Classical fluctuations in dissipative quantum systems

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(Received 30 December 1985)

Dissipative quantum systems such as an unstable field, thermal field, and cosmological particle production are investigated. Equations of motion for each appropriate mean field are revealed to be Langevin type. The derived correlation of the random field turns out not to be Gaussian nor white in general. A relation between a popular quantum average and the statistical correlation is also clarified.

I. INTRODUCTION

Recent work on the early stage of the Universe has revealed the possibility that the density fluctuations we observe now originate from the zero-point oscillation of initial quantum fields.[1] Moreover, some people try to describe the entire Universe by a single wave function based on quantum mechanics.[2] People say that a free propagator of some quantum field in de Sitter space itself describes long-wavelength classical fluctuations. But no statistical property can be derived from quantum mechanics without observation or a coordinate process. In the same way, a wave function of the Universe that obeys the Wheeler-DeWitt equation cannot undergo metamorphosis into a classical distribution function without any process of observation. If we adopt the quantum-mechanical description of the early Universe, we have to clarify how the statistical fluctuations as classical degrees of freedom arise from quantum theory.

The above question reminds us of dissipative back reaction upon classical degrees of freedom by quantum fields. It seems natural to make a conjecture that systems which show frictional motion[3] or relaxation[4] may as well accompany statistical fluctuations since both dissipation and fluctuation stem from information loss. In this paper, we investigate how manifestly classical fluctuations arise in dissipative quantum systems, such as a model of unstable particles, finite-temperature quantum field theory,[5,6] and anisotropy damping due to particle production in the early Universe.

II. DERIVATION OF THE LANGEVIN EQUATION

Let us begin our study by an unstable real scalar field which spontaneously decays into massless fermions. We cannot predict where and when the decay takes place and the momentum of the produced fermions is uncertain. This unpredictability is the very origin of the appearance of the classical (real) fluctuations. We would like to concentrate on the dynamics of the instantaneous vacuum expectation value of the scalar field (mean field) while disregarding fermion dynamics which is thought to be very complicated and unpredictable in a long time scale. The generating functional for ϕ is

$$Z[J]=Z[J_+,J_-]$$
$$=\int_p \mathscr{D}\phi \int_p \mathscr{D}\bar{\psi} \int_p \mathscr{D}\psi \exp\left[i\int_p \mathscr{L}+i\int_p J\phi\right], \tag{1}$$

where

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu\phi)^2-\frac{m^2}{2}\phi^2-\frac{\lambda}{4!}\phi^4-f\phi\bar{\psi}\psi+i\bar{\psi}\partial\psi\ . \tag{2}$$

$\int_p$ in the exponent means $\int d^3\mathbf{x}$ times the time integration along a loop: from 0 to t and then back to 0 again. The path integral is evaluated on this path. We have to distinguish field variables whose arguments are on the positive-direction and negative-direction time branches. They are denoted by $+$ and $-$, respectively. This choice of an integration contour is essential to an instantaneous vacuum expectation value of ϕ (the ordinary one $\langle 0\text{ in}|\phi|0\text{ out}\rangle$ is completely nonlocal, so it is inadequate for the mean field), and to a derivation of physical retarded effect (the ordinary method using a single-direction contour is based on a boundary condition which does not distinguish past and future). The fermion integration in Eq. (1) becomes

$$F[\phi]=\int_p \mathscr{D}\bar{\psi}\int_p \mathscr{D}\psi \exp\left[i\int_p \bar{\psi}(-f\phi+i\partial)\psi\right]$$
$$=\mathrm{Det}[1-(i\partial)^{-1}f\phi]$$
$$\cong\exp\left[f^2\int d^4x\int d^4x'(\phi_+,\phi_-)_x\begin{bmatrix} S^F_{\mu\nu}(x-x')S^{\nu\mu}_F(x'-x) & -S^+_{\mu\nu}(x-x')S^{\nu\mu}_-(x'-x) \\ -S^-_{\mu\nu}(x-x')S^{\nu\mu}_+(x'-x) & S^{\bar{F}}_{\mu\nu}(x-x')S^{\nu\mu}_{\bar{F}}(x'-x)\end{bmatrix}\begin{bmatrix}\phi_+\\ \phi_-\end{bmatrix}_{x'}\right], \tag{3}$$

