2.18	-1.77 ± 0.28	-2.16	-1.7 ± 0.5	-2.0	-1.8
r_s	3.16 ± 0.11	3.19	3.78 ± 0.35	2.03	$2.5^{+1.0}_{-0.5}$	5.0	2.8
a_t	-2.05 ± 0.03	-1.93	-2.06 ± 0.12	-1.32	-1.5 ± 0.05	-2.2	-1.6
r_t	3.3 ± 0.05	3.35	3.18 ± 0.10	2.31	2.0 ± 0.05	3.5	3.3

for the coefficients. The change in a parameter that will produce a χ^2 change of unity was taken by us as the error in that parameter. The extremely small allowed error in these parameters gives an indication of the stability of our fit. Although the χ^2/DF for the overall fit is 6.3 still it becomes as low as 1.7 if one does not consider only two data points of Kadyk *et al.*,[4] those at momentum 0.3 and 0.5 GeV/c. These two data points give a relatively low value for σ_T as compared with the general trend of the data between the momentum range 0.3 GeV/c and 0.8 GeV/c. Only when more data is available in the region 0.3–0.5 GeV/c will we be in a position to comment on the goodness of our fit in this region. While plotting our curve we have taken explicit care to choose two scales, one for energy up to 0.95 GeV/c, and the other (inset in Fig. 2) for $1 \ll$ momentum ≤ 20 GeV/c, so as to demonstrate explicitly the goodness of our fit. It is clear from the curve that the fit is quite acceptable to as high a momentum range as 20 GeV/c except perhaps the data points[4,22] around 0.3–0.5 GeV/c. In fact, these data points seem to be out of sequence with the general trend of the experimental values over the entire range. By extrapolating our curve to higher energies we observed that σ_T values stabilize at about 30 mb. This seems to be quite acceptable in view of the quark-counting results.[5]

Once we obtain the best-fit curve for σ_T we extrapolated it to lower energies (up to 50 MeV) and used these values as our data points at low energies. We then used Eqs. (1) and (2) to get a good fit to the σ_T data between 0.05 GeV and 0.16 GeV. Here we took note of the fact that Eq. (2), written explicitly in terms of the parameters, becomes

$$\sigma_T=\frac{3\pi}{\left[\frac{1}{a_t}\right]^2+\left[\frac{r_tk^2}{2}\right]^2-\frac{r_tk^2}{a_t}+k^2}+\frac{\pi}{\left[\frac{1}{a_s}\right]^2+\left[\frac{r_sk^2}{2}\right]^2-\frac{r_sk^2}{a_s}+k^2}\,. \tag{21}$$

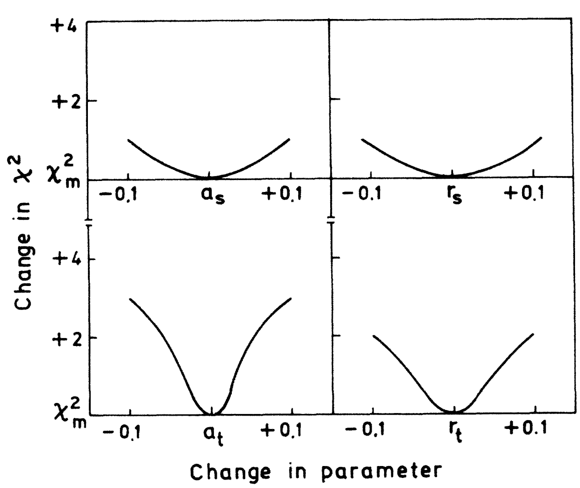

FIG. 3. Change in the parameters vs change in χ^2 for the parameters a_s, r_s, a_t, and r_t.

