Non-Abelian Debye screening: The color-averaged potential

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The non-Abelian analogue of the Debye screening effect is investigated by means of perturbative calculations in high-temperature Yang-Mills theory. It is recalled that the gluon vacuum-polarization tensor, which is gauge dependent, does not directly yield the Debye screening mass beyond leading order. Instead, the correlation function of two Polyakov loops, representing the color-averaged potential energy of a static quark-antiquark pair, is examined as a possible means of defining a gauge-invariant Debye mass. Though apparently satisfactory to leading order, an examination of higher orders reveals two problems: (i) an unexpected breakdown of perturbation theory in the electrostatic sector and (ii) the emergence of the magnetostatic mass gap as the dominant decay mass of this correlation. These results might be the signal for a breakdown of gauge symmetry in the plasma phase and indicate the need for examining a gauge-invariant electrostatic correlation function that is free from color averaging.

I. INTRODUCTION

Investigations of the conjectured "quark-gluon plasma" phase of hadronic matter can provide valuable information about the strong interaction. The high densities and temperatures relevant to its formation could occur not only in some astrophysical contexts but are also expected to be attained in the relativistic heavy-ion collision experiments likely to be performed in the near future. On the theoretical side, the properties of quantum chromodynamics at finite temperature and density ("hot QCD") have been of great interest in recent years. Monte Carlo simulations, complemented by analytical methods, have led to much qualitative and quantitative insight into the nature of the deconfinement—chiral-symmetry-restoration phase transition. Perturbative analyses have been carried out at temperatures and/or densities well above the transition, the rationale being the smallness of the QCD coupling constant in this regime. (For reviews of hot QCD, see Refs. 1 and 2.)

The Debye screening of static color charges and the possible screening of color magnetic fields are the sort of effect one would like to access perturbatively. It was soon realized, however, that the severe infrared divergences in hot QCD must cause perturbation theory to eventually break down. Thus, it was argued[3] that the $O(g^6)$ term in the thermodynamic potential receives contributions from an infinite set of Feynman graphs and so cannot be computed perturbatively. The same holds for the magnetic screening mass, the square of which starts out at $O(g^4)$, and sufficiently high-order corrections to the Debye screening mass. One ought, nevertheless, to be able to compute both the thermodynamic potential and the Debye screening mass at sufficiently low orders. The former has been well investigated (for references see Ref. 1); here we shall concentrate on the latter.

Debye screening refers to the long-distance shielding of electric charge by plasma excitations, which convert the $1/R$ Coulomb potential into the $\exp(-mR)/R$ Yukawa-type potential. The phenomenon was originally discussed in the context of the theory of electrolytes by Debye and Huckel,[4] but the concept is of course much more general and may be applied in a variety of situations. The equilibrium properties of an ordinary (i.e., Abelian) plasma are described field theoretically by quantum electrodynamics at a finite temperature and chemical potential. The Debye screening mass m is obtained as the static infrared limit of the time-time component of the photon vacuum-polarization tensor $\Pi_{44}(k_4=0,\mathbf{k}\to 0)$, which is gauge invariant. The naive method of computing the non-Abelian Debye screening mass is to mimic the procedure followed for Abelian plasmas and take the same limit of the gluon vacuum polarization. At first this seems to work: at the one-loop level $\Pi_{44}^{ab}(k_4=0,\mathbf{k}\to 0)$ turns out to be gauge invariant and yields the leading-order Debye mass.[1] But this convenient situation is short lived:[5,6] beyond the one-loop level, the gauge dependence of the non-Abelian vacuum-polarization tensor manifests itself even in the static infrared limit. Thus, the naive procedure for extracting m breaks down, and one has to turn to something more refined.

To obtain a physical quantity such as the Debye screening mass, one must consider an appropriate *gauge-invariant* correlation function. The obvious choice is the Polyakov loop correlation (PLC), which represents[7] the free energy of a heavy quark-antiquark pair with no color correlation (each source being separately averaged over color space). In the lowest order, this yields the same Debye mass as the naive calculation mentioned above. Corrections to this leading value could then presumably be obtained by computing the PLC to higher orders. This we attempt to do in the present paper.

At the next-to-leading order, however, we encounter an unexpected breakdown of perturbation theory in the electrostatic sector, which seems to throw a shadow on the entire idea of perturbatively computing the Debye screening mass. Moreover, examination of higher orders reveals that this apparently electrostatic correlation is in fact dominated by the mass gap of the *magnetostatic* sector. We are led to the conclusion that the color averaging in the PLC (which results in the correlation being dominated by two-gluon exchange) tends to wash out the Debye screening effect. We ought instead to seek a gauge-invariant correlation function dominated by single-gluon exchange, which would result in a cleaner manifestation of Debye screening.

In Sec. II we review the gauge dependence of the "naive" Debye mass (i.e., as calculated directly from the gluon vacuum polarization). In Sec. III we compute the PLC to leading order and in Sec. IV to higher orders; details of the calculation are given in the Appendix. Our conclusions are presented in Sec. V.

II. GLUON VACUUM POLARIZATION

The phenomenon of non-Abelian Debye screening has remained somewhat obscure due to the lack of a proper gauge-invariant calculation of the screening mass. We begin by reviewing previous calculations of the Debye mass as the static infrared limit of the time-time component of the high-temperature gluon vacuum-polarization tensor. The presence of dynamical quarks is an inessential generalization here, so for simplicity we shall work within the pure SU(N) Yang-Mills theory. The leading-order result for the squared mass is then the gauge-invariant one-loop value:[1]

$$m_E{}^2 = -\Pi_{44}^{(1)}(k_4=0,\mathbf{k}\to 0) = \frac{Ng^2T^2}{3} \ .$$

The same quantity, calculated at two loops, exhibits infrared divergences and gauge dependence.[5,6] The divergences may be eliminated by resumming the above one-loop value, resulting in a nonanalytic $O(g^3)$ correction to Π_{44}. This correction is most conveniently computed using a dimensionally reduced effective theory, "extended quantum chromodynamics in three Euclidean dimensions" (EQCD_3) (Ref. 5). For our present purposes, EQCD_3 may be taken to consist of the Yang-Mills field (A_i, the magnetostatic potential) in three dimensions minimally coupled to a massive adjoint scalar field ($\phi \sim A_4$, the electrostatic potential).

