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Quasipoint vertices

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A class of approximate photon-particle-particle vertex functions including higher-order corrections for particles with spin ≤ 1 is shown to saturate the relevant Ward-Takahashi identities and to possess the Poincaré substructure of the point vertex in certain gauges that is reflective of gauge-covariant photon attachments. This leads to an approximate, higher-spin generalization of the exact scalar-bubble theorem for the persistence of radiation amplitude zeros in the presence of self-energy graphs. This also leads to further insight into the structural features of a spectral-weight ansatz frequently employed in implementing the gauge technique for solving the Schwinger-Dyson equations. The Poincaré substructure of the vertex for the axial-vector coupling of a photon to a spin-$\frac{1}{2}$ particle is determined and some possible applications of this result are discussed.

I. INTRODUCTION

Gauge-covariant[1] point vertices for the attachment of a massless, on-shell gauge boson onto a particle line, whether external or internal to a general Feynman graph, correspond in a definitive way to infinitesimal Poincaré transformations.[2–5] This distinctive, hitherto unnoticed, and not fully understood attribute underlies the radiation theorem[2,3] for S matrices describing gauge-boson emission or absorption in the tree approximation and possibly other, perhaps important, features of gauge theories. When the tree-graph contributions are dominant, the radiation theorem has striking physical consequences that offer the opportunity for the direct testing of the gauge nature of the coupling.

In this paper we investigate the degree to which this Poincaré correspondence is manifested in the full vertex, thus exhibiting a structure more detailed than that following from the usual invariant-amplitude analysis and reflective of the gauge symmetry. We apply our results to obtain higher-order extensions of the radiation theorem along with the elucidation and amplification of some aspects of a type of nonperturbative approximation method in field theory.

The exact vertex includes attachments onto particle lines within closed loops, and while, of course, the point-vertex Poincaré structure is operative for each of these, their dependence upon the loop momenta leads to a very complicated overall effect. Nonetheless, the scalar-bubble theorem[3,6,7] for attachments onto scalars suggests that an identifiable portion of the exact vertex possesses precisely the same simple features of the point vertex, and we will see that, to a certain degree of approximation, and within at least one choice of gauge, this remains true for the Dirac and vector cases as well.

Further analysis of the scalar case reveals that what we refer to as the *quasipoint* piece of the all-orders photon-scalar-scalar vertex is just the Lehmann-spectral-weighted point vertex. This establishes a connection with a well-known prescription[8] for the generation of gauge-invariant approximations to the Schwinger-Dyson equations and suggests possible candidates for quasipoint vertices in the Dirac and vector cases when the naive Ward-Takahashi (WT) identities are still valid. When this is the case, in all three instances (spin ≤ 1), the quasipoint vertices can be identified with the same overall infinitesimal Poincaré transformation as occurs in the point-vertex case. The quasipoint vertices, which reduce without ambiguity to point vertices when higher-order contributions are ignored, include the entire longitudinal parts of the full vertices, which correspond to translations, but only approximate the transverse parts. Moreover, the radiation theorem continues to hold when attachments onto the dressed particle lines are mediated by quasipoint vertices. The structural analysis of the quasipoint vertex thus provides additional motivation for and further insight into an ansatz[8] that has been employed extensively in implementing the so-called gauge technique,[9–11] which is a nonperturbative method for "solving" the WT identity.[12–30]

We confine ourselves to vertices that represent the gauge-covariant attachment of a photon onto a charged-particle line with $m \neq 0$ and spin ≤ 1 including the interesting case of the axial-vector coupling to a Dirac particle. Scalar and spinor quantum electrodynamics (QED) provide unambiguous models for the spin < 1 cases. In a broader context, e.g., when the scalars and fermions are embedded in a more comprehensive gauge model that includes an exact $U(1)_{em}$ gauge symmetry and in dealing with the coupling of photons onto charged vectors, the choice of gauge within the encompassing non-Abelian gauge theory is crucial.[31] The principal reason for this is that the relevant Slavnov-Taylor (ST) identities generally have complicated forms that do not manifest the $U(1)_{em}$ symmetry in the form of the usual ("naive") QED WT identities except in very special gauges. One of these is the covariant unitary gauge that is employed in Refs. 2–5 and 7 and in the present work.

In certain noncovariant gauges, such as the axial gauge, the Slavnov-Taylor identities take on naive WT forms,[32] but then the Poincaré substructure realized in the unitary gauge is subdued. This suppression also occurs both in the linear and nonlinear covariant R_ξ gauges, which include the unitary gauge as a limiting case, even though nonlinear R_ξ gauge choices exist which manifest the $U(1)_{em}$ symmetry in naive Ward identities.[33] Our restriction to the unitary gauge and only to three-vector vertices involving a photon attached to a charged, $m \neq 0$, spin-unity particle limits what we can say concerning the gauge technique in this case and distinguishes our vertex-function structural analysis from that of Baker, and others,[17–20] e.g., for the three-gluon vertex.[34] Our immediate goal in the three-vector case is the identification of an approximate photon vertex that leads to a generalization of the radiation theorem for S matrices with at least one external photon line. In the unitary gauge higher-order Green function corrections may be divergent, even though such contributions should cancel out in the S matrix, so that renormalization problems are likely to accompany any comprehensive attempt to implement the gauge technique in this gauge.

