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On the multiplicity distribution in $e^+e^- \rightarrow$hadrons

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We show that energy-momentum-conservation effects strongly influence the multiplicity distribution, as it is observed in $e^+e^- \rightarrow$hadrons over the full rapidity range. Once these effects are removed, a Koba-Nielsen-Olesen scaling function corresponding to the independent superposition of a small number of geometric cascades is consistent with the data. We predict the multiplicity distribution in the *central* rapidity region, and speculate on its energy dependence.

I. INTRODUCTION

The multiplicity distribution of hadrons produced in high-energy processes provides interesting general information on the production mechanism and has been extensively studied in recent years both experimentally and theoretically.[1] The theoretical approach which seems to grasp the most essential features of the problem is the investigation of the KNO function proposed 14 years ago by Koba, Nielsen, and Olesen.[2] In this approach one studies the KNO function

$$\Psi(z) \equiv \bar{n} P(n) , \tag{1.1}$$

where $z = n/\bar{n}$, with $\bar{n}$ being the average multiplicity and $P(n)$ the observed multiplicity distribution. In many processes, the experimentally determined KNO functions turn out to be only weakly dependent on the energy of the collision. This phenomenon is called KNO scaling.

Recently, a substantial violation of KNO scaling between CERN ISR and collider energies was reported by the UA5 collaboration for $p\bar{p}$ collisions.[3] It was established that the KNO function in $p\bar{p}$ collisions broadens considerably with increasing energy. It was pointed out in Ref. 4 (see also Ref. 5) that one of the important sources of this effect can be the influence of the energy and momentum conservation laws which tend to suppress the large-multiplicity tail. As this suppression is obviously stronger at lower energies, the distribution is expected to broaden with increasing energy, as observed. The analysis of Ref. 4 suggests that the asymptotic KNO function is the same as that found[3,6,7] for particles produced in the central region of rapidity, i.e.,

$$\Psi(z) = 4z\, e^{-2z} . \tag{1.2}$$

(Throughout this paper, the central rapidity region is defined by the requirement that $|y|$ is much smaller than the full rapidity interval available at a given incident energy. In this region the energy and momentum conservation effects are expected to be small.)

This important role of energy and momentum conservation, even at collider energies, implies that such kinematic corrections should be thoroughly examined for any process before a physical interpretation of the multiplicity distribution is attempted.

In the present paper we report an analysis of such kinematic corrections for the process of hadron production in e^+e^- collisions which was recently summarized by the TASSO collaboration.[8] This allows us to estimate (starting from the measured multiplicity distribution in the full rapidity range) the distribution which should be valid in the central region of rapidity and, if KNO scaling holds asymptotically for this process, to predict also the multiplicity distribution at higher energies.

The effects of energy and momentum conservation on multiparticle final states are to some extent model dependent and thus it is necessary to specify the model selected for the analysis. Since we want to invest as little of the physics as possible at this stage of the argument, we have chosen to work in the framework of uncorrelated production of clusters[9,10] or, equivalently, the "bremsstrahlung analogy.[11] The results should be similar for any model which limits the transverse momenta of the final particles and produces a plateau in rapidity.

The resulting analysis of the data of Ref. 8 leads us to the conclusion that the KNO function describing multiplicity distribution in the *central* rapidity region can be approximated by the form

$$\Psi(z) = \frac{k^k}{\Gamma(k)} z^{k-1} e^{-kz} , \tag{1.3}$$

with $2 \lesssim k \lesssim 3$. This form is close to the shape (1.2) observed in $p\bar{p}$ collisions,[3] but much broader than that observed in the full rapidity interval of $e^+e^- \rightarrow$hadrons,[8] which would require $8 \lesssim k \lesssim 9$.

The prediction of energy dependence is more difficult because we do not know *a priori* the energy dependence of k and our analysis is not precise enough to determine it very accurately. Therefore only some speculations on this problem are possible. The situation may be settled when the direct measurements of $\Psi(z)$ in the *central* rapidity region of $e^+e^- \rightarrow$hadrons are available. Such measurements would also provide a crucial test of our approach.

