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# Gravity in mines—An investigation of Newton's law


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The evidence that the value of the Newtonian gravitational constant $G$ inferred from measurements of gravity $g$ in mines and boreholes is of order 1% higher than the laboratory value is hardened with new and improved data from two mines in northwest Queensland. Surface-gravity surveys and more than 14 000 bore-core density values have been used to establish density structures for the mines, permitting full three-dimensional inversion to obtain $G$. Further constraint is imposed by requiring that the density structure give the same value of $G$ for several vertical profiles of $g$, separated by hundreds of meters. The only residual doubt arises from the possibility of bias by an anomalous regional gravity gradient. Neither measurements of gravity gradient above ground level (in tall chimneys) nor surface surveys are yet adequate to remove this doubt, but the coincidence of conclusions derived from mine data obtained in different parts of the world makes such an anomaly appear an improbable explanation. If Newton's law is modified by adding a Yukawa term to the gravitational potential of a point mass $m$ at distance $r$, $V=-(G_\infty m/r)(1+\alpha e^{-r/\lambda})$, then the mine data provide a mutual constraint on the values of $\alpha$ and $\lambda$, although they cannot be determined independently. Our results give $\alpha\approx-0.0075$ if $\lambda\leq 200$ m and $\alpha\approx-0.014$ if $\lambda\geq 10^4$ m, with intermediate values of $\alpha$ between these ranges, but values greater than $\alpha=-0.010$, $\lambda=800$ m appear to be disallowed by a comparison of satellite and land-surface estimates of gravity. Gravity experiments over a range of several kilometers are needed for a better constraint. Recent consideration of the Eötvös experiment in terms of a short-range force dependent upon the nuclear mass defect invites plans for similar experiments at sites where extreme topography ensures that the short-range force is directed at a substantial angle to normal gravity.


## INTRODUCTION

If there is a defect in Newton's law of gravity it is a subtle one, but reasons for postulating a defect have been canvassed for several years. If gravity is to be unified with the other fundamental forces then an experimental lead is needed to identify the range(s) and magnitude(s) of finite-range component(s).[1] At satellite and planetary distances, $10^6$–$10^{13}$ m, the inverse-square law appears unassailable and at greater distances it is difficult to devise definitive observations. There are recent confirmations of the inverse-square law at laboratory scales[2,3] (contradicting an earlier reported discrepancy[4]) although over a much more limited range and with less precision than at planetary distances. The gap between the laboratory and planetary ranges, $1$–$10^6$ m, is only partly and very inadequately filled by geophysical observations. Data available so far are very suggestive of a difference of order 1% between the values of the gravitational constant $G$ obtained in mines, with effective mass separations of 100–1000 m, and in laboratories (at about 0.1 m) and there are no measurements at all at larger scales. Astronomical and geophysical theories all assume the laboratory value of $G$. Since doubt has now arisen there is some incentive to replace this assumption by observations, quite apart from the consideration of unification theories.

The favored form for a modification of Newton's law[1] is the addition of a Yukawa term to the gravitational potential due to a point mass $m$ at distance $r$:

$$V=-\frac{G_\infty m}{r}(1+\alpha e^{-r/\lambda}) , \tag{1}$$

where $\alpha$ is the amplitude of the "short-range" force that is superimposed on normal gravity and $\lambda$ is the effective distance to which it extends. It turns out that, for several reasons that follow, Eq. (1) is difficult to test by measurements in mines. In particular we are not able to separate effectively the parameters $\alpha$ and $\lambda$, and so we present our results in terms of a constraint on the mutual relationship between them. It also happens that a comparison of gravity gradients above and below ground level, which effectively removes extraneous anomalous gradients if the mine data are simply used to determine an assumed unknown but constant value of $G$, becomes difficult to use with Eq.

(1). We therefore conclude this paper by summarizing the further experiments that are needed (and the extent to which we have these experiments in hand ourselves).