Generally, a quasipoint vertex may be regarded to be a gauge-boson vertex that possesses what one chooses to identify as the essential structural characteristics of the point vertex. Of course there is ambiguity in such an identification arising partly from the freedom of the choice of gauge. Our identifications, motivated mainly by the results of Refs. 2–5, stress the Poincaré substructure of the point-vertex function in a particular gauge that is representative of a highly restricted ("minimal") class[1] of gauge-invariant interactions and is thus reflective of renormalizability in addition to gauge symmetry. It is possible that these two attributes possess discernible signatures in general gauges that would facilitate the extension of the quasipoint concept to these circumstances.

II. DECOMPOSITION IDENTITIES

A Green function generally does not decompose into Green functions of lower order. Of course, Green functions containing conserved currents satisfy Ward-Takahasi identities which express their contractions in terms of Green functions with one less external leg and are special cases of what we refer to as *decomposition identities.*

In Refs. 2 and 3 so-called *radiation* decomposition identities for the lowest-order, photon-particle-particle three-point function, with the single-photon propagator removed, are introduced. These identities express the contraction of the vertex with the polarization vector of the on-shell photon in terms of a sum of two single-particle propagators that are each multiplied by mass-independent kinematical factors. We first determine uncontracted forms of these identities that apply even to off-mass-shell photons.[35] The simple WT and radiation decomposition identities then emerge by taking special contractions. In Sec. III we obtain higher-order generalizations of the decomposition identities in the course of identifying quasipoint vertices.

We consider a photon of outgoing momentum q attached to a charged-particle[36] line with momenta p and $p'=p-q$ flowing into and out of the vertex, respectively. Generally, $p^2 \neq m^2$, $p'^2 \neq m^2$, where $m \neq 0$ is the particle mass and $q^2 \neq 0$. For a scalar particle the decomposition identity for the lowest-order three-point function with the photon propagator removed is just the simple resolvent relationship

$$\frac{1}{p'^2-m^2}(p'+p)^\mu \frac{1}{p^2-m^2} = \frac{1}{p'^2-m^2} f_s^\mu - f_s^\mu \frac{1}{p^2-m^2} , \tag{2.1}$$

which in this instance is just partial fractionization, where the scalar radiation factor, f_s^μ, is given by

$$\begin{aligned} f_s^\mu &= \left[p' + \frac{q}{2} \right]^\mu \left[\left[p' + \frac{q}{2} \right] \cdot q \right]^{-1} \\ &= \left[p - \frac{q}{2} \right]^\mu \left[\left[p - \frac{q}{2} \right] \cdot q \right]^{-1} . \end{aligned} \tag{2.2}$$

We note that

$$p' + \tfrac{1}{2} q = p - \tfrac{1}{2} q . \tag{2.3}$$

Equations (2.2) imply that the right-hand side of (2.1) makes reference to the same infinitesimal translations both entering and leaving the vertex. For example, if (2.1) is contracted with a polarization vector ϵ^μ corresponding to an on-shell photon with $\epsilon \cdot q = 0$ and $q^2 = 0$, we recover the radiation decomposition identity,[36]

$$\frac{1}{p'^2-m^2}(p'+p)\cdot\epsilon \frac{1}{p^2-m^2} = \frac{1}{p'^2-m^2} \left[\frac{p'\cdot\epsilon}{p'\cdot q} \right] - \left[\frac{p\cdot\epsilon}{p\cdot q} \right] \frac{1}{p^2-m^2} , \tag{2.4}$$

where the parameter $(p'\cdot\epsilon/p'\cdot q) = (p\cdot\epsilon/p\cdot q)$ characterizes the infinitesimal translation associated with the photon attachment.[2,3] The contraction of (2.1) with q_μ (q^2 arbitrary) yields the WT identity in lowest order:

$$\frac{1}{p'^2-m^2}(p'+p)\cdot q \frac{1}{p^2-m^2} = \frac{1}{p'^2-m^2} - \frac{1}{p^2-m^2} . \tag{2.5}$$

The distinction between (2.4) and (2.5) is evident even in this simple case.

Matters become more interesting for particles with spin. The counterpart of (2.1) for spin $\frac{1}{2}$ is most easily derived[37] by use of the generalized[38] Gordon identity

$$(p+p')_\mu + C_\mu = \gamma_\mu \not{p} + \not{p}' \gamma_\mu , \tag{2.6}$$

where

$$C_\mu = \tfrac{1}{2}[\gamma_\mu, \not{p} - \not{p}'] . \tag{2.7}$$

One finds from (2.6) that

$$\gamma_\mu(p^2 - p'^2) = (p+p')_\mu(\not{p} - \not{p}') + C_\mu \not{p} - \not{p}' C_\mu . \tag{2.8}$$

We then obtain from (2.8) the spinor decomposition identity

$$\frac{1}{\not{p}'-m}\gamma^\mu \frac{1}{\not{p}-m} = \frac{1}{\not{p}'-m} f_D^\mu - f_D^\mu \frac{1}{\not{p}-m} , \tag{2.9}$$

where the Dirac radiation factor

$$f_D^{\mu}=[(p-\tfrac{1}{2}q)^{\mu}+\tfrac{1}{2}C^{\mu}][(p'+q/2)\cdot q]^{-1}$$
$$=[(p-\tfrac{1}{2}q)\cdot q]^{-1}[(p'+\tfrac{1}{2}q)^{\mu}+\tfrac{1}{2}C^{\mu}] \tag{2.10}$$

now represents an infinitesimal Lorentz transformation as well as a translation when contracted with a four-vector. The full Poincaré transformation corresponding to these infinitesimal generators is determined in Ref. 3 and applied to an external plane-wave extension of the radiation theorem in Ref. 4. The radiation decomposition and low-order WT identities are obtained from (2.9) and (2.10) just as in the scalar case.

