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Spontaneous breaking of parity in (2+1)-dimensional QED

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The spontaneous generation of parity- (P) and time-reversal- (T) violating masses in (2+1)-dimensional QED is studied in the large-N limit, where N is the number of two-component complex fermions. Energy considerations of various symmetry-breaking patterns indicate that P and T are not spontaneously broken, even though masses which individually violate these symmetries are dynamically generated.

The study of (2+1)-dimensional QED has attracted a good deal of attention recently. This theory admits a gauge-invariant but P- and T-violating mass term for the photon.[1-3] Furthermore, a mass term for a complex two-component fermion also violates[2] P and T invariance. If one allows a mass for either the photon or the fermion, a mass for the other will be generated perturbatively.[4] Alternatively, one could start with a massless gauge field and a massless fermion and see whether dynamical mass generation occurs. This would lead to the spontaneous breaking of the discrete symmetries P and T.

If one is careful not to violate P and T in regulating the massless theory, a parity-breaking mass will not be generated in any finite order in perturbation theory. Therefore dynamical mass generation has to be investigated in a nonperturbative setting. In this paper we self-consistently solve the Dyson-Schwinger gap equation of the theory to study mass generation. To this end we employ a nonperturbative resummation of perturbation theory using the large-N approximation. Here $\alpha=e^2N$ is kept fixed, where N is the number of two-component complex fermions and e is the dimensionful gauge coupling constant.

The Lagrangian of this model is

$$L=\sum_{i=1}^{N}\bar{\psi}_i(i\partial\!\!\!/-eA\!\!\!/)\psi_i-\tfrac{1}{4}F_{\mu\nu}{}^2, \tag{1}$$

where ψ is a two-component spinor. The three 2×2 Dirac matrices can be written as $\gamma^0=\sigma_3$, $\gamma^1=i\sigma_1$, and $\gamma^2=i\sigma_2$, where σ_i are the Pauli matrices. Mass terms $\mu\epsilon^{\alpha\beta\gamma}F_{\alpha\beta}A_\gamma$ and $m\bar{\psi}\psi$ (either bare or dynamically generated) for the gauge field and the fermion change sign under the following parity transformation:[1-3]

$$\begin{aligned}&A^0(x,y,t)\to A^0(-x,y,t)\ ,\\&A^1(x,y,t)\to -A^1(-x,y,t)\ ,\\&A^2(x,y,t)\to A^2(-x,y,t)\ ,\\&\psi(x,y,t)\to\sigma_1\psi(-x,y,t)\ .\end{aligned}\tag{2a}$$

Under time reversal the transformations are

$$\begin{aligned}&A^0(\mathbf{x},t)\to A^0(\mathbf{x},-t)\ ,\\&\mathbf{A}(\mathbf{x},t)\to -\mathbf{A}(\mathbf{x},-t)\ ,\\&\psi(\mathbf{x},t)\to\sigma_2\psi(\mathbf{x},-t)\ .\end{aligned}\tag{2b}$$

It is also possible to write down a P- and T-conserving mass term for (2+1)-dimensional fermions when there is an even number of fermion doublets. In this case it is more convenient to write the theory in terms of four-component rather than two-component spinors. The mass term $m\bar{\psi}\psi$ for a four-component fermion $\psi=\binom{\psi_1}{\psi_2}$, where ψ_1 and ψ_2 are two-component spinors, is equal to $m\bar{\psi}\psi=m\psi_1^\dagger\sigma_3\psi_1-m\psi_2^\dagger\sigma_3\psi_2$. While a single two-component mass term would be odd[3] under P and T, the four-component $m\bar{\psi}\psi$ is even. Under a parity transformation the spinors transform as $\psi_1\to\sigma_1\psi_2$ and $\psi_2\to\sigma_1\psi_1$, leaving $m\bar{\psi}\psi$ invariant, and similarly for time reversal. This mass term, on the other hand, also violates a kind of three-dimensional chiral symmetry. Recently, the dynamical generation of such a mass in (2+1)-dimensional QED in the large-N (even) limit, where $N/2$ is the number of four-component spinors, has been discussed by two groups.[5,6] Now, of course, $N/2$ four-component fermions are equivalent to N two-component ones. Therefore when a parity-invariant mass m is generated for $N/2$ four-component fermions, $N/2$ of the N two-component fermions acquire a mass m, while the other $N/2$ two-component ones acquire a mass $-m$.

