Renormalization in anti–de Sitter supersymmetry

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The one-loop renormalization of the Wess-Zumino model in four-dimensional anti–de Sitter space $(\mathrm{AdS})_4$ is studied using a manifestly supersymmetric Pauli-Villars regularization procedure. The vacuum expectation value $\langle A \rangle$ of the physical scalar field is linearly divergent, while $\langle F \rangle = 0$. This divergence is canceled by a counterterm in the superpotential which is linear in A. The divergences of two-point functions agree with the nonrenormalization theorems of the flat-space model. However, the divergent one-point function shows that the special notion of "naturalness" in flat-space supersymmetry does not extend to $(\mathrm{AdS})_4$.

I. INTRODUCTION

In this work we extend recent investigations[1] of the properties of supersymmetric field theories in anti–de Sitter (AdS) space. In four dimensions such theories are invariant under the superalgebra OSp(1,4) whose Lie subalgebra SO(3,2) describes the invariance group of anti–de Sitter space. One motivation for these studies is to explore supersymmetry in a context intermediate in complication between flat space and quantum supergravity in which possible clues to the puzzle of the cosmological constant might arise.

The particular topic studied here is the quantum structure of the AdS Wess-Zumino model,[2] and we present results on two related questions.

1. Regularization. Careful calculation of the Green's functions in perturbation theory requires a method of regularization in which the symmetries of the theory are maintained. We use here the Pauli-Villars method in which the regularized action is manifestly supersymmetric. The applicability of Pauli-Villars regularization to the flat-space Wess-Zumino model was stated quite early and the general prescription for superfield perturbation theory[3] was given. The only previous calculations known to us[3,1] involve matrix elements of composite operators such as the stress tensor and conserved currents, which require a formal procedure different from that for the Green's functions of elementary fields.

We present the regularized component field action for the Wess-Zumino model both for flat space and AdS space. A surprising result is that all quartic terms in the Lagrangian vanish due to the Pauli-Villars sum rules, so that quartic vertices do not appear in any calculation. At first sight this seems to be an error since quartic couplings are known to contribute to tree-approximation amplitudes for the physical fields. However, there are cubic couplings between the physical and regulator fields, and it turns out that the effects of the standard quartic coupling is exactly reproduced by Yukawa exchanges of the regulator fields in the infinite-mass limit.

2. One-loop renormalization. One of the most striking features of flat-space supersymmetry is the nonrenormalization theorem[4] which implies that the classical superpotential is not renormalized by perturbative radiative corrections. Our results show that this modified concept of "naturalness" is not true in AdS space. We start with a superpotential containing cubic coupling and mass terms. These are not renormalized at one-loop order, but there is an infinite vacuum expectation value for the scalar field. This must be canceled by a counterterm which is a linear term in the superpotential.

Dimensional arguments strongly suggest that the cutoff-dependent parts of the wave-function and coupling-constant renormalization constants are unchanged by the curved background, but changes in the mass and linear renormalization constants are allowed. Thus it is not clear whether the nonrenormalization of the mass term found at one-loop order will persist to all orders. If so, then the failure of the standard nonrenormalization theorem is confined to one-point functions and is not very drastic. The higher-loop behavior is left as an open question here, although some possible approaches are discussed briefly in Sec. VIII.

II. FOUR-DIMENSIONAL (AdS) GEOMETRY

We start the discussion with the fact that four-dimensional AdS $(\mathrm{AdS})_4$ is the hyperboloid $\eta_{AB}y^Ay^B = -a^2$ embedded in $\mathbb{R}^5$ with Cartesian coordinates y^A, $A=0,1,2,3,4$ and flat metric $\eta_{AB}=(+----+)$. Infinitesimal O(3,2) transformations are realized by Killing vectors

$$K_{AB}=y_A\frac{\partial}{\partial y^B}-y_B\frac{\partial}{\partial y^A} . \tag{1}$$

One can introduce intrinsic coordinates t,ρ,θ,ϕ collectively denoted by x^μ by an explicit formula[5] $y^A(x^\mu)$, such that the induced metric takes the form

$$ds^2=\frac{1}{a^2\cos^2\rho}[dt^2-d\rho^2-\sin^2\rho(d\theta^2+\sin^2\theta 6d\varphi^2)] . \tag{2}$$

The scalar curvature is $R=12a^2$. To formulate supersymmetry one needs Killing spinors $\epsilon(x)$ which satisfy the equations

$$\left[D_\mu + \frac{i}{2} a\gamma_\mu\right]\epsilon(x) = 0 . \tag{3}$$

There are four independent solutions which can be written as $\epsilon(x) = S(x)\xi$ where ξ is a constant Majorana spinor and the matrix $S(x)$ is an explicitly known matrix[4] in the fundamental spinor representation of SO(3,2). Here we need only the properties

$$\begin{aligned} &\bar{S}(x)S(x) = \mathbb{1} , \\ &\bar{S}(x)\gamma^\mu S(x)\partial_\mu = \gamma^a K_{a4} + i\sigma^{ab}K_{ab} \equiv i\Gamma^{AB}K_{AB} , \end{aligned} \tag{4}$$

where $a,b = 0,1,2,3$ and the K_{AB} are the Killing vectors (expressed in intrinsic coordinates). The ten matrices Γ^{AB} generate the spinor representation. For more details of $(\mathrm{AdS})_4$ geometry, see Ref. 5.

III. THE REGULATED WESS-ZUMINO MODEL

The Wess-Zumino model involves chiral multiplets $z(x)$, $\psi(x)$, and $F(x)$ with the transformation rules [with $L,R = \frac{1}{2}(1 \pm \gamma_5)$ and with Killing spinor parameters]:

$$\begin{aligned} &\delta z = \bar{\epsilon} L\psi , \\ &\delta L\psi = L(-i\partial\!\!\!/ z + F)\epsilon , \\ &\delta F = -\bar{\epsilon}(iD\!\!\!\!/ + a)L\psi . \end{aligned} \tag{5}$$

One sees that the auxiliary field $F(x)$ plays its usual role as the indicator of supersymmetry breaking.

We introduce a set of Pauli-Villars regulator multiplets z_i, ψ_i, F_i with kinetic coupling parameters c_i and mass parameters μ_i. Since quadratically divergent integrals occur in the theory, we impose the sum rules

$$\sum_i c_i \mu_i{}^p = 0 , \quad p = 0,1,2 . \tag{6}$$

The physical fields ($i = 0$) have $c_0 = 1$ and $\mu_0 = \mu$. For the regulator fields ($i \geq 1$) we write $\mu_i = \alpha_i M$ where α_i are dimensionless numbers and the limit $M \to \infty$ is taken after calculation of Feynman integrals. Explicit determination of the parameters c_i, α_i is unnecessary.

