Riemannian contributions to the short-ranged velocity-dependent nucleon-nucleon interaction

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A Riemannian curvature-scalar term arises when determining the difference between the velocity-dependent potentials used in the differential Schrödinger equation and in its path-integral Lagrangian representation. Two previous papers have demonstrated that the magnitude of this difference may be within experimental error in nuclear-matter binding-energy calculations, when medium-range and long-range interactions are considered. This paper completes this first series of analyses by focusing on the short-ranged velocity-dependent interactions as parametrized by Lacombe *et al.*

In previous papers,[1,2] it was demonstrated that a velocity-dependent nucleon-nucleon interaction, e.g., arising from nonrelativistic reductions of nucleon spinors and momentum couplings of one-meson-exchange Lagrangians,[3] differs in its differential Schrödinger equation context from its global path-integral context, to the extent that this difference might account for several MeV binding energy per nucleon in a nuclear-matter calculation of the volume term in the Bethe-Weisäcker semiempirical mass formula.

Independently, it is also of extreme interest that this calculation presents the nucleon-nucleon interaction as the first quantum system to be investigated in which these Riemannian contributions are of sufficient magnitude to fall within experimental errors of empirical data. There are presently several derivations in the literature of the determination of the classical or the quantum differential propagator from its path-integral Lagrangian representation.[4-19] These methods also have been applied to classical physics-related problems in neuroscience,[20-26] economics,[27] and artificial intelligence.[28]

A given system may be described by a differential Schrödinger equation containing a velocity-dependent potential with terms of up to second power in the velocity, or equivalently by its path-integral representation, wherein the Hamiltonian or Lagrangian contains momentum dependence or velocity dependence, respectively, in addition to free kinetic energies. Either differential-equation operator-ordering or path-integral time-discretization considerations unambiguously establish the necessity that the potential terms in these two representations differ by terms which turn out to be concisely expressed as the Riemannian curvature scalar in a space with a nonflat metric induced by the velocity dependence, e.g., as derived from the net coefficient of the Laplacian operator in the Schrödinger equation. Even when the velocity dependence of the potential is only of first power in the velocity, operator-ordering or discretization considerations must be taken into account, e.g., to establish minimal coupling of the electromagnetic vector potential.[18]

As any path-integral-type derivation of the nuclear K matrix directly illustrates,[29,30] the nonrelativistically posed nuclear many-body problem (neglecting meson inelasticities,[31] etc.), although it cannot be formulated as a simple collection of two-body Schrödinger equations, does assume from the outset that its two-body potential, before self-energy, ladder partial sums, and other renormalizations are taken, is the same as the potential interaction between two nucleons. If the two-body velocity-dependent potential were fit to the (scattering and deuteron) data by numerically integrating the path-integral for each choice of the parameters to be fit, then this two-body potential would be used in the same (discretized or operator-ordered) way as in the many-body problem. Then, the curvature contribution would still be interesting physics, e.g., occurring in the differential Schrödinger equation if the choice is made to retain a classical Lagrangian,[1,2] and, e.g., wave functions calculated either way would be identical.

However, in practice the two-body potential is more easily fit by numerically integrating the (energy-spectral) differential Schrödinger equation for each choice of the parameters to be fit. Now the Riemannian contribution necessarily appears as difference between the potential used to describe the differential-equation two-body problem and the two-body potential used in *any* path-integral representation, e.g., the nuclear many-body problem, if, e.g., wave functions are to be consistently calculated. As demonstrated here and in previous papers,[1,2] this Riemannian contribution affects the net binding energy of a nucleon in nuclear matter to an extent larger than the empirical error in the Bethe-Weisäcker semiempirical mass formula. Therefore this contribution is a measurable quantity, in that nuclear-matter calculations require this contribution in order to achieve empirical verification.