We shall here generalize this study to allow for parity-violating as well as parity-conserving symmetry-breaking patterns. The Dyson-Schwinger integral equations relevant to the dynamical generation of mass are (in Euclidean notation)

$$p\!\!\!/+\Sigma(p)=\frac{\alpha}{N}\int\frac{d^3k}{(2\pi)^3}D_{\mu\nu}(p-k)\gamma_\nu\frac{k\!\!\!/-\Sigma(k)}{k^2+\Sigma^2(k)}\gamma_\mu\ , \tag{3}$$

$$\Pi_{\mu\nu}(p)=\alpha\int\frac{d^3k}{(2\pi)^3}\mathrm{tr}\left[\gamma_\nu\frac{k\!\!\!/-\Sigma(k)}{k^2+\Sigma^2(k)}\gamma_\mu\frac{p\!\!\!/+k\!\!\!/-\Sigma(p+k)}{(p+k)^2+\Sigma^2(p+k)}\right],$$

where

$$D_{\mu\nu}{}^{-1}=\Delta_{\mu\nu}{}^{-1}-\Pi_{\mu\nu} \tag{4}$$

and $\Delta_{\mu\nu}$ is the free Landau gauge propagator. Σ is the dynamically generated mass matrix for fermions. Here we have approximated the complete gauge vertex by a bare one, invoking the large-N approximation. Fermion wave-function renormalization, which is related to the higher-order vertex by a Ward identity, is also down in $1/N$ and is dropped. The complete gauge propagator can be written in the form

$$D_{\mu\nu}(p)=\frac{(g_{\mu\nu}-p_\mu p_\nu/p^2)}{p^2}\Pi_1(p)+\epsilon_{\mu\nu\lambda}\frac{p_\lambda}{|p|^3}\Pi_2(p)\ , \tag{5}$$

where Π_1 and Π_2 are given by

$$\Pi_1(p)=\frac{1-\Pi_{\rm even}(p)/p^2}{[1-\Pi_{\rm even}(p)/p^2]^2+[\Pi_{\rm odd}(p)/|p|]^2}\;,$$
$$\Pi_2(p)=\frac{\Pi_{\rm odd}(p)/|p|}{[1-\Pi_{\rm even}(p)/p^2]^2+[\Pi_{\rm odd}(p)/|p|]^2}\;, \tag{6}$$

and

$$\Pi_{\mu\nu}(p)=\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right]\Pi_{\rm even}(p)+\epsilon_{\mu\nu\lambda}p_\lambda\Pi_{\rm odd}(p)\;. \tag{7}$$

Therefore, Eq. (3) yields a set of coupled integral equations involving Σ, $\Pi_{\rm even}$, and $\Pi_{\rm odd}$.

The above set of integral equations is too complicated to be amenable to a straightforward analytical study. To solve these equations we make the following approximations. First, we assume that if a dynamical mass $\Sigma(p)$ is generated its magnitude is much less[5] than α. We also assume that $\Sigma(p)$ is approximately constant up to the momentum scale α. This constant mass approximation has yielded reliable qualitative information about mass generation in previous analyses.[5,6] It is supported by analytical studies of Dyson-Schwinger equations which have shown that dynamically generated masses show slow variations with p up to the scale α and are damped above α. Therefore, as a first approximation, we choose to replace the matrix $\Sigma(p)$ by a constant matrix Σ and use α as an ultraviolet cutoff.

We first check the possibility of maximal parity violation by assuming that all the generated masses are of equal magnitude m. So $\Sigma=\mathrm{diag}(m, m, \ldots, m)$. To leading order in $1/N$ all radiative corrections are determined by one-loop graphs. This gives

$$\Pi_{\rm even}(p)=-\frac{\alpha}{8\pi}\left[2m+\frac{p^2-4m^2}{|p|}\arcsin\frac{|p|}{(p^2+4m^2)^{1/2}}\right]\;,$$
$$\Pi_{\rm odd}=\frac{m\alpha}{2\pi|p|}\arcsin\frac{|p|}{(p^2+4m^2)^{1/2}}\;. \tag{8}$$

When $m \ll p$,

$$\Pi_{\rm even}=-\alpha|p|/16,\quad \Pi_{\rm odd}=m\alpha/(4|p|)\;. \tag{9}$$

Therefore for momenta p such that $\alpha \gg p \gg m$,

$$\Pi_1=16|p|/\alpha,\quad \Pi_2=64m/\alpha\;. \tag{10}$$

From Eq. (3), the fermion self-energy at zero momentum is given by

$$\Sigma(0)=\frac{2\alpha}{N}\int\frac{d^3k}{(2\pi)^3}\left[\frac{\Pi_1(k)m}{k^2(k^2+m^2)}-\frac{\Pi_2(k)}{|k|(k^2+m^2)}\right]\;. \tag{11}$$