The action of the regularized model is obtained from the invariant kinetic and interaction Lagrangians

$$\begin{aligned} \mathscr{L}_{\mathrm{kin}} &= \sum_i c_i{}^{-1} \left[\partial_\mu \bar{z}_i \partial^\mu z_i + \frac{i}{2}\bar{\psi}_i D\!\!\!\!/ \psi_i + \bar{F}_i F_i \right. \\ &\qquad \left. + a\bar{z}_i F_i + a\bar{F}_i z_i + 3a^2 \bar{z}_i z_i \right] , \\ \mathscr{L}_{\mathrm{int}} &= \sum_i (F_i W_{,i} + \bar{F}_i \overline{W}_{,i}) \\ &\qquad - \tfrac{1}{2}\sum_{i,j} \bar{\psi}_i (L W_{,ij} + R\overline{W}_{,ij})\psi_j + 3a(W + \overline{W}) , \end{aligned} \tag{7}$$

with superpotential function

$$W(z_i) = \tfrac{1}{2}\sum_i c_i{}^{-1}\mu_i z_i{}^2 + \frac{1}{3\sqrt{2}}\lambda \left[\sum_i z_i\right]^3 . \tag{8}$$

After introducing real scalar and pseudoscalar components $z_i = (A_i + iB_i)/\sqrt{2}$ using $F_i = -az_i - c_i\overline{W}_i$, and eliminating auxiliary fields, we obtain the Lagrangian

$$\begin{aligned} \mathscr{L} = {} & \tfrac{1}{2}\sum_i c_i{}^{-1}[\partial_\mu A_i \partial^\mu A_i + \partial_\mu B_i \partial^\mu B_i + \bar{\psi}_i(iD\!\!\!\!/ - \mu_i)\psi_i - (\mu_i{}^2 - a\mu_i - 2a^2)A_i{}^2 - (\mu_i{}^2 + a\mu_i - 2a^2)B_i{}^2] \\ & - \frac{1}{2}\lambda \left\{ \left[\sum_i \bar{\psi}_i\right] \sum_j (A_j - i\gamma_5 B_j)\left[\sum_k \psi_k\right] + \left[\sum_i \mu_i A_i\right]\left[\left[\sum_j A_j\right]^2 - \left[\sum_j B_j\right]^2\right] \right. \\ & \left. + 2\left[\sum_i \mu_i B_i\right]\left[\sum_j B_j\right]\left[\sum_k A_k\right] \right\} - \tfrac{1}{8}\lambda^2 \left[\sum_i c_i\right] \left[\left[\sum_j A_j\right]^2 + \left[\sum_j B_j\right]^2\right]^2 . \end{aligned} \tag{9}$$

The coefficient of the last term vanishes due to the $p = 0$ sum rule (6), so that quartic couplings can be dropped entirely both in flat space and $(\mathrm{AdS})_4$, as noted in the Introduction.

For perturbative calculations one needs propagators for the fields $A_i(x)$, $B_i(x)$, $\psi_i(x)$. Scalar propagators are functions of a single O(3,2)-invariant variable, and it is convenient to use the chordal distance variable $u = \frac{1}{2}a^2(y^A - y'^A)^2$ which is related to the square of the geodesic distance $\sigma(x,x')$ by $u = 1 - \cosh\{a[-\sigma(x,x')]^{1/2}\}$. Note that σ and u are negative for neighboring spacelike-separated points.

We use the notation

$$\langle A(x)A(x')\rangle \equiv \langle 0\, TA(x)A(x') \,|\, 0\rangle ,$$

etc. It follows from earlier work[1] that the free spinless propagators are given by

$$\langle A_i(x)A_i(x')\rangle = c_i \frac{a^2}{4\pi^2}\frac{d}{du} Q_{\lambda_{A_i}-2}(1 - u + i\epsilon) , \quad \langle B_i(x)B_i(x')\rangle = c_i \frac{a^2}{4\pi^2}\frac{d}{du} Q_{\lambda_{B_i}-2}(1 - u + i\epsilon) , \tag{10}$$

where $Q_\nu(z)$ is the Legendre function of second kind, and $\lambda_{A_i} = 1 + \mu_i/a$ (and $\lambda_{B_i} = 2 + \mu_i/a$) are the lowest-energy eigenvalues of the SO(3,2) representation[5] for the free fields $A_i(x)$ [and $B_i(x)$].

The spinor propagator can be written in terms of $\langle A(x)A(x')\rangle$ or $\langle B(x)B(x')\rangle$ as[1]

$$\begin{aligned}\langle \psi_i(x)\bar{\psi}_i(x')\rangle &= [(i\partial_x+\mu_i+a)\langle A_i(x)A_i(x')\rangle]S(x)\bar{S}(x')\\ &= [(i\partial_x+\mu_i-a)\langle B_i(x)B_i(x')\rangle]\gamma_5 S(x)\bar{S}(x')\gamma_5\end{aligned} \tag{11}$$

which ensures that the $(\mathrm{AdS})_4$ boundary conditions and the supersymmetry Ward identities are satisfied. It is curious that for a massless spinor the propagator contains both even and odd powers of γ^μ whereas chiral invariance would require only odd powers. The reason for this is the boundary conditions which imply that the axial charge is not conserved,[6] although there is a locally conserved axial-vector current. Thus chiral spinors do not exist in $(\mathrm{AdS})_4$. The general features of (11) have been confirmed in a recent analysis[7] involving the more traditional bispinor of parallel transport.

The short-distance behavior[1] of the spinless propagator is determined from the asymptotic expansion

$$\frac{a^2}{4\pi^2}\frac{d}{du}Q_{\lambda-2}(1-u)\underset{u\to 0}{\sim}-\frac{a^2}{8\pi^2}\left[\frac{1}{u}+\frac{1}{2}\left[2+\frac{m^2}{a^2}\right][-\ln(-\tfrac{1}{2}u)+h_0(\lambda)]+O(u\ln u,u)\right], \tag{12}$$

where $\lambda(\lambda-3)=m^2/a^2$ and $m^2=\mu^2\mp a\mu-2a^2$ is the Lagrangian mass for the free scalar (pseudoscalar) field, and $h_0(\lambda)=\psi(1)+\psi(2)-\psi(\lambda)-\psi(\lambda-2)$, where $\psi(\lambda)$ is Euler's function.

We will now use the propagators in one-loop calculations beginning with the more singular one-point function of the scalar field and then the two-point functions. Before proceeding to the quantum calculations we note that the classical stationary point of the potential in (9), namely, $A_i=B_i=0$, also satisfies $F_i=0$ and is therefore supersymmetric. It is the radiative corrections to this classical supersymmetric phase that we will investigate.

IV. THE ONE-POINT FUNCTION

The vacuum expectation value of the field $A_i(x)$, whether a physical or regulator field, can be calculated via the standard Dyson-Wick expansion. To one-loop order this gives

$$\begin{aligned}\langle A_i(x)\rangle &= i\int d^4y[-g(y)]^{1/2}\langle 0|TA_i(x)\mathscr{L}_{\rm int}(y)|0\rangle\\ &= -\frac{i\lambda}{2}\int d^4y[-g(y)]^{1/2}\langle A_i(x)A_i(y)\rangle\sum_j[(\mu_i+2\mu_j)\langle A_j^2(y)\rangle-(\mu_i-2\mu_j)\langle B_j^2(y)\rangle+\langle\bar{\psi}_j(y)\psi_j(y)\rangle]\,.\end{aligned} \tag{13}$$

To obtain the last line we inserted the cubic interaction Lagrangian of (9) and performed Wick contractions. The three terms inside the sum $\sum_j$ are the regulated contributions of scalar, pseudoscalar, and spinor loops. The regulated sum of the zero-separation propagators is well defined and independent of the point y.

The next step is to use the relation

$$\sum_j\langle\bar{\psi}_j(y)\psi_j(y)\rangle=-2\sum_j[(\mu_j+a)\langle A_j(y)^2\rangle+(\mu_j-a)\langle B_j(y)^2\rangle] \tag{14}$$

which has been used previously[1] and follows from the spinor trace of (11) in the short-distance limit. We also define a truncated one-point function $\langle A_i\rangle_{\rm tr}$ in an obvious way and write the result as

$$\langle A_i\rangle_{\rm tr}=-\tfrac{1}{2}\lambda(\mu_i-2a)\sum_j(\langle A_j^2\rangle-\langle B_j^2\rangle)\,. \tag{15}$$

This quantity vanishes in the flat-space limit ($a\to 0$) because scalar and pseudoscalar propagators are equal, but it does not vanish in $(\mathrm{AdS})_4$ as we will see.