The mine method of measuring $G$ was pioneered in the 1850s by Airy[5] and pursued further by von Sterneck, but by the end of the last century it was so clearly inferior to the laboratory methods, especially using torsion balances, that it was lost sight of until revived by us.[6] Our interest was stimulated partly by the calculation of Fujii[7] who estimated values of the constants in Eq. (1), $\alpha=\frac{1}{3}$, $\lambda=10-1000$ m. We happened to be aware of mine-gravity data that made this combination of values appear unacceptable and we began a series of measurements[6] to determine how well modern geophysical methods could constrain the hypothesis of non-Newtonian gravity. We also examined marine-gravity data obtained in oil exploration and data available in the literature on gravity measurements in mines and boreholes for which reliable independently determined densities data were quoted,[8] finding a systematic trend to high values of $G$. It is interesting to note that in our literature searching we stumbled on puzzled comments by two authors[9,10] who had found evidence of a discrepancy between expected and observed variations of gravity with depth, in one case in the sea and in the other in a borehole. In both cases the sign of the discrepancy coincides with our findings.

Our original measurements[6] were made in the mine at Mount Isa, an area of lead-silver-zinc and copper mineralization in northwest Queensland. Subsequently we obtained data also from the mine at Hilton,[11] about 20 km north of Mount Isa, which is much more favorable for our experiment because one of the shafts is well clear of the density inhomogeneities due to mineralization. Both data sets have now been considerably extended and reanalyzed. The final Hilton results are only marginally different from those in the preliminary report[11] and provide the most reliable kilometer-scale estimate of $G$. There are four semi-independent estimates from Mount Isa, where we also have free-air gradient data from the 260-m refinery chimney.

## THEORY OF THE GRAVITY PROFILE

The variation of gravity with depth in the Earth, assuming Newtonian physics and a layered structure, was given by Stacey *et al.*[6] and rederived in more general terms by Dahlen.[12] The assumption of a layered structure is necessarily only an approximation and we now have sufficient data from both the Mount Isa and Hilton mines to avoid using it. Nevertheless a convenient way to proceed in analyzing our data is first to assume an average density throughout the whole mine volume, applying the simple equations for a layered structure, and then iteratively refine the analysis by applying corrections for the localized departures from average density. With a judicious initial choice of average density the convergence is very rapid and a single iteration suffices.

If we assume Newtonian gravity but with an arbitrary and unknown value of the constant $G$, then $G$ is determined from the variation of gravity $g$ with depth $z$,

$$g(z)-g(0)=U(z)-4\pi G X(z)\ , \tag{2}$$

where $U(z)$ is a purely geometrical term, representing the fact that at depth $z$ one is nearer to the center of mass, and incorporating effects of rotation and ellipticity, and $X(z)$ accounts for the reduction in gravity by disallowing the mass outside the level of measurement:

$$U(z)=2\frac{g(0)z}{R}\left[1+\frac{3}{2}\frac{z}{R}-3J_2\left(\tfrac{3}{2}\sin^2\phi_0-\tfrac{1}{2}\right)\right] + 3\omega^2 z(1-\sin^2\phi_0) \tag{3}$$

$$X(z)=\frac{c}{a}\left[1+2\frac{z}{R}+\frac{1}{2}\left[1-\frac{c^2}{a^2}\right]\right]\int_0^z\rho\,dz - \frac{2}{R}\int_0^z \rho z\,dz\ . \tag{4}$$

Here $R$ is the radius of the Earth at the site of measurements, i.e., distance of the surface point from the center, $\phi_0$ is the geocentric latitude, $J_2=1.082\,64\times10^{-3}$ is the inertial ellipticity coefficient, $\omega=7.292\times10^{-5}$ rad sec$^{-1}$ is the angular rotation rate, $a$ and $c$ are the equatorial and polar radii, where $a=6.378\,14\times10^6$ m and $(1-c^2/a^2)=0.006\,694\,4$, and $\rho$ is density. We write to first order in polar flattening

$$R=a\left[1-\frac{1}{2}\left[\frac{a^2}{c^2}-1\right]\sin^2\phi_0\right]+h\ , \tag{5}$$

where $h$ is the height of the surface above sea level.