Our paper is organized as follows. In Sec. II we describe the energy and momentum conservation corrections to multiplicity distribution following from "bremsstrahlung analogy."[11] In Sec. III the theoretical spectra are compared with the data of Ref. 8 and the shape of the distribution in the central rapidity region is derived. In Sec. IV we speculate on the possible energy dependence of the spectra. Our conclusions are summarized in Sec. V. In Appendix A the proof of the equivalence of the formulas for the particle spectrum derived from "bremsstrahlung analogy"[11] and from longitudinal phase space[10] is given. In Appendix B the origin and physical interpretation of Eq. (1.3) are explained.

II. EFFECTS OF ENERGY AND MOMENTUM CONSERVATION

The standard way of estimating to what extent an observed feature of the high-energy data contains real physical information (or if it is merely a reflection of kinematical constraints) is to compare it with the longitudinal phase-space calculation which also includes some fundamental properties of high-energy collisions such as, e.g., limited transverse momenta and the presence of leading particles. We follow this method here, using for our estimates the model of independent emission of clusters[9,10] or, equivalently, the bremsstrahlung analogy.[11] In this model the observed multiplicity spectrum can be approximated by the formula[12]

$$P(n)=\int dz\,\Psi(z)\delta(n-\bar{n}(\lambda))\ , \tag{2.1}$$

where $\lambda=z\lambda_0$ is the height of the inclusive single-cluster spectrum at rapidity $y=0$ [λ fluctuates with the probability distribution $\Psi(z)dz$ where, as we shall show below, $\Psi(z)$ is the KNO function], and $\bar{n}(\lambda)$ is the average number of clusters in events with given λ. (In this formula the fluctuations of multiplicity at fixed λ are neglected. They are expected to be Poisson like[11] and thus they may only introduce some corrections at the low multiplicity end of the spectrum, which is not very essential for our argument. However, they influence the precise determination of k in the central region. This is taken into account in the discussion of Secs. III and IV.)

$n(\lambda)$ is obtained by integrating, over a given rapidity range, the single-cluster inclusive distribution[11,13]

$$\frac{dn}{dy}=\lambda(1-x)^{\lambda}+\lambda x(1-x)^{\lambda-1}\ , \tag{2.2}$$

where the first term describes the produced and the second one the leading clusters. x is the Feynman variable $x=P_{\parallel}/P_{\max}$, and y is the rapidity of the cluster. Equation (2.2) is derived in Appendix A, where also other details of the calculation are given.

It follows from the formula (2.1) that $\Psi(z)$ is the KNO function of the multiplicity distribution in the central rapidity region. Indeed, in this case we have from Eq. (2.2) $\bar{n}(\lambda)\simeq\lambda Y$, with Y being the rapidity interval considered ($x\simeq 0$ in this region). Equation (2.1) then gives

$$P(n)=\frac{1}{\lambda_0 Y}\Psi(n/\lambda_0 Y)\ . \tag{2.3}$$

Since, on the other hand, $\lambda_0 Y$ is easily identified with the average multiplicity in the interval Y (and thus λ_0 with the average height of the single-cluster distribution in the central region) we obtain

$$\bar{n}P(n)=\Psi(n/\bar{n}) \tag{2.4}$$

which completes the proof.

Using Eqs. (2.1), (2.2), and (2.4) one can express the multiplicity distribution in any rapidity interval by the function $\Psi(z)$ describing the multiplicity distribution in the central rapidity region.

III. ANALYSIS OF THE e^+e^- DATA

In Ref. 8 the multiplicity distribution in the central rapidity region was not given. We therefore do not know the function $\Psi(z)$ and are forced to make some assumption about its general shape. We have considered the one-parameter family of $\Psi(z)$ of the form given by Eq. (1.3) and tried to determine the parameter k from the available data in the full rapidity region[8] using Eqs. (2.1)–(2.4). The form (1.3) was suggested by $p\bar{p}$ data [cf. Eq. (1.1)] and by its simple intuitive interpretation, as explained in Appendix B: it corresponds to the distribution created by k-independent form-invariant cascading processes.