However, the recent calculation of this effect[1] is only valid for the medium-range (MR) interaction ($r\sim 4-10$ GeV^{-1}) and the long-range (LR) interaction ($r\sim 10-\infty$ GeV^{-1}), because these are the only ranges within which the nucleon-nucleon interaction was fit to the scattering data and the deuteron.[3,32] Therefore, it is relevant to the more complete understanding or description of this interaction, and it is necessary for future nuclear-matter calculations, to examine the size of this effect for a short-range (SR) interaction ($r\sim 0-4$ GeV^{-1}) fit to empirical data, albeit that this nonrelativistic description of a relativistic region is clearly not unique.

Lacombe *et al.*, in fact, have developed a potential which includes fits to data testing the SR, and which has been parametrized to be velocity dependent,[33] as derived from the observed energy dependence in their previous fits to data.[34]

Here, their SR velocity-dependent potential is examined with respect to the induced Riemannian curvature term. There are additional terms arising from first-order gradients, but these are subject to some interpretation not arising in the curvature contribution which is independent of the ordering given to the momentum operators in the differential equation. It should be noted that this calculation does exhibit a preferred ordering in which the first-order gradient contributions are not present in S states.[1,2]

The calculation of this effect in the combined ^3S and ^1S states is identical to that done previously.[1,2] The $L=0$ S states are most sensitive because of the L^2/r^2 repulsion in higher states. The potential X used here is

$$
\begin{aligned}
X &= V(r)+\frac{1}{2m}[p^2 W(r) p^2], \\
V &= V_0^a + V_1^a, \\
W &= 2\left(\frac{m}{m_0}V_0^b+\frac{m}{m_1}V_1^b\right), \\
V_T^\sigma &= \sum_{j=1}^{12} G_{jT}^\sigma \exp(-\mu_{jT}^\sigma r)/(\mu_{jT}^\sigma r), \\
G_{12T}^\sigma &= -\mu_{12T}\sum_{j=1}^{11} G_{jT}^\sigma/\mu_{jT}^\sigma,
\end{aligned}
\tag{1}
$$