It is convenient to present Eqs. (2)–(4) in a manner that appears to imply that the earth must be ellipsoidally layered at all depths and in particular throughout the surface layers for our analysis to be valid, but that is not the case. With respect to the surface layers we can see that remote material can have no influence by integrating the gravity difference at the opposite faces of a plane circular disc of radius $r$ and thickness $z \ll r$, expanding in powers of $z/r$ (Ref. 13):

$$\Delta g_{\text{layer}}=-4\pi G\rho z\left[1-\frac{1}{2}\frac{z}{r}+O\left[\left[\frac{z}{r}\right]^3\right]\right]\ . \tag{6}$$

Thus for an accuracy of 0.3%, which we claim for the mine experiment, it would not matter if the surface layer disappeared altogether beyond $r=150z$, say 150 km. Given that the average rock densities down to 1 km are unlikely to vary by more than 10% in the range 15–150 km, detailed data out to 15 km suffice. We have density data to 10 km, with no indication of a dramatic change for hundreds of kilometers beyond that. Thus any defect in Eq. (2) does not arise from inhomogeneity of the surface layers. Any difficulty must arise from deep departure from the simple layered density structure. We refer to this problem later.

Since initially we are concerned to know whether a departure from Newtonian gravity is clearly indicated, we use these equations, with arbitrary $G$, to decide whether the value so determined differs significantly or systematically from the laboratory value, $G^* = 6.6726(5)\times 10^{-11}$ $\mathrm{m^3\,kg^{-1}\,sec^{-2}}$ (Ref. 14). Since we are assuming a perfect inverse-square law, we calculate values of $U(z)$ and $X(z)$ for each of the depths at which $g(z)$ is measured, assuming the average density throughout, $\bar{\rho} = 2750\ \mathrm{kg\,m^{-3}}$. The measured values of $g(z)$ are then corrected for the known mass anomalies relative to $\bar{\rho}$, assuming the inverse-square law and $G = G^*$. The corrected values of $[g(z) - g(0)]$ give a value of $G \neq G^*$ by Eq. (2). The calculation is then repeated with the new value of $G$ applied in the corrections, but with a suitable choice of $\bar{\rho}$ the revision to the estimate of $G$ is trivial.

We refer to values of $G$ calculated in this way as "apparent $G$," which cannot, however, be identified with $G_\infty$ in Eq. (1). If the inverse-square law is replaced by Eq. (1), the equation for a mine-gravity profile has additional terms. Assuming $\lambda \ll R$, the gravity variation due to the non-Newtonian term in Eq. (1) is obtained by integrating over a half space (of density $\bar{\rho}$)

$$\Delta[g(z) - g(0)] = -2\pi G_\infty \alpha\lambda\bar{\rho}(1 - e^{-z/\lambda}) . \tag{7}$$

Since the unknown $G_\infty$ occurs also in the Newtonian term this must be revised by substituting in terms of $G^*$

$$G^* = G_\infty \left[1 + \alpha \left[1 + \frac{r^*}{\lambda}\right] e^{-r^*/\lambda}\right] \approx G_\infty (1 + \alpha) , \tag{8}$$

where $r^* = 0.07$ m is the effective mass separation at which the best laboratory value of $G^*$ was obtained[14] and from the result of the experiment by Chen, Cook, and Metherell[2] we know that $\lambda \gg r^*$ for any interesting value of $\alpha$. Noting that by Eq. (1)

$$g_0 = \frac{G_\infty M}{R^2} + 2\pi G_\infty \alpha\lambda\bar{\rho} , \tag{9}$$

where $M$ is the mass of the Earth, and substituting for $G_\infty$ by Eq. (8), we find the difference $\Delta g$ between $[g(z) - g(0)]$ by Eq. (1) with $\lambda \ll R$ and what would be observed if $\alpha = 0$ and perfect Newtonian physics prevailed:

$$\Delta g(z) = \frac{4\pi G^* \bar{\rho}\alpha}{1+\alpha} \left[z - \frac{\lambda}{2}(1 - e^{-z/\lambda})\right] . \tag{10}$$

We refer to $\Delta g$ as the gravity residual. Values are determined by using $G^*$ in Eq. (2) to obtain the theoretical $[g(z) - g(0)]$ for comparison with observations.