Axial-vector currents are relevant where chiral symmetry is a consideration. In spite of this we have included a (bare) mass term in the spinor propagator because our results have applicability in nonchiral symmetric situations and because a mass term is appropriate for the excitations appearing in the Lehmann sum. The axial-vector form of (2.9) is easily deduced from that expression and one finds

$$\frac{1}{\not p'-m}\gamma^{\mu}\gamma_5\frac{1}{\not p-m}=\frac{1}{\not p'-m}f_A^{\mu}-f_A^{\mu}\frac{1}{\not p-m}\ , \tag{2.11}$$

where $\hat{q}=p+p'$ and the modified radiation factor

$$f_A^{\mu}=-\left[\frac{q^{\mu}}{2}+\tfrac{1}{4}[\gamma^{\mu},\hat{\not q}]\right]\left[\frac{\hat{q}\cdot q}{2}\right]^{-1}\gamma_5\ , \tag{2.12}$$

implies a chirality-dependent Poincaré structure with kinematics markedly different from that contained in (2.10). The low-order axial-vector WT identity[39,40] is obtained from (2.11) by contracting it with $\hat{q}$, which is the natural momentum combination in the axial-vector case.[41] The (formal) radiation decomposition identity involves the contracted radiation factor

$$f_A\cdot\epsilon=-\tfrac{1}{4}[\not\epsilon,\hat{\not q}](p\cdot q)^{-1}\gamma_5 \tag{2.13}$$

with no translational term.

Even when dealing with the conventional photon-spinor coupling, (2.11) can be useful in decomposing contributions from successive multiple-photon attachments onto a single line,

$$S(p_1)\gamma^{\mu}S(p_2)\gamma^{\nu}\cdots\gamma^{\lambda}S(p_N)\ ,$$

into a sum of single Dirac propagators, $S(p_i)$. For example, after using (2.9) on $S(p_1)\gamma^{\mu}S(p_2)$, one next encounters terms such as $S(p_1)\gamma^{\mu}\gamma^{\alpha}\gamma^{\beta}S(p_3)$. Since $\gamma^{\mu}\gamma^{\alpha}\gamma^{\beta}$ can be expressed as a linear sum of γ^{λ} and $\gamma^{\rho}\gamma_5$ terms the role of (2.11) in continuing the decomposition is evident.

The point coupling of the photon to a massive vector particle can be characterized in terms of the vertex[42]

$$I_{\gamma\delta}=P_{\gamma\beta}(p',m^2)Y^{\beta\sigma\alpha}(p',q,-p)k_{\sigma}P_{\alpha\delta}(p,m^2)\ , \tag{2.14}$$

where corresponding to our choice of unitary gauge

$$P_{\mu\nu}(p,m^2)=-g_{\mu\nu}+\frac{p_{\mu}p_{\nu}}{m^2}\ , \tag{2.15}$$

$$Y^{\beta\sigma\alpha}(p',q,-p)k_{\sigma}=-[k^{\alpha}(p+q)^{\beta}+k^{\beta}(p'-q)^{\alpha}$$
$$-g^{\alpha\beta}k\cdot(p'+p)]\ , \tag{2.16}$$

and we have contracted the elementary triple-vector vertex with an arbitrary four-vector k_{σ} for the sake of simplicity of presentation. Typically k_{σ} is taken to be q_{σ} to obtain the ("naive") WT identity, or ϵ_{σ} to find the emission amplitude or to determine the radiation decomposition identity, where the covariant, unitary-gauge, free-vector propagator is understood to be[36] $P_{\mu\nu}(p,m^2)(p^2-m^2)^{-1}$.

The vector decomposition identity is found from the following alternative expressions for $I_{\gamma\delta}$:

$$I_{\gamma\delta}=-2P_{\gamma\beta}(p',m^2)\left[t^{\beta}{}_{\delta}+\frac{1}{2m^2}\omega^{\beta\lambda}q_{\lambda}p_{\delta}\right]$$
$$+\frac{(p'^2-m^2)}{m^2}(p'_{\gamma}k_{\delta}+k_{\gamma}p_{\delta})\ , \tag{2.17a}$$

$$I_{\gamma\delta}=2\left[-t_{\gamma}{}^{\alpha}+\frac{1}{2m^2}p'_{\gamma}q_{\lambda}\omega^{\lambda\alpha}\right]P_{\alpha\delta}(p,m^2)$$
$$+\frac{(p^2-m^2)}{m^2}(p'_{\gamma}k_{\delta}+k_{\gamma}p_{\delta})\ , \tag{2.17b}$$

where

$$\omega_{\mu\nu}=q_{\mu}k_{\nu}-k_{\mu}q_{\nu} \tag{2.18}$$

and

$$t_{\mu\nu}=\tfrac{1}{2}(p'+p)\cdot k g_{\mu\nu}-\omega_{\mu\nu}\ . \tag{2.19}$$

