Quasi-Riemannian gravity and spontaneous breaking of the Lorentz gauge symmetry in more than four dimensions

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It is shown that a pure gravity theory in d dimensions, with an action quadratic in torsion and curvature, may lead to a spontaneous breaking of the SO$(1,d-1)$ gauge symmetry. The physical vacuum, corresponding to a minimum of the self-interaction torsion potential, is characterized by a constant nonvanishing torsion background, and in the lowest-order expansion of the gravitational field around this configuration, an effective quasi-Riemannian theory is obtained. In particular, all nine parameters of the corresponding quasi-Riemannian action are determined, as a function of the torsion self-interaction coupling strength, for the tangent space groups SO$(1,d-4)\times$SO(3) and SO$(1,d-2)$. Finally, the possibility that a breaking of the local Lorentz symmetry may be associated to a change of sign of the effective four-dimensional gravitational coupling constant is discussed.

I. INTRODUCTION

In the context of higher-dimensional unified theories, Weinberg[1] has recently suggested the possibility that gravitation may be described in $d>4$ dimensions by generalized theories, called quasi-Riemannian, which are covariant under general-coordinate transformations, but have a local symmetry group G_T other than the d-dimensional Lorentz group SO$(1,d-1)$. The new tangent space group is required to be of the form $G_T=\text{SO}(1,n-1)\times G_T'$, where $4\le n\le d$ and $G_T'\le \text{SO}(d-n)$, and some features of such theories with this generalized gauge symmetry have been investigated in Refs. 2 and 3.

The main advantage offered by quasi-Riemannian theories of gravity is that one can obtain four-dimensional chiral fermions from a higher-dimensional action,[1,3] without adding extra gauge fields to the starting d-dimensional theory,[4] and also without considering a noncompact internal space with finite volume.[5]

However, the choice of G_T with the structure of a direct product of two groups seems to be *ad hoc*,[2] even if it could be motivated by analogy with supersymmetric theories based on graded Lie algebras, which naturally have tangent groups of this form.[2] Moreover the uniqueness features of the usual Riemannian theory are lost,[3] because the construction of a G_T-invariant action involves in general many arbitrary constants, so that some fundamental principle is needed to reduce the number of the independent parameters, as explicitly stressed in Ref. 3.

A possible answer to these problems is to introduce some mechanism which spontaneously breaks the Lorentz gauge symmetry. To this aim, the most direct method to achieve a vacuum background with a local SO$(1,n-1)\times$SO$(d-n)$ invariance is to couple minimally gravity to a self-interacting antisymmetric tensor of rank $d-n$ which develops a nonzero vacuum expectation value [a generalization of the mechanism first suggested by Mallet[6] and based on the presence of a Lorentz vector to break spontaneously the SO(1,3) gauge symmetry in four dimensions]. But this requires the addition of extra nongravitational fields to the starting d-dimensional action, and, like adding gauge fields to obtain chiral fermions, would be a procedure in contrast with the spirit of the original Kaluza-Klein theory, as a truly unified theory should start initially only with pure gravity in d dimensions.[7]

A third-rank antisymmetric tensor, however, may have a natural geometric interpretation, in a non-Riemannian geometrical context, as the totally antisymmetric part of the torsion tensor; moreover, a gravitational Lagrangian quadratic in the curvature tensor automatically provides the self-interaction potential which leads, via the Higgs mechanism, to a nonvanishing constant value of the torsion tensor in the vacuum background. Therefore an entirely geometrical mechanism, which spontaneously breaks the gauge SO$(1,d-1)$ symmetry starting from a d-dimensional theory of pure gravity, may be obtained provided that we relax two assumptions used in Ref. 3 [i.e., torsion-free SO$(1,d-1)$ connection and action linear in the curvature tensor], and we consider a gravitational Lagrangian quadratic in torsion and curvature.

To provide motivations for this approach, it should be mentioned that a quadratic Lagrangian finds a natural justification in the context of the classical Poincaré gauge theory of gravity,[8] or may even be interpreted[9] as representing the effective contribution to the gravitational action arising from quantum fluctuations, at some distance scale close to the Planck length. Recently, it has been shown that curvature-squared terms appear in the field-theory limit of string theories,[10,11] and it is their presence that allows nontrivial compactification on a four-dimensional Minkowski space;[10] moreover it has been conjectured[12] that, in the low-energy limit of the closed Bose string, the complete set of four-derivative terms may be expressed by an action quadratic in curvature, provided that a suitable affine connection with torsion is introduced. Finally, higher (than linear) curvature terms appear in a generalization of the Einstein theory in more than four dimensions, based on a gravitational Lagrangian which consists of a sum of dimensionally continued Euler forms.[11,13–15] In this case, it is important to stress that such a particular combination of curvature terms contributes to the field equations at most second-order terms, in the derivatives of the metric,[13–15] so that the introduction of a quadratic Lagrangian, in dimensions higher than four, does not necessarily imply the presence of higher-derivative equations in the corresponding generalized theory.

Without considering a particular model, the aim of this paper is to discuss the possibility that, starting from a pure gravity theory with propagating torsion and quadratic Lagrangian in d dimensions, a breaking of the Lorentz gauge group $SO(1,d-1)$ can occur, spontaneously induced by a constant nonvanishing torsion background. In this case the lowest-order gravitational excitations of the vacuum configurations are governed by an effective quasi-Riemannian theory, and all nine parameters of the corresponding quasi-Riemannian action constructed in Ref. 3 can be determined as a function of the total number of dimensions and of only one, model-dependent, coupling constant.

The plan of the paper is as follows. In Sec. II the general action for the theory is presented and the vacuum field equations are solved in the case of a constant nonvanishing totally antisymmetric torsion. We consider in particular solutions with the product structure $M_{d-3}\times M_3$, where M_n is an n-dimensional maximally symmetric space, but it is shown that, because of the curvature-squared terms in the action, we can obtain also a solution with the structure $M_4\times M_{d-7}\times M_3$ (in $d\geq 7$) corresponding to a four-dimensional Minkowski vacuum M_4, even if the internal space is not Ricci flat and without fine-tuning of adjustable parameters.

In Sec. III, expanding the gravitational field around the vacuum configuration $M_{d-3}\times M_3$ (keeping the torsion background fixed) we obtain a quasi-Riemannian theory with $SO(1,d-4)\times SO(3)$ as the tangent space group, in which the d-dimensional cosmological constant is vanishing. All the parameters of the theory are determined as a function of the coupling strength of the torsion self-interaction term in the Lagrangian, and we discuss the possibility that a breaking of the Lorentz gauge symmetry may lead to a change of the effective four-dimensional gravitational constant.

In Sec. IV we consider a vacuum configuration characterized by a constant nonvanishing vector part of the torsion, and in this case we obtain a quasi-Riemannian theory with local $SO(1,d-2)$ invariance. Finally, in Sec. V the results obtained are briefly summarized.

For easy reference we use in this paper the same formalism and follow the same notations and conventions as in Ref. 3, with the only difference that the vielbein field is denoted here by $V_M{}^A$, and the anholonomic basis is then $V^A=V_M{}^A dz^M$.

II. VACUUM SOLUTIONS OF THE FIELD EQUATIONS FOR A THEORY OF GRAVITY WITH TORSION AND QUADRATIC LAGRANGIAN

We start considering a theory of pure gravity in d dimensions, covariant under general-coordinate transformations and locally Lorentz invariant. The field variables are the anholonomic basis $V^A=V_M{}^A dz^M$ and the connection one-form $\Omega^{AB}=\Omega_M{}^{AB}dz^M$ for the gauge group $SO(1,d-1)$ (conventions: capital latin indices run from 1 to d, and $A,B,C,D,\ldots$ denote tangent space indices, while $M,N,P,Q,\ldots$ denote holonomic world indices). The linear connection Ω is not required to be torsion-free, so that, unlike in Ref. 3, we have in general

$$dV^A+\Omega^A{}_B\wedge V^B=R^A\neq 0 \ . \tag{2.1}$$

Allowing also the presence of curvature-squared terms (for the motivations see Sec. I) we consider then a model of gravity described by the following simple quadratic action:

$$S=\frac{1}{2\chi}\int[R^{AB}(\Omega)\wedge {}^*V_{AB}+gK^A{}_C\wedge K^{CB}\wedge {}^*V_{AB}$$
$$+2\chi m R^{AB}(\Omega)\wedge {}^*R_{AB}(\Omega)] \ , \tag{2.2}$$

where $R^{AB}(\Omega)$ is the Lorentz curvature two-form:

$$R^{AB}(\Omega)=d\Omega^{AB}+\Omega^A{}_C\wedge\Omega^{CB}$$
$$=\tfrac{1}{2}R_{MN}{}^{AB}(\Omega)dz^M\wedge dz^N \ . \tag{2.3}$$

K^{AB} is the contortion one-form, related to the torsion two-form R^A by

$$-K^A{}_B\wedge V^B=R^A \ . \tag{2.4}$$

χ is the d-dimensional analogue of Newton's constant, g is a dimensionless coupling constant, m has dimensions $(\text{mass})^{4-d}$, $V^{AB}=V^A\wedge V^B$, and finally the Hodge duality operation is defined as (see Ref. 3)

$${}^*V_{AB}=\frac{1}{(d-2)!}\epsilon_{ABC_1\cdots C_{d-2}}V^{C_1}\wedge\cdots\wedge V^{C_{d-2}}$$
$$\equiv\frac{V^{C_1\cdots C_{d-2}}}{(d-2)!}\epsilon_{ABC_1\cdots C_{d-2}} \ , \tag{2.5}$$

$${}^*R_{AB}=\frac{1}{(d-2)!}\frac{1}{2}\epsilon_{EFC_1\cdots C_{d-2}}R^{EF}{}_{AB}V^{C_1\cdots C_{d-2}} \ . \tag{2.6}$$

It should be noted that, while in this section we consider a spontaneous breaking of the Lorentz gauge symmetry induced by the totally antisymmetric part of the torsion tensor, the torsion-squared term we have introduced in the action (2.2) contains, more generally, also the trace of the torsion tensor, so that the same action can be used to discuss also the case of a constant vector torsion background, as we will see in Sec. IV. Note also that the curvature-squared term we consider is not of the ghost-free form, and the action (2.2) is to be regarded here not as a realistic theory, but only as the simplest quadratic model to discuss the general features of this geometric mechanism (a more realistic model could start, for example, from a Lagrangian involving the Gauss-Bonnet term of Ref. 11).