where Det means a determinant over functions whose arguments are on the loop path. A term of lowest order in f is retained in the last line. The S's are fermion propagators:

$$\begin{aligned} S^F_{\mu\nu}(x-x') &= -i\langle T\psi_\mu(x)\bar{\psi}_\nu(x')\rangle\ , \\ S^+_{\mu\nu}(x-x') &= +i\langle\bar{\psi}_\nu(x')\psi_\mu(x)\rangle = S^-_{\mu\nu}(x-x')^*\ , \\ S^{\bar{F}}_{\mu\nu}(x-x') &= -i\langle\bar{T}\psi_\mu(x)\bar{\psi}_\nu(x')\rangle\ . \end{aligned} \tag{4}$$

The matrix structure of the propagator and the vertex reflects a loop path integral of Eq. (1). Equation (3) is rewritten as

$$\begin{aligned} F[\phi] &= \exp\left[-\frac{f^2}{2}\int_0^t dt\int d^3\mathbf{x}\int_0^t dt'\int d^3\mathbf{x}'[\phi_\Delta(x)\Sigma_F^I(x-x')\phi_\Delta(x') - 4i\theta(x_0-x_0')\phi_\Delta(x)\Sigma_F^R(x-x')\phi_c(x')]\right] \\ &= \exp\left[\frac{i}{2}f^2\int d^4x\int d^4x'(\phi_+,\phi_-)_x\begin{bmatrix}\Sigma_F & 0\\ -2\Sigma & \Sigma_{\bar{F}}\end{bmatrix}_{xx'}\begin{bmatrix}\phi_+\\ \phi_-\end{bmatrix}_{x'}\right]\ , \end{aligned} \tag{5}$$

where

$$\begin{aligned} \Sigma_F(x-x') &= -iS^F_{\mu\nu}(x-x')S_F^{\nu\mu}(x'-x)\ , \\ \Sigma_{\bar{F}}(x-x') &= -iS^{\bar{F}}_{\mu\nu}(x-x')S_{\bar{F}}^{\nu\mu}(x'-x)\ , \\ \Sigma(x-x') &= -iS^-_{\mu\nu}(x-x')S_+^{\nu\mu}(x'-x)\ , \\ \Sigma_F^R &= \mathrm{Re}\Sigma_F,\quad \Sigma_F^I = \mathrm{Im}\Sigma_F\ , \end{aligned} \tag{6}$$

and

$$\phi_\Delta = \phi_+ - \phi_-,\quad \phi_c = \tfrac{1}{2}(\phi_+ + \phi_-)\ . \tag{7}$$

The next step is to construct the effective action $\Gamma[\varphi]$. The procedure is almost identical to the ordinary single-time path theory. For example,

$$\varphi(x) = \frac{1}{i}\frac{\delta}{\delta J}\ln Z[J]$$

and

$$\Gamma[\varphi] = W[J[\varphi]] - \int_p J[\varphi]\varphi\ .$$

At tree order, $\exp(i\Gamma[\varphi])$ becomes

$$\begin{aligned} \exp(i\Gamma^{(0)}[\varphi]) = \exp\Bigg\{ & i\int d^4x\left[\left[\tfrac{1}{2}(\partial_\mu\varphi_+)^2 - \frac{m^2}{2}\varphi_+{}^2 - \frac{\lambda}{4!}\varphi_+{}^4\right] - \left[\tfrac{1}{2}(\partial_\mu\varphi_-)^2 - \frac{m^2}{2}\varphi_-{}^2 - \frac{\lambda}{4!}\varphi_-{}^4\right]\right] \\ & + \frac{i}{2}f^2\int d^4x\int d^4x'(\varphi_+,\varphi_-)_x\begin{bmatrix}\Sigma_F & 0\\ -2\Sigma & \Sigma_{\bar{F}}\end{bmatrix}_{xx'}\begin{bmatrix}\varphi_+\\ \varphi_-\end{bmatrix}_{x'}\Bigg\}\ . \end{aligned} \tag{8}$$