Using Eqs. (1.3) and (2.1)–(2.4) we have calculated the expected KNO functions in the full rapidity region at 14 and 34 GeV for several values of the parameter k. They are plotted in Figs. 1(a) and 1(b). One sees that at 14 GeV [Fig. 1(a)] the data are reasonably well described by $k=2$ whereas at 34 GeV $k=3$ seems to be preferred. The corresponding KNO functions for the central region [i.e., Eq. (1.3) with $k=2$ and $k=3$, respectively] are also plotted in Fig. 1 and are seen to be much broader than the ones for the total rapidity range.

This striking effect of strong broadening of the spectrum in the central rapidity region is very characteristic of our analysis (it was already present in the $p\bar{p}$ data[4]) and therefore represents a crucial test of the approach presented here. It would be interesting (and probably not very difficult) to verify this prediction experimentally. [We have checked that this result is insensitive to the presence of leading particles in longitudinal phase space. In this case Eq. (2.2) is replaced by $dn/dy=\lambda(1-x)^{\lambda-1}$ (Ref. 10). The numerical results are slightly modified for small multiplicities, but do not change for $z\gtrsim 0.6$.]

As to the energy dependence of $\Psi(z)$ indicated by the data of Fig. 1, it is a much smaller effect and may well be contained within the error bars of our analysis (which treats the kinematic corrections crudely, particularly at such small energies and multiplicities). The conclusion must thus wait till the direct experimental measurement of $\Psi(z)$ in the central rapidity region is available; in our argument the function $\Psi(z)$ is not known *a priori* and thus one cannot predict the energy dependence at this stage. Some speculations on this subject are nevertheless presented in Sec. IV.

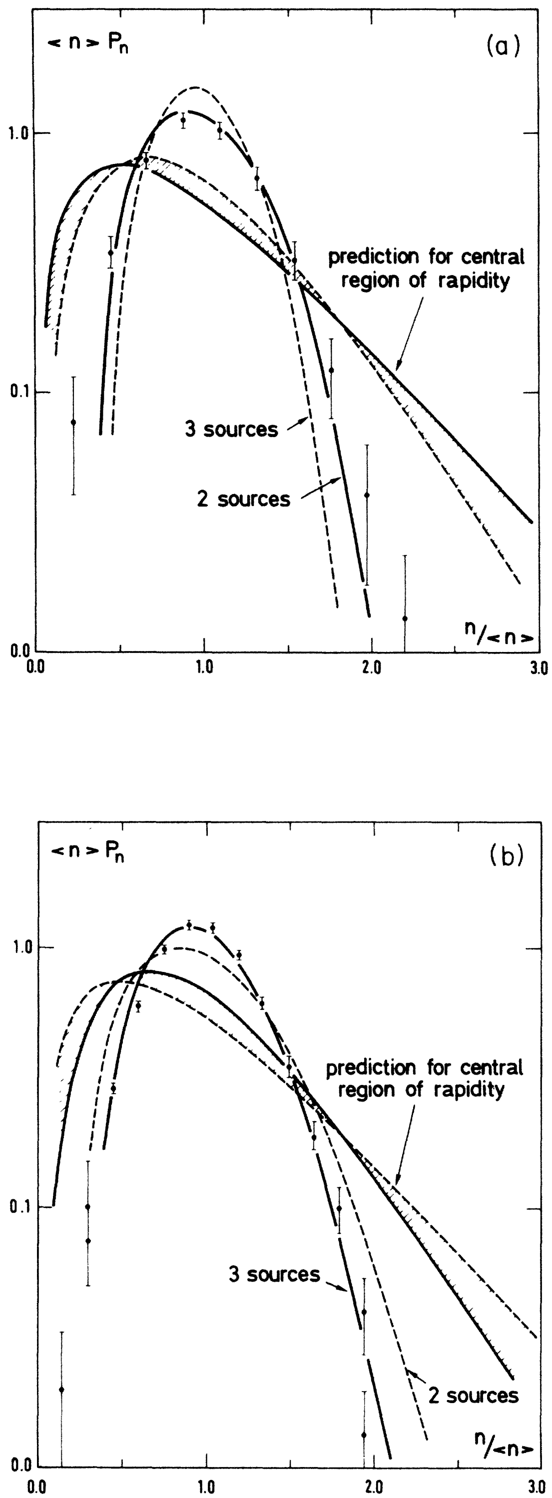

FIG. 1. (a) Hadronic multiplicity distribution ($\langle n \rangle P_n$ vs $n/\langle n \rangle$) in e^+e^- annihilation. The data at 14 GeV (over the full rapidity range) are compared with the theoretical curves which, starting from a KNO scaling function $\Psi(z)$ with two or three sources, incorporate energy-momentum conservation effects in the manner described in the text. Predictions for the central rapidity region are also given in both cases of two or three sources. (b) Same as (a), but with the data at 34 GeV.