In order to separate explicitly the torsion contributions from the Riemannian terms in the action (2.2), we decompose as usual the connection as

$$\Omega^{AB}=\Omega^{0AB}-K^{AB} \ , \tag{2.7}$$

where Ω^0 is the torsion-free part of Ω, which satisfies, from (2.1) and (2.4),

$$D^0V^A \equiv D(\Omega^0)V^A = dV^A + \Omega^{0A}{}_B \wedge V^B = 0 \tag{2.8}$$

and D^0 denotes the exterior covariant derivative for the Riemannian part of the anholonomic connection. The curvature (2.3) becomes then

$$R^{AB}(\Omega) = R^{0AB} - D^0K^{AB} + K^A{}_C \wedge K^{CB} , \tag{2.9}$$

where

$$R^{0AB} = R^{AB}(\Omega^0) = d\Omega^{0AB} + \Omega^{0A}{}_C \wedge \Omega^{0CB} \tag{2.10}$$

is the usual Riemannian curvature, and the action (2.2) can be rewritten

$$S = \frac{1}{2\chi}\int [R^{0AB} \wedge {}^*V_{AB} + 2\chi m(R^{0AB} \wedge {}^*R^0_{AB} + D^0K^{AB} \wedge {}^*D^0K_{AB} - 2R^{0AB} \wedge {}^*D^0K_{AB} - 2K^A{}_C \wedge K^{CB} \wedge {}^*D^0K_{AB}) + 2\chi W(K)] , \tag{2.11}$$

where $W(K)$ is the self-interaction potential d-form for the contortion:

$$W(K) = \left[\frac{1+g}{2\chi}V^{AB} + 2mR^{0AB} + mK^A{}_C \wedge K^{CB}\right] \wedge {}^*(K_A{}^D \wedge K_{DB}) . \tag{2.12}$$

The field equations are obtained by varying the action with respect to the independent variables V^A and K^{AB}. In this way one obtains in general a rather complicated expression in terms of the components of the curvature and of the torsion tensor. However, in this paper we shall look only for solutions of the field equations describing a vacuum background which is the product of maximally symmetric spaces, $\langle R^0 \rangle = \text{const}$, and in which the contortion tensor satisfies

$$\langle D^0K^{AB} \rangle = 0, \quad \langle D^0K_{ABC} \rangle = 0 . \tag{2.13}$$

In this case the equation for the contortion reduces to $\langle \delta W/\delta K \rangle = 0$ (see Appendix A), and the generalized Einstein equations are simply second order, because the Riemannian covariant derivatives of the components of the curvature tensor are vanishing. For this background, writing explicitly in components the equation following from the variation of V^A, we obtain (see Appendix A)

$$\frac{1}{2\chi}\langle 2R^0_M{}^P \rangle = \left\langle mR^0_{MNAB}R^{0NPAB} + 4mR^0_{MN}{}^{AB}K^{[N}{}_{AD}K^{P]D}{}_B + \frac{1+g}{2\chi}4K_{[M}{}^{[N|D}K_{N]D}{}^{P]} + 4mK^{[N}{}_{AD}K^{P]D}{}_B K_{[M}{}^{AE}K_{N]E}{}^B \right\rangle , \tag{2.14}$$

where $R^0_M{}^P = R^0_{MN}{}^{PN}$ is the usual Ricci tensor, and the equation is enclosed in brackets as a reminder that the additional condition (2.13) is to be satisfied, and the curvature tensor $\langle R^0_{MNAB} \rangle$ must be covariantly constant.

It is also convenient to introduce a scalar potential $U(K)$ to express in components the quartic torsion self-interaction. Setting

$$\int W(K) = -2\int V\, d^dz\, U(K) \tag{2.15}$$

where $V = \det(V_M{}^A)$, we find, from Eq. (2.12),

$$U(K) = m(R^0_{MN}{}^{AB} + K_M{}^A{}_C K_N{}^{CB})K^{[M}{}_{AD}K^{N]D}{}_B + \frac{1+g}{2\chi}K_{[M}{}^{MD}K_{N]D}{}^N \tag{2.16}$$

(see also Appendix A).

First of all we consider the case of a vacuum configuration characterized by a constant, nonvanishing value of the totally antisymmetric part of the torsion tensor[16] $\langle K_{ABC} \rangle = \langle K_{[ABC]} \rangle$ (for example, in string theory the field strength H for the two-form potential can be geometrically interpreted as the totally antisymmetric torsion part of the connection,[12,17,18] and vacuum configurations characterized by $H \neq 0$ have been recently considered[12,17,19–21]). The background determined by a nonzero vector part of the torsion will be discussed in Sec. IV.

Looking then for solutions describing the product of two maximally symmetric spaces, $M_{d-3} \times M_3$, respectively, $(d-3)$ and 3 dimensional, we set

$$\langle R^{0\alpha\beta} \rangle = \lambda_{d-3}V^{\alpha\beta}, \quad \langle R^{0ab} \rangle = \lambda_3 V^{ab} , \quad \langle R^{0\alpha a} \rangle = 0 , \tag{2.17}$$

where greek and small latin letters denote tangent space indices running, respectively, from 1 to $d-3$, and from $d-3$ to d, and λ_{d-3}, λ_3 are the cosmological constants of the two spaces. In this case the field equations can be satisfied by

$$\langle K_{abc} \rangle = \sigma_0 \epsilon_{abc}, \quad \langle K_{\alpha BC} \rangle = 0 , \tag{2.18}$$

where σ_0 is a constant which minimizes the potential (2.16). In this background the conditions (2.13) are satisfied (see Appendix B); moreover, from Eq. (2.17) one has

$$\langle R^0_{\mu\nu}{}^{\alpha\beta} \rangle = \lambda_{d-3}(\delta^\alpha_\mu \delta^\beta_\nu - \delta^\alpha_\nu \delta^\beta_\mu) , \quad \langle R^0_{mn}{}^{ab} \rangle = \lambda_3(\delta^a_m \delta^b_n - \delta^a_n \delta^b_m) , \tag{2.19}$$

and then

$$\langle R^0_\mu{}^\nu \rangle = \lambda_{d-3}(d-4)\delta^\nu_\mu, \quad \langle R^0_m{}^n \rangle = 2\lambda_3 \delta^n_m , \tag{2.20}$$

and the field equations (2.14) reduce to

$$\left[\frac{1}{2\chi} + m\lambda_{d-3}\right]\lambda_{d-3} = 0 , \tag{2.21}$$

$$\left[\frac{1}{2\chi}+m\lambda_3\right]\lambda_3=\left[2m\lambda_3+\frac{1+g}{2\chi}\right]\sigma_0{}^2-m\sigma_0{}^4 . \qquad (2.22)$$

The torsion potential (2.16) becomes, setting $K_{ABC} =\sigma\delta^a_A\delta^b_B\delta^c_C\epsilon_{abc}$,

$$V(\sigma)=3m\left[\sigma^4-\sigma^2\left[\frac{1+g}{2\chi m}+2\lambda_3\right]\right] \qquad (2.23)$$

and the torsion field equation for σ=const is simply $\langle\partial U/\partial\sigma\rangle=0$; if the coefficient of σ^2 in Eq. (2.23) is negative, the constant nonvanishing solution $\langle\sigma\rangle=\sigma_0$, where

$$\sigma_0{}^2=\frac{1+g}{4\chi m}+\lambda_3 \qquad (2.24)$$

corresponds to a minimum of U(σ) and breaks spontaneously the local SO($1,d-1$) invariance.

Combining Eqs. (2.22) and (2.24) we then have

$$\lambda_3=-\frac{1}{2\chi m}\frac{1}{4g}(1+g)^2 \qquad (2.25)$$

and

$$\sigma_0{}^2=\frac{1}{2\chi m}\frac{1}{4g}(g^2-1) . \qquad (2.26)$$

Finally, Eq. (2.21) gives two possible values for the cosmological constant of the $(d-3)$-dimensional space: namely,

$$\lambda_{d-3}=0,-\frac{1}{2\chi m} . \qquad (2.27)$$

The vacuum configuration described by Eqs. (2.17) and (2.18) solves then the field equations of the quadratic theory of gravity considered in this paper, provided that λ_{d-3}, λ_3, and σ_0 are related to the parameters χ,m,g of the action (2.2) according to Eqs. (2.25)–(2.27). This vacuum solution with broken Lorentz symmetry may be classically stable only if the nonvanishing value of σ_0 minimizes the potential $U(\sigma)$, and we can see from (2.23) and (2.24) that this occurs, supposing $m>0$, only for $\sigma_0{}^2>0$, that is, using (2.26), for

$$\chi>0,\ g>0,\ g^2>1 , \qquad (2.28)$$

or

$$\chi>0,\ g<0,\ g^2<1 , \qquad (2.29)$$

if the gravitational constant is positive, while for

$$\chi<0,\ g>0,\ g^2<1 , \qquad (2.30)$$

or

$$\chi<0,\ g<0,\ g^2>1 , \qquad (2.31)$$

if the gravitational constant is negative.

To conclude this section, it should be noted that the field equations (2.14) also admit a Ricci-flat solution for the four-dimensional physical space-time even if the internal space is not Ricci flat, unlike the theory considered in Ref. 3. This is due to the curvature-squared term in the action, which leads to a quadratic equation for the cosmological constant in the $(d-3)$-dimensional space, and then to a double solution of the field equations (this point has been recently stressed also by Deser[22]).