From (2.17) one can find the vector counterpart of (2.1), (2.9), and (2.11):

$$\frac{1}{p'^2-m^2}(-I_{\gamma\delta})\frac{1}{p^2-m^2}=\frac{P_{\gamma\beta}(p',m^2)}{p'^2-m^2}(f_V^{(+)})^{\beta}{}_{\delta}$$
$$-(f_V^{(-)})_{\gamma}{}^{\alpha}\frac{P_{\alpha\delta}(p,m^2)}{p^2-m^2}\ , \tag{2.20}$$

where the vector radiation factors are

$$(f_V^{(+)})_{\mu\nu}=\left[t_{\mu\nu}+\frac{1}{2m^2}\omega_{\mu\lambda}q^{\lambda}p_{\nu}\right]\left[\left[p'+\frac{q}{2}\right]\cdot q\right]^{-1}\ , \tag{2.21a}$$

$$(f_V^{(-)})_{\mu\nu}=\left[\left[p-\frac{q}{2}\right]\cdot q\right]^{-1}\left[t_{\mu\nu}-\frac{1}{2m^2}p'_{\mu}q^{\lambda}\omega_{\lambda\nu}\right]\ . \tag{2.21b}$$

The tensor $t_{\mu\nu}$ ($\omega_{\mu\nu}$) is the vector realization of the infinitesimal Poincaré (Lorentz) transformation associated with the photon attachment. It is interesting to note that the naive unitary-gauge WT identities that follow from (2.20) by taking $k^{\sigma}=q^{\sigma}$ correspond to pure translations that are independent of the vertex kinematics in that the radiation factors reduce to $g_{\mu\nu}$ in this case.[43] The Lorentz structure resides solely within the transverse part of the vertex as in the spinor case, but unlike the translation—WT-identity relationship, this structure is not obviously correlated with some property of the exact vertex.[43]

We observe that in contrast with the scalar and Dirac cases, the vector radiation factors differ in their inclusion of purely transverse, m^2-dependent terms that destroy the pure Poincaré structure exhibited for attachments to scalar and Dirac particles. These m-dependent terms vanish for crucial cases $k_\sigma = q_\sigma$, with q^2 arbitrary, and $k_\sigma = \epsilon_\sigma$, for $q^2 = q\cdot\epsilon = 0$. Also, even if $q^2 \neq 0$ for $k_\sigma = \epsilon_\sigma$, these terms do not contribute if the vector particle is coupled to a conserved current.[3]

Equations (2.1), (2.9), (2.11), and (2.20) are the point-vertex decomposition identities and, despite a vast literature concerning various aspects of the elementary point vertex functions, the particular organization they represent is, as far as we are aware, new. They highlight the distinction between the WT and radiation decomposition identities for both the point and higher-order vertex functions. In Sec. III we determine vertex functions that include higher-order corrections and that yet still satisfy decomposition identities with the same basic structure.

III. QUASIPOINT VERTICES

A. Scalar vertex

If we multiply (2.1) by the Lehmann weight function $\rho_s(m^2)$ for the scalar field and integrate over m^2, we find that

$$(p'+p)^\mu \int \frac{dm^2 \rho_s(m^2)}{(p'^2-m^2)(p^2-m^2)} = f_s^\mu[\Delta'(p'^2)-\Delta'(p^2)] , \tag{3.1}$$

where Δ' denotes the full scalar propagator.[44] In this context, the gauge-technique ansatz of Delbourgo and West[8,14] consists in the identification of the quantity

$$\Gamma_P^\mu(p',p)_s \equiv (p'+p)^\mu \Delta'(p'^2)^{-1} \times \left[\int \frac{dm^2 \rho_s(m^2)}{(p'^2-m^2)(p^2-m^2)} \right] \Delta'(p^2)^{-1} \tag{3.2}$$

with the exact scalar-photon vertex function $\Gamma^\mu(p',p)_s$. Since $\Gamma_P^\mu(p',p)_s$ obviously satisfies the naive WT identity, the difference

$$\Gamma_T^\mu(p',p)_s \equiv \Gamma^\mu(p',p)_s - \Gamma_P^\mu(p',p)_s , \tag{3.3}$$

must be purely transverse:

$$(p'-p)_\mu \Gamma_T^\mu(p',p)_s = 0 . \tag{3.4}$$

Because $\Gamma^\mu(p',p)_s$ has the invariant amplitude decomposition[42]

$$\Gamma^\mu(p',p)_s = (p'-p)^\mu f(p'^2,p^2) + (p'+p)^\mu g(p'^2,p^2) , \tag{3.5}$$

we see from (3.2)–(3.5) that

$$\Gamma_P^\mu(p',p)_s = (p'+p)^\mu \left[g(p'^2,p^2) - \frac{(p'-p)^2}{p'^2-p^2} f(p'^2,p^2) \right] . \tag{3.6}$$

Thus the approximation $\Gamma^\mu \simeq \Gamma_P^\mu$, e.g., in the gauge technique, implies the neglect of the transverse part,

$$\Gamma_T^\mu(p',p)_s = \left[(p'-p)^\mu - \frac{(p'+p)^\mu (p'-p)^2}{p'^2-p^2} \right] f(p'^2,p^2) , \tag{3.7}$$

of the exact vertex. The WT identity applied to (3.5) implies that the right-hand side of (3.7) has no kinematical singularity at $p'^2 = p^2$.

