Symmetry breaking in six-dimensional Einstein-Maxwell-σ theory

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The mass spectrum of a six-dimensional gravity theory coupled with the U(1) Maxwell and nonlinear σ fields is analyzed. It is shown that this electroweak-gravity model can have a perturbatively stable ground state and low-mass gauge bosons of SU(2). Except for the graviton, photon, low-mass scalar triplet, and three gauge bosons, all other states acquire masses of the Planck scale.

I. INTRODUCTION

The standard model of Salam and Weinberg using the $SU(2)\times U(1)$ gauge group has succeeded in accounting for a vast range of data related to electroweak interactions. But the Higgs mechanism which is required to provide masses for the gauge bosons is the least attractive aspect of the standard model.[1] There is a variety of theoretical reasons to suppose that this mechanism is an incomplete description of the nature of electroweak symmetry breaking. The Higgs potential of the Lagrangian depends on arbitrary parameters, and the Higgs-boson mass is not fixed by currently measured quantities. Other theories, for example, the technicolor models, are devised to remedy some of the above defects by replacing the Higgs sector with a new set of "techni-fermions" that generates dynamical symmetry breaking.[2] But it is not clear if the new "technicolor" strong gauge interaction really corresponds to nature.

In this paper, we introduce another model for symmetry breaking. We try this by showing that a Higgs phenomenon takes place, giving reasonable masses to the gauge bosons through the compactification mechanism in a Kaluza-Klein-type theory.[3] For this purpose, we introduce the nonlinear σ field and Maxwell field as the matter fields which induce the compactification in six-dimensional gravity theory.

This six-dimensional Einstein-Maxwell-σ model has the merit that it has a simple and close structure to the standard model. The σ field and Maxwell field were used independently to induce the compactification. When the Maxwell field alone is used to obtain a spontaneously compactified solution, the generated spectrum contains massless gauge vectors corresponding to the local symmetry of SU(2) (Ref. 4). Contrary to this, dimensional reduction induced by the nonlinear σ model was shown to give Planck's mass to the gauge vectors. This scheme was studied first by Omero and Percacci and revised by Gell-Mann and Zwiebach.[5] By introducing these two fields simultaneously, we can obtain massive gauge bosons not of the Planck-mass scale. The other low-lying states in this model are the massless graviton and photon and a low-mass scalar. All the other states have masses of the Planck scale and thus beyond the range of current or foreseeable accelerators. The σ model acts mainly as the trigger of the spontaneous symmetry breaking while the Maxwell field is responsible for the dimensional reduction from six to four dimensions.

The use of harmonic expansion on the two-sphere in analyzing the spectrum shows that this model has no tachyons, showing that the above compactification is stable and has no negative-metric ghost states. This internal manifold has also been advocated as the possible source of chiral fermions.[4] Even though our model to be discussed is not intended to be a realistic physical theory, we hope that all the above features will make our model a good starting point in constructing a realistic electroweak-gravity model.

II. BACKGROUND CLASSICAL SOLUTION

Now for the details. Our analysis is very similar to that of Randjbar-Daemi, Salam, and Strathdee.[4] The six-dimensional Einstein-Maxwell-σ field theory with a cosmological constant is characterized by the action

$$S=-\int d^6Z\sqrt{-g}\left[\frac{1}{K^2}R+\tfrac{1}{4}F_{MN}F^{MN}+\lambda+\frac{1}{2t}g^{MN}\partial_M\phi^\mu\partial_N\phi^\nu h_{\mu\nu}\right], \quad (1)$$

where R denotes the curvature scalar, and

$$F_{MN}=\partial_M A_N-\partial_N A_M\ . \quad (2)$$

The scalar fields $\phi^\mu(x)$, $\mu=1,2$ are thought of as coordinates of a two-sphere S^2 with metric $h_{\mu\nu}$. Our conventions are as follows. We write the coordinates Z^M as (x^m,y^μ). Thus upper case indices take the value $0,1,\ldots,6$ while lower case latin indices take the value 0, 1, 2, and 3. The lower case greek indices are used as coordinates and frame labels of S^2. The signature is $-+\cdots+$ and $R_{LM}=R^K{}_{LMK}$, $R^K{}_{LMN}=\partial_M\Gamma^K{}_{LM}-\cdots$.