In fact, following Ref. 3, and decomposing the index set $\{\alpha\}$ as $\{\dot\alpha\}\cup\{\bar\alpha\}$, where $\dot\alpha=1,2,\ldots,4$ and $\bar\alpha=5,6,\ldots,d-3$, we can look for solutions with the product structure $M_4\times M_{d-7}\times M_3$ setting

$$\langle R^{0\dot\alpha\dot\beta}\rangle=\lambda_1 V^{\dot\alpha\dot\beta},\quad \langle R^{0\bar\alpha\bar\beta}\rangle=\lambda_2 V^{\bar\alpha\bar\beta},\quad \langle R^{0ab}\rangle=\lambda_3 V^{ab} . \qquad (2.32)$$

Following the same procedure as before we obtain, instead of Eq. (2.21), two equations for λ_1 and λ_2,

$$\left[\frac{1}{2\chi}+m\lambda_1\right]\lambda_1=0 , \qquad (2.33)$$

$$\left[\frac{1}{2\chi}+m\lambda_2\right]\lambda_2=0 , \qquad (2.34)$$

while the equations for λ_3 and σ_0 are not modified. We can obtain then a Minkowski four-dimensional background, corresponding to the solution

$$\lambda_1=0,\ \lambda_2=-\frac{1}{2\chi m},\ \lambda_3=-\frac{1}{2\chi m}\frac{1}{4g}(1+g)^2 , \qquad (2.35)$$

even if the internal space is not Ricci flat ($\lambda_2\neq0$, $\lambda_3\neq0$), and without fine-tuning of adjustable parameters.

III. THE LOWEST-ORDER EFFECTIVE ACTION WITH SO($1,d-4$)×SO(3) LOCAL INVARIANCE

Choosing as the physical vacuum the solution of the field equations which minimizes the Higgs potential (see Sec. II), and expanding the gravitational field around this configuration, keeping fixed the torsion background (2.18), we are led to shift the definition of curvature

$$R^{0AB}=\langle R^{0AB}\rangle+\bar R^{AB} \qquad (3.1)$$

so that the physical vacuum corresponds to $\langle\bar R^{AB}\rangle=0$.

In Sec. II it has been shown that, because of the curvature-squared term in the action, one has two possible solutions for $\langle R^{0AB}\rangle$ [see Eq. (2.27)]. We must consider separately then the two cases

$$\text{(a)}\ \lambda_{d-3}=0,\ \lambda_3=-(1+g)^2/8\chi mg , \qquad (3.2)$$

and

$$\text{(b)}\ \lambda_{d-3}=-\frac{1}{2\chi m},\ \lambda_3=-(1+g)^2/8\chi mg . \qquad (3.3)$$

In both cases we obtain, to lowest order as an effective action, the action constructed in Ref. 3 for a quasi-Riemannian theory of gravity with SO($1,d-4$)×SO(3) as the tangent space group.

Case (a)

Consider first the case in which the $(d-3)$-dimensional space is Ricci flat, corresponding to Eq. (3.2). In this background $\langle R^{0\alpha\beta}\rangle=0$, and Eq. (3.1) becomes

$$R^{0AB}=\lambda_3\delta^A_a\delta^B_b V^{ab}+\bar R^{AB} . \qquad (3.4)$$

Expanding the action (2.11) on the fixed torsion background $K^{AB}=\langle K^{AB}\rangle$ we find that, to lowest order, the terms linear in D^0K do not contribute (see Appendix B), so that the action (2.11) becomes

$$S=\int\left[\left(\frac{\lambda_3}{2\chi}+m\lambda_3{}^2\right)V^{ab}\wedge{}^*V_{ab}+\frac{2U_0}{d(d-1)}V^{AB}\wedge{}^*V_{AB}\right]$$
$$+\int\left[\frac{1}{2\chi}\bar{R}^{AB}\wedge{}^*V_{AB}+2m\lambda_3\bar{R}^{ab}\wedge{}^*V_{ab}+2m\bar{R}^{AB}\wedge{}^*\langle K_A{}^C\wedge K_{CB}\rangle\right]$$
$$+m\int\bar{R}^{AB}\wedge{}^*\bar{R}_{AB}+m\int D^0\langle K^{AB}\rangle\wedge{}^*D^0\langle K_{AB}\rangle\ ,\tag{3.5}$$

where $U_0=\langle U\rangle=U(\sigma_0)=-3m\sigma_0{}^4$ is the constant value of the torsion potential in vacuum [see Eqs. (2.23) and (2.24)].

It is interesting to note that the first integral in (3.5), representing the contribution to the action of the total effective cosmological constant, is vanishing. In fact, using the definition of duality operation (2.5), one has

$$\int V^{ab}\wedge{}^*V_{ab}=\frac{6}{d(d-1)}\int V^{AB}\wedge{}^*V_{AB}\tag{3.6}$$

and Eq. (2.22) can be rewritten

$$\left(\frac{1}{2\chi}+m\lambda_3\right)\lambda_3=-\tfrac{1}{3}U_0\tag{3.7}$$

so that the two terms in the integral cancel exactly against each other, and we have a theory with zero cosmological term. Moreover, using (2.18) and (2.24),

$$2m\bar{R}^{AB}\wedge{}^*\langle K_A{}^C\wedge K_{CB}\rangle=2m\bar{R}^{AB}\wedge{}^*V^{EF}\langle K_{[E\,|\,A}{}^CK_{F]CB}\rangle=-2m\sigma_0{}^2\bar{R}^{ab}\wedge{}^*V_{ab}=-2m\left(\lambda_3+\frac{1+g}{4\chi m}\right)\bar{R}^{ab}\wedge{}^*V_{ab}\ .\tag{3.8}$$

Therefore the action (3.5) becomes simply

$$S=m\int\bar{R}^{AB}\wedge{}^*\bar{R}_{AB}+\frac{1}{2\chi}\int(\bar{R}^{\alpha\beta}\wedge{}^*V_{\alpha\beta}-g\bar{R}^{ab}\wedge{}^*V_{ab}+2\bar{R}^{\alpha a}\wedge{}^*V_{\alpha a})+m\int D^0\langle K^{AB}\rangle\wedge{}^*D^0\langle K_{AB}\rangle\ .\tag{3.9}$$

The lowest-order action describes an effective theory of gravity with local $G_T=\text{SO}(1,d-4)\times\text{SO}(3)$ gauge invariance, expressed however in terms of the connection Ω^0 and curvature of the original SO$(1,d-1)$ group. Following Ref. 3, the Lorentz connection Ω^0 can be decomposed as

$$\Omega^{0AB}=\omega^{AB}+\bar{\omega}^{AB}\ ,\tag{3.10}$$

where ω is the G_T connection, and $\bar{\omega}$ is a one-form, transforming covariantly under G_T, which can be interpreted as the contortion for the G_T connection: in fact Eq. (2.8) becomes

$$dV^A+\omega^A{}_B\wedge V^B=-\bar{\omega}^A{}_B\wedge V^B\ .\tag{3.11}$$

In the case we are considering, V^α and V^a transform, respectively, as SO$(1,d-4)$ and SO(3) vectors, and we have the decomposition[3]

$$\omega^{\alpha\beta}=\Omega^{0\alpha\beta},\quad \omega^{ab}=\Omega^{0ab},\quad \omega^{\alpha a}=0\ ,$$
$$\bar{\omega}^{\alpha\beta}=0,\quad \bar{\omega}^{ab}=0,\quad \bar{\omega}^{\alpha a}=\Omega^{0\alpha a}\ .\tag{3.12}$$

Starting then from the definition (2.10) of the SO$(1,d-1)$ curvature, one can find easily the decomposition of the terms appearing in the action (3.9):

$$\bar{R}^{\alpha\beta}=d\Omega^{0\alpha\beta}+\Omega^{0\alpha}{}_A\wedge\Omega^{0A\beta}=R^{\alpha\beta}(\omega)+\bar{\omega}^\alpha{}_b\wedge\bar{\omega}^{b\beta}\ ,\tag{3.13}$$

where

$$R^{\alpha\beta}(\omega)=d\omega^{\alpha\beta}+\omega^\alpha{}_\gamma\wedge\omega^{\gamma\beta}\tag{3.14}$$

is the SO$(1,d-4)$ curvature two-form;

$$\bar{R}^{ab}=d\Omega^{0ab}+\Omega^{0a}{}_A\wedge\Omega^{0Ab}=R^{ab}(\omega)+\bar{\omega}^a{}_\gamma\wedge\bar{\omega}^{\gamma b}\ ,\tag{3.15}$$

where

$$R^{ab}(\omega)=d\omega^{ab}+\omega^a{}_c\wedge\omega^{cb}\tag{3.16}$$

is the SO(3) curvature two-form; and finally

$$\bar{R}^{\alpha a}=d\Omega^{0\alpha a}+\Omega^{0\alpha}{}_B\wedge\Omega^{0Ba}=d\bar{\omega}^{\alpha a}+\omega^\alpha{}_\beta\wedge\bar{\omega}^{\beta a}+\omega^a{}_b\wedge\bar{\omega}^{\alpha b}\equiv D(\omega)\bar{\omega}^{\alpha a}\ ,\tag{3.17}$$

where $D(\omega)$ is the G_T-covariant exterior derivative.