To derive the EQCD_3 action starting from that of hot QCD, one begins by noting that the only modes that are significant in the infrared (i.e., for momenta much smaller than the temperature) are the static modes with vanishing Matsubara frequencies.[8] Thus we are interested only in Green's functions with static external legs, and must integrate out the nonstatic loops. This integration can be performed perturbatively for the following reasons: (i) the effective coupling constant is $g^2(T)$, which is small for $T \gg \Lambda_{\mathrm{QCD}}$; (ii) there are no infrared divergences in the nonstatic sector. For our present purposes, we need evaluate the nonstatic effects only at the one-loop level. These are[5] the generation of a mass term for the electrostatic potential and the renormalization of the bare QCD coupling constant into the running coupling defined at momentum scale T. The magnetostatic potential remains massless.

The static modes of hot QCD are then described to a good approximation by the effective Euclidean action

$$S_{\mathrm{EQCD}_3} = \int d^3\mathbf{x}[\tfrac{1}{2}\mathrm{Tr}F^2(A) + \mathrm{Tr}(D\phi)^2 + m_E{}^2\mathrm{Tr}\phi^2] \ .$$

where

$$F_{ij}(A) = \partial_i A_j - \partial_j A_i + iG[A_i, A_j] \quad (A_i = A_i^{(D=4)}/\sqrt{T}) \ ,$$

$$D_i\phi = \partial_i\phi + iG[A_i,\phi] \quad (\phi = A_4^{(D=4)}/\sqrt{T}) \ ,$$

$$G = T^{1/2}g(T) \ ,$$

$$m_E{}^2 = NG^2T/3 \ .$$

[Strictly speaking, one should add to $m_E{}^2$ a mass counterterm $\delta m^2 = -2NG^2\int d^3\mathbf{q}/(2\pi)^3\mathbf{q}^2$, which arises together with $m_E{}^2$ from the integration over nonstatic modes and is a result of the incomplete cancellation of ultraviolet infinites when only nonstatic modes are included; it will be canceled by a similar divergence coming from the static modes of EQCD_3. Such divergences are set to zero in the dimensional regularization scheme which we shall use, so we do not consider them explicitly here.]

To calculate the $O(g^3)$ term in the static infrared limit of Π_{44}, all we need compute is the one-loop scalar self-energy in EQCD_3. We use an O(3)-covariant ξ gauge, in order to exhibit the gauge dependence of the result. The only relevant graph is the second graph in Fig. 2 which leads to the following result:

$$-\Pi_{44}(k_4=0,\mathbf{k}\to 0) = m_E{}^2 + \xi\frac{Ng^2Tm_E}{4\pi} - \frac{Ng^2T}{2\pi}\left[\frac{\mathbf{k}^2 - m_E{}^2}{|\mathbf{k}|}\arctan(|\mathbf{k}|/m_E) + m_E\right]$$

$$= \left[m_E{}^2 + \xi\frac{Ng^2Tm_E}{4\pi}\right] - \frac{2Ng^2T}{3\pi m_E}\mathbf{k}^2 + O\left[\frac{\mathbf{k}^4}{Tm_E}\right] \ .$$

The electrostatic propagator is then

$$iD_{44}(k_4=0,\mathbf{k}\to 0) = \frac{-i}{\mathbf{k}^2 - \Pi_{44}} = \frac{1}{1 - \dfrac{2Ng^2T}{3\pi m_E}}\,\frac{-i}{\mathbf{k}^2 + m_E{}^2\left[1 + \dfrac{Ng^2T}{4\pi m_E}(\xi + \frac{8}{3})\right]} \ .$$

Since $m_E \sim gT$, the correction to $m_E{}^2$ is $O(g^3)$; an appropriate choice of gauge makes it vanish. This strongly suggests that the $O(g^3)$ term could in fact be a gauge artifact.

Regarding attempts to compute the Debye mass from the polarization tensor in a specific gauge, it should be emphasized that physical quantities must first be defined gauge invariantly before one can set out to compute them. Otherwise one risks computing something different. Thus, the static infrared limit of Π_{44} cannot in general be regarded as the physical Debye mass. However, the computation of Π_{44} in a particular gauge may have a direct physical meaning, provided one can relate the Π_{44} so calculated to the physical definition in terms of a gauge-invariant correlation function.

A. Periodic Wilson loop

At zero temperature, one "makes the gluon propagator gauge invariant" by considering it to be the dominant exchange in a Wilson-loop computation. Thus, the static Wilson loop,[9,10] which yields the singlet $\bar{q}q$ potential, has been useful in perturbative calculations of heavy quarkonia. One may try the same tactics at finite temperature, defining a direct finite-temperature analogue of the static Wilson loop, which we call the "periodic Wilson loop:"

$$w[S(R)] \equiv \left\langle \widetilde{\mathrm{Tr}} P \exp\left[-ig \oint_{S(R)} dx_\mu A_\mu\right]\right\rangle$$

$$(\widetilde{\mathrm{Tr}} \equiv \mathrm{Tr}/\mathrm{Tr}1 = \mathrm{Tr}/N)\ .$$

In analogy with the zero-temperature case, we take the contour $S(R)$ to have temporal extent β and spatial extent R. The contour is not unique because any arbitrary spacelike curve may be chosen to connect the ends at $\tau = 0,\beta$.