IV. SPECULATIONS ON THE ENERGY DEPENDENCE OF THE SPECTRUM

Taking into account the data shown in Fig. 1, we shall consider two possibilities: (i) k is energy independent [and so is $\Psi(z)$]; (ii) k increases slowly with energy [and thus $\Psi(z)$ shrinks].

If k is energy independent, one can easily predict the energy dependence of the KNO function in the full rapidity region, using the formulas of Sec. II. In Fig. 2 the KNO function expected at 100 GeV is plotted. One sees that the distribution is expected to broaden substantially with respect to the present data.

If, on the other hand, the indication of energy dependence of $\Psi(z)$, suggested by the data of Fig. 1, is considered seriously, it opens the way to an interesting speculation. Let us start by recalling that Eq. (1.3) is the limiting form of the distribution of particles emitted by k-independent "random" sources.[6,7] The data of Fig. 1 suggest that the number of independent sources of particles in the process $e^+e^- \to$hadrons increases with increasing energy of the collision. It is thus tempting to identify these sources with quark and gluon jets[14] which, as is well known,[8] dominate the particle production process in e^+e^- collisions. This identification, if accepted, has several interesting consequences.

(a) The distribution of multiplicity within one jet must correspond to that of a "random" source. As shown in Appendix B, this implies the distribution of the "geometrical" type, e.g.,

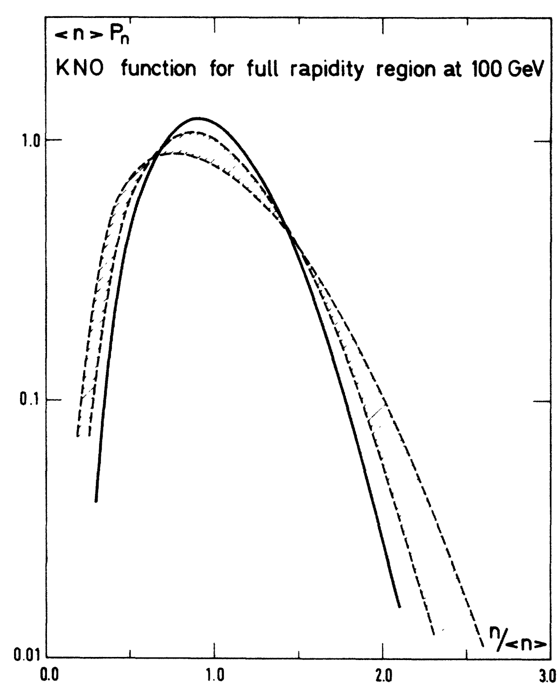

FIG. 2. Prediction of the multiplicity distribution at 100 GeV over the full rapidity interval, depending on whether the number of sources is two or three (the shaded area) or increases to four (the single curve).

$$P(n)=(1-\lambda)\lambda^{n-1},\quad n\geq 1\ . \tag{4.1}$$

[Since we are interested in the limit of large n, the first few terms in Eq. (4.1) are not essential for the argument presented here.]

(b) As shown in Appendix B, the spectrum (4.1) describes a specific example of a cascade process in which the shape of the distribution is *invariant* with respect to the number of generations in the cascade. This property, if indeed verified for the quark and gluon jets, should put an interesting constraint on the (still poorly known) process of hadronization of partons.

(c) If k is interpreted as the number of jets, we expect it to increase approximately logarithmically with energy. If this is the case, one has $k\approx 4$ at 100 GeV and one can again give prediction for the multiplicity distribution in the full rapidity region, using the formulas of Sec. II. It is shown in Fig. 2. Comparing Figs. 2 and 1(b) one notices that in this case ($k=4$) the distribution at 100 GeV is not far from the data at 34 GeV; i.e., there is apparent scaling between those two energies.