We cannot directly write down the equation of motion for φ like $\delta\Gamma[\varphi]/\delta\varphi(x) = -J(x) = 0$, because this equation becomes complex due to Σ_F and Σ whose imaginary part (discontinuity) represents the instability of $\varphi(x)$. (Note that ϕ has been assumed to be real.) However, the difficulty is completely avoided if we regard that the complexity arises from a random perturbation onto the φ field. Equation (8) can be rewritten as

$$\exp(i\Gamma^{(0)}[\varphi]) = \int \mathcal{D}\xi P(\xi)\exp(iS_{\text{eff}}[\varphi,\xi])\ , \tag{9}$$

where

$$P(\xi) = \frac{\exp\left[-\frac{1}{2}\int d^4x\int d^4x'\xi(x)(f^2\Sigma_F^I)_{xx'}{}^{-1}\xi(x')\right]}{\int\mathcal{D}\xi\exp\left[-\frac{1}{2}\int d^4x\int d^4x'\xi(x)(f^2\Sigma_F^I)_{xx'}{}^{-1}\xi(x')\right]}\ , \tag{10}$$

$$\begin{aligned} S_{\text{eff}}[\varphi,\xi] = & \int d^4x\left[\left[\tfrac{1}{2}(\partial_\mu\varphi_+)^2 - \frac{m^2}{2}\varphi_+{}^2 - \frac{\lambda}{4!}\varphi_+{}^4\right] - \left[\tfrac{1}{2}(\partial_\mu\varphi_-)^2 - \frac{m^2}{2}\varphi_-{}^2 - \frac{\lambda}{4!}\varphi_-{}^4\right]\right] \\ & + 2f^2\int d^4x\int d^4x'\theta(x_0-x_0')\varphi_\Delta(x)\Sigma_F^R(x-x')\varphi_c(x') + \int d^4x\,\xi(x)\varphi_\Delta(x)\ . \end{aligned} \tag{11}$$

We can interpret the field $\xi(x)$ as a classical random field which is Gaussian but not white because the weight $P(\xi)$ in the summation by $\xi(x)$ has a pure Gaussian form. In fact defining a statistical average by

$$\langle \cdots \rangle_s = \int \mathscr{D}\xi \cdots P(\xi) , \tag{12}$$

the correlations of the random field are written as

$$\begin{aligned} &\langle \xi(x_1)\xi(x_2)\cdots\xi(x_{2n+1})\rangle_s = 0 , \\ &\langle \xi(x_1)\xi(x_2)\rangle_s = f^2\Sigma_F^I(x_1-x_2) \\ &\qquad = \frac{f^2}{12\pi}\int\frac{d^4p}{(2\pi)^4}e^{-ip(x_1-x_2)}p^2\theta(p^2) , \\ &\langle \xi(x_1)\xi(x_2)\xi(x_3)\xi(x_4)\rangle_s = f^4[\Sigma_F^I(x_1-x_2)\Sigma_F^I(x_3-x_4) \\ &\qquad + \Sigma_F^I(x_1-x_3)\Sigma_F^I(x_2-x_4) \\ &\qquad + \Sigma_F^I(x_1-x_4)\Sigma_F^I(x_2-x_3)] , \\ &\cdots . \end{aligned} \tag{13}$$

They show that the random field $\xi(x)$ is Gaussian but not white.