Define the "string operator"

$$W_\Gamma(\mathbf{R},\mathbf{0}) \equiv P \exp\left[-ig \int_\Gamma dx_i A_i(x)\right]\ ,$$

where the contour Γ runs from $\mathbf{0}$ to $\mathbf{R}$ at fixed τ. In static gauge [$\partial_4 A_4 = 0$ (Refs. 11 and 5)]; see also the next section], we can rewrite $w[S(R)]$ in terms of the EQCD_3 scalar field $\phi = A_4\sqrt{\beta}$:

$$e^{-\beta V_\Gamma(R)} = \langle \widetilde{\mathrm{Tr}} e^{ig\sqrt{\beta}\phi(0)} W_\Gamma(\mathbf{0},\mathbf{R}) e^{-ig\sqrt{\beta}\phi(\mathbf{R})} W_\Gamma(\mathbf{R},\mathbf{0})\rangle\ ,$$

$V_\Gamma(R)$ being the potential defined by $w[S(R)]$. Expanding the exponentials, keeping only the terms linear in β, we find

$$V_\Gamma(R) = -g^2 \langle \widetilde{\mathrm{Tr}} \phi(\mathbf{0}) W_\Gamma(\mathbf{0},\mathbf{R}) \phi(\mathbf{R}) W_\Gamma(\mathbf{R},\mathbf{0})\rangle + \cdots\ ,$$

i.e., at high temperature the potential is essentially the EQCD_3 scalar propagator, made gauge invariant by the introduction of strings.

At leading order, the strings may be set to unity and the potential is given by the bare ϕ propagator:

$$V_\Gamma^{(0)}(R) = -\frac{g^2}{2N}\langle \phi^a(\mathbf{0})\phi^a(\mathbf{R})\rangle = -g^2 \frac{(N^2-1)}{2N}\frac{e^{-m_E R}}{4\pi R}\ .$$

This is precisely the Debye screened version of the leading zero temperature Coulomb potential.[9]

At the next order, however, the strings begin to contribute. What we therefore achieve by enclosing the gluon propagator in a Wilson loop is merely the trading of gauge dependence for path dependence. This problem of nonuniqueness does not arise at zero temperature, since the strings lie at $t = \pm\infty$ and reduce to unity for arbitrary paths.

We learn, then, that the gauge-invariant correlation function in terms of which we define the Debye mass must be disconnected in the sense of being a true two-point function; there must be "no strings attached" to its ends to make it gauge invariant.

B. Comment on the use of temporal gauge

It is instructive to consider again the leading-order contribution to the periodic Wilson loop in an arbitrary gauge. The following result can easily be shown to hold in any gauge whatsoever, provided only that periodic boundary conditions are satisfied:

$$w[S(R)] = 1 + \frac{(N^2-1)g^2}{2NT}\int \frac{d^3\mathbf{k}}{(2\pi)^3}(1-\cos\mathbf{k}\cdot\mathbf{R}) \times D_{44}^{(0)}(k_4=0,\mathbf{k}) + \cdots\ ,$$

where $iD_{\mu\nu}^{(0)}$ is the free gluon propagator. Thus, $D_{44}^{(0)}(k_4=0)$ must be gauge invariant (up to pieces of vanishing Fourier transform). Indeed, in any of the usual gauges (ξ covariant, Coulomb, static, . . .) one has $D_{44}^{(0)}(k_4=0) = -1/\mathbf{k}^2$. In the $A_4 = 0$ gauge, on the other hand, $D_{44}^{(0)} = 0$.

In Ref. 12, the static infrared limit of Π_{44} in the temporal ($A_4 = 0$) gauge has been claimed to be the physical Debye mass, despite the well-known incompatibility of the temporal gauge with the periodic boundary conditions one uses at finite temperature (see, e.g., Ref. 1: the finite-temperature analogue of the temporal gauge is in fact seen to be not the temporal but the static gauge). The formalism in Ref. 12 happens to yield the correct value for the leading Debye mass and this is taken by the authors to indicate its basic soundness; they therefore go on to compute higher-order corrections. No attempt appears to have been made to relate this calculation to that of a gauge-invariant correlation function. The agreement at leading order seems to us to be fortuitous; the above periodic Wilson-loop calculation shows that not all leading results are correctly reproduced and therefore there is good reason to question the higher-order ones.

III. POLYAKOV LOOP CORRELATION

Our aim is to try and calculate the $O(g^3)$ correction Δm^2 to a properly defined Debye mass. We shall read off the Debye mass from the exponential falloff of a suitably chosen gauge-invariant electrostatic correlation function at infrared separation. The natural choice, which we shall now consider, is the Polyakov loop correlation (PLC).

The "Polyakov loop operator"[13] (also referred to in the literature by the names "Wilson string," "Wilson line," or "thermal Wilson loop") at a spatial point $\mathbf{x}$ is defined as

$$\Omega(\mathbf{x})\equiv P\exp\left[-ig\int_0^\beta d\tau\, A_4(\mathbf{x},\tau)\right]\quad(\beta\equiv 1/T)\ ,$$

which under periodic gauge transformations $U(\mathbf{x},\tau)$ transforms as

$$\Omega(\mathbf{x})\ \overset{U(\mathbf{x},\tau)}{\longrightarrow}\ U(\mathbf{x},\beta)\Omega(\mathbf{x})U^\dagger(\mathbf{x},0)\ .$$

Since $U(\mathbf{x},\beta)=U(\mathbf{x},0)$, $\mathrm{Tr}\Omega(\mathbf{x})$ is a gauge-invariant operator. The gauge-invariant resummation of m is most conveniently performed in the *static gauge*,[5,11] defined by $\partial_4 A_4=0$, in which we have the simple expression

$$\Omega(\mathbf{x})=e^{-ig\beta A_4(\mathbf{x})}=e^{-ig\sqrt{\beta}\phi(\mathbf{x})}\ ,$$

ϕ being the adjoint scalar field in EQCD$_3$.