The value of the regularized zero-separation propagator sums can be obtained from the short-distance limit of (12) which gives

$$\langle A_i\rangle_{\rm tr}=\frac{\lambda}{32\pi^2}(\mu_i-2a)\sum_j c_j[\mu_j(\mu_j-a)h_0(\lambda_{A_j})-\mu_j(\mu_j+a)h_0(\lambda_{B_j})]\,. \tag{16}$$

The shift property $\psi(1+z)=\psi(z)+z^{-1}$ and the sum rules (6) can now be used to obtain the simple final result

$$\langle A_i\rangle_{\rm tr}=\frac{a\lambda}{8\pi^2}(\mu_i-2a)\sum_j c_j\mu_j\psi\left[\frac{\mu_j}{a}\right]. \tag{17}$$

Using the asymptotic formula $\psi(z)=\ln z+O(z^{-1})$, one can easily see that the regulator contribution to the sum is linearly divergent as the cutoff $M\to\infty$.

We now note that $\langle B_i\rangle$ vanishes trivially; there are no contributing boson loops because of parity conservation and the trace in the fermion loop vanishes. However, to interpret the result (17) clearly it is important to compute the vacuum expectation value of the auxiliary field F_i. From the expectation value of the equation of motion,

$$-F_i=az_i+\mu_i\bar{z}_i+c_i\lambda\left[\sum_j\bar{z}_j\right]^2\Big/\sqrt{2}\,, \tag{18}$$

we find

$$-\sqrt{2}\langle F_i\rangle=(a+\mu_i)\langle A_i\rangle-\tfrac{1}{2}c_i\lambda\sum_j(\langle A_j^2\rangle-\langle B_j^2\rangle)\,. \tag{19}$$

To proceed further we must relate $\langle A_i\rangle$ to $\langle A_i\rangle_{\rm tr}$ using

$$i\int d^4y\sqrt{-g}\,\langle A_i(x)A_i(x')\rangle=c_i[(\mu_i-2a)(\mu_i+a)]^{-1} \tag{20}$$

which follows by naive manipulation of the equation

$$(\Box+m^2)\langle A(x)A(x')\rangle=-i(-g)^{-1/2}\delta(x,x') .$$

From (13), (15), and (20), we see that $(a+\mu_i)\langle A_i\rangle$ completely cancels the second term in (19), so that the regularized value of $\langle F_i\rangle$ vanishes. Hence supersymmetry is preserved by the one-loop radiative corrections.

An alternate completely local method to show that $\langle F_i\rangle=0$ is to compute $\langle A_i\rangle$ from the vacuum expectation value of the regularized operator field equation

$$c_i^{-1}(\Box+\mu_i^2-a\mu_i-2a^2)\langle A_i(x)\rangle=\left\langle\frac{\delta\mathcal{L}_{\rm int}}{\delta A_i(x)}\right\rangle . \quad (21)$$

The term $\Box\langle A_i\rangle=0$ because of O(3,2) invariance. The right-hand side of (21) is a regulated expression involving operator bilinears, and these can be evaluated using Wick contractions. After use of (14), one finds an expression for $\langle A_i\rangle$ which can be substituted in (19) and again leads to $\langle F_i\rangle=0$.

Our computations have led to the result that $\langle A_i\rangle_{\rm tr}$ is nonvanishing (and infinite as $M\to\infty$) but $\langle F_i\rangle$ vanishes. Thus no symmetry of the initial Lagrangian is violated by the one-loop vacuum expectation value, and we can cancel the ultraviolet divergence and maintain supersymmetry by adding a counterterm to the superpotential which is linear in the z_i. Thus we modify the superpotential (8) by adding the term $\Delta W=v\sum_i z_i$. After eliminating auxiliary fields via $-F_i=v+az_i+c_i\overline{W},i$, we find that the new interaction Lagrangian is

$$\mathcal{L}_{\rm int}+\Delta\mathcal{L}_{\rm int}=\mathcal{L}_{\rm int}-\sqrt{2}\,v\sum_i(\mu_i-2a)A_i . \quad (22)$$

Note that a possible counterterm quadratic in the boson fields vanishes because of the sum rules (6). The truncated one-point function $\langle A_i\rangle_{\rm tr}$ is equal to $\delta\Gamma/\delta A_i$ where Γ is the effective action. If we compare (22) with (15) or (17) we see that the Lagrangian seems to have been waiting for the counterterm in (22), and we can cancel $\langle A_i\rangle_{\rm tr}$ by taking

$$v=-\frac{\lambda}{2\sqrt{2}}\sum_j(\langle A_j^2\rangle-\langle B_j^2\rangle) . \quad (23)$$

Both the infinite (cutoff dependent) and finite parts of $\langle A_i\rangle$ are canceled by this choice, and this is required in order to interpret the theory as an operator theory in Fock space.[8]

Readers should note that $\langle F_i\rangle$ still vanishes after addition of the counterterm, since one can compute using the relation $-F_i=v+az_i+c_i\overline{W}_{,i}$. We now have $\langle z_i\rangle=0$ because of the new counterterm vertices, while v as given in (23) now cancels $\langle c_i\overline{W}_{,i}\rangle$ directly.

The philosophy underlying the procedure just discussed is simply the idea of "naturalness" in nonsupersymmetric flat-space field theories. We found a divergence which was consistent with the symmetries of the theory with the couplings μ and λ, and to cancel it we added a term to the action which was also consistent with those symmetries and might have been added *ab initio*. The conventional nonrenormalization theorems imply a modified view of "naturalness" in supersymmetric theories. The superpotential is not renormalized, so even if one starts from a theory which is not the most general one consistent with symmetries, one is not forced to change it. Our results show that this is not true in $(\rm AdS)_4$ supersymmetry, and it is in this sense that the conventional nonrenormalization theorems fail.

V. THE BOSONIC TWO-POINT FUNCTIONS

Truncated one-loop two-point functions are simply the products of free propagators in configuration space and the regulated expressions are nonsingular at short distance. Therefore it is difficult to recognize directly the effective singular quantities, $\delta(x,x')$ and $\Box_x\delta(x,x')$ to be canceled by counterterms for mass and wave-function renormalization. To circumvent this difficulty we use the adiabatic expansion technique of Bunch and Parker[9] which permits the use of momentum-space methods to determine counterterms.

For the free scalar field $\phi(x)$ in a general curved background, i.e., for the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial^\nu\phi-\tfrac{1}{2}(\overline{m}^2-\xi R)\phi^2 .$$

Bunch and Parker study the propagator $\langle\phi(x)\phi(x')\rangle$ using normal coordinates $z^\mu=e_a{}^\mu(x')z^a$ about the point x'^μ, so that $g_{\mu\nu}(x')z^\mu z^\nu=\eta_{ab}z^az^b=\sigma(x,x')$. They obtain an approximate Fourier representation [their (4-20) is rewritten in our Lorentzian signature conventions, with $kz=\eta_{ab}k^az^b$],

$$\langle\phi(x)\phi(x')\rangle=i\int\frac{d^4k}{(2\pi)^4}e^{-ikz}\left[\frac{1}{k^2-\overline{m}^2+i\epsilon}+\frac{(\frac{1}{3}-\xi)R}{(k^2-\overline{m}^2+i\epsilon)^2}-\frac{2}{3}\frac{R_{\mu\nu}k^\mu k^\nu}{(k^2-\overline{m}^2+i\epsilon)^3}+O(k^{-5})\right]$$
$$=i\int\frac{d^4k}{(2\pi)^4}e^{-ikz}\left[\frac{1}{k^2-\overline{m}^2+i\epsilon}+\frac{(\frac{1}{6}-\xi)R}{(k^2-\overline{m}^2+i\epsilon)^2}-\frac{1}{6}\frac{R\overline{m}^2}{(k^2-\overline{m}^2+i\epsilon)^3}+O(R^2k^{-6})\right] , \quad (24)$$

where in the second line we use $R_{\mu\nu}=\frac{1}{4}Rg_{\mu\nu}$ which is appropriate for an Einstein space.

