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Lorentz invariance in the mean-field approximation

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Problems relating to the Lorentz boost in the mean-field approach are investigated. We consider mainly the case in which the mean-field potential is time-independent. Fundamental assumptions and formulas are given. It is stressed that the Lorentz invariance is in principle recovered by taking account of the residual interaction, and it is discussed how to construct a moving hadron state consisting of confined quarks and/or antiquarks. A brief summary of matrix elements for quark operators sandwiched by one-hadron states is given, which provides us with an intelligible example to see the relationship of the present approach to other ones.

I. INTRODUCTION

There have appeared a lot of papers on the covariant description of the relation between the approximate quark modes in hadrons at rest and those in moving hadrons.[1–16] Such an approximation is the mean-field approximation or the Hartree-Fock-type potential approach,[1,2,6,8–18] which has succeeded in describing various static and sometimes dynamical properties of hadrons. If one wants, however, to construct in the mean-field (MF) approximation a realistic hadron which consists of "confined" quarks, the description depends in some way on **R**, the "center" of the MF potential of a hadron. For example, in the actual calculations based on the MIT bag model,[19] the approximation of "spherical cavity" with its center at the origin of coordinate has been adopted. When the MF potential is time independent, it is not straightforward to construct in a Lorentz-covariant way a confined quark system, in which quarks do not spread over the whole three-dimensional space.

The purpose of the present paper is to investigate problems relating to the Lorentz boost of the MF approximation. We consider mainly the case in which the MF potential is time independent, emphasize the essential role of the residual interaction to restore the Lorentz covariance, and explain how to construct a state vector of moving hadrons. In order to clarify the problem, the present paper includes a pedagogical part and a brief explanation of some explicit forms of matrix elements for quark operators sandwiched by one-hadron states. All of them will serve to understand the meaning and limitation of the MF approximation, and also provide us with an intelligible example to see the relationship of our formalism to other ones.

Some parts of our approach can be applied also to a wider region of subjects, such as the dynamical processes of phase transitions, pair creations on phase boundaries, and so on,[20] because in the mean-field approximation particle pictures associated with different mean-field potentials are related to each other in terms of the "Bogoliubov transformation."[21]

In Sec. II three typical versions of the MF approximation are summarized, and we make an assumption which plays a basic role in this approximation. In Sec. III the MF approximation for hadron systems is investigated, where we explain the role of the residual interaction to recover the Lorentz covariance in the case of the time-independent MF potential. Next we explain how to construct the state vector describing a hadron at rest and then a moving one. Approximate formulas for Lorentz boost of quark operators are given. A concrete example of matrix elements for quark operators between one-hadron states is given. In the appendixes, mathematical details of related problems are given.

After we obtain the basic tools of calculating concrete matrix elements of physical operators between moving hadron states, it becomes possible to explain the meaning of the various approaches proposed,[4–6,9–11,16,22–24] which is the task of a subsequent paper.[25]

II. GENERALIZED MEAN-FIELD APPROXIMATION

Let us consider here our starting Lagrangian L which depends on matter fields Φ under consideration and also on other fields $\Phi_1, \Phi_2, \ldots$:

$$L(\Phi;\Phi_1,\Phi_2,\ldots) . \tag{2.1}$$

Define the *generalized mean-field Lagrangian* $L^S_{\rm MF}(\Phi)$ for a fixed state S:

$$L^S_{\rm MF}(\Phi)=L_{\rm kin}(\Phi)-\Phi(x)^\dagger V_S(x)\Phi(x) . \tag{2.2}$$

Here we have assumed that there exists a Lagrangian (2.2) for a fixed state S with a c-number potential $V_S(x)$, and the residual part of L [(2.1)]

$$L_{\rm res}=L(\Phi;\Phi_1,\Phi_2,\ldots)-L^S_{\rm MF}(\Phi) \tag{2.3}$$

never produces any bilinear terms of $\Phi(x)$ and $\Phi(x)^\dagger$ in the state S; that is, the c-number generalized mean-field potential $V_S(x)$ is determined in a self-consistent way. Although this potential depends generally on the space $\mathbf{x}$ and time t, we often encounter the cases where a system under consideration can be described approximately in terms of $V_S(x)$ with a simpler space-time dependence:

(i) $V_S(x)=\text{const}$ for all space-time regions , (2.4a)

(ii) $V_S(x)=V(\mathbf{x})$, independent of t , (2.4b)

(iii) $V_S(x)=V(t)$, independent of $\mathbf{x}$. (2.4c)

Several examples described in the following would make our discussion understandable.

In the Nambu–Jona-Lasinio–type model,[26] the Lagrangian is expressed in terms of spinor fields ψ and $\bar\psi$ with four-Fermi interaction terms such as[27]

$$L(\psi,\bar\psi)=-\bar\psi\partial\psi+g[(\bar\psi\psi)^2-(\bar\psi\gamma_5\psi)^2] +g'[(\bar\psi\gamma_\mu\psi)^2+(\bar\psi\gamma_\mu\gamma_5\psi)^2] . \tag{2.5}$$

For the case where the state S is the vacuum, under the condition of a sufficiently strong coupling constant, $g>g_c$, we have as an example of type (i)

$$L^V_{\rm MF}(\psi,\bar\psi)=-\bar\psi(\partial+m)\psi \tag{2.6}$$

with

$$m=-2g\langle\bar\psi(x)\psi(x)\rangle_0 =-2g\langle\bar\psi(0)\psi(0)\rangle_0 ; \tag{2.7}$$

the second equality is guaranteed by the translational invariance of the Lagrangian itself and the vacuum.

When we proceed further to composite systems of Fermi particles, we have in general[2]

$$L^S_{\rm MF}(x)=-\bar\psi(x)[\partial+m^S(x)+m^P(x)i\gamma_5 +m^V_\mu(x)i\gamma^\mu+m^A_\mu(x)i\gamma_5\gamma^\mu]\psi(x) , \tag{2.8}$$

where

$$m^S(x)=-2g\langle S\,|\,\bar\psi(x)\psi(x)\,|\,S\rangle,\ \text{etc.} ; \tag{2.9}$$

the expectation values are taken between composite states under consideration. These quantities should be self-consistently determined, and become type (ii) under certain approximations. For more specialized systems with the parity and three-space rotation invariances, we have

$$m^S(x)=m^S(|\mathbf{x}|),\quad m^V_0(x)\equiv m^V(|\mathbf{x}|) , \tag{2.10}$$

all other types of $m^{(\ )}(x)$'s being equal to zero. Such a case has been considered by Kugo, Tanaka, one of the present authors (M.B.) and co-workers;[13–15] there are several examples of this type, e.g., the cavity approximation of the bag model,[19,1] the potential model,[16] and so on. Friedberg and Lee[17] proposed the model Lagrangian for the nucleon as a bound-quark system by introducing the scalar field $\sigma(x)$:

$$L(\psi,\bar\psi,\sigma)=-\bar\psi\partial\psi-\tfrac{1}{2}(\partial_\rho\sigma)^2-g\sigma\bar\psi\psi-V(\sigma) . \tag{2.11}$$

For the state with no nucleon, $\langle\sigma\rangle_0$ is determined by the minimum of the potential $V(\sigma)$,

$$V'(\sigma)=0 \quad \text{for } \sigma=\sigma_0=\langle\sigma\rangle_0 , \tag{2.12}$$

and we have

$$L^{{\rm no}\,N}_{\rm MF}(\psi,\bar\psi)=-\bar\psi(\partial+g\sigma_0)\psi . \tag{2.13}$$

For the one-nucleon state, we have

$$L^N_{\rm MF}(\psi,\bar\psi)=-\bar\psi[\partial+v(x)]\psi , \tag{2.14}$$

where $v(x)\equiv g\langle N\,|\,\sigma(x)\,|\,N\rangle$ is determined by solving the self-consistent coupled equation

$$[\partial+v(x)]\psi(x)=0 , \tag{2.15a}$$

$$\frac{1}{g^2}\Box v(x)-\frac{\delta V(v(x)/g)}{\delta v(x)}=\rho(x) , \tag{2.15b}$$

with $\rho(x)\equiv\langle N\,|\,\bar\psi(x)\psi(x)\,|\,N\rangle$.

Our final example[20] is the model Lagrangian depending on the Higgs field $\phi(x)$ which plays an important role in spontaneous breakdown of gauge symmetry:

$$L(\phi)=-\partial_\rho\phi^\dagger\partial^\rho\phi-V(\phi,\phi^\dagger) \tag{2.16}$$

with , e.g.,

$$V(\phi)=-\mu^2\phi^\dagger\phi+\lambda(\phi^\dagger\phi)^2 . \tag{2.17}$$

Here, let us consider the expanding hot universe with temperature T. When T is high enough, the vacuum of the universe is realized at the point $\phi=0$, and as the temperature goes down to some critical value T_c, the local minimum of the potential V appears and a phase transition occurs. Then, we divide $\phi(x)$ into two parts as

$$\phi(x)=\phi_0(t)+\tilde\phi(\mathbf{x},t) . \tag{2.18}$$

Here the mean field $\phi_0(t)$ is determined by the equation of motion

$$\ddot\phi_0(t)+3\left[\frac{\dot R}{R}\right]\dot\phi_0=\left[\frac{\partial V}{\partial\phi}\right]\phi_0 , \tag{2.19}$$

in which an expanding universe is described in terms of the Robertson-Walker metric

$$ds^2=dt^2-R(t)^2d\mathbf{x}^2 , \tag{2.19'}$$

and we have for the quantum fluctuating part $\tilde{\phi}(\mathbf{x},t)$

$$L_{\rm MF}(\tilde{\phi})=-\partial_\rho\tilde{\phi}^\dagger\partial^\rho\tilde{\phi}-\tilde{\phi}^\dagger m^2[\phi_0(t)]\tilde{\phi}\ . \tag{2.20}$$

Now, in all the models mentioned above, in order that the MF approximate Lagrangians are workable for describing the physical systems under consideration, we take the following assumption.