Now let us derive the equation of motion for $\varphi(x)$. For two independent variables $\varphi_\Delta(x)$ and $\varphi_c(x)$, we obtain the following equations of motion:

$$\begin{aligned} &\delta S_{\text{eff}}/\delta\varphi_\Delta(x) + J_c(x) = 0 , \\ &\delta S_{\text{eff}}/\delta\varphi_c(x) + J_\Delta(x) = 0 , \end{aligned} \tag{14}$$

where $J_\Delta = J_+ - J_-$ and $J_c = \frac{1}{2}(J_+ + J_-)$. To identify physical degrees of freedom, we shall require the normalization of the generating functional:[7]

$$Z[J_\Delta, J_c]\,|_{J_\Delta=0} = 1 . \tag{15}$$

We have put $J_+ = J_-$ because the actual time axis is unique. From Eq. (15) we have

$$\varphi_\Delta(x)\,|_{J_\Delta=0} = \frac{1}{i}\delta\ln Z[J_\Delta, J_c]/\delta J_c(x)\,|_{J_\Delta=0} = 0 . \tag{16}$$

Accordingly we get an equation of motion for $\varphi_c(x)$:

$$\begin{aligned} (\Box + m^2)\varphi_c(x) + \frac{\lambda}{3!}\varphi_c{}^3(x) \\ -2f^2\int_0^t dt'\int d^3\mathbf{x}'\,\Sigma_F^R(x-x')\varphi_c(x') \\ -J_c(x) - \xi(x) = 0 . \end{aligned} \tag{17}$$

This is exactly the Langevin equation. The third term describes the retarded effect while the last term describes the instantaneous statistical perturbation from the random field $\xi(x)$ whose correlations are shown in Eq. (13). Because of the random field $\xi(x)$, spatial and temporal inhomogeneity of the field $\varphi_c(x)$ develops even if $\varphi_c(x)$ had been prepared to be homogeneous. We may identify this effect as a spontaneous translational invariance breaking. Of course the inhomogeneity may vanish if one averages over $\xi(x)$, but the point here is that we can extract manifestly classical fluctuations from the original quantum system. The variable $\xi(x)$ is thought to represent degrees of freedom of fermion which we have already integrated out. Therefore we need not take account of the dynamics of $\xi(x)$.

III. LOOP CORRECTIONS—FRICTION

Higher-loop contributions to the effective action are evaluated similarly. For example, one-loop contribution becomes

$$\exp(i\Gamma^{(1)}[\varphi]) = \exp\left\{-\tfrac{1}{2}\mathrm{Tr}\ln\left[1+\begin{bmatrix} G_F & G_+ \\ G_- & G_{\bar F}\end{bmatrix}\begin{bmatrix}\frac{\lambda}{2}\varphi_+{}^2 & 0 \\ 0 & -\frac{\lambda}{2}\varphi_-{}^2\end{bmatrix}\right]\right\} , \tag{18}$$

where

$$\begin{aligned} &G_F = -i\langle T\phi(x)\phi(x')\rangle , \\ &G_+ = -i\langle \phi(x')\phi(x)\rangle = -G_-^* , \\ &G_{\bar F} = -i\langle \bar T\phi(x)\phi(x')\rangle . \end{aligned} \tag{19}$$

Up to two-loop order and in lowest order in λ, the Langevin equation for $\varphi_c(x)$ becomes

$$\begin{aligned} (\Box + m^2)\varphi_c(x) + \frac{\lambda}{3!}\varphi_c{}^3(x) - 2f^2\int_0^t dt'\int d^3\mathbf{x}'\Sigma_F^R(x-x')\varphi_c(x') - \xi(x) \\ + \frac{\lambda^2}{2}\varphi_c(x)\int_0^t dt'\int d^3\mathbf{x}'\,\mathrm{Im}G_F^2(x-x')\varphi_c{}^2(x') - 2\xi^{(1)}(x)\varphi_c(x) \\ + \frac{\lambda^2}{3}\int_0^t dt'\int d^3\mathbf{x}'\,\mathrm{Im}G_F{}^3(x-x')\varphi_c(x') - J_c(x) = 0 . \end{aligned} \tag{20}$$