The attachment of an on-shell photon onto a fully dressed scalar line entails the contraction

$$\epsilon\cdot\Gamma(p',p)_s = (p'+p)\cdot\epsilon g(p'^2,p^2) = \epsilon\cdot\Gamma_P(p',p)_s . \tag{3.8}$$

Given (3.8) and the observation, following from the combination of (3.1) and (3.2), that Γ_P^μ satisfies an all-orders version of (2.1),

$$\Delta'(p'^2)\Gamma_P^\mu(p',p)_s \Delta'(p^2) = \Delta'(p'^2) f_s^\mu - f_s^\mu \Delta'(p^2) , \tag{3.9}$$

the extension of the proof of the radiation theorem to include exact scalar bubbles is immediate.[3,6,7]

The function $\Gamma_P^\mu(p',p)$ is an example of what we refer to as a *quasipoint vertex*. Generally such a vertex is required to have the same structural features possessed by the point vertex with regard to Poincaré transformations and the satisfaction of the appropriate WT (or ST) identity. Moreover, when higher-order corrections are ignored a quasipoint vertex must reduce to the point vertex in a consistent fashion. Finally, although this requirement may be redundant, a quasipoint vertex must not possess any kinematical singularities;[18,28] clearly $\Gamma_P^\mu(p',p)_s$ has none.

B. Spinor vertices

Evidently the Lehmann weighting of the decomposition identities for spin ≤ 1 suggests itself as a means of generating quasipoint vertices. For spin > 0, however, the occurrence of more than one spectral function complicates this procedure.

The Dirac case[8,11,14] is relatively straightforward because the full Dirac propagator, $S'(p)$, admits of an alternative Lehmann representation in terms of a single, scalar weight function $\rho_D(m)$:

$$S'(p) = \left[\int_{-\infty}^{-m_D} + \int_{m_D}^{\infty} \right] dm \frac{\rho_D(m)}{\not{p}-m} , \tag{3.10}$$

where m_D is the Dirac particle mass.[44] From (2.9) we then obtain

$$S'(p')\Gamma_P^\mu(p',p)_D S'(p) = S'(p') f_D^\mu - f_D^\mu S'(p) , \tag{3.11}$$

where

$$S'(p')\Gamma_P^\mu(p',p)_D S'(p) \equiv \int dm\, \rho_D(m) \frac{1}{\not{p}'-m} \gamma^\mu \frac{1}{\not{p}-m} \tag{3.12}$$

defines the Dirac quasipoint vertex denoted by $\Gamma_P^\mu(p',p)_D$.

As in the scalar case the difference between $\Gamma_P^\mu(p',p)_D$ and the exact Dirac vertex $\Gamma^\mu(p',p)_D$ must be purely transverse,

$$\Gamma_T^\mu(p',p)_D \equiv \Gamma^\mu(p',p)_D - \Gamma_P^\mu(p',p)_D \ , \qquad (3.13)$$

$$(p'-p)_\mu \Gamma_T^\mu(p',p)_D = 0 \ , \qquad (3.14)$$

although even for an on-shell photon with polarization ϵ we have

$$\epsilon_\mu \Gamma_T^\mu(p',p) \neq 0 \ , \qquad (3.15)$$

in general. Thus, the extension of the scalar-bubble theorem to the Dirac case that follows from (3.11) is not exact, but necessitates the expression of the photon attachment in terms of $\epsilon_\mu \Gamma_P^\mu(p',p)_D$, which represents an approximation to the exact ϵ-contracted vertex.

Also, in contrast with the scalar case, $\Gamma_P^\mu(p',p)_D$ is not purely longitudinal, as is evident from the expression

$$\Gamma_P^\mu(p',p) = \frac{(p'+p)^\mu}{p^2-p'^2}[S'(p)^{-1} - S'(p')^{-1}] + \frac{1}{p^2-p'^2}[C^\mu S'(p)^{-1} - S'(p')^{-1} C^\mu] \ , \qquad (3.16)$$

where the commutator

$$C^\mu = \tfrac{1}{2}[\gamma^\mu, \not{q}] \ , \qquad (3.17)$$

is obviously transverse. The purely longitudinal term in (3.16), which saturates the WT identity, possesses kinematical singularities at $p^2 - p'^2$, so a transverse piece is required to cancel these.[18] It is useful to rewrite (3.16) in a form that manifests the absence of such singularities even though we are assured that they are not present from the form of the left-hand side of (3.12) and the existence of $(S')^{-1}$.