The free energy of an isolated static quark at location $\mathbf{x}$ is given by $\langle\tilde{\mathrm{Tr}}\Omega(\mathbf{x})\rangle$, while the correlation of two Polyakov loops separated by a distance R determines the free energy $F(R)$ of a quark-antiquark pair according to the equation

$$\exp[-\beta F(R)]=\langle\tilde{\mathrm{Tr}}\Omega^\dagger(\mathbf{0})\tilde{\mathrm{Tr}}\Omega(\mathbf{R})\rangle$$
$$(\tilde{\mathrm{Tr}}\equiv\mathrm{Tr}/\mathrm{Tr}1=\mathrm{Tr}/N)\ .$$

By retaining only the connected part, one obtains[7,10,14] the color average of the $q\bar{q}$ potential energy in the singlet and adjoint channels:

$$\frac{1}{N^2}\{\exp[-\beta V_1(R)]+(N^2-1)\exp[-\beta V_{\mathrm{adj}}(R)]\}=\langle\tilde{\mathrm{Tr}}\Omega^\dagger(\mathbf{0})\tilde{\mathrm{Tr}}\Omega(\mathbf{R})\rangle_c\equiv C_{\mathrm{PL}}(R)\ .$$

In the static gauge, the Polyakov loop operators may be expanded in terms of the EQCD$_3$ field ϕ, and we get

$$C_{\mathrm{PL}}(R)=1+\frac{g^4\beta^2}{4}\langle\tilde{\mathrm{Tr}}\phi^2(\mathbf{0})\tilde{\mathrm{Tr}}\phi^2(\mathbf{R})\rangle_c+\frac{g^6\beta^3}{36}\langle\tilde{\mathrm{Tr}}\phi^3(\mathbf{0})\tilde{\mathrm{Tr}}\phi^3(\mathbf{R})\rangle_c-\frac{g^6\beta^3}{24}\langle\tilde{\mathrm{Tr}}\phi^4(\mathbf{0})\tilde{\mathrm{Tr}}\phi^2(\mathbf{R})\rangle_c+\text{higher orders}\ .$$

Notice that there is no single-gluon-exchange term, since the generators of SU(N) are all traceless.

Leading-order results

We compute at the tree level the EQCD$_3$ correlation functions occurring in the above equation. The bare ϕ-ϕ propagator in EQCD$_3$ is just $\exp(-m_E R)/4\pi R$, and we need only do the group-theoretical traces; see the Appendix for an example. The results are

$$\langle\tilde{\mathrm{Tr}}\phi^2(\mathbf{0})\tilde{\mathrm{Tr}}\phi^2(\mathbf{R})\rangle_c^{(0)}=\frac{N^2-1}{2N^2}\left[\frac{e^{-m_E R}}{4\pi R}\right]^2,$$

$$\langle\tilde{\mathrm{Tr}}\phi^3(\mathbf{0})\tilde{\mathrm{Tr}}\phi^3(\mathbf{R})\rangle_c^{(0)}=\frac{N^2-1}{2N^2}\frac{3(N^2-4)}{4N}\left[\frac{e^{-m_E R}}{4\pi R}\right]^3,$$

$$\langle\tilde{\mathrm{Tr}}\phi^4(\mathbf{0})\tilde{\mathrm{Tr}}\phi^2(\mathbf{R})\rangle_c^{(0)}=\frac{N^2-1}{2N^2}\frac{2N^2-3}{N}\left[-\frac{m_E}{4\pi}\right]\times\left[\frac{e^{-m_E R}}{4\pi R}\right]^2,$$

where $m_E{}^2\equiv Ng^2T^2/3$. The three-gluon-exchange term above can be neglected to leading order. The contribution of the third correlation above is down by a factor $O(g^3)$ compared to the first and may also be neglected. We therefore find that the Polyakov loop correlation is given to leading order by

$$C_{\mathrm{PL}}(R)=1+\frac{\beta^2}{2!}\frac{N^2-1}{4N^2}\left[\frac{g^2e^{-m_E R}}{4\pi R}\right]^2+\cdots\ .$$

On the other hand, in terms of the singlet and adjoint potentials we have the expansion

$$C_{\mathrm{PL}}(R)=\frac{1}{N^2}\{\exp[-\beta V_1(R)]+(N^2-1)\exp[-\beta V_{\mathrm{adj}}(R)]\}=1+\frac{\beta^2 V_1{}^2(R)}{2(N^2-1)}+\cdots\ ,$$

where we have used the absence of a term linear in β to eliminate V_{adj}. Comparing the two expressions for $C_{\mathrm{PL}}(R)$, we find the leading values for the singlet and adjoint potentials

$$V_1^{(0)}=-(N^2-1)V_{\mathrm{adj}}^{(0)}=-g^2\frac{N^2-1}{2N}\frac{e^{-m_E R}}{4\pi R}\ .$$

We thus identify m_E as the leading value for the Debye mass m; note that $V_1^{(0)}$ is the same as the leading value of V_Γ obtained in the previous section.

IV. MEAN-SQUARE CORRELATION

We have seen in the preceding section that the correlation of two Polyakov loops is dominated by two-gluon exchange, the relevant contribution being proportional to the following EQCD$_3$ correlation function, which we shall call the "mean-square correlation," denoted by $f_{\mathrm{ms}}(R)$:

$$\langle\tilde{\mathrm{Tr}}\phi^2(\mathbf{0})\tilde{\mathrm{Tr}}\phi^2(\mathbf{R})\rangle\equiv f_{\mathrm{ms}}(R)=f_{\mathrm{ms}}^{(0)}(R)+f_{\mathrm{ms}}^{(2)}(R)+\cdots\ .$$

The leading value of $f_{\mathrm{ms}}(R)$, depicted graphically in Fig. 1(a), was shown to be

$$f_{\rm ms}^{(0)}(R)=\frac{N^2-1}{2N^2}\left[\frac{e^{-m_E R}}{4\pi R}\right]^2 ,$$

where m_E is the mass parameter for the electrostatic potential ϕ in the EQCD_3 Lagrangian, arising from the nonstatic modes of hot QCD; the full Debye mass m is the same as m_E to leading order.