Here, $\xi^{(1)}(x)$ is a new type of random field which couples with $\varphi_c(x)$ linearly. The statistical average should be modified:

$$\langle \cdots \rangle_s = \int \mathscr{D}\xi P(\xi)\int\mathscr{D}\xi^{(1)}P^{(1)}(\xi^{(1)})\cdots , \tag{21}$$

where

$$P(\xi)=\exp\left[-\tfrac{1}{2}\int d^4x\int d^4x'\xi(x)\left[f^2\Sigma_F^I+\frac{\lambda^2}{3}\mathrm{Re}G_F{}^3\right]^{-1}_{xx'}\xi(x')\right]\Big/N\ ,$$
$$P^{(1)}(\xi^{(1)})=\exp\left[-\tfrac{1}{2}\int d^4x\int d^4x'\xi^{(1)}(x)\left[\frac{\lambda^2}{8}\mathrm{Re}G_F{}^2\right]^{-1}_{xx'}\xi^{(1)}(x')\right]\Big/N^{(1)}\ ; \tag{22}$$

N and $N^{(1)}$ are normalizations. Then we get

$$\langle\xi(x)\xi^{(1)}(x')\rangle_s=0\ ,$$
$$\langle\xi(x)\xi(x')\rangle_s=f^2\mathrm{Im}\Sigma_F(x-x')+\frac{\lambda^2}{3}\mathrm{Re}G_F{}^3(x-x')\ , \tag{23}$$
$$\langle\xi^{(1)}(x)\xi^{(1)}(x')\rangle_s=\frac{\lambda^2}{8}\mathrm{Re}G_F{}^2(x-x')\ .$$

In Eq. (22), the weights are manifestly positive although they may not be normalizable. In that case, we have to investigate higher-order terms in λ.

In Eq. (20), the terms proportional to $\mathrm{Im}G_F{}^2$ and $\mathrm{Im}G_F{}^3$ give correct[3,8] friction coefficients under the quasiadiabatic approximation:

$$\frac{\lambda^2}{2}\varphi_c{}^2(x)\int_0^t dt'\int d^3\mathbf{x}'\mathrm{Im}G_F{}^2(x-x')\varphi_c{}^2(x')=\mathrm{const}\times\varphi_c{}^3(x)+F_2(x)\dot{\varphi}_c(x)+\cdots\ , \tag{24}$$

$$\frac{\lambda^2}{3}\int_0^t dt'\int d^3\mathbf{x}'\mathrm{Im}G_F{}^3(x-x')\varphi(x')=\mathrm{const}\times\varphi_c(x)+F_3(x)\dot{\varphi}_c(x)+\cdots\ , \tag{25}$$

where friction coefficients are linearly expressed in terms of random-field correlations:

$$F_2(x)=\frac{\lambda^2}{8}\varphi_c{}^2(x)\int\frac{d^3\mathbf{p}}{(2\pi)^3}\int d^4x\,\omega_{\mathbf{p}}{}^{-6}e^{-i\omega_{\mathbf{p}}t+i\mathbf{p}\cdot\mathbf{x}}\langle\xi(x)\xi(0)\rangle_s+O(\lambda^4)\ , \tag{26}$$