Let us express the inverse propagator in the form[18]

$$S'(p)^{-1} = F(p^2)\not{p} + G(p^2) \ , \qquad (3.18)$$

where $F(p^2)$ and $G(p^2)$ have no spinor structure. Then one finds that

$$\Gamma_P^\mu(p',p)_D = \Gamma_{BC}^\mu(p',p) + \left[\frac{F(p^2)-F(p'^2)}{2(p^2-p'^2)}\right](C^\mu \not{p} + \not{p}' C^\mu) + \left[\frac{G(p^2)-G(p'^2)}{p^2-p'^2}\right] C^\mu \ , \qquad (3.19)$$

where

$$\Gamma_{BC}^\mu(p',p) = (p+p')^\mu(\not{p}+\not{p}')\left[\frac{F(p^2)-F(p'^2)}{2(p^2-p'^2)}\right] + \gamma^\mu \frac{F(p^2)+F(p'^2)}{2} + (p+p')^\mu \frac{G(p^2)-G(p'^2)}{(p^2-p'^2)} \qquad (3.20)$$

is the gauge-technique ansatz proposed by Ball and Chiu[18] which is obviously free of kinematical singularities. The remaining purely transverse terms[45] on the right-hand side of (3.19) are also free of kinematical singularities. The preceding calculation is important in that it illustrates quite vividly that the requirements of freedom from kinematical singularities and a consistent lowest-order limit do not constrain the transverse parts too strongly.[28]

Unlike the scalar case, there seems to be no obvious association of $\Gamma_P^\mu(p',p)_D$ with the invariant amplitude structure of $\Gamma^\mu(p',p)_D$. The degree to which Γ_P^μ constitutes a well-defined *part* of $\Gamma^\mu(p',p)_D$ would appear to rest upon the legitimacy of a general representation[30]

$$S'(p')\Gamma^\mu(p',p)_D S'(p) = \int dm\, \rho_D(m) g^\mu(p',p;m) \ , \qquad (3.21)$$

where the difference

$$g^\mu(p',p;m) - \frac{1}{\not{p}'-m}\gamma^\mu \frac{1}{\not{p}-m} = O(\alpha) \ , \qquad (3.22)$$

is purely transverse.

Axial-vector vertices can arise in spinor QED in different ways. In ordinary QED with massive vector-coupled charged fermions one refers to the vertex $\Gamma^\mu(p',p)_{5D}$, defined in terms of the axial-vector current $\bar{\psi}\gamma^\mu\gamma_5\psi$, which satisfies a WT identity that involves the pseudoscalar vertex $\Gamma(p',p)_{5D}$, propagator $-\gamma_5$ products, and anomalous terms.[40] Delbourgo and West[8,14] consider a model for the dynamical breaking of chiral symmetry in axial-vector-coupled spinor QED that is constrained to have no anomalies. In this instance only the propagator $-\gamma_5$ products enter into the WT identity for $\Gamma^\mu(p',p)_{5D}$. Quasipoint axial-vector vertices, $\Gamma_P^\mu(p',p)_{5D}$ can be constructed in at least the first of these models as we next demonstrate. We define a quasipoint axial-vector vertex, $\Gamma_P^\mu(p',p)_{5D}$, by

$$S'(p')\Gamma_P^\mu(p',p)_{5D} S'(p) \equiv \int dm\, \rho_D(m) \frac{1}{\not{p}'-m}\gamma^\mu\gamma_5 \frac{1}{\not{p}-m} \ . \qquad (3.23)$$

Then from (2.11) we see that

$$S'(p')\Gamma_P^\mu(p',p)_{5D} S'(p) = S'(p') f_A^\mu - f_A^\mu S'(p) = S'(p')\Gamma_P^\mu(p',p)_D\, \gamma_5 S(p) \ , \qquad (3.24)$$

from which we infer that $\Gamma_P^\mu(p',p)_{5D}$ has no kinematical singularities. It is easy to check that $\Gamma_P^\mu(p',p)_{5D}$ satisfies the naive WT identity (without an anomalous term) provided that

$$S'(p')\Gamma_P(p',p)_{5D} S'(p) \equiv \int dm\, \rho_D(m) \frac{1}{\not{p}'-m}(m/m_0)\gamma_5 \frac{1}{\not{p}-m} \qquad (3.25)$$

is also identified as the associated quasipoint pseudoscalar vertex, where m_0 is the bare fermion mass.

In Refs. 8 and 14 the gauge technique for axial-vector QED is affected by the identification ($q=p-p'$)

$$S'(p')\Gamma^{\mu}_{P(A)}(p',p)_{5D}S(p) \equiv \int dm\,\rho_A(m)\frac{1}{\not p'-m}\left[\gamma_\mu\gamma_5+\frac{2q_\mu}{q^2}m\gamma_5\right]\frac{1}{\not p-m}\ , \tag{3.26}$$

where the pole at $q^2=0$ is related to the dynamical breakdown of chiral symmetry and $\rho_A(m)$ is the fermion spectral function in this model. The ansatz (3.26) subtracts off from (3.23) the contribution, in which the m_0 factor is canceled out, that (3.25) makes to the WT for $\Gamma^{\mu}_{P}(p',p)_{5D}$, thus assuring that $\Gamma^{\mu}_{P(A)}(p',p)_{5D}$ is appropriate for massless bare fermions. This explicates the basic differences between the two models and also shows that the quasipoint concept is more ambiguous for $\Gamma^{\mu}_{P(A)}(p',p)_{5D}$ than for $\Gamma^{\mu}_{P}(p',p)_{5D}$.