Assumption I. The Heisenberg operator of matter fields under consideration, $\Phi(x)$, can always be expanded in terms of a complete set of functions $\{\phi_n(x)\}$ which are eigensolutions of the equation of motion derived from $L^S_{\rm MF}$, (2.2):

$$\Phi(x)=\sum \alpha_n\phi_n(x)\ . \tag{2.21}$$

This assumption makes it easy to get the relation between the particle modes of different mean fields, which is generally expressed in terms of the Bogoliubov transformation.[21]

Some important comments are in order. First, the set of eigenfunctions of $L^S_{\rm MF}$, $\{\phi_n(x)\}$, does not always belong to the same Hilbert space as that of L [(2.1)] itself. One example is the case of type (i) in which we have an infinite number of mutually orthogonal Hilbert spaces corresponding to different values of constant mean fields. Another one is seen in the MIT bag model[19], where the potential $V(\mathbf{x})=\infty$ outside a domain R, where the wave functions $\phi_n^{(R_1)}(\mathbf{x})$'s can never be expanded in terms of $\phi_n^{(R_2)}(\mathbf{x})$'s if $R_1\cap R_2=0$. Still in these cases the Bogoliubov transformation itself can be defined in a usual manner, although special care should be paid in treating the Bogoliubov transformation.[21] Assumption I is valid in so far as the space-time volume (for the former example) or the potential (for the latter one) is kept finite; thus it is probable that the Heisenberg field is expanded in terms of $\{\phi_n(x)\}$ at least approximately. Second, it sometimes occurs that the MF Lagrangian $L^S_{\rm MF}$ is not invariant under the whole symmetry group G of the starting Lagrangian L. This situation is obviously seen from the cases of type (ii) or (iii); the Lorentz invariance of L is violated in the level of $L^S_{\rm MF}$. Even in the simple Nambu—Jona-Lasinio—type MF Lagrangian (2.5), for example, the chiral symmetry of the original Lagrangian is well known to be violated when an explicit mass term $\bar{\psi}m\psi$ is introduced as in $L^V_{\rm MF}$ [(2.6)]. Such a situation is naturally understood if one notes that $L^S_{\rm MF}$ is determined for a certain state S which is not always invariant under transformations belonging to the symmetry groups of the starting Lagrangian. One of the most important tasks to be considered in the MF approach is to find the way to recover the underlying symmetry in describing physical states and physical observables. Of course, the required symmetry can be *in principle* restored by taking account of whole effects of $L_{\rm res}$. But in practice we are sometimes forced to manage the problem in an approximate way through recovering the invariance at least formally. One such method usually adopted in nuclear physics is the generator coordinate method.[28,2,13] We will explain the above-mentioned method for special hadronic systems in case of the MF potential of the type (ii) in the following sections.

III. MF APPROXIMATION FOR HADRON SYSTEMS, AND THE ROLE OF RESIDUAL INTERACTION

Let us confine ourselves to considering a composite quark system confined in a hadron. In the present status of understanding the confinement mechanism, various effective-Lagrangian approaches are of methodological significance. In the following we make the MF approximation of type (ii), and see how it becomes essential to take into account the residual interaction in order to recover finally the Lorentz invariance of the theory.

A. Time-independent MF approximation

When one hadron is in the state at rest, we can imagine that quarks in the hadron are well described by using an MF potential of type (ii). From this naive consideration, we are led to the following definition.

Definition I. Define the time-independent MF potential

$$V(\mathbf{x},t)=V^{(\mathbf{R})}(\mathbf{x})\ . \tag{3.1}$$

The potential $V^{(\mathbf{R})}(\mathbf{x})$ is the MF potential at center $\mathbf{R}$. (In Ref. 9 the phrase "rest frame" is used in connection with the time-independent potential.) For the sake of practical use, it is often very convenient to consider the case where the following assumption is approximately valid.

Assumption II. The MF potential $V^{(\mathbf{R})}(\mathbf{x})$ is written as $v(|\mathbf{x}-\mathbf{R}|)$, which is spherically symmetric, and the eigenfunctions of equation

$$[\partial\!\!\!/+v(|\mathbf{x}-\mathbf{R}|)]\psi(x)=0 \tag{3.2}$$

for the quark field $\psi(x)$ are characterized by a set of relevant quantum numbers including the total angular momentum, $n=(E_n;j_k,m,\ldots)$ (Refs. 29 and 13).

The approximate quark field satisfying (3.2) is denoted by $\psi^{(\mathbf{R})}(x)$ and expanded in terms of eigenfunctions (U_m, V_n) as

$$\psi^{(\mathbf{R})}(x)=\sum_n[a_n(\mathbf{R})U_n(\mathbf{x}-\mathbf{R})e^{-iE_nx_0}+b_n(\mathbf{R})^\dagger V_n(\mathbf{x}-\mathbf{R})e^{iE_nx_0}]\ , \tag{3.3}$$

which is just what was introduced in Ref. 13.

B. Time dependence of quark operators

First notice that, in accordance with our assumption I, the Heisenberg field $\psi(x)$ appearing in the Lorentz-invariant Lagrangian L can be expressed as

$$\psi(x)=\sum_n[a_n(\mathbf{R};x_0)U_n(\mathbf{x}-\mathbf{R})e^{-iE_nx_0}+b_n(\mathbf{R};x_0)^\dagger V_n(\mathbf{x}-\mathbf{R})e^{iE_nx_0}] \tag{3.4}$$

with the coefficient operators $a_n(\mathbf{R};x_0)$ and $b_n(\mathbf{R};x_0)$ which depend on time x_0 due to the residual interaction $L_{\rm res}$, while the approximate mean field $\psi^{(\mathbf{R})}(x)$ in (3.3) is further specified with a time parameter R_0 besides $\mathbf{R}$ as follows:

$$\psi^{(R)}(x)=\sum [a_n(R)U_n(\mathbf{x}-\mathbf{R})e^{-iE_nx_0} + b_n(R)^\dagger V_n(\mathbf{x}-\mathbf{R})e^{iE_nx_0}] \tag{3.5}$$

with $(R)=(\mathbf{R},R_0)$. This specification means that we take into account the boundary condition in such a way that $\psi^{(R)}(x)$ coincides with the Heisenberg field $\psi(x)$ given by (3.4) at $x_0=R_0$, i.e.,

$$\psi^{(R)}(x)=\psi(x) \quad \text{at } x_0=R_0 , \tag{3.6}$$

or equivalently

$$a_n(R)=a_n(\mathbf{R};x_0=R_0) ,$$
$$b_n(R)=b_n(\mathbf{R};x_0=R_0) . \tag{3.7}$$

It will be proved that properties of the operators $\psi^{(\mathbf{R})}(x)$ and $\{a_m(R),\, b_n(R)\}$ are useful in the following consideration.

The field $\psi(x)$ at an arbitrary x_0 is related to $\psi^{(R)}(x)$ as

$$\psi(x)=U^{(R)}(x_0)\psi^{(R)}(x)U^{(R)}(x_0)^{-1} \tag{3.8}$$

or

$$\begin{Bmatrix} a_n(\mathbf{R},x_0) \\ b_n(\mathbf{R},x_0) \end{Bmatrix} = U^{(R)}(x_0) \begin{Bmatrix} a_n(R) \\ b_n(R) \end{Bmatrix} U^{(R)}(x_0)^{-1} . \tag{3.9}$$

Here, $U^{(R)}(x_0)$ is the unitary operator satisfying the equation

$$i\frac{dU^{(R)}(x_0)}{dx_0}=-H_{\rm res}(x_0)U^{(R)}(x_0) \tag{3.10a}$$

[see Appendix A, Eq. (A7)] with the boundary condition

$$U^{(R)}(x_0=R_0)=I , \tag{3.10b}$$

where $H_{\rm res}(x_0)$ is the residual interaction Hamiltonian, or

$$H_{\rm res}(x_0)=P_0-H_{\rm MF}(x_0) , \tag{3.11a}$$

$$H_{\rm MF}(x_0)=\int_{R_0=x_0} d\mathbf{x}{:}\psi^{(R)}(x)^\dagger \times[-i\alpha\cdot\partial+v(\mathbf{x}-\mathbf{R})\beta]\psi^{(R)}(x){:} , \tag{3.11b}$$

where P_0 is the total Hamiltonian. Note that the time development of $\psi(x)$ is determined by P_0, and that of $\psi^{(R)}(x)$ is controlled by $H_{\rm MF}(R_0)$, i.e.,

$$\psi^{(R)}(x)=D_{\rm MF}(x_0,R_0)\psi^{(R)}(\mathbf{x},R_0)D_{\rm MF}(x_0,R_0)^{-1} \tag{3.12a}$$

with

$$D_{\rm MF}(x_0,R_0)\equiv\exp[iH_{\rm MF}(R_0)(x_0-R_0)] . \tag{3.12b}$$

From (3.12a) we obtain

$$\begin{Bmatrix} a_n(R) \\ b_n(R) \end{Bmatrix} e^{-iE_n(x_0-R_0)} = D_{\rm MF}(x_0,R_0) \begin{Bmatrix} a_n(R) \\ b_n(R) \end{Bmatrix} D_{\rm MF}(x_0,R_0)^{-1} . \tag{3.13}$$

Details are given in Appendix A.