To determine the $O(g^3)$ correction to m^2, we write

$$m^2=m_E{}^2+\Delta m^2$$

and resum Δm^2 by the usual trick of adding it to the free part of the EQCD_3 Lagrangian and subtracting it from the interaction part. The value of Δm^2 is then chosen so that after adding one-loop corrections (we are talking about loops in EQCD_3 here), the asymptotic form of $f_{\rm ms}(R)$ remains unchanged from the leading expression, but with m replacing m_E. In what follows, we first calculate the one-loop contribution and find the surprising result that for large distances it overwhelms the tree-level value, thus causing Δm^2 to be undetermined. We next examine higher-order contributions and find that the decay of $f_{\rm ms}(R)$, an apparently electrostatic correlation function, is in fact governed by the *magnetostatic* mass.

A. One-loop contribution

The one-loop contribution to $f_{\rm ms}(R)$ is represented by the two graphs shown in Fig. 1(b),

$$f_{\rm ms}^{(2)}(R)=(\mathrm{I})+(\mathrm{II}) ,$$

corresponding to the expressions

$$(\mathrm{I})=\frac{N^2-1}{2N^2}\left[\frac{e^{-mR}}{4\pi R}\right]\int\frac{d_3\mathbf{k}\,e^{-i\mathbf{k}\cdot\mathbf{R}}}{(\mathbf{k}^2+m^2)^2}\left[2\Delta m^2+2Ng^2T\int\frac{d_3 l(2k+l)_i(2k+l)_j}{[(\mathbf{k}+l)^2+m^2]l^2}\left[\delta_{ij}+(\xi-1)\frac{l_il_j}{l^2}\right]\right] ,$$

$$(\mathrm{II})=\frac{N^2-1}{2N^2}Ng^2T\int\frac{d_3\mathbf{k}_1d_3\mathbf{k}_2d_3l\,e^{-i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{R}}}{(\mathbf{k}_1{}^2+m^2)(\mathbf{k}_2{}^2+m^2)[(\mathbf{k}_1+l)^2+m^2][(\mathbf{k}_2+l)^2+m^2]}\times\frac{(2k_1+l)_i(2k_2+l)_j}{l^2}\left[\delta_{ij}+(\xi-1)\frac{l_il_j}{l^2}\right] .$$

Here $d_3\mathbf{k}\equiv d^3\mathbf{k}/(2\pi)^3$ and we have used a covariant gauge, ξ being the gauge parameter. It is easily verified that the gauge dependences of (I) and (II) cancel out, and we may choose ξ at will. Scaling $f_{\rm ms}^{(2)}(R)$ with respect to $f_{\rm ms}^{(0)}(R)$, we write

$$f_{\rm ms}^{(2)}(R)=f_{\rm ms}^{(0)}(R)[f_{\rm I}(R)+f_{\rm II}(R)] ,$$

where we have, in the Feynman ($\xi=1$) gauge,

$$f_{\rm I}=e^{mR}4\pi R\int\frac{d_3\mathbf{k}\,e^{-i\mathbf{k}\cdot\mathbf{R}}}{(\mathbf{k}^2+m^2)^2}\left[2\Delta m^2+2Ng^2T\int\frac{d^3l(2\mathbf{k}+l)^2}{[(\mathbf{k}+l)^2+m^2]l^2}\right] ,$$

$$f_{\rm II}=e^{2mR}(4\pi R)^2Ng^2T\int\frac{d_3\mathbf{k}_1d_3\mathbf{k}_2d_3l\,e^{-i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{R}}(2\mathbf{k}_1+l)\cdot(2\mathbf{k}_2+l)}{(\mathbf{k}_1{}^2+m^2)(\mathbf{k}_2{}^2+m^2)[(\mathbf{k}_1+l)^2+m^2][(\mathbf{k}_2+l)^2+m^2]l^2} .$$

The integrals in $f_{\rm I}$ and $f_{\rm II}$ can be evaluated exactly; details of the calculation are to be found in the Appendix. Here we present the results for two limiting cases.

1. *Massless limit—$m \ll 1/R$*

This limit corresponds to removing the Debye mass resummation and makes the bare electrostatic propagator massless. Accordingly, infrared divergences show up in the results, signaling the need for such resummation. If $m^2\to 0$, then $\Delta m^2\to -m_E{}^2$, and we find that

$$f_{\rm I}=-\frac{m_E{}^2R}{m}-\frac{Ng^2TR}{\pi}[\ln 2mR+\gamma-\tfrac{3}{4}+mR\ln 2mR+O(mR)] ,$$

$$f_{\rm II}=\frac{Ng^2TR}{\pi}\left[1-\frac{\pi^2}{16}+O(mR)\right] .$$

Adding the two results, we get

$$(f_{\rm I}+f_{\rm II})=-\frac{m_E{}^2R}{m}-\frac{Ng^2TR}{\pi}\left[\ln 2mR+\gamma+\frac{\pi^2}{16}-\tfrac{7}{4}+O(mR\ln mR)\right] .$$

The linear and logarithmic divergences as $m\to 0$, previously encountered at two loops in the hot QCD vacuum-polarization tensor,[5] are here seen to have a gauge-invariant meaning. Thus mass resummation in the electrostatic sector is essential.