C. Vector vertices

The covariant part of the vector propagator, $\Delta'_{\mu\nu}(p)$, has a unitary-gauge spectral representation

$$\Delta'_{\mu\nu}(p)=\int\frac{dm^2}{p^2-m^2}[P_{\mu\nu}(p,m^2)\rho_1(m^2)+g_{\mu\nu}\rho_2(m^2)]\ , \tag{3.27}$$

which involves two spectral functions. This circumstance, along with the appearance of the transverse m^2-dependent terms in (2.20), complicates matters considerably as compared to the scalar and spinor cases.[46,47]

Our principal guideline in constructing a vector quasipoint vertex is that it reflect the Poincaré structure of the point vertex as closely as possible. This would seem to suggest that $g_{\mu\nu}\rho_2(m^2)$ enter into the spectral weighting in conjunction with the same radiation factors as does $P_{\mu\nu}(p,m^2)\rho_1(m^2)$. Accordingly, let us tentatively define a quasipoint vector vertex, $\widetilde{\Gamma}_P(p',p)_V$, by

$$\Delta'_{\mu\alpha}(p')\widetilde{\Gamma}^{\alpha\beta}_{P}(p',p)_V\Delta'_{\beta\nu}(p)\equiv\int dm^2\rho_1(m^2)\left[\frac{1}{p'^2-m^2}(-I_{\mu\nu})\frac{1}{p^2-m^2}\right]+\int dm^2\rho_2(m^2)\left[\frac{g_{\mu\alpha}}{p'^2-m^2}(f_V^{(+)})^{\alpha}{}_{\nu}-(f_V^{(-)})_{\mu}{}^{\alpha}\frac{g_{\alpha\nu}}{p^2-m^2}\right]. \tag{3.28}$$

Then we see from (2.20) and (2.21) that

$$\Delta'_{\mu\alpha}(p')\widetilde{\Gamma}^{\alpha\beta}_{P}(p',p)_V\Delta'_{\beta\nu}(p)=\Delta'_{\mu\alpha}(p')f^{\alpha}{}_{\nu}-f_{\mu}{}^{\alpha}\Delta'_{\alpha\nu}(p)+\left[\frac{1}{p^2-p'^2}\right][\widetilde{\delta}_{\mu\beta}(p')p_\nu-p'_\mu\widetilde{\delta}_{\alpha\nu}(p)]\omega^{\alpha\lambda}q_\lambda\ , \tag{3.29}$$

where

$$\widetilde{\delta}_{\mu\nu}(p)=\int\frac{dm^2}{(p^2-m^2)m^2}[P_{\mu\nu}(p,m^2)\rho_1(m^2)+g_{\mu\nu}\rho_2(m^2)]\ , \tag{3.30}$$

$$f_{\mu\nu}=\frac{2t_{\mu\nu}}{p^2-p'^2}\ . \tag{3.31}$$

Generally, even assuming the existence of the inverse propagators, the right-hand side of (3.28), and thus $\widetilde{\Gamma}^{\alpha,\beta}_{L}(p',p)_V$, possesses kinematical singularities at $p^2=p'^2$ and is therefore an unsatisfactory candidate for a quasipoint vertex. The source of the problem clearly lies in the spectral weighting over $\rho_2(m^2)$ in (3.28), since the $\rho_1(m^2)$ integral does not give rise to singularities at $p^2=p'^2$. We can alleviate this difficulty by using only the mass-independent portion of the radiation factors in the $\rho_2(m^2)$ weighting.

Thus, using f in place of $f_V^{(+)}$ and $f_V^{(-)}$ in the ρ_2 integral in (3.28) we can define a quasipoint vector vertex that fulfills all of the requirements we have stipulated for such an object:

$$\Delta'_{\mu\alpha}(p')\Gamma^{\alpha\beta}_{P}(p',p)_V\Delta'_{\beta\nu}(p)\equiv\Delta'_{\mu\alpha}(p')f^{\alpha}{}_{\nu}-f_{\mu}{}^{\alpha}\Delta'_{\alpha\nu}(p)+\frac{1}{p^2-p'^2}[\delta_{\mu\alpha}(p')p_\nu-p'_\mu\delta_{\alpha\nu}(p)]\omega^{\alpha\lambda}q_\lambda\ , \tag{3.32}$$

where now

$$\delta_{\mu\nu}(p)=\int\frac{dm^2}{(p^2-m^2)m^2}P_{\mu\nu}(p,m^2)\rho_1(m^2)\ . \tag{3.33}$$

The $p^2=p'^2$ singularities in each of the terms in (3.32) cancel by virtue of the decomposition (2.20) and the spectral representation (3.27). The terms containing the $\delta(p)$ functions are purely transverse and also vanish when $k^\sigma=\epsilon^\sigma(q)$ and $q^2=\epsilon\cdot q=0$, which is exactly the circumstance required for the proof of the radiation theorem. This last evidently holds with the photon attachments onto the full vector propagators represented by $\Gamma^{\alpha\beta}_{P}(p'p)_V$.