Moreover, in the configuration (2.8), in which

$$\langle K_{ab}\rangle=\sigma_0\epsilon_{abc}V^c\tag{3.18}$$

and all the other components of K are vanishing, we have

$$D^0\langle K^{\alpha\beta}\rangle=0\ ,\tag{3.19}$$
$$D^0\langle K^{ab}\rangle=D(\omega)K^{ab}=-\sigma_0\epsilon_c{}^{ab}\bar{\omega}^c{}_\alpha\wedge V^\alpha\ ,\tag{3.20}$$
$$D^0\langle K^{\alpha a}\rangle=\bar{\omega}^\alpha{}_c\wedge K^{ca}=\sigma_0\epsilon^{ca}{}_b\bar{\omega}^\alpha{}_c\wedge V^b\ .\tag{3.21}$$

Therefore the lowest-order contribution to the action from the contortion kinetic term can be expressed as a sum of bilinears in $\bar{\omega}$, that is,

$$m D^0\langle K^{AB}\rangle \wedge {}^*D^0\langle K_{AB}\rangle = m(D^0\langle K^{ab}\rangle \wedge {}^*D^0\langle K_{ab}\rangle + 2D^0\langle K^{a\alpha}\rangle \wedge {}^*D^0\langle K_{a\alpha}\rangle)$$
$$= 2m\sigma_0{}^2[\bar\omega^c{}_\alpha \wedge V^\alpha \wedge {}^*(\bar\omega_{c\beta}\wedge V^\beta) + \bar\omega^\alpha{}_c \wedge V_b \wedge {}^*(\bar\omega_\alpha{}^c \wedge V^b - \bar\omega_\alpha{}^b \wedge V^c)] \ . \tag{3.22}$$

The lowest-order effective action (3.9), considering the small curvature limit $|\chi m R| \ll 1$ in which the contributions of the curvature-squared terms can be neglected, then becomes

$$S = \frac{1}{2\chi}\int \Bigg[R^{\alpha\beta}(\omega)\wedge {}^*V_{\alpha\beta} - gR^{ab}(\omega)\wedge {}^*V_{ab} + \bar\omega^\alpha{}_b \wedge \bar\omega^{b\beta}\wedge {}^*V_{ab} - g\bar\omega^a{}_\gamma \wedge \bar\omega^{\gamma b}\wedge {}^*V_{ab} + 2D(\omega)\bar\omega^{a\alpha}\wedge {}^*V_{a\alpha}$$
$$+ \frac{1}{2g}(g^2-1)[\bar\omega^c{}_\alpha \wedge V^\alpha \wedge {}^*(\bar\omega_{c\beta}\wedge V^\beta) + \bar\omega^\alpha{}_c \wedge V_b \wedge {}^*(\bar\omega_\alpha{}^c \wedge V^b - \bar\omega_\alpha{}^b \wedge V^c)] \Bigg] \tag{3.23}$$

[note that also the $D(\omega)\bar\omega$ term in the action is bilinear in $\bar\omega$, as can be easily seen integrating by parts and using Eq. (3.11)].

In order to compare this effective theory with the quasi-Riemannian theory of Ref. 3, it is convenient to express the action (3.23) explicitly in components. A straightforward computation leads to

$$R^{\alpha\beta}(\omega)\wedge {}^*V_{\alpha\beta} = -V\, d^dz\, R_{\alpha\beta}{}^{\alpha\beta} \ , \tag{3.24}$$

$$R^{ab}(\omega)\wedge {}^*V_{ab} = -V\, d^dz\, R_{ab}{}^{ab} \ , \tag{3.25}$$

where $R_{\alpha\beta}{}^{\alpha\beta}$ and $R_{ab}{}^{ab}$ are, respectively, the SO(1,$d-4$) and SO(3) scalar curvatures:

$$\bar\omega^\alpha{}_b \wedge \bar\omega^{b\beta}\wedge {}^*V_{\alpha\beta} = -V\, d^dz\, 2\bar\omega_{[\alpha}{}^{\beta b}\bar\omega_{\beta]}{}^\alpha{}_b \ , \tag{3.26}$$

$$\bar\omega^a{}_\gamma \wedge \bar\omega^{\gamma b}\wedge {}^*V_{ab} = -V\, d^dz\, 2\bar\omega_{[a}{}^{b\beta}\bar\omega_{b]}{}^a{}_\beta \ , \tag{3.27}$$

$$D(\omega)\bar\omega^{a\alpha}\wedge {}^*V_{a\alpha} = V\, d^dz\, 2(\bar\omega_{[\alpha}{}^{\beta b}\bar\omega_{\beta]}{}^\alpha{}_b + \bar\omega_{[a}{}^{b\beta}\bar\omega_{b]}{}^a{}_\beta) \ , \tag{3.28}$$

$$\bar\omega^c{}_\alpha \wedge V^\alpha \wedge {}^*(\bar\omega_{c\beta}\wedge V^\beta) + 2\bar\omega^\alpha{}_c \wedge V_b \wedge {}^*(\bar\omega_\alpha{}^{[c}\wedge V^{b]}) = -V\, d^dz(3\bar\omega_{\beta ac}\bar\omega^{\beta ac} - \bar\omega_{\beta ac}\bar\omega^{\alpha\beta c} + 2\bar\omega_{bac}\bar\omega^{bac} + \bar\omega_c{}^c{}_\alpha \bar\omega_b{}^{b\alpha}) \ . \tag{3.29}$$

Following the notation of Ref. 3, we define the symmetric trace-free part of $\bar\omega$ as

$$\bar\omega_{\{\alpha\beta\}a} = \bar\omega_{(\alpha\beta)a} - \frac{1}{d-3}\eta_{\alpha\beta}\bar\omega_\gamma{}^\gamma{}_a, \quad \bar\omega_{\{ab\}\beta} = \bar\omega_{(ab)\beta} - \tfrac{1}{3}\delta_{ab}\bar\omega_c{}^c{}_\beta \tag{3.30}$$

(curly brackets denote symmetrization) such that $\eta^{\alpha\beta}\bar\omega_{\{\alpha\beta\}a} = 0 = \delta^{ab}\bar\omega_{\{ab\}\beta}$. The action (3.23) can be rewritten then finally as

$$S = -\frac{1}{2\chi}\int V\, d^dz \Bigg[R_{\alpha\beta}{}^{\alpha\beta} - gR_{ab}{}^{ab} + \left[1 + 2\frac{g^2-1}{g}\right]\bar\omega_{[\beta\gamma]a}\bar\omega^{[\beta\gamma]a} + \left[\frac{g^2-1}{g} - 1\right]\bar\omega_{\{\beta\gamma\}a}\bar\omega^{\{\beta\gamma\}a}$$
$$+ \left[\frac{d-4}{d-3} + \frac{g^2-1}{g(d-3)}\right]\bar\omega_{\beta}{}^\beta{}_a \bar\omega_\gamma{}^{\gamma a} + \left[2 + g + \frac{g^2-1}{g}\right]\bar\omega_{[bc]\alpha}\bar\omega^{[bc]\alpha}$$
$$+ \left[\frac{g^2-1}{g} - 2 - g\right]\bar\omega_{\{bc\}\alpha}\bar\omega^{\{bc\}\alpha} + \left[\tfrac{2}{3}(g+2) + \frac{5}{3}\frac{g^2-1}{2g}\right]\bar\omega_b{}^b{}_\alpha \bar\omega_c{}^{c\alpha} \Bigg] \ . \tag{3.31}$$

Comparing this expression with the general form of the quasi-Riemannian action constructed in Ref. 3, which depends on nine arbitrary parameters $c_1, \ldots, c_9$, we can conclude that the mechanism of spontaneous breaking of the Lorentz symmetry we have considered, based on a gravitational self-interaction quadratic in torsion and curvature, leads in this case to a quasi-Riemannian theory with local SO(1,$d-4$)$\times$SO(3) gauge invariance. In the low-energy limit, we obtain in particular the action of Ref. 3, in which all nine parameters can be expressed in terms of only one independent coupling constant, and in this case we find, in units such that $1/2\chi = 1$,

$$\begin{aligned} &c_1 = 1, \quad c_2 = -g, \quad c_3 = 1 + 2\frac{g^2-1}{g} \ , \\ &c_4 = \frac{g^2-1}{g} - 1, \quad c_5 = \frac{d-4}{d-3} + \frac{g^2-1}{g(d-3)} \ , \\ &c_6 = 2 + g + \frac{g^2-1}{g}, \quad c_7 = \frac{g^2-1}{g} - 2 - g \ , \\ &c_8 = \frac{2}{3}(g+2) + \frac{5}{3}\frac{g^2-1}{2g}, \quad c_9 = 0 \ . \end{aligned} \tag{3.32}$$

Case (b)

The second possible solution of the vacuum field equations (2.14), with a constant totally antisymmetric torsion background, corresponds to a nonvanishing curvature also in the $(d-3)$-dimensional space, $\lambda_{d-3}=-1/2\chi m$, see Eq. (3.3). In this case we must set

$$R^{0AB}=\lambda_{d-3}\delta^A_\alpha\delta^B_\beta V^{\alpha\beta}+\lambda_3\delta^A_a\delta^B_b V^{ab}+\bar{R}^{\,AB}$$

and expanding the action (2.11) as before, we find again that the terms linear in D^0K do not contribute, and, using the field equations (2.21) and (2.22) and the identity (3.6), that the total effective cosmological constant is vanishing.

Using also Eq. (3.8) we obtain then, to lowest order, the action

$$S=m\int(\bar{R}^{\,AB}\wedge{}^*\bar{R}_{AB}+D^0\langle K^{AB}\rangle\wedge{}^*D^0\langle K_{AB}\rangle)+\frac{1}{2\chi}\int(-\bar{R}^{\,\alpha\beta}\wedge{}^*V_{\alpha\beta}-g\bar{R}^{\,ab}\wedge{}^*V_{ab}+2\bar{R}^{\,\alpha a}\wedge{}^*V_{\alpha a})\,. \tag{3.33}$$

It is interesting to note, comparing this equation with the corresponding action (3.9) obtained in the previous case, the sign difference of the term containing the $SO(1,d-4)$ scalar curvature, $\bar{R}^{\,\alpha\beta}\wedge{}^*V_{\alpha\beta}$.