where $V_{0,1}^{a,b}$ represent the 48 terms of the S-state

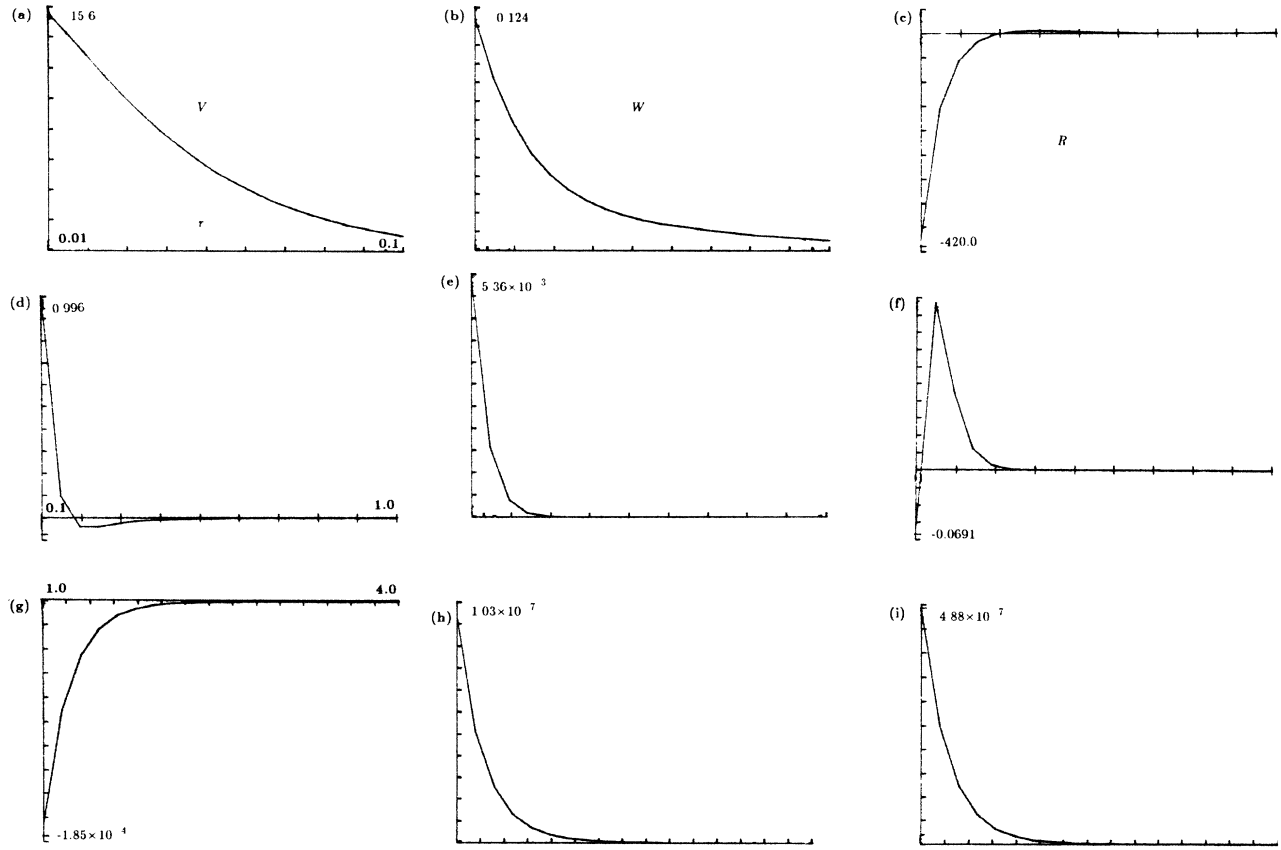

FIG. 1. The abscissa in all figures is the r axis. (a), (b), and (c) represent $0.01\leqslant r\leqslant 0.1$ GeV^{-1}; (d), (e), and (f) represent $0.1\leqslant r\leqslant 1.0$ GeV^{-1}; (g), (h), and (i) represent $1.0\leqslant r\leqslant 4.0$ GeV^{-1}. The ordinate units of the potentials are in GeV. The figures in the left-hand column represent the static contribution V; the figures in the center column represent the velocity-dependent contribution W; the figures in the right-hand column represent the W-induced curvature R. See Table I for representative values of these potentials.

potentials defined by Lacombe *et al.*[33] (The previous calculation of this effect for the MR and LR interactions[1] required including 159 terms representing the exchanged set of mesons $\{\pi,\eta,\sigma,\rho,\omega,\phi\}$ in momentum-transfer space,[3] which expands out to more terms when cutoffs are included and when expressed in coordinate space.) The subscripts refer to the isospin states, e.g., $^3S_1$ is a $T=0$ state, and $^1S_0$ is a $T=1$ state. The average nucleon mass m is taken here to be $2m_1m_2/(m_1+m_2)=0.938\,582\,2$. The units of r have been converted to GeV^{-1} by using $\hbar c=0.197\,329$ GeV fm.

The Schrödinger equation with a velocity-dependent potential is written as

$$
\begin{aligned}
i\frac{\partial\psi}{\partial t}&=\left(\frac{1}{m}p^2+V+\frac{1}{2m}(p^2W+Wp^2)\right)\psi\\
&=\left(\frac{1}{m}g^{ij}p_ip_j+V-\frac{1}{2m}W_{,ii}-\frac{i}{m}W_{,i}p_i\right)\psi\,,\\
(\cdots)_{,i}&=\partial_{x^i}(\cdots)=\partial(\cdots)/\partial x^i\,, \qquad (2)\\
p^2&=\eta^{ij}p_ip_j\,,\\
g^{ij}&=(1+W)\eta^{ij}\,,
\end{aligned}
$$

where η^{ij} is the (usual) flat-space contravariant metric, g^{ij} is the induced contravariant metric in the space curved by W, and the summation convention has been invoked. As mentioned above, in general, the partial-wave reduced equations contain contributions to the coefficient of p_i in addition to the $W_{,i}$ contribution here in Eq. (2). The p_i and Wp^2 terms have been treated consistently in previous scattering, deuteron, and nuclear-matter calculations.[3,32]