Comparing with the boson mass terms of (9), we see that $\xi=\frac{1}{6}$ and $\overline{m}^2\to\mu_i^2\mp a\mu_i$ for the scalar $A_i(x)$ [and pseudoscalar $B_i(x)$]. Thus we obtain the approximate boson propagators:

$$\left.\begin{matrix}\langle A_i(x)A_i(x')\rangle\\ \langle B_i(x)B_i(x')\rangle\end{matrix}\right\}=ic_i\int\frac{d^4k}{(2\pi)^4}e^{-ikz}\left[\frac{1}{k^2-\mu_i^2\pm a\mu_i}-\frac{2a^2(\mu_i^2\mp a\mu_i)}{(k^2-\mu_i^2\pm a\mu_i)^3}+O(a^4k^{-6})\right] . \quad (25)$$

For the fermion propagators, we insert (25) in (11). The virtue of this is that the correct supersymmetric Ward-identity relations between Bose and Fermi propagators are manifestly maintained, although the price of virtue is that special techniques will be required to handle fermion-loop diagrams.

The justification of the use of the approximate propagators for the calculation of the ultraviolet properties of the theory is that they satisfy two related criteria.

(a) The approximate expressions match the terms in the short-distance expansion of the exact propagators, both for bosons and fermions, up to regular terms.

(b) The approximate expressions coincide with the power-series expansion in a^2 of the exact propagators up to an order beyond which, on dimensional grounds, there can be no modification of the renormalization structure of the flat-space theory.

The second term in (25) turns out not to contribute to the ultraviolet-divergent renormalization constants. One might suspect this since it is a regular term at short distance in both the Bose and Fermi propagators. However, we have kept this term in our calculations because it is still required by criterion (b). It also appears that the criteria (a) and (b) would not be satisfied if the adiabatic fermion propagator of Bunch and Parker (see also Ref. 10) were used directly, because the chiral projections of the exact and adiabatic propagators differ by terms which are singular at short distances.

In the adiabatic method one calculates the self-energy

$$\Pi(x,x')=\int \frac{d^4k}{(2\pi)^4}e^{-ikz}\Pi(k^2) \tag{26}$$

essentially by momentum-space techniques using the approximate propagator above. Ultraviolet divergences as $M\rightarrow\infty$ are contained in the coefficients P and Q of the Taylor series $\Pi(k^2)=P+Qk^2+O(k^4)$, and one identifies the corresponding counterterms via

$$\int \frac{d^4k}{(2\pi)^4}e^{-ikz}(P+Qk^2)=P\delta^4(z)-Q\eta^{ab}\frac{\partial}{\partial z^a}\frac{\partial}{\partial z^b}\delta^4(z)=\frac{1}{[-g(x)]^{1/2}}[P\delta^4(x,x')-Q(\Box_x-\tfrac{1}{3}R)\delta(x,x')] \ . \tag{27}$$

The result[10] involving $\Box-\frac{1}{3}R=\Box-4a^2$ can be verified by multiplying the terms involving Q by a function of compact support and integrating over the manifold. The term $\frac{1}{3}R$ comes from the normal coordinate dependence of the volume factor $[-g(z)]^{1/2}$.

Now that we have set up and justified the adiabatic method of calculation, we will apply it to compute the one-loop corrections to the two-point function $\langle 0\,|\,\langle TA(x)A(x')\,|\,0\rangle$. We again use the Dyson-Wick expansion:

$$\langle 0\,|\,\langle TA(x)A(x')\,|\,0\rangle_{1\,\text{loop}}=\frac{i^2}{2}\int d^4z_1\,d^4z_2[-g(z_1)]^{1/2}[-g(z_2)]^{1/2}\langle 0\,|\,\langle TA(x)A(x')\mathscr{L}_{\text{int}}(z_1)\mathscr{L}_{\text{int}}(z_2)\,|\,0\rangle \ . \tag{28}$$

After performing the Wick contractions using the vertices of (9) we find

$$\begin{aligned}\langle 0\,|\,\langle TA(x)A(x')\,|\,0\rangle_{\text{1-loop}}=-\frac{\lambda^2}{2}\int d^4z_1\sqrt{-g}\,\langle A(x)A(z_1)\rangle\int d^4z_2\sqrt{-g}\,\langle A(x')A(z_2)\rangle&\\ \times\sum_{k,l}\{(\mu^2+4\mu\mu_l+2\mu_l{}^2+2\mu_l\mu_k)&\\ \times\langle A_k(z_1)A_k(z_2)\rangle\langle A_l(z_1)A_l(z_2)\rangle&\\ +(\mu^2-4\mu\mu_l+2\mu_l{}^2+2\mu_l\mu_k)&\\ \times\langle B_k(z_1)B_k(z_2)\rangle\langle B_l(z_1)B_l(z_2)\rangle&\\ -\text{Tr}[\langle\psi_k(z_1)\bar\psi_k(z_2)\rangle\langle\psi_l(z_1)\bar\psi_l(z_2)\rangle]\}& \ .\end{aligned} \tag{29}$$

The three terms clearly correspond to scalar, pseudoscalar, and spinor loops. We have omitted tadpole diagrams since we have already introduced the counterterms which cancel them.

For further discussion we define the truncated two-point function or self-energy by

$$\begin{aligned}i\Pi(x,x')=-\frac{\lambda^2}{2}\sum_{k,l}\{&(\mu+\mu_k+\mu_l)^2\langle A_k(x)A_k(x')\rangle\langle A_l(x)A_l(x')\rangle\\ &+(\mu-\mu_l-\mu_l)^2\langle B_k(x)B_k(x')\rangle\langle B_l(x)B_l(x')\rangle-\text{Tr}[\langle\psi_k(x)\bar\psi_k(x')\rangle\langle\psi_l(x)\bar\psi_l(x')\rangle]\}\end{aligned} \tag{30}$$

in which (29) has been rewritten symmetrically in k and l.

Let us first study the spinor loop. The Ward identity (11) and the property (4) imply that

$$\bar{S}(x)\langle\psi_k(x)\bar{\psi}_k(x')\rangle S(x')(-a\Gamma^{AB}K_{AB_x}+\mu_k+a)\langle A_k(x)A_k(x')\rangle\ ,$$
$$\bar{S}(x)\gamma_5\langle\psi_k(x)\bar{\psi}_k(x')\rangle S(x')(x)=(-a\Gamma^{AB}K_{AB_x}-\mu_k+a)\langle B_k(x)B_k(x')\rangle\ . \tag{31}$$

With these relations, the spinor trace becomes

$$\mathrm{Tr}\langle\psi_k(x)\bar{\psi}_k(x')\rangle\langle\psi_l(x)\bar{\psi}_l(x')\rangle=\tfrac{1}{2}\mathrm{Tr}[(-a\Gamma^{AB}K_{AB_x}+\mu_k+a)\langle A_k(x)A_k(x')\rangle(-a\Gamma^{CD}K_{CD_{x'}}+\mu_l+a)\langle A_l(x')A_l(x)\rangle]$$
$$+\tfrac{1}{2}\mathrm{Tr}[(-a\Gamma^{AB}K_{AB_x}-\mu_k+a)\langle B_k(x)B_k(x')\rangle(-a\Gamma^{CD}K_{CD_{x'}}-\mu_l+a)\langle B_l(x')B_l(x)\rangle]\ , \tag{32}$$

where K_{AB_x} (or $K_{CD_{x'}}$) indicates a differential operator with respect to the intrinsic coordinates x^μ (or x'^μ). However, the biscalar propagators can equally well be regarded as functions of the embedding coordinates y^A and y'^A. Indeed [see (10)] they are simply functions of the chordal distance $u=\frac{1}{2}a^2(y^A-y'^A)^2$. Thus we introduce the simple notation $\langle A_k(x)A_k(x')\rangle=a_k(u)$ and $\langle B_k(x)B_k(x')\rangle=b_k(u)$. Then using (1) we have