Since it is from this term that one obtains, after dimensional reduction, the usual four-dimensional Einstein Lagrangian of general relativity [i.e., the SO(3,1) scalar curvature], it follows that in this case a negative value of χ is required to avoid that, in four dimensions, the effective low-energy gravitational interaction is repulsive.

This implies however that the original action (2.2), describing gravity before the process of spontaneous symmetry breaking, contains a gravitational coupling constant with the "wrong" sign, $\chi<0$, or, in other words, describes "antigravity," and it is the transition associated with the breaking of the d-dimensional Lorentz symmetry that leads to the change of sign of the effective four-dimensional coupling constant (as regards this point, it should be mentioned that the possibility of antigravity in the early Universe, in relation with the spontaneous breaking of a local gauge symmetry, was first considered by Linde;[23] moreover, a reversal of sign of the effective gravitational coupling, induced by a breaking of the Lorentz symmetry, is not in disagreement with the possibility of obtaining, in the early Universe, an inflationary phase of accelerated expansion in the context of a theory of gravity which is not locally Lorentz invariant[24]).

In any case, what is to be stressed is that, according to the mechanism of spontaneous breaking discussed in this paper, it is the value of the parameter g, governing the strength of the torsion-squared term in the gravitational Lagrangian, which determines if there is a reversal of sign in the effective gravitational coupling constant as a consequence of the $SO(1,d-1)$ breaking.

In fact, if $-1<g<0$, or $g>1$, then the torsion potential has a minimum for $\chi>0$ [see (2.28) and (2.29)], and then we must choose the vacuum configuration corresponding to case (a) (i.e., $\lambda_{d-3}=0$) to avoid a repulsive four-dimensional interaction in the low-energy limit. On the contrary, if $g<-1$, or $0<g<1$, the minimum is obtained for $\chi<0$ [see (2.30) and (2.31)], and then the physically acceptable solution corresponds to case (b) ($\lambda_{d-3}=-1/2\chi m$), so that one has antigravity before the Lorentz invariance is broken. And since a negative value of χ may introduce ghosts into the theory, this seems to suggest that a realistic model should be characterized by a torsion self-interaction appearing in the action with an effective strength g varying in the range $-1<g<0$, $g>1$.

To conclude this section we can determine the values of the parameters of the low-energy quasi-Riemannian effective theory corresponding to this last case. Starting from the lowest-order action (3.33), and following exactly the same procedure as before [i.e., decomposing the Lorentz connection and curvature according to the new $SO(1,d-4)\times SO(3)$ tangent space group, keeping fixed the torsion background (2.18), and defining an effective coupling constant $\chi'=-\chi>0$], we obtain

$$S=\frac{1}{2\chi'}\int\Big\{R^{\alpha\beta}(\omega)\wedge{}^*V_{\alpha\beta}+gR^{ab}(\omega)\wedge{}^*V_{ab}+\bar{\omega}^{\alpha}{}_{b}\wedge\bar{\omega}^{b\beta}\wedge{}^*V_{\alpha\beta}+g\bar{\omega}^{a}{}_{\gamma}\wedge\bar{\omega}^{\gamma b}\wedge{}^*V_{ab}-2D(\omega)\bar{\omega}^{\alpha a}\wedge{}^*V_{\alpha a}-\frac{g^2-1}{2g}[\bar{\omega}^{c}{}_{\alpha}\wedge V^{\alpha}\wedge{}^*(\bar{\omega}_{c\beta}\wedge V^{\beta})+\bar{\omega}^{\alpha}{}_{c}\wedge V_b\wedge{}^*(\bar{\omega}_{\alpha}{}^{c}\wedge V^{b}-\bar{\omega}_{\alpha}{}^{b}\wedge V^{c})]\Big\}\,. \tag{3.34}$$

Written explicitly in components we have

$$S=-\frac{1}{2\chi'}\int V\,d^dz\Big[R_{\alpha\beta}{}^{\alpha\beta}+gR_{ab}{}^{ab}-\Big(3+2\frac{g^2-1}{g}\Big)\bar{\omega}_{[\beta\gamma]a}\bar{\omega}^{[\beta\gamma]a}+\Big(3-\frac{g^2-1}{g}\Big)\bar{\omega}_{\{\beta\gamma\}a}\bar{\omega}^{\{\beta\gamma\}a}-\Big(\frac{3(d-4)}{d-3}+\frac{g^2-1}{g(d-3)}\Big)\bar{\omega}_{\beta}{}^{\beta}{}_{a}\bar{\omega}^{\gamma a}_{\gamma}-\Big(g+2+\frac{g^2-1}{g}\Big)\bar{\omega}_{[bc]\alpha}\bar{\omega}^{[bc]\alpha}+\Big(g+2-\frac{g^2-1}{g}\Big)\bar{\omega}_{\{bc\}\alpha}\bar{\omega}^{\{bc\}\alpha}-\Big(\tfrac{2}{3}(g+2)+\frac{5}{3}\,\frac{g^2-1}{2g}\Big)\bar{\omega}_{b}{}^{b}{}_{\alpha}\bar{\omega}_{c}{}^{c\alpha}\Big]\,. \tag{3.35}$$

The nine parameters of Ref. 3 are then determined as (in units $1/2\chi'=1$)

$$\begin{aligned} &c_1=1,\ c_2=g,\ c_3=-3-2\frac{g^2-1}{g}\ ,\\ &c_4=3-\frac{g^2-1}{g},\ c_5=-3\frac{d-4}{d-3}-\frac{g^2-1}{g(d-3)}\ ,\\ &c_6=-2-g-\frac{g^2-1}{g},\ c_7=2+g-\frac{g^2-1}{g}\ ,\\ &c_8=-\frac{2}{3}(2+g)-\frac{5}{3}\frac{g^2-1}{2g},\ c_9=0\ . \end{aligned} \tag{3.36}$$

IV. BROKEN LORENTZ SYMMETRY WITH A CONSTANT VECTOR TORSION BACKGROUND

Starting again from the d-dimensional Lorentz-invariant action (2.2), in this section we consider a vacuum configuration characterized by a constant, nonvanishing value of the vector part ϕ^A of the torsion tensor, and we show that in this case one can obtain, in the low-energy limit, an effective action for a quasi-Riemannian theory of gravity with local SO$(1,d-1)$ invariance.

Supposing that only the vector ϕ^A contributes to the torsion background, to discuss the solution of the vacuum field equations we can set

$$K^{AB}=V^A\phi^B-V^B\phi^A\ , \tag{4.1}$$

that is,

$$K_C{}^{AB}=\delta_C{}^A\phi^B-\delta_C{}^B\phi^A \tag{4.2}$$

so that the torsion potential (2.16) becomes simply

$$U(\phi)=-\frac{1+g}{2\chi}\tfrac{1}{2}\phi^2(d-1)(d-2)+m(2R_A{}^B\phi^A\phi_B-R\phi^2)+\tfrac{1}{2}m\phi^4(d-1)(d-2) \tag{4.3}$$

($\phi^2=\phi^A\phi_A$) and the generalized Einstein equations (2.14) reduce to

$$\frac{1}{2\chi}\langle 2R^{0}_{M}{}^{P}\rangle=\Big\langle mR^0_{MNAB}R^{0NPAB}+\frac{1+g}{2\chi}2(d-2)(\phi^2\delta_M{}^P-\phi_M\phi^P)+4m(\phi^2R^0_M{}^P-R^0_M{}^A\phi^P\phi_A+R^0_{MN}{}^{AP}\phi^N\phi_A)+2m(d-2)(\phi^2\phi_M\phi^P-\delta_M{}^P\phi^4)\Big\rangle\ . \tag{4.4}$$

The vacuum field equations for the metric and the torsion can be satisfied by choosing a background configuration corresponding to a maximally symmetry space with $d-1$ dimensions, that is, by setting

$$\begin{aligned}&\langle R^{0\alpha\beta}\rangle=\lambda V^{\alpha\beta},\ \langle R^{0\alpha\underline{d}}\rangle=0\ ,\\ &\langle\phi^\alpha\rangle=0,\ \langle\phi^{\underline{d}}\rangle=\sigma_0\ ,\end{aligned} \tag{4.5}$$

where greek indices run from 1 to $d-1$, the underlined index $\underline{d}$ means the fixed value corresponding to the total number of dimensions we are considering (for example, in $d=7$ we have $R^{\alpha\underline{d}}=R^{\alpha 7}$, $\phi^{\underline{d}}=\phi^7$), and σ_0 is a constant which minimizes the potential (4.3).

With the choice (4.5), the conditions (2.13) are satisfied: in fact from (4.1) and (4.5) we have

$$\langle K^{\alpha\beta}\rangle=0,\ \langle K^{\alpha\underline{d}}\rangle=\sigma_0V^\alpha\ , \tag{4.6}$$

and since

$$D^0K^{AB}=dK^{AB}+\Omega^{0A}{}_C\wedge K^{CB}+\Omega^{0B}{}_C\wedge K^{AC} \tag{4.7}$$

it follows that

$$\langle D^0K^{\alpha\beta}\rangle=\langle\Omega^{0\alpha}{}_{\underline{d}}\wedge K^{\underline{d}\beta}+\Omega^{0\beta}{}_{\underline{d}}\wedge K^{\alpha\underline{d}}\rangle\ , \tag{4.8}$$

$$\begin{aligned}\langle D^0K^{\alpha\underline{d}}\rangle&=\langle dK^{\alpha\underline{d}}+\Omega^{0\alpha}{}_\beta\wedge K^{\beta\underline{d}}\rangle\\ &=\sigma_0\langle dV^\alpha+\Omega^{0\alpha}{}_\beta\wedge V^\beta\rangle\\ &=-\sigma_0\langle\Omega^{0\alpha}{}_{\underline{d}}\wedge V^{\underline{d}}\rangle\end{aligned} \tag{4.9}$$

[we have used Eq. (2.8)]. But the maximal symmetry of the background implies $\langle\Omega^{0\alpha}{}_{\underline{d}}\rangle=0$; therefore Eqs. (4.8) and (4.9) are both vanishing. Moreover, the only nonvanishing components of $\langle K_{ABC}\rangle$ are $\langle K_\gamma{}^{\alpha\underline{d}}\rangle=\sigma_0\delta^\alpha_\gamma$ and

$$\langle D^0K_\gamma{}^{\alpha\underline{d}}\rangle=\sigma_0(\Omega^{0\beta}_\gamma\delta_\beta{}^\alpha+\Omega^{0\alpha}{}_\beta\delta_\gamma{}^\beta)\equiv 0 \tag{4.10}$$

(because, for a Lorentz connection, $\Omega^{0\alpha\beta}=-\Omega^{0\beta\alpha}$). Therefore also $\langle D^0K_{ABC}\rangle=0$.