C. State vectors of composite system

Next we will give formal expressions of the hadron state vector with definite values of the four-momentum (P_μ). First note the familiar displacement character of the operators (P_μ),

$$[P_\mu,\psi(x)]=i\frac{\partial}{\partial x^\mu}\psi(x) , \tag{3.14}$$

from which we have for arbitrary displacement $(\epsilon_\rho)=(\boldsymbol{\epsilon},\epsilon_0)$

$$D(\epsilon)\begin{Bmatrix} a_n(R) \\ b_n(R) \end{Bmatrix} D(\epsilon)^{-1} = \begin{Bmatrix} a_n(R+\epsilon) \\ b_n(R+\epsilon) \end{Bmatrix} e^{-iE_n\epsilon_0} , \tag{3.15}$$

where

$$D(\epsilon)\equiv\exp(-iP_\mu\epsilon^\mu) . \tag{3.16}$$

Now according to the standard generator-coordinate method,[2,13,28] we construct the following hadron state which can be identified as the state *at rest:*

$$|H_A\rangle\equiv\mathcal{N}_A\int d\mathbf{R}\,dR_0 e^{-i\alpha_A R_0}|H_A;R\rangle \tag{3.17}$$

with some normalization factor $\mathcal{N}_A$, where $|H_A;R\rangle$ is defined with the use of appropriate quark configurations $H_A(a^\dagger,b^\dagger)$ by

$$|H_A;R\rangle=H_A(a(R)^\dagger,b(R)^\dagger)\,|h(R)\rangle; \tag{3.18}$$

the *hadronic vacuum* $|h(R)\rangle$ is defined by

$$a_n(R)\,|h(R)\rangle=b_m(R)\,|h(R)\rangle =0 \quad \text{for any } n,m . \tag{3.19}$$

α_A in (3.17) is a parameter, the meaning of which is to be explained later in this subsection. Note that in the MF approximation each (anti) quark in (3.18) behaves as an independent particle, and the configuration of the hadronic state (3.18) is so arranged that it is a linear combination of the same power of polynomials of $a_m^\dagger$'s and/or $b_m^\dagger$'s (concrete examples are found in Refs. 12 and 13). By using

$$|H_A;\mathbf{R},R_0\rangle=e^{-i\mathbf{P}\cdot\mathbf{R}}|H_A;\mathbf{O},R_0\rangle , \tag{3.20}$$

(3.17) is written formally as

$$|H_A\rangle=\mathcal{N}_A(2\pi)^3\delta(\mathbf{P})\int dR_0 e^{-i\alpha_A R_0}|H_A,\mathbf{O},R_0\rangle , \tag{3.21}$$

which means that $|H_A\rangle$ in (3.17) can be regarded as a hadron state *at rest.* Note that we have implicitly assumed that

$$D(\boldsymbol{\epsilon},0)\,|h(R)\rangle=|h(\mathbf{R}+\boldsymbol{\epsilon},R_0)\rangle . \tag{3.22}$$

Under this condition, we can prove

$$D(\epsilon)\,|h(R)\rangle=e^{iE_h\epsilon_0}|h(R+\epsilon)\rangle , \tag{3.23}$$

where E_h is equal to β_0, defined by (A16) in Appendix A.

The parameter α_A in (3.17) is determined in such a way that the relevant hadron H_A described by the state (3.17) has the collective energy caused by the residual interaction, which gives rise to mass correction in the MF approximation. By using (3.15) and (3.23), we obtain

$$D(\epsilon)|H_A\rangle=\mathcal{N}_A\int d\mathbf{R}dR_0e^{-i\alpha_AR_0}\left\{\exp\left[i\left(\sum_{i\in H_A}E_{ni}+E_h\right)\epsilon_0\right]\right\}|H_A(R+\epsilon)\rangle$$
$$=\left\{\exp\left[i\left(\sum_{i\in H_A}E_{ni}+E_h+\alpha_A\right)\epsilon_0\right]\right\}|H_A\rangle\ , \qquad (3.24)$$

which indicates that the state $|H_A\rangle$ can be regarded as a true eigenstate of the total four momenta P_μ's with eigenvalues

$$(\mathbf{p},p_0)=\left[\mathbf{0},m_A=\sum_{i\in H_A}E_{ni}+E_h+\alpha_A\right]. \qquad (3.25)$$

In other words, making use of the exact operators $a_n(R)$'s and $b_n(R)$'s at $x_0=R_0$, we can construct the hadron state *at rest*, $|H_A\rangle$ (3.17), with the proper eigenvalue of the total momenta, when α_A is so determined as to reproduce the correct relation (3.25).

The hadron states in the MF approximation, $|H_A\rangle_{\rm MF}$ are obtained by adopting the R_0-independent operators $\{a_m(\mathbf{R}),b_n(\mathbf{R})\}$, which amounts to setting $\{a_m(\mathbf{R},R_0=0)\equiv a_m(\mathbf{R}),\ b_n(\mathbf{R},R_0=0)\equiv b_n(\mathbf{R})\}$, and to neglecting the effect due to $H'_{\rm res}$. Here $H'_{\rm res}(R_0)$ is equal to $H_{\rm res}(R_0)$ minus $H'(R_0)$ [see (B15) in Appendix B]; $H'(R_0)$ is defined so as to satisfy

$$H'(R_0)|h(R)\rangle=E_h^{(0)}|h(R)\rangle$$

and

$$[H'(R_0),a_m(R)]=[H'(R_0),b_n(R)]$$
$$=0\ \text{ for any }(m,n)\ . \qquad (3.26)$$

$|H_A\rangle_{\rm MF}$ is defined by

$$|H_A\rangle_{\rm MF}=N_A\int d\mathbf{R}H_A(a\mathbf{R})^\dagger,b(\mathbf{R})^\dagger)|h(\mathbf{R})\rangle\ . \qquad (3.27)$$

This state has the momentum eigenvalue $\mathbf{p}=0$ under the condition that $|h(\mathbf{R})\rangle$ satisfies (3.22). Also, as explained in Appendix B, the state

$$|H_A;\mathbf{R}\rangle\equiv H_A(a(\mathbf{R})^\dagger,b(\mathbf{R})^\dagger)|h(\mathbf{R})\rangle \qquad (3.28)$$

has the eigenvalue of $H_{\rm MF}(R_0=0)+H'(R_0=0)$ which is equal to

$$\sum_{i\in H_A}E_{ni}+E_h^{(0)}\equiv m_A^{(0)} \qquad (3.29)$$

[see (B19) in Appendix B], which can be regarded as an approximate mass of the hadron H_A in the zeroth order with respect to $H'_{\rm res}$.

The higher-order corrections due to $H'_{\rm res}$ are in principle obtained as explained in Appendix B by taking account of the detailed structure of each term of the total Hamiltonian P_0 and also of the hadronic vacuum. It is, however, hardly possible to see those detailed structures, and we have to treat α_A in (3.17) as a phenomenologically adjustable parameter since we have little knowledge of the residual interaction. In the variational method, α_A is taken so as to give the minimal energy-expectation value.[2]

Throughout the following considerations, therefore, we assume that $|H_A\rangle_{\rm MF}\equiv|H_A;\mathbf{p}=\mathbf{0}\rangle$, (3.27), describes approximately well the hadron state at rest ($\mathbf{p}=\mathbf{0}$) with its mass m_A; i.e.,

$$(\mathbf{P},P_0)|H_A;\mathbf{p}=\mathbf{0}\rangle=(\mathbf{0},m_A^{(0)}\simeq m_A)|H_A;\mathbf{p}=\mathbf{0}\rangle\ . \qquad (3.30)$$

Then we can construct the momentum eigenstate $|H_A;\mathbf{p}\rangle$ which satisfies approximately the on-shell relation by Lorentz boosting $U(\mathbf{v}_p)$:

$$|H_A;\mathbf{p}\rangle=U(\mathbf{v}_p)|H_A;\mathbf{p}=\mathbf{0}\rangle \qquad (3.31)$$

with the energy of this state $p_0\simeq(\mathbf{p}^2+m_A^{(0)2})^{1/2}$ and $\mathbf{v}_p=\mathbf{p}/p_0$. From (3.30) and (3.31), we can derive a formula[12] which relates the state (3.31) to the three-momentum eigenstate which has been adopted in the MF approach[13,15] and used in various calculations:[14,15,16]

$$|\mathbf{p},H_A\rangle=N'_A\int d\mathbf{R}e^{i\mathbf{p}\cdot\mathbf{R}}|H_A;\mathbf{R}\rangle\ . \qquad (3.32)$$

A summary of relevant formulas is given in Appendix C.

D. Lorentz boost of quark operators

When we calculate various types of matrix elements of quark operators, such as

$$j_\rho(x)=i\bar\psi(x)\gamma_\rho\psi(x)\ ,$$
$$H_I(x)=\bar\psi(x)O_\rho\psi(x)\bar\psi(x)O^\rho\psi(x)\ ,$$

etc., between states $\langle H_A;\mathbf{q}|$ and $|H_B;\mathbf{p}\rangle$, it is necessary to obtain the Lorentz boost of quark operators.

With the Lorentz-boosting operator $U(L)$, which corresponds to the Lorentz transformation $x'_\mu=L_\mu{}^\nu x_\nu$, the operator $\psi(x)$ transforms as

$$\psi'(x')=U(L)^{-1}\psi(x')U(L)=\Lambda(L)\psi(x)\ , \qquad (3.33a)$$

where

$$\Lambda(L)^{-1}\gamma_\mu\Lambda(L)=L_\mu{}^\nu\gamma_\nu,\ \ \Lambda(L)^\dagger=\gamma_4\Lambda(L)^{-1}\gamma_4\ . \qquad (3.33b)$$

In general, the generators of Lorentz transformation are formally expressed in terms of the energy-momentum tensors $T_{\mu\nu}$'s which depend on the whole Lagrangian. We have introduced the MF Lagrangian $L_{\rm MF}$ of type (ii) which contains the potential $V^{(\mathbf{R})}(\mathbf{x})$ with a center $\mathbf{R}$ and without a special manipulation is not Lorentz covariant; then the residual interaction is needed to recover the Lorentz covariance. We show in Appendix B how the covariance can be restored by taking into account the residual interaction.