Finally, as in the Dirac case, the degree to which $\Gamma^{\alpha\beta}_{P}(p'p)_V$ represents a well-defined part of the exact vector vertex, $\Gamma^{\alpha\beta}(p',p)_V$, appears related to the validity of a counterpart of the representation (3.21) and (3.22). Otherwise, all that we can be certain of is that, by virtue of our construction, the difference between the two functions

$$\Gamma_T^{\alpha\beta}(p',p)_V=\Gamma^{\alpha\beta}(p',p)_V-\Gamma_P^{\alpha\beta}(p,p)_V\ , \tag{3.34}$$

possesses no kinematical singularities, is purely transverse, viz.,

$$[\Gamma_T^{\alpha\beta}(p'p)_V]_{k=q}=0\ , \tag{3.35}$$

and vanishes in the free-particle limit $[\rho_2=0,\rho_1\rightarrow(\rho_1)_{\text{free}}]$. As in the Dirac case, $\Gamma_T^{\alpha\beta}(p',p)_V$ spoils the possibility of radiation zeros because

$$[\Gamma_T^{\alpha\beta}(p',p)_V]_{k=\epsilon}\neq 0\ , \tag{3.36}$$

in general, even when $q^2=\epsilon\cdot q=0$.

In summary, (3.32) represents both a unitary-gauge gauge-technique type of ansatz for the case of a photon gauge-symmetrically coupled to a charged vector particle as well as a vertex function that includes higher-order corrections for which the radiation theorem still remains valid.

IV. SUMMARY AND CONCLUDING REMARKS

The usual invariant-amplitude analysis of vertex functions rests on the Poincaré transformation properties of the underlying fields, but for vertices involving gauge bosons there is a further Poincaré substructure reflecting the gauge-covariant nature of the coupling. This substructure is manifested simply only by a portion of the exact vertex, and then only in certain gauges, that we have termed the *quasipoint vertex*. We have confined ourselves to photon-particle-particle vertices with $m\neq 0$ and spin ≤ 1.

A quasipoint vertex necessarily includes the entire longitudinal part of the exact vertex function as a consequence of the Poincaré substructure requirement, but it only approximates the transverse piece. The Ward-Takahashi identity is therefore satisfied identically, a circumstance that is correlated with only the translational component of the Poincaré substructure. The purely Lorentz part occurs only in the transverse piece of the quasipoint vertex. For spin >0, transverse parts are required in order to cancel kinematical singularities as well as to ensure a proper point-vertex limit in the absence of higher-order corrections; the Poincaré substructure requirement provides a useful guideline to their minimal content particularly in the complicated vector case.

We have shown that the identity of the Poincaré substructure in the point and quasipoint vertices leads to an extension of the radiation theorem that includes photon attachments onto certain classes of self-energy graphs for Dirac and vector particles extending an earlier result for scalars. The supersymmetric extension of the radiation theorem raises interesting questions concerning the generalization of these results and about the structural characteristics of photino-particle-particle vertices.[48,49]

Explicit forms of the quasipoint vertices for spin ≤ 1 are determined using a Lehmann spectral-weighting method introduced originally in conjunction with the so-called gauge technique for solving the Schwinger-Dyson equations.

Finally, we have established the chirality dependent Poincaré substructure of the point vertex in the spinor case when coupling is axial vector. We have pointed out how the identities that express this substructure can be exploited to obtain a decomposition of multiphoton attachments via ordinary vector couplings onto a spinor line into a sum of effective single photon attachments.

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[1]This is taken to mean that the only allowed (covariant) derivatives that appear in the interaction Lagrangian are none for Dirac, single for scalar, single for vector, and double (Higgs type) for scalars arising in Yang-Mills trilinear forms, and products of these forms. This is consistent with the conventional minimal coupling prescriptions with no anomalous magnetic term for $s=\frac{1}{2}$ and $\kappa=1$ for $s=1$.

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[34]See also the critical analyses in Refs. 28 and 30 for detailed discussions.

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[36]Unless noted explicitly to the contrary we set the charge equal to unity. Also, we ignore the overall factors of i that are needed for the conventional propagator identifications.

[37]A derivation generalizing the one employed in Ref. 3 is more complicated but introduces other interesting identities. For example, one finds that Eqs. (5.17) of Ref. 3 hold if γ^μ is substituted for $\not\epsilon$ in the relation and

$$\left[p-\frac{q}{2}\right]^\mu = \left[p'+\frac{q}{2}\right]^\mu \quad \text{for } p'\cdot\epsilon = p\cdot\epsilon\ .$$

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[41]This can be trivially rearranged into the more customary form of the axial WT identity (cf. Ref. 40).

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[43]This remark applies to the scalar and Dirac cases as well.

[44]It is assumed that we are using a covariant gauge. See Ref. 14 for the development of the spectral-weighting gauge technique in the noncovariant axial gauge. See the discussion in Sec. I concerning the possible generality of our results beyond simple scalar QED.

[45]The fact that Γ^μ_P and Γ^μ_{BC} differ by purely transverse terms is noted by King (Ref. 27), who also proposes additional transverse terms in order to overcome difficulties in the gauge technique that are generated by the overlapping divergences which accompany the choices Γ^μ_P or Γ^μ_{BC} as the vertex function ansatz. In this regard see also R. Delbourgo and R. Zhang, J. Phys. A **17**, 3593 (1984).

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