$$\Gamma^{AB}K_{AB_x}\langle A_k(x)A_k(x')\rangle=2\Gamma^{AB}y_A\frac{\partial}{\partial y^B}a_k(u)=2a^2\Gamma^{AB}y_A(y_B-y_B')a_k'(u)=-2a^2\Gamma^{AB}y_Ay_B'a_k'(u)\ , \tag{33}$$

where $a_k'(u)$ is the derivative with respect to u. Using this and its analogue for $\langle B_k(x)B_k(x')\rangle$ we can rewrite (32) as

$$\mathrm{Tr}\langle\psi_k(x)\bar{\psi}_k(x')\rangle\langle\psi_k(x')\bar{\psi}_k(x)\rangle=2a^6\mathrm{Tr}(\Gamma^{AB}\Gamma^{CD})y_Ay_B'y_C'y_D[a_k'(u)a_l'(u)+b_k'(u)b_l'(u)]$$
$$+2(\mu_k+a)(\mu_l+a)a_k(u)a_l(u)+2(\mu_k-a)(\mu_l-a)b_k(u)b_l(u)\ . \tag{34}$$

The trace can be calculated from the implicit definition (4) of Γ^{AB} with the result $\mathrm{Tr}(\Gamma^{AB}\Gamma^{CD})=\eta^{AD}\eta^{BC}-\eta^{AC}\eta^{BD}$. Using this, the constraint $y^Ay_A=a^{-2}$, and the definition of u, we obtain

$$\mathrm{Tr}\langle\psi_k(x)\bar{\psi}_k(x')\rangle\langle\psi_l(x')\bar{\psi}_l(x)\rangle=2a^2u(2-u)[a_k'(u)a_l'(u)+b_k'(u)b_l'(u)]$$
$$+2(\mu_k+a)(\mu_l+a)a_k(u)a_l(u)+2(\mu_k-a)(\mu_l-a)b_k(u)b_l(u)\ . \tag{35}$$

The self-energy (30) now becomes

$$i\Pi(u)=\frac{\lambda^2}{2}\left\{2a^2u(2-u)\sum_{k,l}[a_k'(u)a_l'(u)+b_k'(u)b_l'(u)]-\sum_{k,l}\{[(\mu+\mu_k+\mu_l)^2-2(\mu_k+a)(\mu_l+a)]a_k(u)a_l(u)\right.$$
$$\left.+[(\mu-\mu_k-\mu_l)^2-2(\mu_k-a)(\mu_l-a)]b_k(u)b_l(u)\}\right\}\ . \tag{36}$$

We will now discuss the first term in (36). It is clear that only this term will give us divergences proportional to q^2. Let us only work out the details for the A fields, the result for the B fields can then be obtained by simply replacing $\mu_k\to-\mu_k$. The adiabatic propagators (10) are actually functions of the geodesic distance $\sigma^{1/2}(x,x')=z$, so we can write $a_k(u)=\bar{a}_k(z)$, etc., and compute the derivative via the chain rule

$$a_k'(u)=\frac{dz}{du}\bar{a}_k(z)=-\frac{1}{a[u(u-2)]^{1/2}}\frac{d}{dz}\bar{a}_k(z)\ , \tag{37}$$

where we have used $u=1-\cosh[a(-z^2)^{1/2}]$. It is now convenient to make a Euclidean rotation in (25) and perform the angular integration to obtain

$$\bar{a}_k(z)=-\frac{c_k}{4\pi^3}\int k^3dk\,\sin^2x\,dx\,e^{ikz\cos x}a_k(k^2)=\frac{c_k}{(2\pi)^2}\int k^3dk\frac{J_1(kz)}{kz}a_k(k^2)\ ,$$
$$a_k(k^2)\equiv\frac{1}{k^2+\mu_k{}^2-a\mu_k}-\frac{2a^2(\mu_k{}^2-a\mu_k)}{(k^2+\mu_k{}^2-a\mu_k)^3}\ , \tag{38}$$

and we have used a standard integral representation of the Bessel function $J_1(kz)$. If we now use

$$\frac{d}{dx}\frac{J_1(x)}{x}=-\frac{J_2(x)}{x} \tag{39}$$

we find for the derivative of the propagator

$$\frac{d}{du}a_k(u)=\frac{-c_k}{a[u(u-2)]^{1/2}}\frac{1}{(2\pi)^2}\int k^3dk\frac{J_2(kz)}{z}a_k(k^2) \tag{40}$$

and therefore for

$$2a^2(2-u)a_k'(u)a_l'(u)=-2\frac{c_k c_l}{(2\pi)^4}\frac{1}{z^2}\int dk\, k^3 J_2(kz)a_k(k^2)\int dp\, p^3 J_2(pz)a_l(p^2)\ . \tag{41}$$

On the one hand, we are pleased by this expression, since the factor $u(2-u)$ was canceled; on the other hand, we ended up with Bessel functions of order 2 and factors of $1/z$. These factors seem not to allow us to undo the angular integrals and go back to momentum space. But we employ a little trick to achieve this goal. Observe simply that the relations

$$\frac{d}{dk}k^2 J_2(k_z)=k^2 z J_1(kz)\ ,\quad \frac{1}{z}\frac{d}{dk}\frac{J_1(kz)}{k}=-\frac{1}{k}J_2(kz)\ , \tag{42}$$

hold. Using these two relations and integration by parts we easily see that

$$\begin{aligned}\int dk\, k^2 J_2(kz)ka_k(k^2)&=\int dk\, k^2 J_2(kz)\frac{d}{dk}A_k(k^2)\\ &=-\int dk\frac{d}{dk}[k^2 J_2(kz)]A_k(k^2)=-\int dk\, k^2 z J_1(kz)A_k(k^2)\end{aligned} \tag{43}$$

and

$$\int dp\frac{J_2(pz)}{p}p^4 a_l(p^2)=-\int dp\frac{1}{z}\frac{d}{dp}\left[\frac{J_1(pz)}{p}\right]p^4 a_l(p^2)=\int dp\frac{1}{z}\frac{J_1(pz)}{p}\frac{d}{dp}[p^4 a_l(p^2)]\ ,$$

where we implicitly defined $A_l(k^2)$ to be the integral of $ka_l(k^2)$. End-point contributions from the partial integration cancel in the self-energy by the sum rules (6).

We now reassemble things and restore the angular integral ending up with a nice expression involving only Lorentzian-signature Fourier transforms, viz.,

$$\begin{aligned}2a^2u(2-u)a_k'(u)a_l'(u)&=2\frac{c_k c_l}{(2\pi)^4}\left[\int dk\, k^2\frac{J_1(kz)}{z}A_k(k^2)\right]\left[\int dp\, p^2\frac{J_1(pz)}{z}\frac{1}{p^3}\frac{d}{dp}[p^4 a_l(p^2)]\right]\\ &=2c_k c_l\left[\int\frac{d^4k}{(2\pi)^4}e^{-ikz}\left[\ln(-k^2+\mu_k{}^2-a\mu_k)+\frac{a^2(\mu_k{}^2-a\mu_k)}{(k^2-\mu_k{}^2+a\mu_k)^2}\right]\right.\\ &\quad\times\int\frac{d^4p}{(2\pi)^4}e^{-ipz}\left[\frac{2}{p^2-\mu_l{}^2+a\mu_l}-\frac{p^2}{(p^2-\mu_l{}^2+a\mu_l)^2}\right.\\ &\quad\left.\left.-\frac{4a^2(\mu_l{}^2-a\mu_l)}{(p^2-\mu_l{}^2+a\mu_l)^3}+\frac{6p^2a^2(\mu_l{}^2-a\mu_l)}{(p^2-\mu_l{}^2+a\mu_l)^4}\right]\right]\ .\end{aligned} \tag{44}$$