The classical equations of motion from the action are

$$R_{MN}-\tfrac{1}{2}g_{MN}R=-\frac{K^2}{2}(T_{MN}-\lambda g_{MN})\ ,$$
$$F^{MN}{}_{;M}=\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}F^{MN})=0\ , \quad (3)$$
$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}g^{MN}\partial_N\phi^\mu)+\Gamma^\mu{}_{\nu\delta}\partial_M\phi^\nu\partial_N\phi^\delta g^{MN}=0\ ,$$

with the energy-momentum tensor

$$T_{MN}=\frac{1}{t}h_{\mu\nu}(\partial_M\phi^\mu\partial_N\phi^\nu-\tfrac{1}{2}g_{MN}g^{PQ}\partial_P\phi^\mu\partial_Q\phi^\nu)+F_{ML}F_N{}^L-\tfrac{1}{4}g_{MN}F^2\ . \quad (4)$$

Here $\Gamma^\mu{}_{\nu\delta}$ is the Levi-Civita connection of the σ-field internal space and a semicolon denotes the standard covariant derivative. The classical solution with the compactification structure $M^4\times S^2$ is obtained by[4,5]

$$\begin{aligned}&\bar g_{MN}dZ^M dZ^N=\eta_{mn}dx^m dx^n+a^2(d\theta^2+\sin^2\theta\, d\phi^2)\ ,\\&\bar A_\mu(y)dy^\mu=\frac{n}{2e}(\cos\theta\mp 1)d\phi\ ,\\&\bar\phi^\mu(y)=y^\mu\ ,\\&h_{\mu\nu}[\phi(y)]=\bar g_{\mu\nu}(y)\ ,\end{aligned}\quad (5)$$

where a is the radius of the internal space to be determined. We use two coordinate patches to express the monopole configuration of the Maxwell field. Note that the physical coordinate and internal space of the σ field are both described by S^2, enabling the homotopic classification of $\pi_2(S^2)=Z$. The solution in (5) has the nontrivial $Z=1$ winding number, which with the nontrivial topology of vector bundles of the classical Maxwell field over compactified dimension, may be invoked to ensure both the stability of the compactification as well as the appearance of the massless chiral fermions in the four-dimensional space-time.[6]

By substituting these vacuum expectation values into (3), one could find the following algebraic equations from the first equation of (3) when M,N takes four-dimensional or internal coordinates, respectively:

$$\begin{aligned}&\frac{1}{a^2}=\frac{K^2}{8}\bar F^2+\frac{K^2}{2t}+\frac{\lambda}{2}K^2\ ,\\&0=\tfrac{1}{4}\bar F^2-\lambda\ ,\end{aligned}\quad (6)$$

where

$$\bar F^2=\frac{n^2}{2e^2a^4}=\frac{4\delta^2}{a^2K^2}\ . \quad (7)$$

The newly defined δ has the value of almost one as will be shown in Sec. III. The radius of internal space a to be of order one in Planck's units takes the value

$$\frac{1}{a^2}=\frac{K^2\bar F^2}{4}+\frac{K^2}{2t}\approx\frac{8e^2}{n^2K^2}-\frac{K^2}{2t} \quad (8)$$

with $8e^2/n^2K^2 \gg K^2/2t$. As is clear from the following analysis, the term $K^2/2t$, even though small, is important in obtaining the desired symmetry breaking.

III. FLUCTUATION ANALYSIS

We shall now examine the fluctuation on this background to derive the spectrum of masses for the scalar, vector, and tensor excitations. Consider the small fluctuation expansion of the action around the classical solution

$$\begin{aligned}&g_{MN}=\bar g_{MN}+Kh_{MN}\ ,\\&A_M=\bar A_M+V_M\ ,\\&\phi^\mu=y^\mu+Z^\mu\ ,\end{aligned}\quad (9)$$

and expand the action up to terms quadratic in the fluctuation fields h_{MN}, V_M, and Z^μ.