For the configuration (4.5), the field equation (4.4) reduces to

$$\left[\frac{1}{2\chi}+m\lambda\right]\lambda=\left[\frac{1+g}{2\chi}+2m\lambda\right]\sigma_0{}^2-m\sigma_0{}^4 \tag{4.11}$$

and the potential (4.3), setting $\phi^A=\delta^A_{\underline{d}}\sigma$, is

$$U(\sigma)=\frac{m}{2}(d-1)(d-2)\left[\sigma^4-\sigma^2\left[\frac{1+g}{2\chi m}+2\lambda\right]\right]\ . \tag{4.12}$$

The constant nonvanishing solution $\langle\sigma\rangle=\sigma_0$,

$$\sigma_0{}^2=\frac{1+g}{4\chi m}+\lambda \tag{4.13}$$

of the torsion field equation $\langle\partial U/\partial\sigma\rangle=0$ breaks spontaneously the SO$(1,d-1)$ symmetry, and in this case, combining Eqs. (4.11) and (4.13), we can express λ and $\sigma_0{}^2$ as a function of g,χ,m, according to Eqs. (2.25) and (2.26), just like in the case of a totally antisymmetric torsion background previously considered. This vacuum solution corresponds to a minimum of the potential (4.12) only if the coefficient of σ^2 is negative, that is only if the conditions (2.28)–(2.31) are satisfied.

Expanding the gravitational field around this fixed torsion configuration, we set

$$R^{0AB}=\lambda\delta^A_\alpha\delta^B_\beta V^{\alpha\beta}+\bar{R}^{AB} \tag{4.14}$$

and starting from the action (2.11) we find, to lowest order,

$$S=\int\left[\left[\frac{1}{2\chi}+m\lambda\right]\lambda V^{\alpha\beta}\wedge{}^*V_{\alpha\beta}+2\frac{U_0}{d(d-1)}V^{AB}\wedge{}^*V_{AB}\right]$$
$$+\int\left[\frac{1}{2\chi}\bar{R}^{AB}\wedge{}^*V_{AB}+2m\lambda\bar{R}^{\alpha\beta}\wedge{}^*V_{\alpha\beta}+2m\bar{R}^{AB}\wedge{}^*\langle K_A{}^C\wedge K_{CB}\rangle\right]$$
$$+m\int(\bar{R}^{AB}\wedge{}^*\bar{R}_{AB}+D^0\langle K^{AB}\rangle\wedge{}^*D^0\langle K_{AB}\rangle)-2m\int D^0\langle K^{AB}\rangle\wedge{}^*(R^0_{AB}+\langle K_A{}^C\wedge K_{CB}\rangle)\,, \tag{4.15}$$

where $U_0=\langle U\rangle=U(\sigma_0)$ is the constant vacuum value of the potential (4.12), i.e.,

$$U_0=-\frac{m}{2}(d-1)(d-2)\sigma_0{}^4\,. \tag{4.16}$$

The first integral in (4.15), representing the cosmological term, is vanishing: in fact we have the identity

$$\frac{1}{(d-1)(d-2)}\int V^{\alpha\beta}\wedge{}^*V_{\alpha\beta}=\frac{1}{d(d-1)}\int V^{AB}\wedge{}^*V_{AB} \tag{4.17}$$

and the field equation (4.11), using (4.13) and (4.16), can be rewritten

$$\left[\frac{1}{2\chi}+m\lambda\right]\lambda=m\sigma_0{}^4=-\frac{2U_0}{(d-1)(d-2)} \tag{4.18}$$

so that the two contributions to the cosmological constant cancel against each other.

Moreover, the last integral in Eq. (4.15) is vanishing to lowest order: in fact, for the constant torsion background (4.6)

$$\langle K_\alpha{}^B\wedge K_{B\beta}\rangle=-\sigma_0{}^2V_{\alpha\beta},\quad \langle K_\alpha{}^B\wedge K_{B\underline{d}}\rangle=0 \tag{4.19}$$

and the terms linear in D^0K become, neglecting trilinears in the connection,

$$D^0\langle K^{AB}\rangle\wedge{}^*(R^0_{AB}+\langle K_A{}^C\wedge K_{CB}\rangle)=(\lambda^2-\sigma_0{}^2)D^0\langle K^{\alpha\beta}\rangle\wedge{}^*V_{\alpha\beta} \tag{4.20}$$

so that, integrating by parts and using (2.8), its contribution to the action integral (4.15) is vanishing.

Finally we note that, from Eqs. (4.19) and (4.13), we obtain

$$2m\bar{R}^{AB}\wedge{}^*\langle K_A{}^C\wedge K_{CB}\rangle=-\left[\frac{1+g}{2\chi}+2m\lambda\right]\times\bar{R}^{\alpha\beta}\wedge{}^*V_{\alpha\beta}\,. \tag{4.21}$$

The lowest-order action reduces then to

$$S=\frac{1}{2\chi}\int(-g\bar{R}_{\alpha\beta}\wedge{}^*V_{\alpha\beta}+2\bar{R}^{\alpha\underline{d}}\wedge{}^*V_{\alpha\underline{d}})+m\int\bar{R}^{AB}\wedge{}^*\bar{R}_{AB}+m\int D^0\langle K^{AB}\rangle\wedge{}^*D^0\langle K_{AB}\rangle\,. \tag{4.22}$$

As in the case considered previously, the constant coefficient of the SO(1,$d-2$) scalar curvature must be positive, in order to avoid obtaining, in four dimensions, an effective gravitational coupling constant with the wrong sign, which would lead in the low-energy limit to a repulsive interaction instead of an attractive one. Therefore this model of gravity with broken Lorentz symmetry may be physically acceptable (attractive low-energy gravity and stable vacuum configuration) only if $\chi>0$ and $-1<g<0$ or, alternatively, $\chi<0$ and $0<g<1$ (in this second case the breaking of the Lorentz gauge symmetry is associated with a reversal of sign in the effective coupling constant).

The action (4.22) has a local $G_T=$SO(1,$d-2$) invariance. Decomposing the Lorentz connection Ω^0 into the G_T connection ω and the one-form $\bar{\omega}$ (transforming covariantly under G_T) according to Eqs. (3.10) and (3.11), in this case we have

$$\omega^{\alpha\beta}=\Omega^{0\alpha\beta},\quad \omega^{\alpha\underline{d}}=0,\quad \bar{\omega}^{\alpha\beta}=0,\quad \bar{\omega}^{\alpha\underline{d}}=\Omega^{0\alpha\underline{d}}\,, \tag{4.23}$$

and the curvature terms may be decomposed as

$$\bar{R}^{\alpha\beta}=R^{\alpha\beta}(\omega)+\bar{\omega}^\alpha{}_{\underline{d}}\wedge\bar{\omega}^{\underline{d}\beta}\,, \tag{4.24}$$

$$\bar{R}^{\alpha\underline{d}}=D(\omega)\bar{\omega}^{\alpha\underline{d}} \tag{4.25}$$

[see also (3.13) and (3.17)], where

$$R^{\alpha\beta}(\omega)=d\omega^{\alpha\beta}+\omega^\alpha{}_\gamma\wedge\omega^{\gamma\beta} \tag{4.26}$$

is the SO(1,$d-2$) curvature two-form, and $D(\omega)$ denotes in this case the SO(1,$d-2$)-covariant derivative:

$$D(\omega)\bar{\omega}^{\alpha\underline{d}}=d\bar{\omega}^{\alpha\underline{d}}+\omega^\alpha{}_\beta\wedge\bar{\omega}^{\beta\underline{d}}\,. \tag{4.27}$$

The lowest-order contribution of the contortion kinetic term, appearing in the action (4.22), can be expressed as a sum of bilinears in $\bar{\omega}$. In fact, from Eqs. (4.6), (4.8), and (4.9) we have

$$D^0\langle K^{\alpha\beta}\rangle=-\sigma_0(\bar{\omega}^\alpha{}_{\underline{d}}\wedge V^\beta-\bar{\omega}^\beta{}_{\underline{d}}\wedge V^\alpha)\,, \tag{4.28}$$

$$D^0\langle K^{\alpha\underline{d}}\rangle=-\sigma_0\bar{\omega}^\alpha{}_{\underline{d}}\wedge V^{\underline{d}}\,. \tag{4.29}$$

Therefore

$$mD^0\langle K^{AB}\rangle\wedge{}^*D^0\langle K_{AB}\rangle=m(D^0\langle K^{\alpha\beta}\rangle\wedge{}^*D^0\langle K_{\alpha\beta}\rangle+2D^0\langle K^{\alpha\underline{d}}\rangle\wedge{}^*D^0\langle K_{\alpha\underline{d}}\rangle)$$
$$=2m\sigma_0{}^2[\bar{\omega}^{[\alpha}{}_{\underline{d}}\wedge V^{\beta]}\wedge{}^*(\bar{\omega}_{\alpha\underline{d}}\wedge V_\beta-\bar{\omega}_{\beta\underline{d}}\wedge V_\alpha)+\bar{\omega}^\alpha{}_{\underline{d}}\wedge V^{\underline{d}}\wedge{}^*(\bar{\omega}_{\alpha\underline{d}}\wedge V^{\underline{d}})]\,. \tag{4.30}$$