To progress further we introduce $q=p+k$, let k go to $-k$, and rewrite (44) as

$$\begin{aligned}2a^2u(2-u)a_k'(u)a_l'(u)&=2\frac{c_k c_l}{(2\pi)^4}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\int\frac{d^4k}{(2\pi)^4}\left[\ln(-k+\mu_k{}^2-a\mu_k)+\frac{a^2(\mu_k{}^2-a\mu_k)}{(k^2-\mu_k{}^2+a\mu_k)^2}\right]\\ &\quad\times\left[\frac{2}{(q+k)^2-\mu_l{}^2+a\mu_l}-\frac{(q+k)^2}{[(q+k)^2-\mu_l{}^2+a\mu_l]^2}-\frac{4a^2(\mu_l{}^2-a\mu_l)}{[(q+k)^2-\mu_l{}^2+a\mu_l]^3}\right.\\ &\quad\left.+\frac{6(q+k)^2a^2(\mu_l{}^2-a\mu_l)}{[(q+k)^2-\mu_l{}^2+a\mu_l]^4}\right]\ .\end{aligned} \tag{45}$$

This contains Feynman integrals of a moderately unusual type, and a lot of them. However, power counting implies that the integrals involving the a^2 term in the left-hand factor give finite expressions as $M\to\infty$. It is less clear whether the product of $\ln(-k^2+\mu_k{}^2-a\mu_k)$ with the a^2 terms in the right-hand factor are ultraviolet finite, and we have evaluated these integrals explicitly and determined that they are finite as $M\to\infty$. The details are too tedious to record here.

The remaining integrals without explicit factors of a^2 can be handled using the identity

$$\begin{aligned}&\ln(-k^2+\mu_k{}^2-a\mu_k)\left[\frac{2}{(q+k)^2-\mu_l{}^2+a\mu_l}-\frac{(q+k)^2}{[(q+k)^2-\mu_l{}^2+a\mu_l]^2}\right]\\ &\quad=\frac{1}{k^2-\mu_k{}^2+a\mu_k}\frac{k(q+k)}{(q+k)^2-\mu_l{}^2+a\mu_l}-\frac{\partial}{\partial k^\mu}\left[\tfrac{1}{2}\ln(-k^2+\mu_k{}^2-a\mu_k)\frac{(q+k)^\mu}{(q+k)^2-\mu_l{}^2+a\mu_l}\right]\ .\end{aligned} \tag{46}$$

We may then write, finally,

$$2a^2u(2-u)a_k'(u)a_l'(u)=c_kc_l\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-\mu_k{}^2+a\mu_k}\frac{2k(q+k)}{(q+k)^2-\mu_l{}^2+a\mu_l}\,, \tag{47}$$

since the surface term in (46) vanishes at infinity. Note that (47) is precisely the expression one would get in flat space, except for the $a\mu_k$ terms in the mass denominators.

We are now ready to write an expression for the complete self-energy (30), including (47), the mass terms from the fermion trace in (35), and the boson loops. The contributions of the term proportional to a^2 in the boson propagator (25) are finite by power counting, so we discard these terms. The result is

$$\begin{aligned} i\Pi(x,x')=-\frac{\lambda}{2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\sum_{k,l}c_kc_l\int\frac{d^4k}{(2\pi)^4}\Bigg\{&[\mu^2+\mu_k{}^2+\mu_l{}^2+2(\mu-a)(\mu_l+\mu_k)-2a^2-2k(k+q)]\\ &\times\frac{1}{k^2-\mu_k{}^2+a\mu_k}\frac{1}{(k+q)^2-\mu_l{}^2+a\mu_l}\\ &+[\mu^2+\mu_k{}^2+\mu_l{}^2-2(\mu-a)(\mu_l+\mu_k)-2a^2-2k(k+q)]\\ &\times\frac{1}{k^2-\mu_k{}^2-a\mu_k}\frac{1}{(k+q)^2-\mu_l{}^2-a\mu_l}\Bigg\} \end{aligned} \tag{48}$$

Further simplification is achieved by expanding the propagators in powers of a

$$\frac{1}{k^2-\mu_k{}^2+a\mu_k}=\frac{1}{k^2-\mu_k{}^2}-\frac{a\mu_k}{(k^2-\mu_k{}^2)^2}+\cdots\,,\quad \frac{1}{k^2+\mu_k{}^2+a\mu_k}=\frac{1}{k^2-\mu_k{}^2}+\frac{a\mu_k}{(k^2-\mu_k{}^2)^2}+\cdots\,. \tag{49}$$

We see that the contributions of the terms of linear and higher orders in a are either finite by power counting or finite after cancellation between the scalar and pseudoscalar contributions. Dropping these terms we can rewrite (48) as

$$i\Pi(x,x')=-\frac{\lambda}{2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\sum_{k,l}c_kc_l\int\frac{d^4k}{(2\pi)^4}2[\mu^2+\mu_k{}^2+\mu_l{}^2-2a^2-2(k+q)k]\frac{1}{k^2-\mu_k{}^2}\frac{1}{(k+q)^2-\mu_l{}^2}\,. \tag{50}$$

With the help of Feynman-parameter techniques we may evaluate the momentum integral. The $\mu_k{}^2+\mu_l{}^2$ terms in the numerator cancel with other terms from the momentum shift. The result is

$$i\Pi(x,x')=i\lambda^2\frac{1}{(4\pi)^2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}(q^2+\mu^2-2a^2)\sum_{k,l}c_kc_l\int_0^1dx\,\ln\left[\frac{1}{\mu^2}x(1-x)q^2+x\mu_k{}^2+(1-x)\mu_l{}^2\right]. \tag{51}$$

This is the final form for the self-energy. We can evaluate the Feynman parameter integral (at $q^2=0$, so that only finite terms are neglected), and we find

$$i\Pi(x,x')=i\int\frac{d^4q}{(2\pi)^4}e^{-iqz}(q^2+\mu^2-2a^2)\frac{\lambda^2}{32\pi^2}\sum_{k,l}c_kc_l\frac{\mu_k{}^2+\mu_l{}^2}{\mu_k{}^2-\mu_l{}^2}\ln\frac{\mu_k{}^2}{\mu_l{}^2}\,, \tag{52}$$

where ultraviolet-finite terms have been omitted. Using (27) we rewrite this as

$$i\Pi(x,x')=i(-\Box+2a^2+\mu^2)\frac{\delta(x,x')}{[-g(x)]^{1/2}}\frac{\lambda^2}{32\pi^2}\sum_{k,l}c_kc_l\frac{\mu_k{}^2+\mu_l{}^2}{\mu_k{}^2-\mu_l{}^2}\ln\frac{\mu_k{}^2}{\mu_l{}^2}\,, \tag{53}$$

where the sum over regulator masses diverges logarithmically as $M\to\infty$.

We have also evaluated the self-energy of the pseudoscalar $B(x)$ in a computation similar to that described above. The result is that the infinite part of the self-energy is identical to that of the scalar in (53). There are differences in the finite terms which are expected since not even in the tree approximation do $\langle A(x)A(x')\rangle$ and $\langle B(x)B(x')\rangle$ coincide.