The contribution from the Einstein and Maxwell fields is essentially the same as that of Ref. 4, while that of scalars is given by Gell-Mann and Zweibach[5] with some notational and metric change. Add the following source terms to the fluctuation action:

$$S_{\text{source}}=\int d^6Z\sqrt{-g}\left[\tfrac{1}{2}T_{AB}h^{AB}+J_AV^A-\frac{1}{t}I_\mu Z^\mu\right]\ . \quad (10)$$

The equations of motion that follow from $S+S_{\text{source}}$ are

$$\begin{aligned}&\nabla^2(h_{AB}-\tfrac{1}{2}g_{AB}h)+(\bar R_{AC}h_B{}^C+\bar R_{BC}h_A{}^C)+\frac{K^2}{t}(g_{\mu\nu}h^\delta{}_\delta-h_{\mu\nu})+K\bar F_B{}^C(V_{C;A}-V_{A;C})+K\bar F_A{}^C(V_{C;B}-V_{B;C})\\&\qquad -Kg_{AB}\bar F^{DC}V_{C;D}+\frac{K}{t}(Z_{B;A}+Z_{A;B})-\frac{K}{t}g_{AB}Z_\mu{}^{;\mu}=-T_{AB}\ ,\\&\nabla^2V_A+\bar R_{AB}V^B+K\nabla_c(h_{AB}\bar F^{BC}+h_B{}^C\bar F_A{}^B-\tfrac{1}{2}h\bar F_A{}^C)=-J_A\ ,\\&\nabla^2Z_\mu-\bar R_{\mu\nu}Z^\nu-Kh_{\mu L}{}^{;L}+\tfrac{1}{2}Kh_{;\mu}=I_\mu\ .\end{aligned}\quad (11)$$

In the course of the above derivation, we use the harmonic Lorentz gauge condition. Now the sources T_{AB}, J_A, and I_μ are not independent; Eqs. (11) are compatible only if they are suitably constrained by

$$T^{aB}{}_{;B}=0,\quad T^{\mu B}{}_{;B}=KF^{\mu B}J_B-\frac{K}{t}I^\mu,\quad J^A{}_{;A}=0\ . \quad (12)$$

The next step is to solve the linear equations for h, V, and Z in terms of T, J, and I, using Eqs. (11). It is convenient to Fourier transform the dependence on the four-dimensional space, which we take to be flat. Furthermore, one expands all fields and sources in harmonics of the internal space $S^2=SU(2)/U(1)$. The expansion method in coset space is discussed by Salam and Strathdee in Refs. 4 and 7. In the following, we use the same notation as in Ref. 4. The fields are decomposed into irreducible representations of the SO(2) rotations, labeled by the "isohelicity" λ. In particular, the isohelicity of the scalar fields are ± 1 (Refs. 5 and 8).

We can now extract the following equations for the harmonic components, where we suppress the isospin labels:

$$\left[\partial^2-\frac{l^2+l-2}{a^2}\right]h_{\pm\pm}-\frac{K^2}{t}h_{\pm\pm}+\frac{2Ki}{at}\left[\frac{(l-1)(l+2)}{2}\right]^{1/2}Z_\pm=-T_{\pm\pm}\ ,$$

$$\left[\partial^2-\frac{l^2+l}{a^2}\right]h^a{}_a+\frac{4}{a^2}h_{+-}-\frac{2K^2}{t}h_{+-}+\frac{2\delta}{a^2}\sqrt{l(l+1)}(V_+-V_-)=2T_{+-}\ ,$$

$$\left[\partial^2-\frac{l^2+l}{a^2}\right]h_{a\pm}\pm\frac{i\sqrt{2}\delta}{a}\partial_aV_\pm\pm\frac{\delta\sqrt{l(l+1)}}{a^2}V_a+\frac{K}{t}\partial_aZ_\pm=-T_{a\pm}\ ,$$