2. *Long-distance limit—$R \gg 1/m$*

This is the appropriate limit for extracting the Debye mass. Having made the $mR\to\infty$ approximation, the idea is to choose the mass shift Δm^2 such that the higher-order correction $f_{\rm ms}^{(2)}(R)$ is negligible compared to the tree-level value $f_{\rm ms}^{(0)}(R)$. We obtain, in the long-distance limit,

$$f_{\mathrm{I}}=mR\left\{\frac{\Delta m^2}{m^2}+\frac{Ng^2T}{2\pi m}\left[-\ln 2mR+\tfrac{3}{2}-\gamma+\frac{1}{2mR}+O\left[\frac{1}{m^2R^2}\right]\right]\right\},$$

$$f_{\mathrm{II}}=mR\frac{Ng^2T}{2\pi m}\left[\frac{\ln mR+\gamma}{2}+\frac{3\ln mR}{4mR}+\frac{6\gamma+4\ln 2-5}{8mR}+O\left[\frac{\ln mR}{m^2R^2}\right]\right],$$

$$(f_{\mathrm{I}}+f_{\mathrm{II}})=\frac{R}{m}\left[\Delta m^2+\frac{Ng^2Tm}{4\pi}(3-\gamma-2\ln 2)\right]+\frac{Ng^2T}{4\pi m}\left[-mR\ln mR+\frac{3\ln mR}{2}+\frac{6\gamma+4\ln 2-1}{4}+O\left[\frac{\ln mR}{mR}\right]\right].$$

We find that the leading logarithms in graphs I and II do not cancel, and thus Δm^2 is in fact undefined. The contribution of the second-order graphs for $f_{\mathrm{ms}}(R)$ therefore overwhelms the tree-level value, and we are faced with an unexpected breakdown of perturbation theory in the electrostatic sector.

B. Higher orders

At higher orders, we are in for more surprises. Consider the fourth-order contribution to $f_{\mathrm{ms}}(R)$. The relevant graphs are shown in Fig. 1(c). The last four graphs represent the exchange of two *magnetostatic* gluons. Since a possible magnetostatic mass gap can be no larger than $O(g^2T)$, we realize that the Polyakov loop correlation is in fact dominated by the magnetostatic sector of the theory.

V. CONCLUSION

The non-Abelian Debye screening mass appears to be an ill-defined concept in perturbation theory. Naively, one would calculate it by mimicking the procedure followed in Abelian theories, viz., by taking the static infrared limit of the vacuum-polarization tensor. This method yields, at the one-loop level, the correct gauge-invariant value of $m_E{}^2=Ng^2T^2/3$. At higher orders, the gauge dependence of the non-Abelian vacuum-polarization tensor forces one to depart from the naive method and turn instead to a manifestly gauge-invariant definition. In this paper, we studied the most natural such generalization, based on the correlation of two Polyakov loops which represents the color-averaged static quark-antiquark potential. We found that to leading order this yielded the same Debye mass as the above one-loop result, but were unable to compute corrections to this value for the following reasons: (i) The next-order contribution overwhelms the leading one at distances larger than the Debye screening length; (ii) higher-order graphs are dominated by the magnetostatic mass gap, which at $O(g^2T)$ is smaller than the $O(gT)$ Debye screening length. Thus we are no better off than we were at the start.

Since $C_{\mathrm{PL}}(R)$ represents a complicated combination of quark-antiquark potentials in the singlet and adjoint channels, it might be conjectured that any Debye screening taking place in either or both of these channels is being washed out by the color averaging. In order to project out the channels in $C_{\mathrm{PL}}(R)$, we must consider a correlation function which is free from color averaging and therefore dominated by single-gluon exchange. This is the correlation of the *eigenvalues* (rather than the trace) of the operator Ω; it is directly related to the singlet potential.[14]

The breakdown in perturbation theory implies that the leading value of the Debye screening mass applies only to a limited range of distances. In the deep infrared, i.e., distances on the order of (g^2T) and beyond, it may be necessary to resum large orders of perturbation theory to produce an effective coupling constant in terms of which the theory could be expanded. Alternatively, one might imagine that the infrared divergences cause a dynamical breakdown in the gauge symmetry of the static sector, leading to an expansion about the wrong vacuum in the above calculations.

The calculation of the singlet potential and the question of symmetry breakdown are intimately connected, and shall be taken up in a forthcoming paper.[14]

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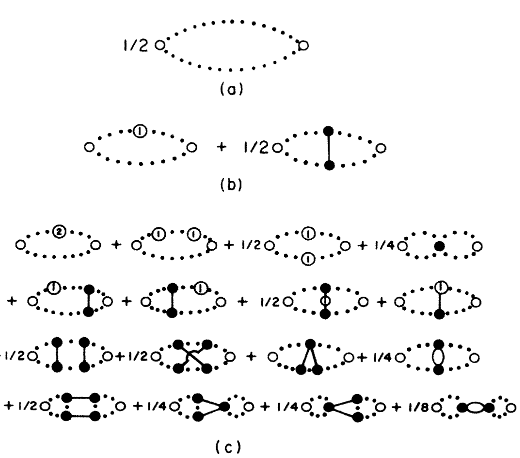

FIG. 1. Graphs contributing to the mean-square correlation function in EQCD$_3$: (a) zeroth order (tree level), (b) second order, (c) fourth order. Dotted lines are electrostatic (ϕ) propagators, solid lines are magnetostatic (A_i) propagators and the bubbles marked 1 and 2 are one-loop (see Fig. 2) and two-loop self-energy insertions; for EQCD$_3$ Feynman rules, see Ref. 5.

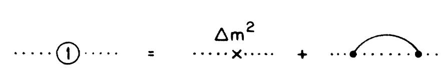

FIG. 2. The one-loop electrostatic self-energy in EQCD$_3$.