In the lowest order of the MF approximation, we can express explicitly the boosted quark operators $U(\mathbf{v})\{a_n(\mathbf{R})$ or $b_m(\mathbf{R})\}U(\mathbf{v})^{-1}$ in terms of $\{a_m(\mathbf{R})$'s,

$b_n(\mathbf{R})$'s} by using (3.33a) and assumption I; here, $U(\mathbf{v})\equiv U(L(\mathbf{v}))$. Let us now describe it in the following.

The concrete form of $\Lambda(L(\mathbf{v}))$ in (3.33a) is expressed as

$$\Lambda(L(\mathbf{v}))\equiv\Lambda(\mathbf{v})=\exp[w\mathbf{v}\cdot\boldsymbol{\alpha}/(2v)] , \tag{3.34}$$

with $\boldsymbol{\alpha}=i\gamma_4\boldsymbol{\gamma}$ and $\tanh w=v/c$; w is the Lorentz-rotation angle of the boost $U(\mathbf{v})$:

$$([L(\mathbf{v})x]_\rho)=(x_\perp,x_{\parallel}\cosh w+x_0\sinh w ,\; x_{\parallel}\sinh w+x_0\cosh w) . \tag{3.35}$$

If the $H'_{\rm res}(R_0=0)$ part in P_0 is neglected, we obtain from (3.15) and (3.13)

$$\begin{aligned} a_m(\mathbf{R},y_0)e^{-iE_my_0}&=D(y_0)a_m(\mathbf{R})D(y_0)^{-1}\\ &\doteq D_{\rm MF}(y_0)a_m(\mathbf{R})D_{\rm MF}(y_0)^{-1}\\ &=a_m(\mathbf{R})e^{-iE_my_0} , \end{aligned} \tag{3.36}$$

where $D_{\rm MF}(y_0)\equiv\exp[iH_{\rm MF}(R_0=0)y_0]$; the sign $\doteq$ means the equality when $H'_{\rm res}$ is neglected. From (3.33a), we find for $x'_\rho=L_\rho{}^\lambda x_\lambda$

$$\begin{aligned} U(\mathbf{v})a_n(\mathbf{R},x_0)U(\mathbf{v})^{-1}e^{-iE_nx_0}&=\sum_m\int d\mathbf{x}\,U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}U_m(\mathbf{x}'-\mathbf{R})e^{-iE_mx'_0}a_m(\mathbf{R},x'_0)\\ &\quad+\sum_{\hat m}\int d\mathbf{x}\,U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}V_m(\mathbf{x}'-\mathbf{R})e^{iE_{\hat m}x'_0}b_{\hat m}(\mathbf{R},x'_0)^\dagger , \end{aligned} \tag{3.37a}$$

$$\begin{aligned} U(\mathbf{v})b_{\hat n}(\mathbf{R},x_0)^\dagger U(\mathbf{v})^{-1}e^{iE_{\hat n}x_0}&=\sum_m\int d\mathbf{x}\,V_{\hat n}(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}U_m(\mathbf{x}'-\mathbf{R})e^{-iE_mx'_0}a_m(\mathbf{R},x'_0)\\ &\quad+\sum_{\hat m}\int d\mathbf{x}\,V_{\hat n}(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}V_{\hat m}(\mathbf{x}'-\mathbf{R})e^{iE_{\hat m}x'_0}b_{\hat m}(\mathbf{R},x'_0)^\dagger . \end{aligned} \tag{3.37b}$$

By applying the approximation (3.36) to the right- (RHS) as well as left-hand sides of (3.37a) and (3.37b), we obtain as a formal expression

$$U(\mathbf{v})\alpha(\mathbf{R},x_0)U(\mathbf{v})^{-1}\doteq F(\mathbf{R},\mathbf{v})_{x_0x'_0}\alpha(\mathbf{R},x'_0) , \tag{3.38a}$$

where

$$\alpha(\mathbf{R},x_0)^T\equiv(a_1(\mathbf{R})e^{-iE_1x_0},\ldots,a_n(\mathbf{R})e^{-iE_nx_0},\ldots,b_{\hat 1}(\mathbf{R})^\dagger e^{iE_{\hat 1}x_0},\ldots,b_{\hat m}(\mathbf{R})e^{iE_{\hat m}x_0},\ldots) , \tag{3.38b}$$

$$F(\mathbf{R},\mathbf{v})_{x_0x'_0}\equiv\begin{bmatrix}F(\mathbf{R},\mathbf{v})_{x_0x'_0:nm} & F(\mathbf{R},\mathbf{v})_{x_0x'_0:n\hat m}\\ F(\mathbf{R},\mathbf{v})_{x_0x'_0:\hat nm} & F(\mathbf{R},\mathbf{v})_{x_0x'_0:\hat n\hat m}\end{bmatrix} , \tag{3.38c}$$

$$F(\mathbf{R},\mathbf{v})_{x_0,x'_0:nm}\equiv\int dx'_\perp U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}U_m(\mathbf{x}'-\mathbf{R}) , \tag{3.38d}$$

etc.

The sum over x'_0 on the RHS of (3.38a) means the integral over $x_{\parallel}$. (Note $x'_0=x_0\cosh w+x_{\parallel}\sinh w$.) Full expressions of the F matrix and its properties are given in Appendix D. Equation (3.38a) can be rewritten as

$$U(\mathbf{v})\alpha(\mathbf{R},0)U(\mathbf{v})^{-1}\doteq F(\mathbf{R},\mathbf{v})\alpha(\mathbf{R},0) , \tag{3.39a}$$

$$F(\mathbf{R},\mathbf{v})\equiv\begin{bmatrix}F(\mathbf{R},\mathbf{v})_{nm} & F(\mathbf{R},\mathbf{v})_{n\hat m}\\ F(\mathbf{R},\mathbf{v})_{\hat nm} & F(\mathbf{R},\mathbf{v})_{\hat n\hat m}\end{bmatrix} , \tag{3.39b}$$

$$F(\mathbf{R},\mathbf{v})_{nm}\equiv\int d\mathbf{x}\,U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}U_m(\mathbf{x}'-\mathbf{R})e^{-iE_mx'_0},\ \text{etc.} , \tag{3.39c}$$

$$\begin{bmatrix}x'_0\\ x'_{\parallel}\end{bmatrix}=\begin{bmatrix}\cosh w & \cosh w\\ \sinh w & \cosh w\end{bmatrix}\begin{bmatrix}x_0=0\\ x_{\parallel}\end{bmatrix} . \tag{3.39d}$$

From (3.39a) we get formally

$$U(\mathbf{v})\doteq\exp[-\alpha^\dagger(\mathbf{R},0)\ln F(\mathbf{R},\mathbf{v})\alpha(\mathbf{R},0)] . \tag{3.40}$$

Note that, when one performs successive Lorentz transformation, one should start always with Eq. (3.38a). It is clear that the straightforward application of (3.39a) leads us to incorrect relations. This can be understood easily by tracing back in what way the RHS of (3.38a) satisfies the properties required from, e.g.,

$$U(\mathbf{v})U(-\mathbf{v})=1 , \tag{3.41a}$$

$$U(\mathbf{v})U(\mathbf{v})^\dagger=U(\mathbf{v})^\dagger U(\mathbf{v})=1 . \tag{3.41b}$$

This is examined in Appendix D.

E. Remark on matrix elements of quark operators between one-baryon states

Until now, we have explained how to construct, in case of the time-dependent MF potential (2.4b), a moving hadron state which consists of confined quarks and/or antiquarks, and also considered the Lorentz boost of the quark operators. Thus, it would be instructive and pedagogical to show explicit forms of matrix elements of quark operators. So, in this subsection, we give summarizing remarks on matrix elements of quark operators sandwiched by one-hadron states, the details being left in a subsequent paper.[25]

To be concrete, we consider matrix elements of quark current operators between one low-lying baryon state:

$$\langle B_C;\mathbf{q}\,|\,J_\rho(0)\,|\,B_D;\mathbf{p}\rangle = N_C(q)N_D(p)\int d\mathbf{r}\int d\mathbf{x}\, e^{-i(\mathbf{p}+\mathbf{q})\cdot\mathbf{r}/2}e^{-i\mathbf{k}\cdot\mathbf{x}} \times\langle B_C;\mathbf{S}=\mathbf{0}\,|\,U(\mathbf{v}_q)^{-1}e^{i\mathbf{P}\cdot(\mathbf{x}+\mathbf{r}/2)}J_\rho(0)e^{-i\mathbf{P}\cdot(\mathbf{x}-\mathbf{r}/2)}U(\mathbf{v}_p)\,|\,B_D;\mathbf{S}=\mathbf{0}\rangle\ . \tag{3.42}$$

Here, the expression of the state vector (C7) is used; $k_\rho = q_\rho - p_\rho$; $N_C(q) = N_C(0)/(1-v_q{}^2)^{1/2}$. The quark current J_ρ has generally a form

$$J_\rho(x) = \sum_{\underline{b},\underline{d}} g_{\underline{b}\underline{d}} J_\rho^{\underline{b}\,\underline{d}}(x)\ , \tag{3.43}$$

where $g_{\underline{b}\,\underline{d}}$'s are numerical coefficients, and

$$J_\rho^{\underline{b}\,\underline{d}}(x) = \bar{\psi}_{,\underline{b}}(x)O_\rho\psi_{,\underline{d}}(x) \tag{3.44}$$

with an appropriate γ matrix O_ρ. The quark spinor $\psi_{,\underline{b}}(x)$ describes the quark field with a set of the color and flavor quantum numbers denoted by $\underline{b}$. Then we can obtain

$$\begin{aligned}\langle B_C;\mathbf{q}\,|\,J_\rho^{\underline{b}\,\underline{d}}(0)\,|\,B_D;\mathbf{p}\rangle &= N_C(q)N_D(p)\int d\mathbf{r}\int d\mathbf{x}\, e^{-i(\mathbf{q}+\mathbf{p})\mathbf{r}/2}e^{i\mathbf{k}\cdot\mathbf{x}}\langle B_C;\mathbf{S}=\mathbf{0}\,|\,\bar{\psi}_{,\underline{b}}(L(-\mathbf{v}_q)x_-)\Lambda(v_q)^{-1}U(\mathbf{v}_q)^{-1}\\ &\quad\times e^{i\mathbf{P}\cdot\mathbf{r}}U(\mathbf{v}_p)O_\rho\Lambda(\mathbf{v}_p)\psi_{,\underline{d}}(L(-\mathbf{v}_p)x_+)\,|\,B_D;\mathbf{S}=\mathbf{0}\rangle\end{aligned} \tag{3.45}$$

with $x_\pm = x \pm r/2$ and $(x_\pm)_0 = 0$.