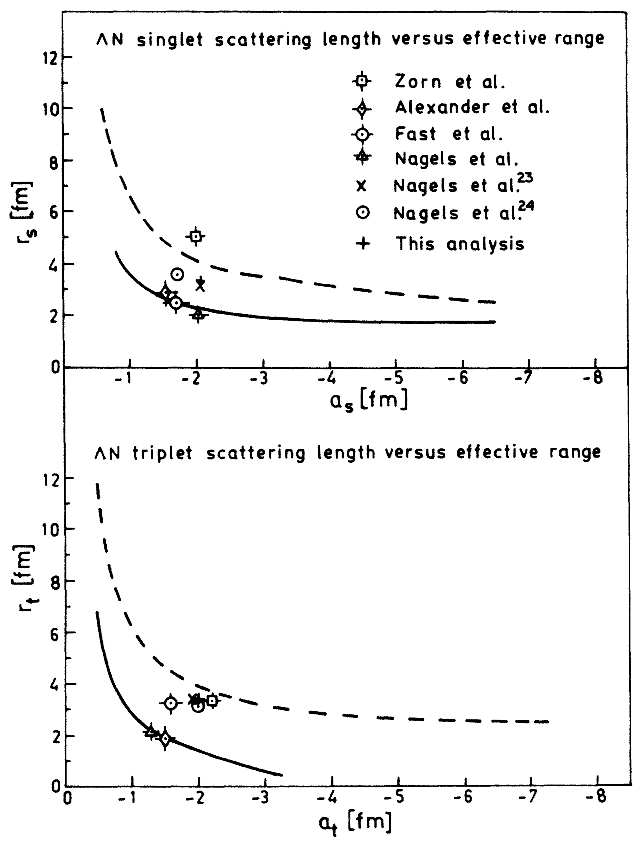

FIG. 4. Correlation curve for a_t-r_t and a_s-r_s with the results of different authors (Refs. 1, 2, 12, 14, 23, and 24) along with our analysis.

Although the interference terms between a and r will be effective for the estimation of the parameters, they can at best fix the relative sign between a and r. To remove this ambiguity we assume that both a_s and a_t are negative, as there is so far no evidence regarding the existence of a Λp bound state. Our best-fit values for the low-energy parameters are given in Table I along with those of some of the earlier workers[1,2,12,14,23,24] for comparison. The change in a parameter that will produce a change of unity in the total χ^2 (Fig. 3) has been reported by us as the error in that parameter.

It is clear from Table I that there is some form of unanimity regarding the sign and value of these parameters. Here four comments are in order. (i) A plot (Fig. 4) of our values for these parameters over the correlation curves[8,12,25] between a_s, r_s and a_t, r_t shows that our values lie inside the curves. This suggests that such a correlation may possibly be used as an input in the analysis of σ_T. (ii) In our analysis we also have $|a_s| > |a_t|$ in agreement with Nagels, Rijken, and de Swart. (iii) In our values $r_t > r_s$, showing that the triplet spin state is less attractive than the singlet spin state. (iv) Our zero-energy σ_T value is 550 mb which is more or less of the same order as obtained by earlier workers.[1,2,12,14,23,24] In conclusion we note that in our parametrization we did not observe any cusp at the ΣN threshold and this agrees well with the experimental observations of Hauptman.[22] Thus the existence of such a cusp as seen by earlier workers could be due to the triplet hard-core radii chosen by them. Further from Eq. (16) we obtained $f_{\text{Re}}(s,0)$ and $f_{\text{Im}}(s,0)$ and calculated ρ for P_Λ up to 20 GeV/c, where

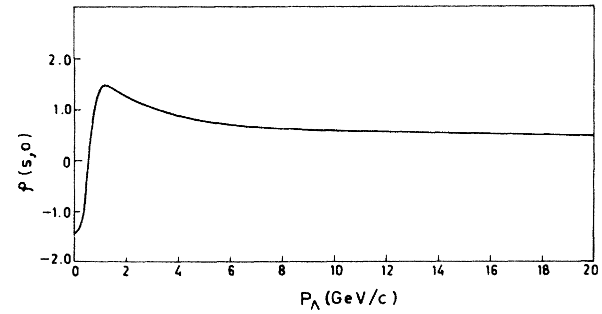

FIG. 5. The ratio of the real to the imaginary part of the forward elastic-scattering amplitude.

$$\rho = \frac{f_{\text{Re}}(s,0)}{f_{\text{Im}}(s,0)} . \tag{22}$$

Our values are plotted in Fig. 5. This ratio approaches zero for high energy as is expected. In the absence of any published numbers it is not possible to make a direct comparison with experimental values.

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