$$F_3(x)=\frac{\lambda^2}{16}\int\frac{d^3\mathbf{p}_1}{(2\pi)^3}\int\frac{d^3\mathbf{p}_2}{(2\pi)^3}\int\frac{d^3\mathbf{p}_3}{(2\pi)^3}(2\pi)^3\delta\left[\sum_{i=1}^{3}\mathbf{p}_i\right]$$
$$\times\frac{3\omega_{\mathbf{p}_1}+\omega_{\mathbf{p}_2}+\omega_{\mathbf{p}_3}}{\omega_{\mathbf{p}_1}{}^3\omega_{\mathbf{p}_2}\omega_{\mathbf{p}_3}(\omega_{\mathbf{p}_1}+\omega_{\mathbf{p}_2}+\omega_{\mathbf{p}_3})^3}\int d^4x\,e^{-i\omega_{\mathbf{p}_1}t+i\mathbf{p}_1\cdot\mathbf{x}}\langle\xi(x)\xi(0)\rangle_s+O(\lambda^4) \tag{27}$$

and

$$\omega_{\mathbf{p}_i}=(\mathbf{p}_i{}^2+m^2)^{1/2}\ .$$

However, we cannot directly count the term proportional to $\mathrm{Re}\Sigma_F$ for friction because this term always exists irrespective of dissipativity of the system. In fact, a quasiadiabatic approximation on this term gives us the mass and wave-function renormalization but not friction. A similar term appears which cannot be counted for by friction in the model of cosmological anisotropy relaxation. Up to now, we have been concerned about the self-energy part. We can, as well, take account of the complexity in the vertex part and higher-point functions. Then the random field is in general not even Gaussian:

$$\exp(i\Gamma[\varphi])=\int\mathscr{D}\xi P(\xi)\exp\left[i\,\mathrm{Re}\Gamma[\varphi]+i\int\xi\varphi\right]\ , \tag{28}$$

where

$$P(\xi)=\int\mathscr{D}\varphi\exp\left[-\mathrm{Im}\Gamma[\varphi]-i\int\xi\varphi\right]\Big/\exp(-\mathrm{Im}\Gamma[0])\ . \tag{29}$$

Unfortunately, we have not yet succeeded in proving the positivity of this $P(\xi)$.

IV. STATISTICAL FLUCTUATIONS OF THE MEAN FIELD

A relation between a pure statistical correlation and an ordinary vacuum average should be elucidated. The generating functional in Eq. (1) is now expressed as

$$Z[J]=\int\mathscr{D}\xi P(\xi)\exp(i\tilde{W}[J])=\exp(iW[J])\ , \tag{30}$$

$$\exp(i\tilde{W}[J])=\int\mathscr{D}\phi\exp\left[iS_{\mathrm{eff}}[\phi,\xi]+i\int(\phi_\Delta J_c+\phi_c J_\Delta)\right]\ , \tag{31}$$

where $S_{\mathrm{eff}}[\phi,\xi]$ is defined by Eq. (11) and ξ is introduced in Eq. (9). Thus, our pure statistical correlation is expressed as

$$\langle\varphi_c(x)\varphi_c(y)\rangle_s-\langle\varphi_c(x)\rangle_s\langle\varphi_c(y)\rangle_s$$
$$=\frac{1}{i}\frac{\delta}{\delta J_\Delta(x)}\frac{1}{i}\frac{\delta}{\delta J_\Delta(y)}iW[J]\Big|_{J=0}-\left\langle\frac{1}{i}\frac{\delta}{\delta J_\Delta(x)}\frac{1}{i}\frac{\delta}{\delta J_\Delta(y)}i\tilde{W}[J]\right\rangle_s\Bigg|_{J=0}\ , \tag{32}$$

while the ordinary vacuum average of $\phi_+(x)\phi_+(y)[=\langle\langle\phi_+(x)\phi_+(y)\rangle\rangle]$ becomes

$$\langle\langle\phi_+(x)\phi_+(y)\rangle\rangle=\frac{1}{i}\frac{\delta}{\delta J_+(x)}\frac{1}{i}\frac{\delta}{\delta J_+(y)}iW[J]\,|_{J=0} \quad (33)$$

which is mainly different from the former by the second term of Eq. (32).