With the aim of comparing (3.45) with the matrix elements derived by other authors,[4-6,8-11,22-24] we take

$$\psi(L(-\mathbf{v}_p)x_+)\,|_{(x_+)_0=0} \doteq \psi^{(\mathbf{R}=0)}(L(-\mathbf{v}_p)x_+)\,|_{(x_+)_0=0}\ , \tag{3.46}$$

and also the same relation with $\mathbf{q}$ and x_- substituted for $\mathbf{p}$ and x_+ in (3.46). In the following we choose the Breit frame with $\mathbf{q} = -\mathbf{p} = \mathbf{k}/2$, and use the Lorentz-rotation angle $\mathbf{w}_{\mathbf{k}/2} = \Omega = \operatorname{arctanh}[k/(2E_B(k/2))]$ in order to specify the relevant Lorentz transformation:

$$E_B(k/2) = [(k/2)^2 + m_B{}^2]^{1/2}\ ;$$

it is enough for our purpose to take the same mass m_B for all the low-lying baryons. Then we obtain

$$\begin{aligned}\left\langle B_C;\frac{\mathbf{k}}{2}\,\middle|\,J_\rho^{\underline{b}\,\underline{d}}(0)\,\middle|\,B_D;\frac{-\mathbf{k}}{2}\right\rangle &\doteq N_C(k/2)N_D(k/2)\int d\mathbf{r}\int d\mathbf{x}\, e^{i\mathbf{k}\cdot\mathbf{x}}\sum_{b-\underline{b}}\sum_{d-\underline{d}}\bar{U}_{,b}([L(-\Omega)x_-]_i)\Lambda(-\Omega)O_\rho\Lambda(-\Omega)U_{,d}([L(\Omega)x_+]_i)\\ &\quad\times e^{iE_b(L(-\Omega)x_-)_0}e^{-iE_d(L(\Omega)x_+)_0}\,|_{(x_+)_0=(x_-)_0=0}\\ &\quad\times\langle B_C;\mathbf{S}=\mathbf{0}\,|\,a_b(0)^\dagger U(-\Omega)e^{i\mathbf{P}\cdot\mathbf{r}}U(-\Omega)a_d(0)\,|\,B_D;\mathbf{S}=\mathbf{0}\rangle + \cdots\ ,\end{aligned} \tag{3.47}$$

where $\cdots$ means the contribution from the antiquark modes. In the following we use the concrete form of the "bag" state (3.28) for the low-lying baryons written as[13,15]

$$|\,B_C;\mathbf{S}=\mathbf{0}\rangle = \sum_{\{b_j\}} B^C_{b_1b_2b_3}a_{b_1}(0)^\dagger a_{b_2}(0)^\dagger a_{b_3}(0)^\dagger\,|\,h(0)\rangle\ . \tag{3.48}$$

In order to go further, we adopt the following two approximations. The first is to neglect the antiquark-mode contribution in (3.47), and the second is the "local approximation."[13,15] The validity of the first approximation is not evident, which will be examined in a subsequent paper.[25] As to the second we would like to stress that this approximation is implicitly used in most of the usual calculations based on not only bag models but also other models such as the Skyrmion model recently investigated by many authors.[30] Matrix-element calculations done without recourse to the second approximation are found in Ref. 31. In our formalism, the local approximation amounts to take[25]

$$\frac{1}{[V_C(0)V_D(0)]^{1/2}}\int d\mathbf{r}f(\mathbf{r})\rightarrow f(0)\ , \tag{3.49a}$$

where $V_C(0)$ is defined as

$$V_C(0)\equiv\int d\mathbf{r}\langle B_C;\mathbf{S}=\mathbf{0}\,|\,e^{-i\mathbf{P}\cdot\mathbf{r}}\,|\,B_C;\mathbf{S}=\mathbf{0}\rangle\ . \tag{3.49b}$$

$$\left\langle B_C;\frac{\mathbf{k}}{2}\left|J_p^{bd}(0)\right|B_D;\frac{-\mathbf{k}}{2}\right\rangle\simeq 18[4E_C(k/2)E_D(k/2)]^{1/2}$$
$$\times\sum_{b-\underline{b},b_2b_3}\ \sum_{d-\underline{d},d_2d_3}B^C_{bb_2b_3}B^D_{dd_2d_3}\int d\mathbf{x}\exp[i(k-2E_0\sinh\Omega)x_{||}]$$
$$\times\overline{U}_{,b}(\mathbf{x}')\Lambda(-\Omega)O_p\Lambda(-\Omega)U_{,d}(\mathbf{x}')\left[\prod_{j=2}^{3}F(0,2\Omega)_{b_jd_j}\right]$$
$$\times\langle h(0)\,|\,U(2\Omega)\,|\,h(\mathbf{0})\rangle \tag{3.50}$$

with $\mathbf{x}'=\mathbf{x}_\perp+\mathbf{x}_{||}\cosh\Omega$. This is to be compared with the formula (53) derived by Betz and Goldflam.[9]

Now we are in a position to make a remark on the behavior of (3.50) in the low-momentum-transfer region. By examining the first-order term in (3.50) with respect to k/m_B, it is easy to see that the baryon magnetic moments include two kinds of correction terms to the static values. Those are the "retardation" and the "spin precession" terms,[4] the former and the latter of which come from the factor $\exp(-2iE_0x_{||}\sinh\Omega)$ and $\Lambda(-\Omega)O_p\Lambda(\Omega)$, respectively. This result is in accordance with that obtained in Refs. 4 and 9, but is different from that derived by Hwang[5,22] and Chizhov and Dorokhov,[23] where only the spin-precession term appears.

As to the second-order corrections with respect to k/m_B, we obtain as the charge radius of the proton the following formula:[25]

$$\langle r_c{}^2\rangle^{\text{recoil}}=\left[1-\frac{2E_0}{m_N}+\frac{3E_0{}^2}{m_N{}^2}\right]\langle r_c{}^2\rangle^{\text{static}}+\frac{3}{2m_N{}^2}-6\frac{d}{dk^2}\langle h(\mathbf{0})\,|\,U(2\Omega)\,|\,h(\mathbf{0})\rangle\Bigg|_{k=0}\ . \tag{3.51}$$

Apart from the last term coming from the Lorentz-deformed hadronic vacuum, the above formula is the same as derived in Ref. 9. It should be noted that, apart from $\langle h(0)|U(2\Omega)|h(0)\rangle$, there is a certain difference between (3.50) and Eq. (53) in Ref. 9. This difference comes from different ways of evaluating the spectator contribution to the matrix element. Really, we can derive Eq. (53) in Ref. 9 by using some special manipulation.[25] It can be proved,[25] however, that apart from $\langle h(\mathbf{0})\,|\,U(2\Omega)\,|\,h(\mathbf{0})\rangle$ the same matrix elements are obtained from (3.50) and Eq. (53) in Ref. 9 to the *second* order with respect to k/m_B, and the difference appears only from terms with the order equal to or higher than $(k/m_B)^3$, which depends on the way one evaluates the spectator part. Therefore, behaviors of terms higher than $(k/m_B)^2$ are ambiguous, and this suggests the limit of applicability of the various MF approaches including the present one.

It should be remarked lastly that the factor relevant to the hadronic vacuum such as $\langle h(\mathbf{0})\,|\,U(2\Omega)|h(\mathbf{0})\rangle$ in (3.50) appears, which is characteristic of our formalism, and will be further investigated in the subsequent paper.[25]

VI. SUMMARY

We have investigated problems concerning the Lorentz boost in the mean-field approach. We have concentrated our considerations especially on quark modes in the time-independent mean field. We emphasized the role played by the residual interaction in order to recover the Lorentz invariance which the starting Lagrangian has. The Lorentz boost forces us to deal with the time development of the quark operators, which is governed by the residual interaction. Since at present we scarcely have knowledge of the latter, we have to drop somewhere in matrix element calculations the time evolution of the operators due to the residual interaction. We explained the problems brought about in the approximate treatments of boosting quark modes.

Once we clarify the concept of the mean-field approximation and provide basic tools for describing boosted quark modes in a moving hadron or for evaluating matrix elements of physical operators, it becomes transparent what types of approximation have been made by various authors.[4–6,9,11,22–24,32] In the present paper we gave a concrete example of matrix elements for quark operators between one-hadron states, and made a remark on the relation of our approach to other ones. In a subsequent paper,[25] full explanations will be given and we clarify the meaning or interpretation of wave functions[3,6,9,11,16,22–24] which have been proposed by taking account of certain kinds of relativistic effects.