The arc length in isotropic form is defined by

$$ds^2=(1+W)^{-1}(dr^2+r^2d\theta^2+r^2\sin^2\theta\,d\phi^2)\,. \qquad (3)$$

This can be put into "standard form,"[35] which is more convenient for further calculation:

$$
\begin{aligned}
ds^2&=g_{ij}\,dx^i\,dx^j\\
&=A(r')dr'^2+r'^2(d\theta^2+\sin^2\theta\,d\phi^2)\,,\\
r'^2&=[1+W(r)]^{-1}r^2\,, \qquad (4)\\
A(r')&=\left(1-\frac{r}{2}W(r)_{,r}[1+W(r)]^{-1}\right)^{-2}\,,\\
(\cdots)_{,r}&=\partial(\cdots)/\partial r\,.
\end{aligned}
$$

Using the Ricci tensor R_i^j,

$$
\begin{aligned}
R_{r'}^{r'}&=(r'A^2)^{-1}A_{,r'}\,,\\
R_\theta^\theta&=R_\phi^\phi=r'^{-2}+(2r'A^2)^{-1}A_{,r}-(r'^2A)^{-1}\,, \qquad (5)\\
A_{,r}&=A_{,r}r_{,r'}\,,
\end{aligned}
$$

yields the curvature scalar $R(r)$,

$$R(r)=\frac{2}{r'^2}\left(1-\frac{1}{A(r')}+\frac{r'A(r')_{,r'}}{A(r')^2}\right)\,. \qquad (6)$$

As Fig. 1 and Table I demonstrate, the curvature contribution R is larger than the velocity-dependent W throughout the range of r. This R contribution is a much larger percentage of the W contribution than was found for the MR and LR regions calculated previously.[1,2] Examining the range $r>0.1$ GeV^{-1}, the SR W terms are much smaller than the static V terms by a factor of 10^{-3}, and the net effect is to keep R less than V in this range. However, within $r<0.01$ GeV^{-1}, R is not suppressed to 0 as $r\to0$, as are V and W, by the constraint on $G^{\sigma}_{12\,T}$ in Eq. (1). Therefore, it would seem appropriate for future fits to add an additional constraint on $G^{\sigma}_{11\,T}$, as was done for tensor and spin-orbit potentials,[33] if these potentials are to be used as well for nuclear-matter calculations. Therefore, within the range $0.05\leqslant r\leqslant0.1$ GeV^{-1}, R is appreciable relative to V.

TABLE I. Values of V, W, and R, in units of GeV, for representative values of r in the SR, in units of GeV^{-1}.

r	V	W	R
0.001	17.8	0.225	-11 155.0
0.005	16.9	0.172	-1 481.0
0.01	15.6	0.124	-420.0
0.05	5.60	0.0175	2.80
0.1	0.996	5.36×10^{-3}	-0.0691
0.5	-3.42×10^{-3}	8.56×10^{-7}	-9.65×10^{-6}
1.0	-1.85×10^{-4}	1.03×10^{-7}	4.88×10^{-7}
5.0	-2.94×10^{-11}	1.67×10^{-14}	8.21×10^{-14}

It now is clear that these Riemannian corrections are small, but not negligible.[1,2] For example, if a velocity-dependent potential were fit to data using the Schrödinger equation, this potential would have to include the curvature term. The parameters of this fit most likely would absorb this affect so that the net functional form would be only slightly affected. However, in calculating the nuclear-matter K matrix, which is derived from a Lagrangian or Hamiltonian path-integral representation, this curvature term would *not* appear, thereby directly affecting the binding energy and saturation properties. A correction on the order of 1% in the potential could mean on the order of a 1 MeV correction to the binding energy, which is itself determined by a cancellation of the kinetic energy and the self-consistent nuclear-matter potential ~100 MeV, especially since this correction is influential in the steep part of the potential which most sensitively determines this cancellation.[1]

Thus, it is still an open question as to what will be the net effect of using a nucleon-nucleon potential, including these Riemannian contributions, to calculate self-consistent Brueckner type nuclear matter.

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