VI. THE SPINOR TWO-POINT FUNCTION

We now study the fermion self-energy beginning again with the Dyson-Wick expansion and a formula analogous to (28) with the interaction term

$$\mathscr{L}_{\rm int}=-\tfrac{1}{2}\lambda\left[\sum_i\psi_i\right]\left[\sum_j A_j-i\gamma_5B_j\right]\left[\sum_k\psi_k\right].$$

We then obtain the self-energy

$$i\Sigma(x,x')=-\lambda^2\sum_{j,k}[\langle A_j(x)A_j(x')\rangle\langle\psi_k(x)\bar\psi_k(x')\rangle-\langle B_j(x)B_j(x')\rangle\gamma_5\langle\psi_k(x)\bar\psi_k(x')\rangle\gamma_5]\,. \tag{54}$$

This is a bispinor, i.e., the left spinor index responds to local Lorentz transformation at x and the right spinor index to local Lorentz transformations at x'. We take care to preserve this property in our calculations. Using (11) we rewrite the previous expression as

$$i\Sigma(x,x')=-\lambda^2\sum_{j,k}[\langle A_j(x)A_j(x')\rangle(i\partial_x+\mu_k+a)\langle A_k(x)A_k(x')\rangle + \langle B_j(x)B_j(x')\rangle(i\partial_x-\mu_k+a)\langle B_k(x)B_k(x')\rangle]S(x)\bar{S}(x') . \quad (55)$$

The quantity within the square brackets is a second-rank spinor at x, and it is this quantity to which we apply the normal-coordinate methods. Our treatment is thus somewhat different from Refs. 9 and 10.

In our conventions[11] the inverse metric can be expanded in normal coordinates $z^\mu \equiv e_a{}^\mu(x')z^a=\delta_a{}^\mu z^a$ as

$$g^{\mu\nu}(x)=\delta^{\mu\nu}+\tfrac{1}{3}R^\mu{}_c{}^\nu{}_d(x')z^cz^d \quad (56)$$

and we can choose local frames so that

$$e_a^\mu(x)=\delta_a{}^\mu+\tfrac{1}{6}R^\mu{}_{cad}(x')z^cz^d . \quad (57)$$

Thus the differential operator which occurs in (55) can be expressed in normal coordinates as

$$\gamma^\mu\frac{\partial}{\partial x^\mu}=\gamma^a e_a{}^\nu(x)\frac{\partial}{\partial z^\nu}=\gamma^a(\delta_a{}^b+\tfrac{1}{6}R^b{}_{cad}z^cz^d)\frac{\partial}{\partial z^b} .$$

Thus we can insert the adiabatic approximate boson propagators (25) in (55) and write

$$i\Sigma(x,x')=-\lambda^2\sum_{jk}c_jc_k\left\{\int\frac{d^4k}{(2\pi)^4}e^{-ikz}a_j(k^2)\int\frac{d^4p}{(2\pi)^4}e^{-ipz}[\gamma^a(\delta_a{}^b+\tfrac{1}{6}R^b{}_{cad}z^cz^d)p_b+\mu_k+a]a_k(p^2) + \int\frac{d^4k}{(2\pi)^4}e^{-ikz}b_j(k^2)\int\frac{d^4p}{(2\pi)^4}e^{-ipz}[\gamma^a(\delta_a{}^b+\tfrac{1}{6}R^b{}_{cad}z^cz^d)p_b-\mu_k+a]b_k(p^2)\right\}S(x)\bar{S}(x') . \quad (58)$$

The z^cz^d terms can be traded for $(\partial/\partial p^c)\partial/\partial p^d$, and they yield Feynman integrals which are convergent by power counting. Similar remarks apply to the contribution of the a^2-dependent terms, including the $\pm a\mu$ in the mass denominators. We therefore drop all such terms, let $k=q-p$, and rewrite (58) as

$$i\Sigma(x,x')=\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Sigma(q)S(x)\bar{S}(x') \quad (59)$$

with

$$\begin{aligned}\Sigma(q)&=-\lambda^2\sum_{j,k}c_jc_k\int\frac{d^4p}{(2\pi)^4}\frac{(\not p+\mu_k+a)+(\not p-\mu_k+a)}{[(q-p)^2-\mu_j{}^2](p^2-\mu_k{}^2)}\\ &=-\lambda^2(\not q+2a)\int_0^1dx\int\frac{d^4l}{(2\pi)^4}\frac{1}{[l^2+x(1-x)q^2-x\mu_j{}^2-(1-x)\mu_k{}^2]^2}\\ &=i(\not q+2a)\frac{\lambda^2}{16\pi^2}\sum_{j,k}c_jc_k\int_0^1dx\,\ln[-x(1-x)q^2+x\mu_j{}^2+(1-x)\mu_k{}^2]\\ &=i(\not q+2a)\frac{\lambda^2}{32\pi^2}\sum_{j,k}c_jc_k\frac{\mu_j{}^2+\mu_k{}^2}{\mu_j{}^2-\mu_k{}^2}\ln\frac{\mu_j{}^2}{\mu_k{}^2} .\end{aligned} \quad (60)$$

In the various steps we have used standard manipulations of Feynman integrals. In the x integral we set $q^2=0$, thus neglecting finite terms. We insert this in (59) and obtain the divergent part of the self-energy

$$\begin{aligned}i\Sigma(x,x')&=i\left[\left[i\gamma^a\frac{\partial}{\partial z^a}+2a\right]\delta(z)\right]S(x)\bar{S}(x')\frac{\lambda^2}{32\pi^2}\sum_{j,k}c_jc_k\frac{\mu_j{}^2+\mu_k{}^2}{\mu_j{}^2-\mu_k{}^2}\ln\frac{\mu_j{}^2}{\mu_k{}^2}\\ &=i\not D\left[\frac{S(x)\bar{S}(x')\delta(x,x')}{[-g(x)]^{1/2}}\right]\frac{\lambda^2}{32\pi^2}\sum_{j,k}c_jc_k\frac{\mu_j{}^2+\mu_k{}^2}{\mu_j{}^2-\mu_k{}^2}\ln\frac{\mu_j{}^2}{\mu_k{}^2} .\end{aligned} \quad (61)$$

In the last step we have reintroduced the original coordinates and used the Killing spinor property (3).

VII. COUNTERTERMS FOR TWO-POINT FUNCTIONS

We now show that the divergent parts of the boson and fermion self-energies, $\Pi(x,x')$ in (53) and $\Sigma(x,x')$ in (61) can be canceled by adding supersymmetric counterterms for the physical fields of the Lagrangian (9). If we scale the fields A_0, B_0, Ψ_0, and the parameter $\mu=\mu_0$ according to

$$A_0(x)\to Z_A{}^{1/2}A(x)\ ,\ \ B_0(x)\to Z_B{}^{1/2}B(x)\ ,\ \ \psi_0(x)\to Z_\psi{}^{1/2}\psi(x)\ ,\ \ \mu_0\to Z_\mu\mu\ , \tag{62}$$

and let $Z_A=1+\lambda^2\delta z_A$, etc., we find the counterterm Lagrangian

$$\begin{aligned}\lambda^{-2}\mathscr{L}_{\rm ct}=&\tfrac{1}{2}A[\delta z_A(-\Box+2a^2)-(\delta z_A+2\delta z_\mu)\mu^2+a\mu(\delta z_A+\delta z_\mu)]A\\&+\tfrac{1}{2}B[\delta z_B(-\Box+2a^2)-(\delta z_B+2\delta z_\mu)\mu^2-a\mu(\delta z_B+\delta z_\mu)]B+\tfrac{1}{2}\bar\psi[\delta z_\psi i\not{D}-(\delta z_\psi+\delta z_\mu)\mu]\psi\ .\end{aligned} \tag{63}$$

By taking the second variational derivative with respect to $A(x)$, we see that the divergences in (53) can be canceled if and only if we take

$$\delta z_A=-\delta z_\mu=-\frac{1}{32\pi^2}\sum_{j,k}c_jc_k\frac{\mu_j{}^2+\mu_k{}^2}{\mu_j{}^2-\mu_k{}^2}\ln\frac{\mu_j{}^2}{\mu_k{}^2}\ . \tag{64}$$

The same steps apply to the B-field self-energy, which is identical to (53), and the divergence is canceled only if $\delta Z_A=\delta Z_B$.

