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Velocity of sound in SU(3) lattice gauge theory

R. V. Gavai and A. Gocksch
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973
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Using Monte Carlo methods we calculate the velocity of sound in both the confining and deconfining phases of QCD. The results are consistent with the theoretical expectations based on an ideal gas of glueballs and gluons, respectively.

In recent years it has become increasingly more clear, that at sufficiently high temperatures and/or densities quarks and gluons will undergo a transition from a phase in which they are permanently bound in color-neutral objects (i.e., mesons, baryons, and glueballs) to a new phase in which they are essentially free. From a fundamental point of view of QCD, the theory of the strong interactions, one expects certain observables usually associated with confinement or chiral symmetry to undergo a rapid change in magnitude once a critical temperature or density has been surpassed. These are strictly nonperturbative effects and nonperturbative methods are called for. Most of our knowledge about QCD at finite temperature therefore comes from lattice Monte Carlo calculations. (QCD at finite baryon density remains to a large extent unexplored.) Early calculations performed in the quenched approximation indicated that QCD has a first-order phase transition. The effects of dynamical fermions has been taken into account only recently. Although there is still some controversy surrounding the order of the transition the following picture seems to emerge from these calculations:[1] At a temperature of about 200–300 MeV the theory, in some sense, "deconfines" and the spontaneously broken chiral symmetry is restored. Both these phenomena take place at the same temperature. These results are of course of great importance for the physics of heavy-ion collisions and the early Universe. Just from the possible existence of a first-order transition alone one can speculate about remarkable cosmological consequences.[2]

It is expected that the high temperatures and/or densities needed can be reached in relativistic heavy-ion collisions,[3] allowing one to investigate new interesting phenomena. In order to obtain a detailed space-time picture of the quark-gluon plasma produced in such collisions a theoretical model is needed and one usually employs relativistic hydrodynamics for this purpose. This seems appropriate since in a nucleus such as uranium the quark mean free path can easily be estimated to be ~ 0.2 fm, which is small compared to the typical size of such a nucleus, ~ 15 fm. The hydrodynamic equations do, however, involve parameters that are *a priori* unknown and which contain the memory of the underlying theory, QCD. One such quantity to which we address ourselves in this paper is the velocity of sound in the quark-gluon "fluid." The knowledge of its dependence on the temperature is of crucial importance in the solution of the hydrodynamic equations. It is the purpose of this Rapid Communication to demonstrate how lattice Monte Carlo calculations can supply this information. Our study is in a sense preliminary: We restricted ourselves to the pure gauge theory to check the feasibility of the method. We find it encouraging that our results are in agreement with naive theoretical expectations.

We simulated the following partition function:

$$Z=\int [dU]e^{-S(U)} \ , \qquad (1)$$

where

$$S(U)=\beta \sum_{n_0=0}^{n_\tau-1} \sum_{n_i=0}^{n_\sigma-1} \sum_{\mu,\nu} \mathrm{tr}[U_\mu(n)U_\nu(n+\mu)U_\mu^\dagger(n+\nu)U_\nu^\dagger(n)+\mathrm{H.c.}] \ . \qquad (2)$$

Here β is related to the bare coupling constant by $\beta=6/g^2$ and $n_\tau=(1/T)a$, where T is the temperature and a the lattice spacing. The thermodynamics of lattice QCD has been studied in detail in Ref. 4. We will need the following expressions for energy density ϵ and pressure P:

$$\epsilon=\frac{3\beta}{a^4}[\bar{p}_S-\bar{p}_T+g^2C_S(\bar{p}_S-\bar{p})+g^2C_T(\bar{p}_T-\bar{p})] \ , \qquad (3)$$

$$\epsilon-3P=\Delta \ . \qquad (4)$$

C_S and C_T in (3) are constants that can be calculated in perturbation theory.[5] $\bar{p}_S$ and $\bar{p}_T$ denote the average spacelike and timelike plaquettes, respectively. $\bar{p}$ denotes the average plaquette obtained on a $n_\sigma{}^4$ lattice. The quantity Δ in (4) is given by

$$\Delta=18a^{-3}\frac{\partial g^{-2}}{\partial a}(\bar{p}_S+\bar{p}_T-2\bar{p}) \ . \qquad (5)$$

Δ has been called the "interaction measure."[4] It represents

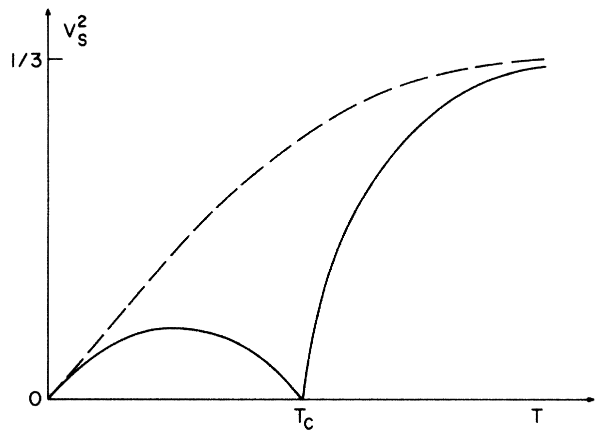

FIG. 1. Schematic representation of theoretical expectations for V_S^2 as a function of T.