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APPENDIX: CALCULATION OF THE MEAN-SQUARE CORRELATION FUNCTION

We calculate the mean-square correlation function $f_{\rm ms}(R)$ to $O(G^2=g^2T)$. The relevant graphs are shown in Figs. 1 and 2. At tree level we have

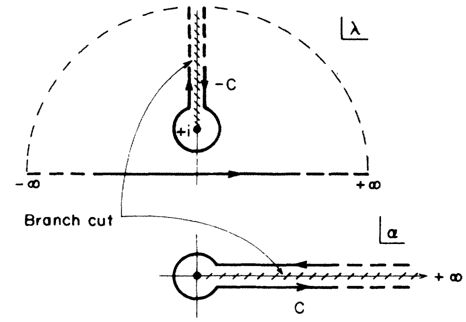

FIG. 3. The contour C in the λ and α planes.

$$f_{\rm ms}^{(0)}(R)=\langle \widetilde{\rm Tr}\phi^2(\mathbf{0})\widetilde{\rm Tr}\phi^2(\mathbf{R})\rangle_c^{(0)}=\frac{1}{N^2}{\rm Tr}(T^aT^b){\rm Tr}(T^{a'}T^{b'})\langle\phi^a(\mathbf{0})\phi^b(\mathbf{0})\phi^{a'}(\mathbf{R})\phi^{b'}(\mathbf{R})\rangle_c^{(0)}$$
$$=\frac{1}{N^2}\frac{\delta^{ab}}{2}\frac{\delta^{a'b'}}{2}(\delta^{aa'}\delta^{bb'}+\delta^{ab'}\delta^{ba'})\langle\phi\phi\rangle_R{}^2$$
$$=\frac{N^2-1}{2N^2}\left[\frac{e^{-m_ER}}{4\pi R}\right]^2 .$$

At the second order we must calculate the integrals $f_{\rm I}$ and $f_{\rm II}$ defined in the text. We shall do this in a general covariant gauge. We have the following decomposition:

$$f_{\rm I,II}(\text{general }\xi\text{ gauge})=f_{\rm I,II}(\text{Landau gauge, }\xi=0)+f_{\rm I,II}^{\xi} ,$$

where $f_{\rm I,II}^{\xi}$ are the pieces proportional to ξ and are readily calculated to be

$$-f_{\rm I}^{\xi}=f_{\rm II}^{\xi}=\xi\frac{Ng^2TR}{4\pi} .$$

We thus need to calculate $f_{\rm I,II}$ in Landau gauge. In each case, our strategy will be to do the integrations over the exponential factors last.

1. Calculation of $f_{\rm I}$

We have in the Landau gauge

$$f_{\rm I}=\frac{R\Delta m^2}{m}+e^{mR}32\pi RNg^2T\int\frac{d_3\mathbf{k}\,e^{-i\mathbf{k}\cdot\mathbf{R}}(\mathbf{k}^2\delta_{ij}-k_ik_j)}{(\mathbf{k}^2+m^2)^2}\int\frac{d_3l\,l_il_j}{[(\mathbf{k}+\mathbf{l})^2+m^2]l^4} .$$

Doing the integration over l and the angular integration over $\mathbf{k}$, we get

$$f_{\rm I}=R\left[\frac{\Delta m^2}{m}+\frac{Ng^2T}{2\pi}\right]+R\frac{Ng^2T}{2\pi}\frac{2e^{mR}}{i\pi mR}\int_{-\infty}^{+\infty}\frac{d\lambda\,e^{imR\lambda}(\lambda^2-1)\arctan\lambda}{(1+\lambda^2)^2},\quad \lambda\equiv|k|/m .$$

We now use the identity

$$\arctan\lambda=\frac{1}{2i}\ln\frac{i-\lambda}{i+\lambda} ,$$

and set $\lambda=i(\alpha+1)$ to get

$$f_{\rm I}=mR\left[\frac{\Delta m^2}{m^2}+\frac{Ng^2T}{2\pi m}\right]+mR\frac{Ng^2T}{2\pi m}\left[\frac{i}{\pi mR}\right]\int_C\frac{d\alpha\,e^{-mR\alpha}(\alpha^2+2\alpha+2)}{(\alpha^2+2\alpha)^2}\ln\left[\frac{-\alpha}{2+\alpha}\right] ,$$

where the contour C is shown in Fig. 3. On performing the contour integration we obtain

$$f_{\rm I}=mR\left[\frac{\Delta m^2}{m^2}+\frac{Ng^2T}{2\pi m}\right]+mR\frac{Ng^2T}{2\pi m}[-\ln 2mR+(1-\gamma)-e^{2mR}{\rm Ei}(-2mR)] ,$$

where ${\rm Ei}(x)$ is the exponential-integral function.[15]

2. Calculation of $f_{\rm II}$

In the Landau gauge, $f_{\rm II}$ is given by

$$f_{\rm II}=e^{2mR}(4\pi R)^2 4Ng^2T\int\frac{d_3\mathbf{k}_1 d_3\mathbf{k}_2 d_3\boldsymbol{l}\, e^{-i(\mathbf{k}_1-\mathbf{k}_2)\cdot R}[\boldsymbol{l}^2\mathbf{k}_1\cdot\mathbf{k}_2-(\boldsymbol{l}\cdot\mathbf{k}_1)(\boldsymbol{l}\cdot\mathbf{k}_2)]}{(\mathbf{k}_1{}^2+m^2)(\mathbf{k}_2{}^2+m^2)[(\mathbf{k}_1+\boldsymbol{l})^2+m^2][(\mathbf{k}_2+\boldsymbol{l})^2+m^2]\boldsymbol{l}^4}\ .$$