To conclude this section, let us repeat that the argument presented here is of speculative character. Nevertheless, it shows that a measurement of the multiplicity distribution at higher energies (particularly in the central rapidity region) shall be very interesting for studying the processes of hadronization.

V. CONCLUSIONS

We have analyzed kinematic corrections to the multiplicity distribution observed in the process $e^+e^-\to$hadrons at 14 and 34 GeV. Our conclusions can be summarized as follows.

(i) The observed shape of the multiplicity distribution in the full rapidity interval is strongly influenced by effects of energy and momentum conservation. The measurements of the KNO function in the central region of rapidity, where the kinematic constraints are minimized, would thus give much more direct physical information on the particle production process.

(ii) The KNO function in the central rapidity region is predicted to be of the form

$$\Psi(z)=\frac{k^k}{\Gamma(k)}z^{k-1}e^{-kz}\ , \tag{5.1}$$

with $2\lesssim k\lesssim 3$ in the (15–34)-GeV range and thus much broader than the one observed in the full rapidity interval.[8] Checking this prediction would provide a most crucial test for our argument.

(iii) If the form (5.1) is confirmed by the data, it means that the KNO function in the central rapidity region is not very different from that observed in pp collisions.[3] It follows that fluctuations in impact parameter (which cannot play any role in e^+e^- collisions) do not influence significantly the multiplicity distribution in hadronic collisions.[4] This seems consistent with the color-exchange picture of particle production,[15] but inconsistent with some of the current models.[16]

(iv) Possible energy dependence and physical interpretation of the obtained results depends crucially on precise determination of the KNO function in the central rapidity region, which was not possible by the indirect method employed in the present paper. Some speculations on this subject are nevertheless given in Sec. IV. Conclusions in this respect must wait till more complete data are available.

Note added in proof. The recent measurements of multiplicity distribution in central region of $e^+e^-\to$hadrons at 29 GeV [M. Derrick *et al.* (unpublished)] confirms our predictions, as shown in Fig. 1(b).

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APPENDIX A: DISTRIBUTION OF PRODUCED AND LEADING CLUSTERS FOR DEFORMED LONGITUDINAL PHASE SPACE

In this appendix we derived the Stodolsky-Benecke formula (2.2) from longitudinal phase space, using the method of Ref. 10. Let us define the function

$$\Omega_\lambda(Q_0,Q)\equiv\sum_n\frac{\lambda^n}{n!}\int\left[\prod_{i=1}^n e^{\alpha\epsilon_i}\frac{dp_i}{\epsilon_i}\right]\delta(Q_0-\epsilon_1-\cdots-\epsilon_n)\delta(Q-p_1-\cdots-p_n)\ . \tag{A1}$$

This function is proportional to the sum of longitudinal phase-space integrals of the type

$$\frac{1}{n!}\int[\,|M_n|^2]\left[\prod_{i=1}^n\frac{dp_i}{\epsilon_i}\right]\delta(Q_0-\epsilon_1-\cdots-\epsilon_n)\delta(Q-p_1-\cdots-p_n)\ , \tag{A2}$$

where matrix elements $|M_n|^2$ are given by

$$|M_n|^2=\lambda^n\prod_{i=1}^n e^{\alpha\epsilon_i}\ . \tag{A3}$$

This is an uncorrelated jet model with longitudinal distribution $e^{\alpha\epsilon}$.

De Groot observed that the presence of δ function in integrals (1) and (2) allows us to take the factor

$$\prod_{i=1}^n e^{\alpha\epsilon_i}=\exp[\alpha(\epsilon_1+\cdots+\epsilon_n)]$$

out of the integral and thus gives

$$\Omega_\lambda(Q_0,Q)=e^{\alpha Q_0}\sum_n \frac{\lambda^n}{n!}\int \prod_{i=1}^{n}\frac{dp_i}{\epsilon_i}\delta\left[Q_0-\sum \epsilon_i\right]\delta\left[Q-\sum p_i\right]\equiv e^{\alpha Q_0}C_\lambda(Q_0,Q)\ . \tag{A4}$$

The function $C_\lambda(Q_0,Q)$ was calculated by De Groot in the high-energy limit. We shall use here only the leading term (which corresponds to the Stodolsky formula for $\alpha=0$). In this limit one has

$$C_\lambda(Q_0,Q)=A(\lambda)(Q_0{}^2-Q^2)^{\lambda-1}=A(\lambda)(Q_0-Q)^{\lambda-1}(Q_0+Q)^{\lambda-1}\ . \tag{A5}$$

The function $\Omega_\lambda(Q_0,Q)$ can be used to express the inclusive distributions if multiparticle production is described by amplitudes given by Eq. (A3).