The integral is naturally cut off by α in the ultraviolet and by m in the infrared. By requiring $\Sigma(0)=m$ we get

$$m\sim\frac{m}{N\pi^2}\int_m^\alpha dk\left[\frac{16}{k}-\frac{64}{k}\right]\;. \tag{12}$$

Therefore,

$$m\sim\alpha\exp(N\pi^2/48)\;. \tag{13}$$

This result contradicts the initial assumption that $m \ll \alpha$ and arises because Π_2 contributes to Eq. (12) with opposite sign to Π_1 and with greater magnitude. We could also modify our initial assumption that $m \ll \alpha$ and look for self-consistent solutions with $m \gg \alpha$. In this case we find that $m\sim\alpha/(N\pi^2)$ which again contradicts the initial assumption $m \gg \alpha$.

These results indicate that dynamical masses of the maximal parity-violating kind are not generated. We have not yet excluded the possibility that a mass of order α is generated dynamically. In this case the approximations made in our analysis do not hold, and one has to resort to a numerical solution of the integral equations. However, a solution with m/α of order one is unlikely in the large-N limit, since in this case there will be no large dynamical hierarchy to compensate the $1/N$ suppression on the right-hand side of the Dyson-Schwinger equation.

We now go on to consider the possibility that $N-L$ fermions acquire a positive mass m and L fermions acquire a negative mass $-m$:

$$m_i=\begin{cases} m, & i=1,\ldots,N-L\;,\\ -m, & i=N-L+1,\ldots,N\;.\end{cases}$$

For $\alpha \gg p \gg m$,

$$\Pi_1=\frac{16|p|}{\alpha}\;,$$
$$\Pi_2=\frac{64}{\alpha N}\sum_{i=1}^{N}m_i=\frac{64}{\alpha N}(N-2L)m\;.$$

Going through the same analysis as before and using the same approximations we find

$$m_i\sim\frac{1}{N\pi^2}\int_m^\alpha dk\left[\frac{16m_i}{k}-\frac{64}{kN}(N-2L)m\right]\;. \tag{14}$$

For $i=1,\ldots,N-L$, Eq. (14) gives, for $m\neq0$,

$$1\sim\frac{1}{N\pi^2}\left[16-\frac{64}{N}(N-2L)\right]\ln\frac{\alpha}{m}\;, \tag{15}$$

while for $i=N-L+1,\ldots,N$,

$$1\sim\frac{1}{N\pi^2}\left[16+\frac{64}{N}(N-2L)\right]\ln\frac{\alpha}{m}\;. \tag{16}$$

Equations (15) and (16) are consistent only if $L/N=\frac{1}{2}+O(1/N)$. In this case

$$m=c\alpha\exp(-N\pi^2/16)\;, \tag{17}$$

where c is a positive number ~ 1 and, at least in the $1/N$ approximation, half the fermions acquire positive mass m and the other half negative mass $-m$.

We know from previous analysis[6] of the effective potential that the symmetry-breaking solution [Eq. (17)] is energetically preferred to the symmetric one ($m=0$). So there are fermion masses dynamically generated which individually violate parity and time-reversal invariance. However, the fact that half of the fermions acquire positive mass m and half of them acquire negative mass $-m$ ensures that parity and time reversal are not spontaneously broken overall, at least in the $1/N$ expansion.

The lack of parity-violating solutions to the Dyson-Schwinger equation means that any parity-odd configuration does not correspond to a stationary point of the composite operator effective potential, at least in the large-N limit. Therefore all vacuum configurations conserve parity. Vafa and Witten[7] showed in their general analysis, based on inequalities of operator expectation values, that the energy is minimized by the parity-conserving configuration consisting of half the fermions acquiring equal positive masses m and half equal negative masses $-m$. Our result agrees with their conclusion and elucidates the dynamics of symmetry breaking in large-N QED_3. A detailed analysis of the precise mechanism that leads to parity conservation is currently under investigation.

While this paper was being typed we received a copy of a report by Stam[8] which reaches similar conclusions. We also learned that Rao and Yahalom[9] are studying issues related to the ones discussed here.

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