$$\left[\partial^2-\frac{l^2+l}{a^2}\right](h_{ab}+g_{ab}h_{+-})-g_{ab}\frac{\delta}{a^2}\sqrt{l(l+1)}(V_+-V_-)+g_{ab}\frac{K}{t}\frac{i}{a}\left[\frac{l(l+1)}{2}\right]^{1/2}(Z_++Z_-)=-T_{ab}+\tfrac{1}{2}g_{ab}T^C{}_C\ ,$$

$$\left[\partial^2-\frac{l^2+l}{a^2}\right]V_a+\frac{\delta}{a^2}\sqrt{l(l+1)}(h_{a+}-h_{a-})=-J_a\ , \tag{13}$$

$$\left[\partial^2-\frac{l^2+l}{a^2}\right]V_\pm\pm i\frac{\sqrt{2}\delta}{a^2}\partial^bh_{b\pm}\mp\frac{\delta\sqrt{l(l+1)}}{a^2}(h_{+-}-\tfrac{1}{2}h^a{}_a)=-J_\pm\ ,$$

$$\left[\partial^2-\frac{l^2+l-2}{a^2}\right]Z_\pm=I_\pm\ .$$

The solutions of these equations are then substituted into the functional $S+S_{\text{source}}$. We now examine the action near the poles which result after the substitution. Of course, it is important to make use of the conservation law (12) to extract the physical part of the spectrum.

Let us now consider massive states. Choosing a coordinate frame, we take $p_a=(p_0,0,0,0)$. The pole terms in $S+S_{\text{source}}$ arrange into towers of spin-two, spin-one, and spin-zero states:

$$I\propto\sum_{l\geq 0}(I_0^{(+)}+I_0^{(-)}+I_2)+\sum_{l\geq 1}(I_1^{(+)}+I_1^{(-)})+\sum_{l\geq 2}I_0\ . \tag{14}$$

The explicit forms are

$$I_0=\frac{1}{p^2+M_0{}^2}\frac{1-\delta^2}{l(l+1)-2\delta^2}\left[\left|T_{++}+\frac{iaK}{\sqrt{2}t}\frac{(l^2+l-2)^{1/2}}{1-\delta^2}I_+\right|^2+\left|T_{--}+\frac{iaK}{\sqrt{2}t}\frac{(l^2+l-2)}{1-\delta^2}I_-\right|^2\right],$$

$$I_0^{(\pm)}=\frac{1}{p^2+M_{0\pm}{}^2}\frac{\pm1}{4\delta(\delta^2+7l^2+7l)\pm4(\delta^2+l^2+l)[\delta^2+12l(l+1)]^{1/2}}\frac{1}{[\delta^2+12l(l+1)]^{1/2}}$$
$$\times\Big|\{\delta^2\pm\delta[\delta^2+12l(l+1)]^{1/2}+2l(l+1)\}(T_{+-}-\tfrac{1}{2}T^a{}_a)+\{5\delta\pm[\delta^2+12l(l+1)]^{1/2}\}\sqrt{l(l+1)}(J_+-J_-)$$
$$-2\sqrt{(l-1)l(l+1)(l+2)}(T_{++}+T_{--})+2\sqrt{2}\frac{aKi}{t}\sqrt{l(l+1)}(I_++I_-)\Big|^2\ , \tag{15}$$

$$I_1^{(\pm)}=\frac{1}{p^2+M_{1\pm}{}^2}\tfrac{1}{4}\,|\,T_{i+}-T_{i-}\mp\sqrt{2}J_i\,|^2\ ,$$

$$I_2=\frac{1}{p^2+M_2{}^2}\tfrac{1}{2}\,|\,T_{ij}-\tfrac{1}{3}\delta_{ij}T_{kk}\,|^2\ ,$$

where the nonzero masses are given by

$$M_0{}^2=\frac{l(l+1)-2\delta^2}{a^2}\ ,$$

$$M_{0\pm}{}^2=\frac{1}{2a^2}\{2l(l+1)+\delta^2\pm\delta[\delta^2+12l(l+1)]^{1/2}\}\ , \tag{16}$$

$$M_{1\pm}{}^2=\frac{1}{a^2}[l(l+1)\pm\delta\sqrt{2l(l+1)}]\ ,$$

$$M_2{}^2=\frac{l(l+1)}{a^2}\ .$$

We can see that the low-mass gauge bosons acquire an equal mass of $M_{1-}(l=1)=\sqrt{2-2\delta}/a$, due to the remaining SU(2) global symmetry, while the local symmetry is broken by the σ field. Using (7) and (8), it corresponds to $M_{1-}{}^2\approx K^2/2t=8\pi G/t$. The smallest mass of the scalar particles is

$$M_{0-}(l=1)=\frac{[4+\delta^2-\delta(\delta^2+2t)^{1/2}]^{1/2}}{\sqrt{2}a}\ .$$