The cross section to find leading clusters with momenta L_1 and L_2 in the c.m. frame (and any number of produced clusters) is

$$\begin{aligned} d\sigma(x_1,x_2)&=B\Omega_\lambda(2E-L_{10}-L_{20};-L_1-L_2)dL_1dL_2\\ &=BA(\lambda)\exp[\alpha(2E-L_{10}-L_{20})]\tfrac{1}{4}(2E)^{2\lambda}(1-x_1)^{\lambda-1}(1-x_2)^{\lambda-1}dx_1dx_2\ . \end{aligned}\tag{A6}$$

Here we denote by $2E$ the total c.m. energy and introduce the scaling variables

$$x_i=L_{i0}/E\approx |L_i|/E\ . \tag{A7}$$

The constant B is to be determined from the normalization condition

$$\int d\sigma=\sigma\ , \tag{A8}$$

where σ is the total inelastic cross section.

Similarly, the inclusive cross section to find a produced cluster with four-momentum (ϵ,k) and two leaching clusters with momenta L_1,L_2 is

$$\begin{aligned} d\sigma&=B\Omega_\lambda(2E-L_{10}-L_{20}-\epsilon,-L_1-L_2-\epsilon)dL_1dL_2\frac{dk}{\epsilon}\lambda e^{\alpha\epsilon}\\ &=B\frac{A(\lambda)}{4}(2E)^{2\lambda}\lambda e^{ax}\exp[a(2-x_1-x_2-x)](1-x_1-x)^{\lambda-1}(1-x_1)^{\lambda-1}dx_1dx_2\ . \end{aligned}\tag{A9}$$

The condition (A8) can be exploited using Eq. (A6). We have

$$\begin{aligned} \sigma&=B\frac{A(\lambda)}{4}(2E)^{2\lambda}\int_0^1 dx_1\int_0^1 dx_2 e^{a(1-x_1)}e^{a(1-x_2)}(1-x_1)^{\lambda-1}(1-x_2)^{\lambda-1}\\ &=B\frac{A(\lambda)}{4}(2E)^{2\lambda}[\phi(a,\lambda)]^2\ , \end{aligned}\tag{A10}$$

where

$$\phi(a,\lambda)=\int_0^1 e^{az}z^{\lambda-1}dz \tag{A11}$$

and $a\equiv\alpha E$.

Thus the distribution of leading clusters is

$$\frac{d\sigma}{\sigma}=\frac{1}{[\phi(a,\lambda)]^2}e^{a(2-x_1-x_2)}(1-x_1)^{\lambda-1}(1-x_2)^{\lambda-2}dx_1dx_2 \tag{A12}$$

and for the inclusive distribution of produced clusters we obtain

$$\begin{aligned} \frac{d\sigma}{\sigma}&=\frac{1}{[\phi(a,\lambda)]^2}\lambda e^{ax}\frac{dk}{\epsilon}\int_0^1 dx_2(1-x_2)^{\lambda-1}e^{a(1-x_2)}\int_0^{1-x}dx_1(1-x-x_1)^{\lambda-1}e^{a(1-x-x_1)}\\ &=\lambda e^{ax}(1-x)^\lambda\frac{dk}{\epsilon}\frac{\phi(a(1-x),\lambda)}{\phi(a,\lambda)}\ . \end{aligned}\tag{A13}$$

Finally, let us note that for $a=0$,

$$\phi(0,\lambda)=1/\lambda \tag{A14}$$

as is easily seen from Eq. (A11). Thus for $a=0$ we have from (A12)

$$\frac{d\sigma}{\sigma}=\lambda(1-x)^{\lambda-1}dx \tag{A15}$$

for each of the leading clusters. Furthermore, formula (A13) reduces in this case to

$$\frac{d\sigma}{\sigma}=\lambda(1-x)^\lambda\frac{dk}{\epsilon}=\lambda(1-x)^\lambda dy\ . \tag{A16}$$

Thus for $a=0$ we obtain the formulas of the Stodolsky bremsstrahlung model.