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APPENDIX A: RELATIONS BETWEEN REPRESENTATIONS, AND REMARK ON (3.21)

The Heisenberg field $\psi(x)$ satisfies

$$[P_\rho,\psi(x)]=i\frac{\partial\psi(x)}{\partial x^\rho}\ , \qquad \text{(A1)}$$

where P_ρ's are the total energy-momentum operators,

$$\psi(\mathbf{x},x_0)=D(x_0-R_0)\psi(\mathbf{x},R_0)D(x_0-R_0)^{-1}\ , \qquad \text{(A2)}$$

with

$$D(x_0)\equiv\exp(iP_0x_0)\ .$$

On the other hand, we have the time development of the approximate field operator $\psi^{(\mathbf{R},R_0)}(x)$ which is set equal to $\psi(x)$ at $x_0=R_0$ [see (3.6)]

$$\psi^{(R)}(x)=D_{\rm MF}(x_0,R_0)\psi^{(R)}(\mathbf{x},R_0)D_{\rm MF}(x_0,R_0)^{-1}\ , \qquad \text{(A3)}$$

where

$$D_{\rm MF}(x_0,R_0)\equiv\exp[iH_{\rm MF}(R_0)(x_0-R_0)]\ , \qquad \text{(A4)}$$

$H_{\rm HF}(R_0)$ is defined by (3.11b). Substituting (A3) into (A2), we obtain

$$\psi(x)=U^{(R_0)}(x_0)\psi^{(R)}(x)U^{(R_0)}(x_0)^{-1}\ , \qquad \text{(A5)}$$

where

$$U^{(R_0)}(x_0)\equiv D(x_0-R_0)D_{\rm MF}(x_0,R_0)^{-1}\ . \qquad \text{(A6)}$$

$U^{(R_0)}(x_0)$ can be regarded as the unitary operator connecting the Heisenberg and the *interaction* representations, and satisfying

$$i\frac{dU^{(R_0)}(x_0)}{dx_0}=-H_{\rm res}(x_0)U^{(R_0)}(x_0) \qquad \text{(A7)}$$

with the boundary condition

$$U^{(R_0)}(R_0)=1\ . \qquad \text{(A8)}$$

It may be needless to say that $H_{\rm res}(x_0)$ is given by

$$H_{\rm res}(x_0)=D(x_0-R_0)H_{\rm res}(R_0)D(x_0-R_0)^{-1}\ . \qquad \text{(A9)}$$

Next we consider the R_0 dependence of $\{a_m(R), b_n(R)\}$. By noting

$$D(\epsilon)\psi(\mathbf{x},R_0)D(\epsilon)^{-1}=\psi(\mathbf{x}+\boldsymbol{\epsilon},R_0+\epsilon_0) \qquad \text{(A10)}$$

with $D(\epsilon)=\exp(-iP_\rho\epsilon^\rho)$, and using (3.5), we have

$$D(\epsilon)\psi^{(R)}(\mathbf{x},R_0)D(\epsilon)^{-1}=\psi^{(R+\epsilon)}(\mathbf{x}+\boldsymbol{\epsilon},R_0+\epsilon_0)\ , \qquad \text{(A11)}$$

from which we find

$$D(\epsilon)\begin{Bmatrix}a_n(R)\\ b_n(R)\end{Bmatrix}D(\epsilon)^{-1}=e^{-iE_n\epsilon_0}\begin{Bmatrix}a_n(R+\epsilon)\\ b_n(R+\epsilon)\end{Bmatrix}\ . \qquad \text{(A12)}$$

From (A12) and the definition of the hadronic vacuum $|h(R)\rangle$, (3.19), we have for any (m,n)

$$0=D(\epsilon)\begin{Bmatrix}a_m(R)\\ b_n(R)\end{Bmatrix}D(\epsilon)^{-1}D(\epsilon)|h(R)\rangle=\begin{Bmatrix}e^{-iE_m\epsilon_0}\ \ a_m(R+\epsilon)\\ e^{-iE_n\epsilon_0}\ \ b_n(R+\epsilon)\end{Bmatrix}D(\epsilon)|h(R)\rangle\ , \qquad \text{(A13)}$$

from which one obtains

$$D(\epsilon)|h(R)\rangle=e^{if(R,\epsilon)}|h(R+\epsilon)\rangle\ , \qquad \text{(A14)}$$

where $f(R,\epsilon)$ is a real phase. If $f(R,\epsilon)$ is independent of R and linear with respect to ϵ, we have

$$D(\epsilon^{(1)})D(\epsilon^{(2)})=D(\epsilon^{(2)})D(\epsilon^{(1)})=D(\epsilon^{(1)}+\epsilon^{(2)})\ .$$

Then we obtain

$$D(\epsilon)|h(R)\rangle=e^{-i\beta_\rho\epsilon^\rho}|h(R+\epsilon) \qquad \text{(A15)}$$

with R-independent β_ρ. If the hadronic vacuum on which the hadron states are constructed is taken to satisfy

$$D(\epsilon)|h(R)\rangle=e^{i\beta_0\epsilon_0}|h(R+\epsilon)\rangle\ , \qquad \text{(A16)}$$

the contribution from $|h(R)\rangle$ to the hadron three-momentum $\mathbf{p}$ is zero.

It may be useful to note that from (A12) and (3.5) we find

$$D(\epsilon)\psi^{(R)}(x)D(\epsilon)^{-1}=\psi^{(R+\epsilon)}(x+\epsilon)\ . \qquad \text{(A17)}$$

APPENDIX B: R_0 DEPENDENCE OF $\{a_m(R),\, b_n(R)\}$, and $|h(R)\rangle$

From (A16) and (A12) [or (3.23) and (3.15)] one obtains

$$\frac{\partial}{\partial R_0}\,|h(R)\rangle = i(P_0-\beta_0)\,|h(R)\rangle \tag{B1}$$

and

$$\frac{\partial a_n(R)}{\partial R_0} = i[P_0,a_n(R)] + iE_n a_n(R) \tag{B2}$$

$$= i[H_{\rm res}(R_0),a_n(R)]\ . \tag{B3}$$

Defining

$$\underline{a}_n(R) \equiv a_n(R)e^{-iE_nR_0}\ , \tag{B4}$$

we find from (B2)

$$\frac{\partial}{\partial R_0}\underline{a}_n(R) = i[P_0,\underline{a}_n(R)]\ , \tag{B5}$$

By setting

$$a_m(\mathbf{R}) \equiv a_m(\mathbf{R},0) \tag{B6}$$

we obtain the equation equivalent to (A12)

$$\underline{a}_n(R) = D(R_0)a_n(\mathbf{R})D(R_0)^{-1} \tag{B7}$$

or

$$a_n(R) = U^{(0)}(R_0)a_n(\mathbf{R})U^{(0)}(R_0)^{-1} \tag{B8}$$

with

$$U^{(0)}(R_0) = D(R_0)D_{\rm MF}(R_0)^{-1}$$
$$= [\exp(iP_0R_0)]\,[\exp(-iH_{\rm MF}(x_0=0)R_0)]\ . \tag{B9}$$

[See Eq. (A5).]

As to the hadronic vacuum, we obtain from (B1)

$$|h(R)\rangle = D_{\beta 0}(R_0)\,|h(\mathbf{R})\rangle\ , \tag{B10}$$

where

$$D_{\beta 0}(R_0) \equiv \exp(iR_0(P_0-\beta_0))$$

and

$$|h(\mathbf{R})\rangle \equiv |h(\mathbf{R},0)\rangle\ .$$

We see from (B7) and (B10) together with (3.19)

$$a_m(\mathbf{R})\,|h(\mathbf{R})\rangle = b_n(\mathbf{R})\,|h(\mathbf{R})\rangle$$
$$= 0 \quad \text{for any } (m,n)\ . \tag{B11}$$

Next we consider the approximate MF state

$$|H_A\rangle_{\rm MF} = N_A \int d\mathbf{R}\,|H_A;\mathbf{R}\rangle \tag{B12}$$

with

$$|H_A;\mathbf{R}\rangle = H_A(a(\mathbf{R})^\dagger,b(\mathbf{R})^\dagger)\,|h(\mathbf{R})\rangle\ . \tag{B13}$$

Remembering (A12) and (A16), we find

$$D(\boldsymbol{\epsilon})\,|H_A\rangle_{\rm MF} = |H_A\rangle_{\rm MF}\ ; \tag{B14}$$

therefore, the state $|H_A\rangle_{\rm MF}$ is the eigenstate with $\mathbf{p}=\mathbf{0}$. As for the energy eigenvalue, we decompose P_0 as

$$P_0 = H_{\rm MF}(R_0=0) + H_{\rm res}(R_0=0)$$
$$= H_{\rm MF}(0) + H'(0) + H'_{\rm res}(0)\ , \tag{B15}$$

where $H'(0)$, if it exists, is a part of $H_{\rm res}(0)$ and satisfies

$$H'(0)\,|h(\mathbf{R})\rangle = E_h^{(0)}\,|h(\mathbf{R})\rangle\ , \tag{B16}$$

$$[H'(0),a_m(\mathbf{R})] = [H'(0),b_n(\mathbf{R})]$$
$$= 0 \quad \text{for any } (m,n)\ . \tag{B17}$$