Using

$$\delta^2=\frac{a^2K^2\bar{F}^2}{4}\cong1-\frac{n^2K^4}{16e^2t}\approx1$$

in (7) and (8), the ratio of the scalar mass versus the gauge-boson mass is found to be $2{:}\sqrt{5}$. Of course, our model is not realistic yet and the above values are perhaps devoid of physical significance. It is clear from the above expression that tachyon instabilities are never present, and since all residues are positive definite there is no negative-metric ghost.

To find the massless states, it is necessary to use a frame such as $p_a=(p_0,0,0,p_2)$. When this is done, one finds the terms

$$I \propto \frac{1}{p^2}(\,|T_{12}|^2+\tfrac{1}{4}|T_{11}-T_{22}|^2+|J_1|^2+|J_2|^2)_{l=0} \tag{17}$$

which indicates a massless graviton ($\lambda=\pm 2$) and a "photon" ($\lambda=\pm 1$) as expected.

IV. DISCUSSION

Before discussing our results, the effects of fermion fields on the model are shortly considered. At the classical level fermion fields do not affect the classical solution of the ground state and the question of stability. As is well known, it is possible to obtain chiral fermions under the nontrivial background of the Maxwell field in six-dimensional theory. Let fermions now couple to the scalar fields with the term

$$\frac{1}{2t}\psi_\mu \gamma^A E_A{}^M \Gamma^\mu_{\delta\eta}\partial_M \phi^\delta \psi^\eta \; .$$

This type of coupling can be obtained from the supersymmetric extension of nonlinear σ field.[9] This coupling does not change the isohelicity of fermions and gives small masses to the massless zero mode fermions. The typical value of mass acquired by zero mode fermion is

$$\frac{a^2}{t}\approx M_{1-}(l=1)^2\frac{n^2}{4e^2} \; .$$

The initial motivation of Kaluza was to unify the gravitational and electromagnetic forces. From this point of view, the scheme explained here is rather anti-Kaluza-Klein. But it seems that a higher-dimensional theory of pure gravity cannot be a satisfactory starting point for a realistic four-dimensional field theory. At least fermion fields have to be introduced. In addition we have introduced the Maxwell and nonlinear σ field. One possible interpretation of this additional field given by Gell-Mann and Zwiebach is that the σ field is not thought to be fundamental but rather an effective field theory for composite scalars arising in a supertheory at some energy scales below the Planck mass.[5] Another possibility is that the Maxwell field be obtained from more higher-dimensional pure gravity theory.[10]

To make our model more realistic, we need to split the masses of three gauge bosons and couple them to real photons. One possibility may be the introduction of complex σ field which couples with the Maxwell field. Then it has one more merit, that the complex σ field can couple with fermions supersymmetrically giving more information on the mass spectrum of low-lying fermions.

In conclusion, one can make the three gauge bosons of SU(2) massive by introducing the nonlinear σ field on the six-dimensional Einstein-Maxwell system. It is contrary to the conventional wisdom which hopes that the states massless at the tree level would acquire their small masses via the quantum effect. This electroweak gravity model has smaller parameters than those of the Higgs model and they can be fixed by currently measured quantities. The symmetry-breaking mechanism occurs directly in the Kaluza-Klein scheme and the nonlinear σ field accepts the geometrical interpretation like the Einstein and Maxwell fields. We hope that all these features will make the nonlinear σ field as a possible substitute of the conventional Higgs scalars in spite of its various defects like the non-renormalizability or no realistic spectrum, etc.

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