Time development of the statistical correlation is derived from the Dyson equation. Neglecting self-interaction, we get

$$\Lambda_x\Lambda_y\langle\varphi_c(x)\varphi_c(y)\rangle_s-\langle\xi(x)\xi(y)\rangle_s=0\ , \quad (34)$$

where

$$\Lambda_x\varphi_c(x)=-(\Box+m^2)\varphi_c(x) +2f^2\int d^4x'\theta(x_0-x_0')\Sigma_F^R(x-x')\varphi_c(x').\quad (35)$$

On the other hand, the ordinary vacuum average obeys

$$-(\Box_x+m^2)\langle\langle\phi_+(x)\phi_+(y)\rangle\rangle+2f^2\int d^4x\,\Sigma_F(x-x')\times\langle\langle\phi_+(x')\phi_+(y)\rangle\rangle+\delta^{(4)}(x-y)=0\ . \quad (36)$$

We observe that the source terms are completely different from each other: Quantum fluctuations always exist [Eq. (36)] while statistical fluctuations arise only when the system shows dissipativity [Eqs. (23) and (34)]. We do not have to be pessimistic that the primordial density fluctuations calculated basically from Eq. (36) are too great[1] since the genuine density fluctuations should be related to $\langle\varphi_c(x)\varphi_c(y)\rangle_s$ whose amplitude is proportional to the instability of the system.

V. APPLICATIONS TO THERMAL FIELD THEORY AND COSMIC ANISOTROPY RELAXATION

(1) *Thermal field:* Now let us turn our attention to the real-time finite-temperature quantum field theory.[5,6] We start from a generating functional as in Ref. 6:

$$Z[J]=\mathrm{Tr}\left[e^{-\beta H}T_c\exp\left[i\int_c J\phi\right]\right]\quad(\beta^{-1}=\text{temperature})\ , \quad (37)$$

where c means that the operations should be performed along the time path:

$$\left[t=-T\to T\to T-\frac{i}{2}\beta\to -T-\frac{i}{2}\beta\to -T-i\beta\right]\ .$$

A structure of the propagators becomes similar to the previous one when $T\to\infty$. Then the procedure to derive a Langevin equation for a thermal average of ϕ and a spectrum of the random field, etc., are the same as before. We get Eqs. (20)—(23) but f^2's are set to be zero and $G_F(x-x')$ is now a thermal propagator:[8]

$$G_F(y)=\begin{cases}-\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{y}}}{2\omega}[e^{-iy_0\omega}\cosh^2\theta(\omega-i\Gamma)-e^{iy_0\omega}\cosh^2\theta(-\omega-i\Gamma)]e^{-y_0\Gamma} & \text{for } y_0>0\ ,\\ -\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot\mathbf{y}}}{2\omega}[e^{-iy_0\omega}\sinh^2\theta(\omega+i\Gamma)-e^{iy_0\omega}\sinh^2\theta(-\omega+i\Gamma)]e^{y_0\Gamma} & \text{for } y_0<0\ ,\end{cases} \quad (38)$$

where

$$\Gamma(\mathbf{k})=\frac{\lambda^2(2\pi)^4}{8\omega n_\omega}\prod_{i=1}^{3}\left[\int\frac{d^3\mathbf{k}_i}{(2\pi)^3 2\omega_i}\right](1+n_{\omega_1})n_{\omega_2}n_{\omega_3}\delta^{(3)}(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)\delta(\omega+\omega_1-\omega_2-\omega_3)\ ,$$
$$\sinh^2\theta(\omega)=n_\omega=(e^{\beta\omega}-1)^{-1},\quad \omega_i=(\mathbf{k}_i^{\,2}+m^2)^{1/2}\ . \quad (39)$$

The strongest random field is now $\xi^{(1)}(x)$ and its spatially uniform component $[\xi(t)]$ has the following correlation:

$$\langle\xi(t)\xi(t')\rangle_s=\frac{\lambda^2}{8}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{e^{-2\Gamma|t-t'|}}{4\omega^2}\{2n(1+n)+(1+2n+2n^2)\cos[2(t-t')\omega]-2n(1+n)(1+2n)\beta\Gamma\sin[2(t-t')\omega]\}\ . \quad (40)$$

Friction coefficients are also derived like Eqs. (24) and (25), but they are not expressed linearly in the statistical correlation.[8] This model is a natural extension of Ref. 9 in the sense that now we treat quantum field theory, and random fields as well as frictions are automatically derived.