Then, remembering

$$H_{\rm MF}(R_0=0)\,|h(\mathbf{R})\rangle = 0 \tag{B18}$$

we see

$$[H_{\rm MF}(0)+H'(0)]\,|H_A;\mathbf{R}\rangle$$
$$= \left[\sum_{i\in H_A} E_{ni} + E_h^{(0)}\right]|H_A;\mathbf{R}\rangle\ , \tag{B19}$$

which may be considered to be an approximate mass value $m_A^{(0)}$ of the state $|H_A\rangle_{\rm MF}$. While, we can construct the MF hadron state by using the energy projection as

$$|H_A\rangle \equiv N_A \int d\mathbf{R} \int dR_0 H_A(a(R)^\dagger,b(R)^\dagger)\,|h(R)\rangle\ . \tag{B20}$$

Remembering

$$D(\epsilon)\,|H_A;R\rangle$$
$$= \left\{\exp\left[i\left[\sum_{i\in H_A} E_{ni} + \beta_0\right]\epsilon_0\right]\right\}|H_A;R+\epsilon\rangle\ , \tag{B21}$$

we see the eigenvalue of P_0 for $|H_A\rangle$ is equal to

$$\sum_{i\in H_A} E_{ni} + \beta_0 = m_A^{(0)} + \Delta\beta_0\ ,$$
$$\Delta\beta_0 \equiv \beta_0 - E_h^{(0)}\ . \tag{B22}$$

If $H'_{\rm res}$ can be treated as a perturbation, we obtain from (B1) and (B3)

$$\frac{\partial}{\partial R_0}\,|h(R)\rangle = i[-\Delta\beta_0 + H'_{\rm res}(R_0)]\,|h(R)\rangle\ , \tag{B23}$$

$$\frac{\partial a_n(R)}{\partial R_0} = i[H'_{\rm res}(R_0),a_n(R)]\ , \tag{B24}$$

from which in the first order of $H'_{\rm res}$

$$|h(R)\rangle \simeq e^{-i(R_0-R_0')\Delta\beta_0}\left[1+i\int_{R_0'}^{R_0} dx_0 H'_{\rm res}(x_0)\right]|h(\mathbf{R},R_0')\rangle \ , \tag{B25}$$

$$a_n(R)\simeq a_n(\mathbf{R},R_0')+i\left[\int_{R_0'}^{R_0} dx_0 H'_{\rm res}(x_0),a_n(\mathbf{R},R_0')\right] . \tag{B26}$$

Let us define $|h(R)\rangle_1$ and $a_n(R)_1$ including the first-order correction with respect to $H'_{\rm res}$ as

$$|h(R)\rangle_1 \equiv e^{-iR_0\Delta\beta_0}\left[1+i\int_0^{R_0} dx_0 H'_{\rm res}(x_0)\right]|h(\mathbf{R})\rangle \ , \tag{B27}$$

$$a_n(R)_1 \equiv a_n(\mathbf{R})+i\left[\int_0^{R_0} dx_0 H'_{\rm res}(x_0),a_n(\mathbf{R})\right] . \tag{B28}$$

From (B25) and (A16), we find

$$\begin{aligned} D(\epsilon_0)|h(R)\rangle_1 &= e^{-iR_0\Delta\beta_0}\left[1+\int_{\epsilon_0}^{R_0+\epsilon_0} dx_0 H'_{\rm res}(x_0)\right]e^{i\beta_0\epsilon_0}|h(\mathbf{R},\epsilon_0)\rangle \\ &= e^{i\epsilon_0\beta_0}e^{-i(R_0+\epsilon_0)\Delta\beta_0}\left[1+i\int_0^{R_0+\epsilon_0} dx_0 H'_{\rm res}(x_0)\right]|h(\mathbf{R})\rangle \\ &= e^{i\epsilon_0\beta_0}|h(\mathbf{R},R_0+\epsilon_0)\rangle_1 \ . \end{aligned} \tag{B29}$$

Similarly, from (B26)

$$D(\epsilon_0)a_n(\mathbf{R})D(\epsilon_0)^{-1}=a_n(\mathbf{R},\epsilon_0)e^{-iE_n\epsilon_0}\simeq e^{-iE_n\epsilon_0}\left[a_n(\mathbf{R})+i\left[\int_0^{\epsilon_0} dx_0 H'_{\rm res}(x_0),a_n(\mathbf{R})\right]\right] . \tag{B30}$$

Therefore, to the first order of $H'_{\rm res}$

$$D(\epsilon_0)a_n(R)_1D(\epsilon_0)^{-1}=e^{-iE_n\epsilon_0}\left[a_n(\mathbf{R})+i\left[\int_0^{R_0+\epsilon_0} dx_0 H'_{\rm res}(x_0),a_n(\mathbf{R})\right]\right]=e^{-iE_n\epsilon_0}a_n(\mathbf{R},R_0+\epsilon_0)_1 \ . \tag{B31}$$

We see that, when we construct the state vector

$$|H_A\rangle_1 \equiv \mathcal{N}_A\int d\mathbf{R}\int dR_0 H_A(a(R)_1^\dagger,b(R)_1^\dagger)|h(R)\rangle_1 \ , \tag{B32}$$

the eigenvalue of P_0 for $|H_A\rangle_1$ is nearly equal to (B22).

At the present stage of considerations, we do not know any details of $H_{\rm res}=H'+H'_{\rm res}$; therefore, we cannot examine the second-order correction, and for the moment we cannot but treat $E_h^{(0)}$ and $\beta_0=E_h$ as parameters.

APPENDIX C: RELATION BETWEEN LORENTZ BOOST AND THREE-MOMENTUM PROJECTION

In this appendix, we summarize formulas representing a connection between the state $|H_A;\mathbf{p}\rangle$ obtained from $|H_A;\mathbf{p}=\mathbf{0}\rangle$ by a Lorentz boost and the one constructed from the "bag" state $|H_A;\mathbf{R}\rangle$ through the three-momentum projection. Details have been given in other papers,[12] and the following is only the essence.

The Lorentz boost considered here is

$$\begin{aligned}(x_\alpha)&\to(x'_\alpha)\\ &=(L(\mathbf{v})_\alpha{}^\beta x_\beta)\\ &=\left[x_\perp,\frac{1}{(1-v^2)^{1/2}}(x_\parallel+vx_0)\ ,\right.\\ &\qquad\left.\frac{1}{(1-v^2)^{1/2}}(vx_\parallel+x_0)\right] ,\end{aligned} \tag{C1}$$

where $\perp$ and $\parallel$ mean the components of $\mathbf{x}$ perpendicular and parallel to $\mathbf{v}$, respectively. Then, the total four-momentum operators satisfy under the Lorentz boost $U(\mathbf{v})$

$$\begin{aligned}(U(\mathbf{v})P^\rho U(\mathbf{v})^{-1})&=(P^\lambda L(\mathbf{v})_\lambda{}^\rho)\\ &=\left[P_\perp,\frac{1}{(1-v^2)^{1/2}}(P_\parallel-vP_0)\ ,\right.\\ &\qquad\left.\frac{-1}{(1-v^2)^{1/2}}(-vP_\parallel+P_0)\right] .\end{aligned} \tag{C2}$$

It is a matter of course that, when a one-hadron state with momentum and energy eigenvalues ($\mathbf{p}=\mathbf{0}$, $p_0=m_A$), $|H_A;\mathbf{p}=\mathbf{0}\rangle$, is given, the state obtained by the Lorentz boost $U(\mathbf{v}_p)$

$$U(\mathbf{v}_p)|H_A;\mathbf{p}=\mathbf{0}\rangle \equiv |H_A;\mathbf{p}\rangle \tag{C3}$$

is the eigenstate of the total four-momentum with the eigenvalues

$$(\mathbf{p},E(p))=\left[\frac{m_A\mathbf{v}_p}{(1-v_p{}^2)^{1/2}},\frac{m_A}{(1-v_p{}^2)^{1/2}}\right] . \tag{C4}$$

Then, we can prove the following proposition.

Proposition. The following two conditions are assumed to hold.

(I) The hadron state $|H_A;\mathbf{p}=\mathbf{0}\rangle$ is given as a MF state obtained by superposing "bag" states:

$$|H_A;\mathbf{p}=\mathbf{0}\rangle=N_A(0)\int d\mathbf{R}\,|H_A;\mathbf{R}\rangle \ . \tag{C5}$$

(II)

$$P_0|H_A;\mathbf{R}=\mathbf{0}\rangle=m_A|H_a;\mathbf{R}=\mathbf{0}\rangle \ . \tag{C6}$$

Then, we can prove

$$| H_A;\mathbf{p}\rangle = U(\mathbf{v}_p)\,| H_A;\mathbf{p}=0\rangle = \frac{N_A(0)}{(1-v_p{}^2)^{1/2}}\int d\mathbf{R}\, e^{i(\mathbf{p}-\mathbf{P})\mathbf{R}}U(\mathbf{v}_p)\,| H_A;\mathbf{R}=0\rangle \,. \tag{C7}$$

The proof is straightforward as shown in Ref. 12. Here we add only a remark as follows. When the state vector (3.17) is written as

$$| H_A\rangle = \mathcal{N}_A \int d\mathbf{R}\,| H_A;\mathbf{R}\rangle\rangle \tag{C8}$$

with

$$| H_A;\mathbf{R}\rangle\rangle \equiv \int dR_0 e^{-i\alpha_A R_0}\,| H_A;R\rangle \,,$$

we obtain

$$D(\epsilon_0)\,| H_A;\mathbf{R}\rangle\rangle = \left\{\exp\left[\left[i\sum_{i\in H_A} E_{ni}+E_h+\alpha_A\right]\epsilon_0\right]\right\}| H_A;\mathbf{R}\rangle\rangle \,, \tag{C9}$$

which shows $| H_A;\mathbf{R}\rangle\rangle$ satisfies the condition (C6) if $m_A = \sum_{i\in H_A} E_{ni} + E_h + \alpha_A$ [see (3.25)].