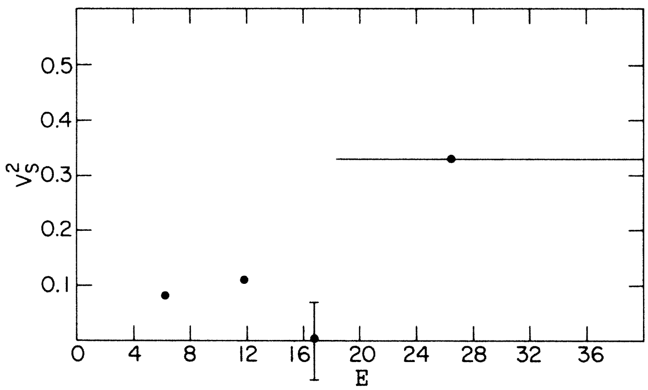

FIG. 3. The velocity of sound squared vs ϵ. The corresponding range in temperature and coupling constant is given in Table I.

the deviations from ideal-gas behavior for which $p=\frac{1}{3}\epsilon$. In our calculation we used the two-loop perturbative value for $a(\partial g^2/\partial a)$:

$$\Delta=-18\left(\frac{11}{8\pi^2}+g^2\frac{51}{64\pi^4}\right)(\bar{p}_S+\bar{p}_T-2\bar{p})/a^4 \ . \tag{6}$$

The velocity of sound is in general defined as

$$V_S{}^2=\frac{\partial P}{\partial \epsilon} \ . \tag{7}$$

Before we go on to show how we calculated (7) on the lattice let us shortly discuss what to expect theoretically. As mentioned earlier, pure SU(3) gauge theory is known to have a first-order transition with a rather large latent heat.[6] At the transition point T_c we expect the velocity of sound to be zero since P is continuous and ϵ is not. In other words, for a whole range of energy densities given by the discontinuity the function $P(\epsilon)$ is constant. In the confined phase we can approximate the thermodynamics at sufficiently small T by a nonrelativistic, ideal gas of glueballs. In this case $\epsilon\rightarrow\rho$, where ρ is the mass density. Taking the derivative (7) at constant entropy one obtains

$$V_S{}^2=\frac{\gamma T}{m_G} \ , \tag{8}$$

where $\gamma=C_p/C_v$ and m_G is the (effective) glueball mass. In the deconfined phase one has an ideal gas of gluons and one expects $V_S{}^2$ to approach its limiting value of $\frac{1}{3}$. These expectations are summarized in Fig. 1. Contrasting the solid line depicting them is a dashed line indicating what one might expect if there were no (first- or second-order) phase transition at all. It may be remarked here that for a first-order phase transition the solid curve in Fig. 1 could have a discontinuity at T_c.[7]

We now proceed to explain how we extracted $V_S{}^2$ from Eq. (4). Differentiating with respect to ϵ yields

$$V_S{}^2=\frac{1}{3}\left(1-\frac{\partial\Delta}{\partial\epsilon}\right) \ . \tag{9}$$

To take the derivative in (9) we converted both Δ and ϵ to physical units using the two-loop renormalization-group formula. Since both quantities are then expressed in terms of physical temperature (in units of Λ_L) we can eliminate T to obtain Δ as a function of ϵ. We display $\Delta\equiv\Delta(\epsilon)$ calculated on a $8^3\times 4$ lattice together with a typical error bar in Fig. 2. The derivative in (9) was then approximately calculated by taking finite differences. $V_S{}^2$ thus obtained is shown in Fig. 3. Our data are summarized in Table I. Before we draw any conclusions let us summarize the assumptions that went into the analysis. We assumed asymptotic scaling in (6) and (9). From Monte Carlo renormalization-group studies and precision measurements of T_c one knows,[8] however, that in the coupling-constant regime considered here, physical