(2) *Anisotropy relaxation*: Next, we investigate classical fluctuations accompanied with the cosmological anisotropy (β^{ij}) relaxation due to particle production.[4] In the Friedmann universe, we consider massless conformal real scalar field ϕ. The initial cosmological anisotropy is subsequently reduced due to ϕ-particle pair production. In Ref. 4, geometry β^{ij} determined by equations of motion is complex. According to the present procedure, we can avoid this complex geometry and moreover we obtain a Langevin equation in which a fluctuating back reaction upon β^{ij} by ϕ-particle production is manifest. A random field ξ_{ij} which appears in the Langevin equation turns out to be Gaussian and white:

$$\langle\xi_{ij}(\eta)\xi_{kl}(\eta')\rangle_s=(1920\pi)^{-1}(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\times\frac{d^4}{d\eta^4}\delta(\eta-\eta')\ , \quad (41)$$

where η and η' are conformal time variables. We emphasize that this back reaction is entirely new and different from ordinary dispersive or dissipative back reactions. Unfortunately, a retarded response term [corresponds to the term proportional to Σ_F^R in Eq. (20)] is completely nonlocal in conformal time and cannot be properly interpreted as friction.

VI. DISCUSSION

In this article we have derived Langevin equations for each appropriate mean field of the system. Classical fluctuations of the mean field stem from quantum fluctuations through radiative corrections and instability of the system whose properties determine spectrum and time evolution of the statistical correlations.

Several applications of the method are possible. (i) In a very strong electromagnetic field $F_{\mu\nu}$, electron-positron pair production may spontaneously take place[10] and the $F_{\mu\nu}$ is thought to be reduced without observation. It seems natural that the relaxation of $F_{\mu\nu}$ accompany fluctuations in $F_{\mu\nu}$. This is also an example of the spontaneous translational invariance breaking. (ii) Nonconvex potentials are widely applied to dynamics of phase transitions.[11] In this case, an effective potential becomes complex (e.g.,

$$\mathrm{Im}\,V_{\mathrm{eff}}[\varphi] \sim \frac{-1}{64\pi}\lambda m^2\varphi^2 ,$$

for the $\lambda\varphi^4$ model with negative mass squared m^2). This means that on the nonconvex region, there appears a random field which triggers the phase transition pushing stochastically the order parameter from zero. This mechanism may shed light on the problem of the identification of a zero mode and fluctuations of an expectation value of a Higgs field in the inflationary universe model.[12]

In the argument of finite-temperature quantum field theory, it was essential to use renormalized propagators to derive dissipative and fluctuating properties. Spectrum of the Hamiltonian which corresponds to a tree Lagrangian is not bounded below and this is only an indication of the instability. By radiative corrections and renormalizations, the instability becomes manifest in the form of dissipation and fluctuation. In this sense, the renormalization induces a change of description of the system from a representation based on stable asymptotic fields into that based on unstable ones. A role of a renormalization as a map which connects reversible and irreversible description of dynamics is now under investigation.

ACKNOWLEDGMENTS

The author would like to thank Professor H. Sato for continuous encouragement and Dr. M. Sasaki for helpful discussions and reading the manuscript. He also wishes to acknowledge valuable discussions with Dr. M. Bando, Dr. H. Kodama, Professor T. Kugo, Professor S. Machida, Dr. I. Ojima, and Dr. M. Sakagami.

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