The action (4.22), in the limit $|\chi m R| \ll 1$ in which the curvature-squared terms may be neglected, then becomes

$$S=\frac{1}{2\chi}\int \Big[-gR^{\alpha\beta}(\omega)\wedge {}^*V_{\alpha\beta}-g\bar{\omega}^{\alpha}{}_{\underline{d}}\wedge\bar{\omega}^{\underline{d}\beta}\wedge {}^*V_{\alpha\beta}+2D(\omega)\bar{\omega}^{\alpha\underline{d}}\wedge {}^*V_{\alpha\underline{d}} + \frac{1}{2g}(g^2-1)[\bar{\omega}^{[\alpha}{}_{\underline{d}}\wedge V^{\beta]}\wedge {}^*(\bar{\omega}_{\alpha\underline{d}}\wedge V_\beta-\bar{\omega}_{\beta\underline{d}}\wedge V_\alpha)+\bar{\omega}^{\alpha}{}_{\underline{d}}\wedge V^{\underline{d}}\wedge {}^*(\bar{\omega}_{\alpha\underline{d}}\wedge V^{\underline{d}})]\Big] . \tag{4.31}$$

In order to compute in this case the coefficients of the corresponding quasi-Riemannian action of Ref. 3, we can rewrite Eq. (4.31) in components. A simple computation leads to

$$R^{\alpha\beta}(\omega)\wedge {}^*V_{\alpha\beta}=-V\,d^dz\,R_{\alpha\beta}{}^{\alpha\beta} , \tag{4.32}$$

$$\bar{\omega}^{\alpha}\underline{d}\wedge\bar{\omega}^{d\beta}\wedge {}^*V_{\alpha\beta}=-V\,d^dz\,2\bar{\omega}_{[\alpha}{}^{\beta\underline{d}}\bar{\omega}_{\beta]}{}^{\alpha}{}_{\underline{d}} , \tag{4.33}$$

$$D(\omega)\bar{\omega}^{\alpha\underline{d}}\wedge {}^*V_{\alpha\underline{d}}=V\,d^dz\,2\bar{\omega}_{[\alpha}{}^{\beta\underline{d}}\bar{\omega}_{\beta]}{}^{\alpha}{}_{\underline{d}} , \tag{4.34}$$

$$\bar{\omega}^{\alpha}{}_{\underline{d}}\wedge V^{\underline{d}}\wedge {}^*(\bar{\omega}_{\alpha\underline{d}}\wedge V^{\underline{d}})+2\bar{\omega}^{[\alpha}{}_{\underline{d}}\wedge V^{\beta]}\wedge {}^*(\bar{\omega}_{\alpha\underline{d}}\wedge V_\beta)=-V\,d^dz[(d-2)\bar{\omega}_{\beta\alpha\underline{d}}\bar{\omega}^{\beta\alpha\underline{d}}+(d-2)\bar{\omega}_{\underline{d}\alpha\underline{d}}\bar{\omega}^{\underline{d}\alpha\underline{d}}+\bar{\omega}_{\beta}{}^{\beta}{}_{\underline{d}}\bar{\omega}_{\alpha}{}^{\alpha\underline{d}}] , \tag{4.35}$$

and defining, as in Ref. 3,

$$\bar{\omega}_{\{\alpha\beta\}\underline{d}}=\bar{\omega}_{(\alpha\beta)\underline{d}}-\frac{1}{d-1}\eta_{\alpha\beta}\omega_{\gamma}{}^{\gamma}{}_{\underline{d}} \tag{4.36}$$

we obtain finally

$$S=-\frac{1}{2\chi}\int V\,d^dz\Big[-gR_{\alpha\beta}{}^{\alpha\beta}+\Big[g+2+\frac{g^2-1}{2g}(d-2)\Big]\bar{\omega}_{[\alpha\beta]\underline{d}}\bar{\omega}^{[\alpha\beta]\underline{d}}+\Big[\frac{g^2-1}{2g}(d-2)-g-2\Big]\bar{\omega}_{\{\alpha\beta\}\underline{d}}\bar{\omega}^{\{\alpha\beta\}\underline{d}} + \Big[\frac{g^2-1}{2g}\frac{2d-3}{d-1}-\frac{d-2}{d-1}\Big]\bar{\omega}_{\alpha}{}^{\alpha}{}_{\underline{d}}\bar{\omega}_{\beta}{}^{\beta\underline{d}}+\frac{1}{2g}(g^2-1)(d-2)\bar{\omega}_{\underline{d}}{}^{d}{}_{\alpha}\bar{\omega}_{\underline{d}}{}^{d\alpha}\Big] . \tag{4.37}$$

Comparing this expression with the action of Ref. 3, we note that for a quasi-Riemannian theory in d dimensions, with tangent space group $SO(1,d-2)$, the terms in the action corresponding to the coefficients c_2,c_6,c_7 are vanishing, because obviously $R_{\underline{d}\underline{d}}{}^{\underline{d}\underline{d}}=\bar{\omega}_{[\underline{d}\underline{d}]\alpha}=\bar{\omega}_{\{\underline{d}\underline{d}\}\alpha}=0$. Introducing an effective coupling constant χ' such that $-1/2\chi'=g/2\chi$, we find that, according to this model of spontaneously broken Lorentz symmetry, the parameters of Ref. 3 are determined as (in units in which $1/2\chi'=1$)

$$\begin{aligned} &C_1=1,\quad C_3=-1-\frac{2}{g}-\frac{g^2-1}{2g}\frac{d-2}{g} ,\\ &C_4=1+\frac{2}{g}-\frac{g^2-1}{2g}\frac{d-2}{g} ,\\ &C_5=\frac{d-2}{g(d-1)}-\frac{g^2-1}{2g}\frac{2d-3}{(d-1)g} ,\\ &C_8=-\frac{g^2-1}{2g}\frac{d-2}{g},\quad C_9=0 . \end{aligned} \tag{4.38}$$

V. CONCLUSION

The model of higher-dimensional gravitational theory considered in this paper, based on the action (2.2), admits solutions of the field equations corresponding to a maximally symmetric ground state $M_{d-3}\times M_3$ or M_{d-1}, according to whether the vacuum is characterized by a nonzero value of the totally antisymmetric part or of the vector part of the torsion tensor.

In both cases, the lowest-order gravitational excitations of these configurations are described by a quasi-Riemannian theory with gauge group $SO(1,d-4)\times SO(3)$ or $SO(1,d-2)$, respectively, and all the arbitrary parameters of the corresponding action constructed in Ref. 3 can be determined as a function of d and g [see Eqs. (3.32), (3.36), and (4.38)].

In the case of a totally antisymmetric torsion one obtains two possible vacuum configurations which, minimizing the torsion potential, breaks spontaneously the $SO(1,d-1)$ gauge invariance. If M_{d-3} is not Ricci flat [see Eq. (3.3)], then the breaking of the Lorentz symmetry produces a reversal of sign in the effective four-dimensional gravitational coupling constant. In this case, however, the theory is physically acceptable (that is, one obtains an attractive four-dimensional low-energy gravitational interaction) and the ground-state configuration may be stable (that is, corresponds to a minimum of the torsion potential) only if $0<g<1$, or $g<-1$. If this condition is not satisfied, then the physical vacuum corresponds to a Ricci flat M_{d-3} [see Eq. (3.2)], and there is no change in the sign of the effective gravitational coupling associated with the transition from $SO(1,d-1)$ to $SO(1,d-4)\times SO(3)$.

In the case of a constant vector torsion background, the physical solution (attractive low-energy gravity and stable vacuum configuration) is obtained for $\chi>0$ and $-1<g<0$, or $\chi>0$ and $0<g<1$. It is only in this second case that the effective gravitational coupling changes sign when the SO$(1,d-1)$ Lorentz group breaks down spontaneously, leading to a quasi-Riemannian theory with SO$(1,d-2)$ local invariance.

Finally, it should be remarked that, in a realistic model, the requirement of the absence of ghosts and tachyons may put further restrictions on the possible form of the gravitational action and on the value of the arbitrary coupling constants.

APPENDIX A: VACUUM FIELD EQUATIONS

Using the definition of duality operation [see Eqs. (2.5) and (2.6)], the action (2.11) can be rewritten explicitly as

$$S=\frac{1}{(d-2)!}\int\left[\frac{R^{0AB}}{2\chi}+\frac{m}{2}R^{0EF}R^{0AB}{}_{EF}+m(D^0K^{EF}-2R^{0EF}-2K^E{}_D\wedge K^{DF})D^{0[A}K^{B]}{}_{EF}\right.$$
$$\left.+(2mR^{0EF}+mK^E{}_D\wedge K^{DF})K^{[A}{}_{ED}K^{B]D}{}_F+\frac{1+g}{2\chi}K^A{}_D\wedge K^{DB}\right]\wedge V^{C_1\cdots C_{d-2}}\epsilon_{ABC_1\cdots C_{d-2}}\,. \tag{A1}$$