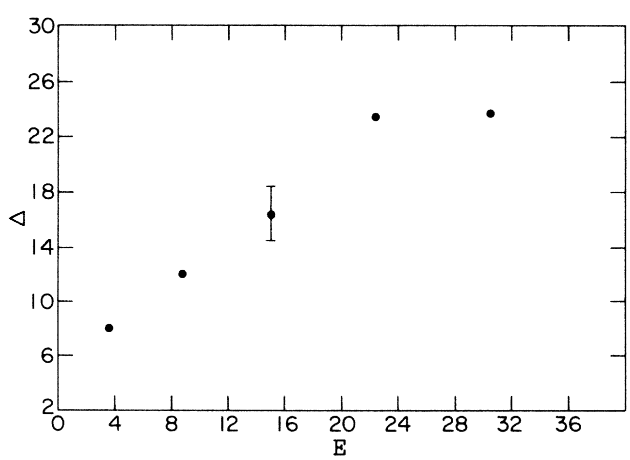

FIG. 2. Δ vs ϵ. Both quantities are in units of $\Lambda_L{}^4 10^{-7}$.

TABLE I. Summary of our data. Estimates of the errors are shown in the figures.

β	$T(\Lambda_L)$	$\epsilon(\Lambda_L{}^4 10^{-7})$	$\Delta(\Lambda_L{}^4 10^{-7})$	$V_S{}^2$
5.66	72.7	3.56	8.02	· · ·
5.665	73.1	6.15	· · ·	0.08
5.67	73.5	8.73	12.0	· · ·
5.685	74.8	11.92	· · ·	0.105
5.70	76.1	15.2	16.4	· · ·
5.705	76.5	18.72	· · ·	0
5.71	76.9	22.33	23.3	· · ·
5.73	78.7	26.36	· · ·	0.33
5.75	80.4	30.38	23.7	· · ·

quantities do not yet scale according to the perturbative formula. It is possible, though, that there is nonasymptotic scaling in the sense that ratios of physical quantities remain constant. As a matter of fact, the combined data of string tension and deconfinement temperature are consistent with a scaling of the form

$$a\frac{dg^{-2}}{da}=-2\bar{b}_0\ , \tag{10}$$

where $\bar{b}_0$ is roughly one half the perturbative value. If one accepts this idea of "prescaling" then the value of $\bar{b}_0$ in (10) parametrizes the systematic error in our *quantitative* determination of $V_S{}^2$. Neglecting for the moment the terms proportional to C_S and C_T in (3) which are small, it is not hard to arrive at the following approximate formula for $\partial\Delta/\partial\epsilon$:

$$\frac{\partial\Delta}{\partial\epsilon}\approx g^2\bar{b}_0\frac{d(\bar{p}_S+\bar{p}_T-2\bar{p})}{d(\bar{p}_S-\bar{p}_T)}\ . \tag{11}$$

(11) shows explicitly how the value of $V_S{}^2$ depends on the precise form of the scaling function as parametrized by $\bar{b}_0$. Since we do not know the true scaling function we chose to work with its asymptotic form. At small temperatures, where $(\partial\Delta/\partial\epsilon)$ is large, this procedure may lead to a value of $V_S{}^2$, which is off by as much as a factor of 2, as one sees clearly from (11). However, (11) also shows that the qualitative dependence of $V_S{}^2$ on the coupling is independent of the details of the scaling. This explains the nice qualitative agreement of our data with the theoretical expectations. As mentioned above, the final derivative was approximated by a finite difference. This introduces a systematic error which at this point is hard to estimate, especially since errors from other sources are perhaps more significant. Clearly, one should get more statistics and above all more points.

The calculations reported here are very time consuming. Since the energy density is a wildly fluctuating quantity we typically ran 3000 iterations for thermalization and 6000 iterations for measurements. The errors shown are twice the naive error which is close to an error estimate obtained by binning the data. The error on $V_S{}^2$ comes from simple error propagation. Despite the obvious uncertainties in our method the agreement with the theoretical expectation is quite nice. At the critical point $V_S{}^2$ clearly drops to a small value consistent with 0 and approaches $\frac{1}{3}$ (the ideal-gas value) very quickly above the transition. It is an amusing exercise and also a consistency check to assume that the first point in Fig. 3 already satisfies Eq. (8). We should, however, drop the factor of γ in the comparison with our measured $V_S{}^2$ since the derivative has been taken at constant T. Equation (8) then yields $m_G\simeq 900$ MeV which is certainly in the right ballpark.

To summarize, we have determined the velocity of sound in pure SU(3) gauge theory at four points. It is unlikely that in the confining phase of the pure gauge theory $V_S{}^2$ exceeds a value of 0.2. At the transition it is negligibly small and the limiting value of $\frac{1}{3}$ is approached very rapidly above the transition. Although more work is clearly needed both for the pure gauge theory and especially in the full theory including dynamical fermions we believe that we have demonstrated that Monte Carlo calculations can be used to extract quantities desired by phenomenologists.

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