The gradient of the gravity residual is

$$\frac{d\Delta g(z)}{dz} = \frac{4\pi G^* \bar{\rho}\alpha}{1+\alpha} [1 - \tfrac{1}{2} e^{-z/\lambda}] \tag{11}$$

from which it is seen that there is an anomalous gradient below the surface for any value of $\lambda$. If $z \ll \lambda$ the factor in square brackets in Eq. (11) is $\frac{1}{2}$ and if $z \gg \lambda$ this factor becomes unity. Thus there is a change in the anomalous gradient by a factor 2 at $z \approx \lambda$, but if the depth range of measurements is either much less than or much greater than $\lambda$ an almost constant anomalous gradient is obtained. It is also of interest to calculate the free-air gradient (above ground level) from Eq. (1), which gives at height $h$ an anomalous term

$$\Delta g(h) = \frac{2\pi G^* \alpha\lambda\bar{\rho}}{1+\alpha} e^{-h/\lambda} . \tag{12}$$

At the surface itself Eq. (12) gives the same anomalous gradient as Eq. (10), so that by Eq. (1) the gradient discontinuity through the surface is precisely what one would expect by Newton's law with $G = G^*$.

## RESULTS FROM THE HILTON MINE

A preliminary report[11] on gravity measurements in the mine at Hilton outlined the geological structure of the area and reported values for the densities of all the major rock units that are represented. The mine exists because of mineralization of one particular formation, Urquhart shale, all other rocks being barren and having much more homogeneous densities. A particular advantage of the Hilton mine for our experiment is that it has two shafts, one, of depth 650 m, designated J53 in the mine coordinate system, penetrating the Urquhart shale, and the other of depth 1000 m, designated P49, 600 m away from J53 and several hundred meters from the nearest mineralization. It is the P49 shaft that is of primary interest to this work, but J53 is valuable because the gravity profile down it provides a check on the Urquhart shale density. The estimate of $G$ from P49 is only slightly affected by the shale density, whereas $G$ from the J53 profile is strongly affected. By adjusting the estimated density so that both gravity profiles give the same value of $G$ we obtain a self-consistent solution that is independent of uncertainty in the Urquhart shale density.

To a good approximation the geological structure is two dimensional, with layers inclined at about 76° to horizontal, striking almost due north-south. In the original analysis[11] this was assumed to be exactly so, but here we present a revision that acknowledges north-south irregularities in the Urquhart shale and adjacent siltstone. These irregularities are barely apparent in surface-gravity survey data that were used to make minor adjustments to the geological model reported by Holding and Tuck[11] but are more obvious on a gravity profile along the 600-m-deep tunnel connecting the two shafts. We also divided the shale into three regions of slightly different densities.

Accuracy of the density data is crucial to the whole experiment and two sources of error are recognized. First, densities of individual bore-core samples are determined only to the nearest 10 $\mathrm{kg\,m^{-3}}$. Although variability is greater than this, extensive sampling reduces the random error below 10 $\mathrm{kg\,m^{-3}}$. Then this limit (0.36%) becomes an upper bound on the possible error in $G$ arising from a systematic error in density measurement. The second problem arises from nonrandom sampling. This is the problem with Urquhart shale. Apart from the localized high densities due to mineralization, the total volume of which is reasonably well known, there are in the shale numerous thin sheared and fractured zones. Because these zones do not produce good core they are not represented in the density sampling. We therefore adopted two extreme models for the Urquhart shale density. The upper limit (model $A$) assumes that the average density is the appropriate average core density (weighted according to the mineralized fraction). The other extreme (model $B$) takes an overgenerous estimate of the fragmented volume as 20% and assumes that this is all 40% less dense than the competent rock, making the average shale density 8% lower than the average core value. Densities of the rocks were carefully sampled to a range of about 1 km, using 2300 core samples, to establish statistical uncertainties below the 10-kg m$^{-3}$ limit. The densities were identified with well-located rock units to give the three-dimensional density structure. Outside this range we relied upon density surveys by the Geology Department of Mount Isa Mines Ltd. to distances greater than 10 km in all directions and the accuracy is believed to be not much less.