Looking for a solution of the field equations which satisfies $\langle D^0R^0_{ABCD}\rangle=0$ and $\langle D^0K_{ABC}\rangle=0$, by varying the action with respect to V^A we can neglect terms containing the covariant derivatives of the curvature and contortion tensor, and we simply obtain

$$\left\langle\left[\frac{1}{2\chi}R^{0AB}+\frac{m}{2}R^{0EF}R^{0AB}{}_{EF}+\frac{1+g}{2\chi}K^A{}_D\wedge K^{DB}+m(2R^{0EF}+K^E{}_C\wedge K^{CF})K^{[A}{}_{ED}K^{B]D}{}_F\right]\wedge V^{C_1\cdots C_{d-3}}\epsilon_{ABC_1\cdots C_{d-2}}\right\rangle=0 \tag{A2}$$

or, in components,

$$\left\langle\left[\frac{1}{4\chi}R^0_{MN}{}^{AB}+\frac{m}{4}R^0_{MN}{}^{EF}R^{0AB}{}_{EF}+\frac{1+g}{2\chi}K_{[M}{}^{AD}K_{N]D}{}^B+m(R^0_{MN}{}^{EF}+K_{[M}{}^{ED}K_{N]D}{}^F)K^{[A}{}_{EL}K^{B]L}{}_F\right]V^{MNP}_{ABC}\right\rangle=0\,, \tag{A3}$$

where the symbol V^{MNP}_{ABC} is defined by

$$\epsilon^{MNPP_1\cdots P_{d-3}}\epsilon_{ABCC_1\cdots C_{d-3}}V^{C_1}_{P_1}V^{C_2}_{P_2}\cdots V^{C_{d-3}}_{P_{d-3}}=-(d-3)!\,VV^{MNP}_{ABC}\,. \tag{A4}$$

Therefore

$$R^0_{MN}{}^{AB}V^{MNP}_{ABC}=(R^0_{MN}{}^{MN}\delta_C{}^P+R^0_{MN}{}^{NP}\delta_C{}^M+R^0_{MN}{}^{PM}\delta_C{}^N-R^0_{MN}{}^{MP}\delta_C{}^N-R^0_{MN}{}^{NM}\delta_C{}^P-R^0_{MN}{}^{PN}\delta_C{}^M)=2R^0V_C{}^P-4R^{0P}_C\,, \tag{A5}$$

where $R^{0P}_C=R^0_{CN}{}^{PN}$, and so on for the other terms, so that Eq. (A3) becomes

$$\left\langle\frac{1}{2\chi}\frac{1}{2}(2R^0V_C{}^P-4R^{0P}_C)+\frac{m}{4}(2V_C{}^PR^0_{MNAB}R^{0MNAB}+4R^{0NPAB}R^0_{CNAB})+\frac{1+g}{2\chi}(2V_C{}^PK_{[M}{}^{MD}K_{N]D}{}^N+4K_{[C}{}^{[N|D}K_{N]D}{}^{|P]})\right.$$
$$+m(2V_C{}^PR^0_{MN}{}^{EF}K^{[M}{}_{ED}K^{N]D}{}_F+4R^0_{CN}{}^{EF}K^{[N}{}_{ED}K^{P]D}{}_F)+m(2V_C{}^PK_{[M}{}^{ED}K_{N]D}{}^FK^M{}_{EC}K^{NC}{}_F$$
$$\left.+4K_{[C}{}^{ED}K_{N]D}{}^FK^{[N}{}_{EL}K^{P]L}{}_F)\right\rangle=0\,. \tag{A6}$$

At this point it is convenient to write in components the contribution to the action of the torsion self-interaction (2.12). We have

$$\int W(K)=\frac{1}{(d-2)!}\int d^dz\left[m(R^0_{MN}{}^{EF}+K_{[M}{}^{ED}K_{N]D}{}^F)K^{[A}{}_{ED}K^{B]D}{}_F\right.$$
$$\left.+\frac{1+g}{2\chi}K_{[M}{}^{AD}K_{N]D}{}^B\right]\epsilon_{ABC_1\cdots C_{d-2}}\epsilon^{MNP_1\cdots P_{d-2}}V^{C_1}_{P_1}\cdots V^{C_{d-2}}_{P_{d-2}}$$
$$=-2\int V\,d^dz\,U(K)\,, \tag{A7}$$

where

$$U(K)=m(R^0_{MN}{}^{EF}+K_M{}^{ED}K_{ND}{}^F)K^{[M}{}_{EC}K^{N]C}{}_F+\frac{1+g}{2\chi}K_{[M}{}^{MD}K_{N]D}{}^N \tag{A8}$$

and then Eq. (A6) can be rewritten as

$$\left\langle \frac{1}{2\chi}\frac{1}{2}(2R^0V_C{}^P-4R_C^{0P})+\frac{m}{4}(2V_C{}^PR^{0MNAB}R^0_{MNAB}+4R^{0NPAB}R^0_{CNAB})+\frac{1+g}{2\chi}4K_{[C}{}^{[N|D}K_{N]D}{}^{|P]}+4mR^0_{CN}{}^{EF}K^{[N}{}_{ED}K^{P]D}{}_F \right.$$
$$\left. +4mK_{[C}{}^{ED}K_{N]D}{}^FK^{[N}{}_{EL}K^{P]L}{}_F+2V_C{}^PU(K)\right\rangle=0\ . \quad \text{(A9)}$$

Tracing gives

$$\left\langle \frac{1}{4\chi}R^0+\frac{m}{4}R^0_{MNAB}R^{0MNAB}+U\right\rangle=0 \quad \text{(A10)}$$

so that, from Eq. (A9), we obtain the field equation (2.14)

$$\left\langle -\frac{1}{2\chi}2R_C^{0P}+mR^{0NPAB}R^0_{CNAB}+\frac{1+g}{2\chi}4K_{[C}{}^{[N|D}K_{N]D}{}^{|P]}+4mR^0_{CN}{}^{EF}K^{[N}{}_{ED}K^{P]D}{}_F+4mK_{[C}{}^{ED}K_{N]D}{}^FK^{[N}{}_{EL}K^{P]L}{}_F\right\rangle=0\ . \quad \text{(A11)}$$

As regards the torsion field equation, by varying the action (2.11) with respect to K we have

$$\delta S=2m\int(D^{0*}D^0K_{AB}-D^{0*}R^0_{AB}+f_{AB})\wedge\delta K^{AB}\ , \quad \text{(A12)}$$

where the three-form f_{AB} is defined so that

$$\delta\left[\frac{W}{2m}-D^0K^{AB}\wedge{}^*(K_A{}^C\wedge K_{CB})\right]=f_{AB}\wedge\delta K^{AB}\ . \quad \text{(A13)}$$

If we are interested in a vacuum configuration described by a maximally symmetric space, $\langle R^0_{AB}\rangle=\lambda V_{AB}$, then $\langle D^{0*}R_{AB}\rangle=0$ [remember Eq. (2.8)]; moreover, if we impose on the solution the conditions (2.13), then the torsion field equation reduces simply to $\langle f_{AB}\rangle=0$, where $\langle f_{AB}\rangle=\delta W/\delta K^{AB}$.

APPENDIX B: VACUUM CONFIGURATION WITH A CONSTANT NONZERO TOTALLY ANTISYMMETRIC TORSION

Since $K^{AB}=K_C{}^{AB}V^C$, in the configuration (2.18) we have

$$\langle K^{\alpha\beta}\rangle=0=\langle K^{\alpha a}\rangle,\quad \langle K^{ab}\rangle=\sigma_0\epsilon_C{}^{ab}V^C\ . \quad \text{(B1)}$$

As

$$D^0K^{AB}=dK^{AB}+\Omega^{0A}{}_C\wedge K^{CB}+\Omega^{0B}{}_C\wedge K^{AC} \quad \text{(B2)}$$

it follows that

$$\langle D^0K^{\alpha\beta}\rangle=0\ , \quad \text{(B3)}$$

$$\langle D^0K^{\alpha a}\rangle=\langle\Omega^{0\alpha}{}_b\wedge K^{ba}\rangle\ , \quad \text{(B4)}$$

$$\langle D^0K^{ab}\rangle=\langle dK^{ab}+\Omega^{0a}{}_c\wedge K^{cb}+\Omega^{0b}{}_c\wedge K^{ac}\rangle$$
$$=\langle D(\omega)K^{ab}\rangle\ , \quad \text{(B5)}$$

where $\omega^{ab}=\Omega^{0ab}$ and $D(\omega)$ are, respectively, the SO(3) connection and covariant derivative.

Using Eq. (2.8), which implies

$$D^0V^a=dV^a+\Omega^{0a}{}_B\wedge V^B=D(\omega)V^a+\Omega^{0a}{}_\gamma\wedge V^\gamma=0 \quad \text{(B6)}$$

and remembering that ϵ_{abc} is an SO(3)-invariant tensor, we obtain then

$$\langle D^0K^{ab}\rangle=-\sigma_0\epsilon_c{}^{ab}\langle\Omega^{0c}{}_\gamma\wedge V^\gamma\rangle\ . \quad \text{(B7)}$$

But the maximal symmetry of the background [see (2.17)] implies $\langle\Omega^{0\alpha b}\rangle=0$; therefore, Eqs. (B4) and (B7) are both vanishing, and $\langle D^0K^{AB}\rangle=0$. Moreover $\langle D^0K_{abc}\rangle=D(\omega)\sigma_0\epsilon_{abc}=0$, so that conditions (2.13) are satisfied.

Expanding the gravitational field on the constant torsion background (B1), and using Eq. (3.4), we have, to lowest order,

$$-2m\int R^{0AB}\wedge{}^*D^0\langle K_{AB}\rangle=-2m\lambda_3\int D^0\langle K^{ab}\rangle\wedge{}^*V_{ab} \quad \text{(B8)}$$

modulo terms trilinear in the connection. Moreover the only nonvanishing components of $K_A{}^C\wedge K_{CB}$ on this background are

$$K_a{}^c\wedge K_{cb}=\sigma_0{}^2\epsilon_{da}{}^c\epsilon_{ecb}V^{de}=-\sigma_0{}^2V_{ab}\ . \quad \text{(B9)}$$

Therefore

$$-2m\int D^0\langle K^{AB}\rangle\wedge{}^*\langle K_A{}^C\wedge K_{CB}\rangle$$
$$=2m\sigma_0{}^2\int D^0\langle K^{ab}\rangle\wedge{}^*V_{ab}\ . \quad \text{(B10)}$$

Integrating by parts, and using (2.8), both the integral (B8) and (B10) are vanishing, so that the terms linear in D^0K, in the action (2.11), do not contribute to the lowest-order expansion.

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