The spinor counterterm is somewhat trickier because we must properly interpret

$$\begin{aligned}&\frac{\delta^2}{\delta\psi'_\alpha(x')\delta\psi_\beta(x)}\tfrac{1}{2}\int d^4y[-g(y)]^{1/2}\bar\psi(y)[\delta z_\psi i\not{D}_y-(\delta z_\psi+\delta z_\mu)\mu]\psi(y)\\&\qquad=\frac{\delta}{\delta\psi'_\alpha(x')}[-g(x)]^{1/2}[\delta z_\psi i\not{D}_x-(\delta z_\psi+\delta z_\mu)\mu]_{\beta\gamma}\psi_\gamma(x)\end{aligned} \tag{65}$$

so that the bispinor transformation properties are respected. Since the Killing spinor matrix $S(x)$ enforces these transformation properties in supersymmetric Ward identities such as (11), it is consistent to define

$$\frac{\delta\psi_\gamma(x)}{\delta\psi'_\alpha(x')}=S_{\gamma\delta}(x)\bar S_{\delta\alpha'}(x')\frac{\delta(x,x')}{\sqrt{-g}}\ . \tag{66}$$

We then see that the divergences of (61) can be canceled only if we take $\delta Z_\psi=-\delta Z_\mu$ as before.

The results indicate that the nonrenormalization theorems of flat-space supersymmetry are obeyed for all two-point functions in one-loop order. There is a common wave-function renormalization δZ for all fields which satisfies $\delta Z=-\delta Z_\mu$ as required. The renormalization constants are independent of the curvature; indeed they coincide with the expressions that would be obtained in a treatment of the flat-space Wess-Zumino model using Pauli-Villars regularization.

It should be noted that dimensional arguments would have permitted the more general replacement $\mu_0\to Z_\mu\mu+\lambda^2\delta Z'a$ where $\delta Z'$ is a dimensionless logarithmically divergent quantity. This could have led to additional counterterms

$$\begin{aligned}\lambda^{-2}\mathscr{L}'_{\rm ct}=&-\tfrac{1}{2}A^2(2\delta z'a\mu-\delta z'a^2)\\&-\tfrac{1}{2}B^2(2\delta z'a\mu+\delta z'a^2)\\&-\tfrac{1}{2}\bar\psi\psi a\delta z'\end{aligned}$$

which were not present in the calculated self-energies. Finally we repeat the earlier remark that explicit calculation of three-point functions is not required, since dimensional analysis implies that their renormalization is not affected by the curved space-time background.

VIII. DISCUSSION

The chief result of this paper is that the special notion of naturalness which occurs in flat-space supersymmetry because of the nonrenormalization theorem does not extend to the Wess-Zumino model in $(\mathrm{AdS})_4$. This effect is rather modest; it occurs only in the one-point function, whereas dimensional arguments would have permitted an effect on the mass renormalization as well. It would be curious to investigate the situation in higher-loop order. The obvious technique to use is the form of superspace perturbation theory[2,4] which is applicable in $(\mathrm{AdS})_4$. This is more complicated than in flat superspace, but there is still a distinction between chiral and full superspace integrals. Well before the development of superfield calculations, the particular form of the renormalization of the Wess-Zumino model was determined by use of functional techniques[12,13] which went beyond supersymmetric Ward identities. It may be possible to generalize these techniques to $(\mathrm{AdS})_4$.

Quantum field theory in AdS space is unusual because of the boundary conditions[14] required for conserved generators[5,1] of OSp(1,4). One place where these have impacted on the present calculations is in the treatment of the spinor propagator as in (11) where there are terms which are singular at short distance which seem to have the character of boundary terms in the heat kernel expansion. It is not clear whether these terms have actually affected the final results on ultraviolet structure.

We have discussed above that these terms imply chirality nonconservation even in the massless Wess-Zumino model. It is interesting to conjecture that a nonzero (and ultraviolet finite) fermion mass is generated at one-loop order. This should be a calculable effect because the one-loop mass counterterm vanishes in the theory with zero "bare mass." To establish this conjecture would require accurate calculations of the finite part of $\Sigma(x,x')$, and possible use of the nonperturbative criterion that physical mass is determined from the pole in the transform of the Lehmann spectral representation which has recently[15] been extended to $(\mathrm{AdS})_4$.

The boundary conditions imply that the fields in $(\mathrm{AdS})_4$ have a definite symmetry under the antipodal reflection $y^A \rightarrow -y^A$ on the hyperboloid. Because of this, propagators are singular not only on the light cone $u(x,x')=0$ which contains coincident points $x=x'$, but also on the reflected light cone $u(x,x')=2$ which contains antipodal points $x_a=x'$. The Legendre functions in (10) are clearly singular at $u=0$ and $u=2$. This implies that propagator wave equations should have both coincident and reflected sources $\delta(x,x')+e^{-i\pi\lambda}\delta(x_a,x')$. The naive manipulations used to derive (20) did not incorporate this feature. We are not sure how to do this properly, although we believe that there is a proper argument which does lead to (20) simply because this result is required in order to agree with the closely related local calculation using (21). One should also expect that the ultraviolet singularities of the two-point functions would involve both coincident and antipodal δ functions, and it is not quite clear how to set up the calculations to incorporate this. The present calculation should not be affected, since there is simply another contributing region. We hope that these questions can be illuminated in the future.

The results reported here can be compared with several recent papers of Bellucci and González. The first of these[16] arrived long after the completion of the calculations of Sec. IV, and contained one-loop calculations of two- and three-point functions in the massless Wess-Zumino model. The second arrived[17] soon after the completion of the calculations of Sec. V and contained one-loop calculations of two- and three-point functions in the massive model. Dimensional regularization and the adiabatic expansion technique were used, and it was concluded that the nonrenormalization theorems of the flat-space theory remain valid. This conclusion appeared to us to be premature and incomplete since one-point functions were omitted from consideration.

Several weeks later we obtained a third paper[18] in which one-point functions (and also two- and three-point functions) are calculated using Pauli-Villars regularization with independent A and F fields. There is a simple methodological error in the relation between $\langle F \rangle$ and the one-particle-irreducible Γ_F [see (22) of Ref. 18] which leads the authors to conclude incorrectly that supersymmetry is spontaneously broken by the radiative corrections while the nonrenormalization theorems are maintained. This completely disagrees with our results.

It is less clear whether there is agreement between the results of calculations involving the spinor propagator. Bellucci and González use the standard adiabatic approximate spinor propagator which differs even at short distances from the propagator (11) which obeys boundary conditions and supersymmetry Ward identities. It appears that the ultraviolet-divergent part of the spinor graphs for $\langle A \rangle$ and $\langle 0 | TA(x)A(x') | 0 \rangle$ do not differ in the two treatments but there is at least an apparent difference in the calculation of $\langle 0 | T\Psi(x)\Psi(x') | 0 \rangle$. Bellucci and González find, in effect, $\Sigma(q) \sim q \ln M^2$ whereas we find, in (60), $\Sigma(q) \sim (q+2a)\ln M^2$. The term involving a can be traced clearly to the a term in $(i\partial+\mu+a)\langle A(x)A(x')\rangle$ of (11) and this is not present in the adiabatic spinor propagator used in Ref. 18. Our treatment of the adiabatic spinor propagator differs from Refs. 9 and 10. There, the local frame is chosen so that the bispinor of parallel transport is unity, and it does appear correct to conclude in this framework that $\Sigma(q) \sim q \ln M^2$ corresponds simply to a spinor wave-function renormalization. What is not clear to us is whether the divergent parts of two-point functions are then consistent with supersymmetry Ward identities, because it is $S(x)\bar{S}(x')$ which plays the role of bispinor of parallel transport in these identities. It is worthwhile to clarify this point because there is an issue of principle involved, namely, do the reflective boundary conditions affect the ultraviolet behavior?

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