On substituting the expansions

$$2\mathbf{k}_1\cdot\mathbf{k}_2=(\mathbf{k}_1{}^2+m^2)+(\mathbf{k}_2{}^2+m^2)-[(\mathbf{k}_1-\mathbf{k}_2)^2+2m^2]\ ,$$

$$2\mathbf{k}_{1,2}\cdot\boldsymbol{l}=[(\mathbf{k}_{1,2}+\boldsymbol{l})^2+m^2]-(\mathbf{k}_{1,2}{}^2+m^2)-\boldsymbol{l}^2\ ,$$

and redefining some of the momenta, we get

$$\begin{aligned}f_{\rm II}=e^{2mR}(4\pi R)^2Ng^2T\Bigg[&4\int\frac{d_3\mathbf{k}_1d_3\mathbf{k}_2d_3\boldsymbol{l}\,e^{-i(\mathbf{k}_1-\mathbf{k}_2)\cdot R}}{(\mathbf{k}_1{}^2+m^2)[(\mathbf{k}_1+\boldsymbol{l})^2+m^2](\mathbf{k}_2{}^2+m^2)\boldsymbol{l}^2}\\&-2\int\frac{d_3\mathbf{k}\,e^{-i\mathbf{k}\cdot\mathbf{R}}(\mathbf{k}^2+2m^2)d_3\mathbf{p}\,d_3\mathbf{q}}{(\mathbf{p}-\mathbf{q})^2(\mathbf{p}^2+m^2)(\mathbf{q}^2+m^2)[(\mathbf{p}+\mathbf{k})^2+m^2][(\mathbf{q}+\mathbf{k})^2+m^2]}\\&-2\left[\int\frac{d_3\mathbf{k}\,e^{-i\mathbf{k}\cdot\mathbf{R}}}{\mathbf{k}^2+m^2}\right]^2\int\frac{d_3\boldsymbol{l}(1-e^{i\boldsymbol{l}\cdot\mathbf{R}})}{\boldsymbol{l}^4}-\int d_3\mathbf{k}\,e^{-i\mathbf{k}\cdot\mathbf{R}}\left[\int\frac{d_3\boldsymbol{l}}{[(\boldsymbol{l}+\mathbf{k})^2+m^2](\boldsymbol{l}^2+m^2)}\right]^2\Bigg]\ .\end{aligned}$$

As in the case of $f_{\rm I}$, we now perform some of the momentum integrations, do the angular integrations over the remaining momentum variables, and express the final momentum integrals as contour integrals over the complex variable α, to obtain

$$\begin{aligned}f_{\rm II}=&-\frac{Ng^2T}{2\pi^2im}\int_C\frac{d\alpha\,e^{-mR\alpha}}{\alpha(2+\alpha)}\ln\left[-\frac{\alpha}{2+\alpha}\right]-\frac{Ng^2TR}{4\pi^2i}\int_C\frac{d\alpha(1+4\alpha+2\alpha^2)e^{-2mR\alpha}}{\alpha(1+\alpha)(2+\alpha)}\ln[-\alpha(2+\alpha)]\\&-\frac{Ng^2TR}{4\pi}+\frac{Ng^2TR}{16\pi^2i}\int_C\frac{d\alpha\,e^{-2mR\alpha}}{\alpha+1}\ln^2\left[-\frac{\alpha}{2+\alpha}\right]\ ,\end{aligned}$$

where we have used

$$\int\frac{d_3\mathbf{p}\,d_3\mathbf{q}}{(\mathbf{p}-\mathbf{q})^2(\mathbf{p}^2+m^2)(\mathbf{q}^2+m^2)[(\mathbf{p}+\mathbf{k})^2+m^2][(\mathbf{q}+\mathbf{k})^2+m^2]}=\frac{\ln(1+\mathbf{k}^2/4m^2)}{16\pi^2\mathbf{k}^2(\mathbf{k}^2+4m^2)}\ ,$$

and the contour C is the same as before. On doing the contour integrations, we finally obtain

$$\begin{aligned}f_{\rm II}=&\frac{Ng^2T}{2\pi m}[\ln 2mR+\gamma-e^{2mR}{\rm Ei}(-2mR)]+\frac{Ng^2TR}{4\pi}[\ln mR+\gamma+2e^{2mR}{\rm Ei}(-2mR)+e^{4mR}{\rm Ei}(-4mR)]\\&-\frac{Ng^2TR}{4\pi}-\frac{Ng^2TR}{4\pi}\int_0^\infty\frac{dx}{x+1}\ln\left[\frac{x+2}{x}\right]e^{-2mRx}\ .\end{aligned}$$

3. Summary of calculations

From the above results we write down $f_{\rm I,II}$ in a general ξ gauge:

$$f_{\rm I}=mR\frac{\Delta m^2}{m^2}+mR\frac{Ng^2T}{2\pi m}\left[-\ln 2mR+\left[2-\frac{\xi}{2}-\gamma\right]-h(2mR)\right]\ ,$$

$$f_{\rm II}=mR\frac{Ng^2T}{2\pi m}\left[\frac{\ln mR+\gamma+\xi-1}{2}-\tfrac{1}{2}g(2mR)+\frac{\ln 2mR+\gamma}{mR}+h(2mR)+\tfrac{1}{2}h(4mR)-\frac{h(2mR)}{mR}\right]\ ,$$

where we have introduced the special functions

$$g(2mR) \equiv \int_0^\infty \frac{dx}{x+1} e^{-2mRx} \ln\left[\frac{x+2}{x}\right] = \begin{cases} \frac{\pi^2}{4} + 4mR \ln 2mR + O(2mR), & mR \ll 1 , \\ \frac{\ln 4mR + \gamma}{2mR} + O\left[\frac{\ln 2mR}{(2mR)^2}\right], & mR \gg 1 , \end{cases}$$

$$h(nmR) \equiv e^{nmR}\text{Ei}(-nmR) = \begin{cases} \ln mR + \gamma + nmR \ln nmR + (\gamma - 1)nmR + \frac{(nmR)^2 \ln nmR}{2} + O((nmR)^2), & mR \ll 1 , \\ -\frac{1}{nmR} + \frac{1}{(nmR)^2} + O\left[\left[\frac{1}{nmR}\right]^3\right], & mR \gg 1 . \end{cases}$$

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