APPENDIX B: THE CASCADING PROCESS

A cascading or branching process starts with a cluster which decays into n particles with probability P_n. Each particle in turn splits into n particles with the same distribution. A cascade of successive generations thus develops, each element in a generation being both particle and cluster.

At the lth generation the probability distribution is given by

$$P_{n;l}=\sum_m P_{m;l-1}\sum_{\substack{p_1,\ldots,p_m\\ \sum p_i=n}} P_{p_1}P_{p_2}\cdots P_{p_m}$$

as a function of $P_{m;l-1}$ and the original distribution P_n. The corresponding generating function is

$$G^{(l)}(z)=\sum z^n P_{n;l}=G^{(l-1)}(G(z)) ,$$

$$G^{(0)}(G(z))=G(z) ,$$

where $G(z)=\sum z^n P_n$ determines the cascading process.

Let us consider the case of a geometrical initial distribution[17] given by

$$P_n=(1-\lambda)\lambda^{n-1},\ \ P_0=0 . \tag{B1}$$

The generating functions are

$$G(z)=\frac{(1-\lambda)z}{1-\lambda z} , \tag{B2}$$

$$G^{(l)}(z)=\frac{(1-\lambda)^l z}{1-[1-(1-\lambda)^l]z} . \tag{B3}$$

The generating function keeps its shape from generation to generation. This is a remarkable feature of a geometric distribution. From the expression of $G^{(l)}(z)$ we obtain

$$P_{n;l}=(1-\lambda)^l[1-(1-\lambda)^l]^{n-1},\ \ P_{0;l}=0 \tag{B4}$$

and

$$\langle n\rangle_l=\frac{1}{(1-\lambda)^l} \tag{B5}$$

$$\frac{\langle n(n-1)\cdots(n-k+1)\rangle_l}{\langle n\rangle_l^k}=k!\left[1-\frac{1}{\langle n\rangle_l}\right]^{k-1} . \tag{B6}$$

The average number of particles in a generation increases with generation number, and the normalized moments increase correspondingly. In the limit where $n\to\infty$, $z=n/\langle n\rangle$ fixed, the distribution becomes scale invariant in the following sense:

$$\lim_{\substack{n\to\infty\\ z\ \text{fixed}}}\langle n\rangle_l P_{n;l}=e^{-z} .$$

Scale invariance is a characteristic feature of branching processes. In the language of Ref. 6 the asymptotic behavior corresponds to emission from one hadronic source. If there are k-independent sources, the generating function is simply the product of k factors equal to $G(z)$. Taking for $G(z)$ the function corresponding to the geometric distribution (1), one has

$$G_k(z)=[G(z)]^k=\frac{(1-\lambda)^k z^k}{(1-\lambda z)^k} ,$$

from which follows, after expansion in z of the denominator,

$$P_n(n\geq k)=\frac{(n-1)!}{(k-1)!(n-k)!}\left[\frac{1-\lambda}{\lambda}\right]^k\lambda^n ,$$

$$P_n(n<k)=0$$

and the average value

$$\langle n\rangle=\frac{k}{1-\lambda} .$$

In terms of $\langle n\rangle$, the multiplicity distribution becomes

$$P_n(n\geq k)=\frac{(n-1)!}{(k-1)!(n-k)!}\frac{1}{(\langle n\rangle/k-1)^k}\times\left[1-\frac{k}{\langle n\rangle}\right]^n .$$

In the limit $n\to\infty$, $z=n/\langle n\rangle$ fixed, this distribution, when multiplied by $\langle n\rangle$, tends to the scaling function

$$\Psi_k(z)=\frac{k^k}{(k-1)!}z^{k-1}e^{-kz} ,$$

where k denotes the number of sources.

The identical asymptotic form is obtained in Ref. 6, where the multiplicity distribution, based on optical considerations, is given by

$$P_n=\frac{(n+k-1)!}{n!(k-1)!}\left[\frac{1}{1+\langle n\rangle/k}\right]^k\left[\frac{1}{1+k/\langle n\rangle}\right]^n .$$

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