APPENDIX D: PROPERTIES OF MATRIX $F(\mathbf{R},\mathbf{v})$, (3.38c)

The concrete form of the matrix $F(\mathbf{R},\mathbf{v})$ is written as

$$U(\mathbf{v})\alpha(\mathbf{R},x_0)U(\mathbf{v})^{-1} \doteq F(\mathbf{R},\mathbf{v})_{x_0x_0'}\,\alpha(\mathbf{R},x_0'),(3.38a) \,, \tag{D1}$$

$$F(\mathbf{R},\mathbf{v})_{x_0x_0':nm} \equiv \int dx_\perp U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}U_m(\mathbf{x}'-\mathbf{R}) \,, \tag{D2a}$$

$$F(\mathbf{R},\mathbf{v})_{x_0x_0':n\hat{m}} \equiv \int dx_\perp U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}V_{\hat{m}}(\mathbf{x}'-\mathbf{R}) \,, \tag{D2b}$$

$$F(\mathbf{R},\mathbf{v})_{x_0x_0':\hat{n}m} \equiv \int dx_\perp V_{\hat{n}}(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}U_m(\mathbf{x}'-\mathbf{R}) \,, \tag{D2c}$$

$$F(\mathbf{R},\mathbf{v})_{x_0,x_0':\hat{n}\hat{m}} \equiv \int dx_\perp V_{\hat{n}}(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v})^{-1}V_{\hat{m}}(\mathbf{x}'-\mathbf{R}) \,. \tag{D2d}$$

It is to be remembered that the sum over x_0' on the RHS of (D1) means the integral over $x_{||}$.

As noted in the last paragraph of Sec. III, we examine how the matrix $F(\mathbf{R},\mathbf{v})$ satisfies the desired properties.

(i) First $U(\mathbf{v})$ should satisfy

$$U(\mathbf{v}(w'))U(\mathbf{v}(w)) = U(\mathbf{v}(w+w')) \,, \tag{D3}$$

where w and w' mean the angles of the Lorentz rotations $x_\rho' = L(\mathbf{v})_\rho{}^\lambda x_\lambda$ and $x_\rho'' = L(\mathbf{v}')_\rho{}^\lambda x_\lambda'$, respectively; here, $\mathbf{v}$ and $\mathbf{v}'\equiv\mathbf{v}(w')$ are taken to be parallel to each other, for simplicity. In terms of F's Eq. (D3) is expressed as

$$F(\mathbf{R},\mathbf{v}(w))_{x_0x_0'}F(\mathbf{R},\mathbf{v}(w'))_{x_0',x_0''} = F(\mathbf{R},\mathbf{v}(w+w'))_{x_0x_0''} \,. \tag{D4}$$

It is proved directly that (D4) holds for $F(\mathbf{R},\mathbf{v})$ given by (D2a)–(D2d)

Proof.

$$[F(\mathbf{R},\mathbf{v}(w))_{x_0x_0'}F(\mathbf{R},\mathbf{v}(w'))_{x_0'x_0''}\alpha(\mathbf{R},\mathbf{x}_0'')]_n = \int d\mathbf{x}\, U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v}(w))^{-1}\int d\mathbf{y}'\left[\sum_j U_j(\mathbf{x}'-\mathbf{R})U_j(\mathbf{y}'-\mathbf{R})^\dagger + \sum_{\hat{j}} V_{\hat{j}}(\mathbf{x}'-\mathbf{R})V_{\hat{j}}(\mathbf{y}'-\mathbf{R})^\dagger\right]$$
$$\times\Lambda(\mathbf{v}(w'))^{-1}\left[\sum_k U_k(\mathbf{y}''-\mathbf{R})a_k(\mathbf{R})e^{-iE_kx_0''} + \sum_k V_{\hat{k}}(\mathbf{y}''-\mathbf{R})b_{\hat{k}}^\dagger(\mathbf{R})e^{iE_{\hat{k}}x_0''}\right], \tag{D5a}$$

where

$$\begin{bmatrix} x_0' \\ x_{||}' \end{bmatrix} = \begin{bmatrix} \cosh w & \sinh w \\ \sinh w & \cosh w \end{bmatrix}\begin{bmatrix} x_0 \\ x_{||} \end{bmatrix},\quad x_\perp' = x_\perp \,, \tag{D5b}$$

$$\begin{bmatrix} x_0'' \\ y_{||}'' \end{bmatrix} = \begin{bmatrix} \cosh w' & \sinh w' \\ \sinh w' & \cosh w' \end{bmatrix}\begin{bmatrix} x_0' \\ y_{||}' \end{bmatrix},\quad y_\perp'' = y_\perp' \,. \tag{D5c}$$

Because of the closure property of $\{U_n(\mathbf{x}),\ V_{\hat{m}}(\mathbf{x})\}$ derived from the canonical commutation relation for $\psi(x)$'s under assumption I, (D5a) is rewritten as

$$(D5a) = \int d\mathbf{x}\, U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda(\mathbf{v}(w))^{-1}\Lambda(\mathbf{v}(w'))^{-1}\left[\sum_k U_k(\mathbf{x}''-\mathbf{R})a_k(\mathbf{R})e^{-iE_kx_0''} + \sum_{\hat{k}} V_{\hat{k}}(\mathbf{x}''-\mathbf{R})b_{\hat{k}}^\dagger(\mathbf{R})e^{iE_{\hat{k}}x_0''}\right] \tag{D6}$$

with $x_\rho''=L(\mathbf{v}(w+w'))_\rho{}^\lambda x_\lambda$, which reduces to

$$(\mathrm{D6})=[F(\mathbf{R},\mathbf{v}(w+w'))_{x_0x_0''}\alpha(\mathbf{R},x_0'')]_n\ . \tag{D7}$$

(ii) Next the commutation relation

$$\{\alpha(\mathbf{R},x_0),\alpha(\mathbf{R},x_0)^\dagger\}=I \tag{D8}$$

does not change under the transformation (3.38a); then we have proven

$$I=U(\mathbf{v})\{\alpha(\mathbf{R},x_0),\alpha(\mathbf{R},x_0)^\dagger\}U(\mathbf{v})^{-1}\doteq F(\mathbf{R},\mathbf{v})_{x_0x_0'}\{\alpha(\mathbf{R},x_0'),\alpha(\mathbf{R},y_0')^\dagger\}F^\dagger(\mathbf{R},\mathbf{v})_{y_0'y_0}\,|_{x_0=y_0}\ , \tag{D9}$$

where

$$F^\dagger(\mathbf{R},\mathbf{v})_{x_0'x_0:nm}\equiv[F(\mathbf{R},\mathbf{v})_{x_0,x_0':nm}]^*,\ \text{ etc.} \tag{D10}$$

This can also be derived directly as follows

$$\begin{aligned}\delta(\mathbf{x}-\mathbf{y})&=U(\mathbf{v})\{\psi(x),\psi(y)^\dagger\}U(\mathbf{v})^{-1}\,|_{x_0=y_0}=\Lambda^{-1}(\mathbf{v})\{\psi(x'),\psi(y')^\dagger\}\Lambda^{-1}(\mathbf{v})^\dagger\\&\doteq\Lambda^{-1}(\mathbf{v})\{\psi^{(\mathbf{R})}(x'),\psi^{(\mathbf{R})}(y')^\dagger\}\Lambda^{-1}(\mathbf{v})^\dagger\end{aligned} \tag{D11}$$

with

$$x_\rho'=L(\mathbf{v})_\rho{}^\mu x_\mu \ \text{ and } y_\rho'=L(\mathbf{v})_\rho{}^\mu y_\mu\ .$$

Multiplying $U_n(\mathbf{x}-\mathbf{R})^\dagger$ and $U_m(\mathbf{y}-\mathbf{R})$ from the left and right, respectively, and integrating $\mathbf{x}$ and $\mathbf{y}$, we obtain

$$\begin{aligned}\delta_{nm}&\doteq\int d\mathbf{x}\,U_n(\mathbf{x}-\mathbf{R})^\dagger\Lambda^{-1}(\mathbf{v})\\&\quad\times\int d\mathbf{y}\left[\sum_{k,j}U_k(\mathbf{x}'-\mathbf{R})\{a_k(R)e^{-iE_kx_0'},a_j(R)^\dagger e^{+iE_jy_0'}\}U_j(\mathbf{y}'-\mathbf{R})^\dagger\right.\\&\qquad\left.+\sum_{\hat{k},\hat{j}}V_{\hat{k}}(\mathbf{x}'-\mathbf{R})\{b_{\hat{k}}(\mathbf{R})e^{iE_{\hat{k}}x_0'},b_{\hat{j}}(\mathbf{R})^\dagger e^{-iE_{\hat{j}}y_0'}\}V_{\hat{j}}(\mathbf{y}'-\mathbf{R})^\dagger\right]\\&\quad\times\Lambda^{-1}(\mathbf{v})^\dagger U_m(\mathbf{y}-\mathbf{R})\,|_{x_0=y_0}\\&=[\text{RHS of (D9)}]_{nm}\ .\end{aligned} \tag{D12}$$

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$$\gamma_j^\dagger=\gamma_j \quad (j=1,2,3), \quad \gamma_0^\dagger=-\gamma_0 ,$$

$$\boldsymbol{\gamma}=i\boldsymbol{\alpha}\beta, \quad \gamma_4=\beta=i\gamma_0 ,$$

$$\gamma_5=-\gamma_1\gamma_2\gamma_3\gamma_4, \quad x_\rho p^\rho=\mathbf{x}\cdot\mathbf{p}-x_0p_0 .$$

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