All gravity measurements were made with LaCoste-Romberg meter G608. Readings were adjusted for the gravity tide, checked for instrument drift in the conventional way, and referenced to a gravity station at Mount Isa airport. Some readings were cross checked with another meter (G20) but no measurable differences were found. Measurements were made not in the shafts but in tunnels or stubs to one side, with the meter mounted on a tripod as near as convenient to the center of the tunnel cross section, to minimize excavation corrections. Positions were obtained from mine survey pegs, believed to be accurate in three coordinates to better than 0.1 m. Surface terrain corrections were applied to all values and a correction for loss of density in a weathered surface layer of variable thickness identified by the surface gravity survey.

By assuming a uniform average density of 2750 kg m$^{-3}$ for the whole mine area and applying corrections to the $[g(z)-g(0)]$ values to account for details of the density structure, we convert $X(z)$ [Eq. (4)] to a straightforward analytical expression, as is $U(z)$ [Eq. (3)]. Then all of the information from which conclusions must be drawn is contained in the corrected gravity data. We make this more obvious by calculating what the gravity values should be according to Newtonian physics with $G=G^*$ and taking differences, which are identified as the gravity residuals (observed minus calculated), for comparison with Eq. (10). Table I lists the gravity residuals for both shafts of the Hilton mine after adjustment of the Urquhart shale density estimate to give equal $G$ for the two shafts by Eq. (2). This requires 0.72 times model $A$ plus 0.38 times model $B$. However, the P49 data are only marginally affected by this density adjustment.

The P49 residuals are plotted in Fig. 1 with two curves given by Eq. (10) with differently selected values of $\alpha$ and $\lambda$. The solid curve is the least-squares fit to all of the data, equally weighted and taking the surface value as one data point, not as a constraint. It is important not to overemphasize the surface data point in the analysis, because its value relative to the others is determined by the densities of the surface layers, which are poorly measured. The data point at 183 m is believed to be displaced by a local density inhomogeneity due to a sharp dip in the weathering horizon that was not accurately modeled. If we omit both this point and the surface point then the best fit is given by the dashed line. The two curves are more or less equally good fits to the data, emphasising the ambiguity in determining $\alpha$ and $\lambda$ separately.

TABLE I. Gravity residuals (observed minus calculated gravity variation with depth) for the two shafts of the Hilton mine (20°34′S, 139°28′E). The J53 data were obtained in a region of density inhomogeneity and are used only to provide a density check. The P49 data may be identified with $\Delta g$ by Eq. (10).

| P49 | | J53 | |
|---|---|---|---|
| $z$ (m) | $\Delta g$ (mGal) | $z$ (m) | $\Delta g$ (mGal) |
| 0 | 0 | 0 | 0 |
| 183.62 | −0.396 | 129.16 | −0.694 |
| 243.45 | −0.323 | 189.54 | −0.823 |
| 363.55 | −0.523 | 249.59 | −0.839 |
| 483.55 | −0.654 | 308.98 | −0.913 |
| 603.65 | −0.961 | 369.02 | −1.042 |
| 723.50 | −1.126 | 489.17 | −1.712 |
| 783.55 | −1.263 | 609.43 | −1.255 |
| 813.55 | −1.357 | | |
| 843.55 | −1.356 | | |
| 963.55 | −1.576 | | |
| 993.54 | −1.614 | | |

If we fix $\lambda$ at each of a series of values and obtain corresponding values of $\alpha$ for best fits to the data we find the relationship between $\alpha$ and $\lambda$ in Fig. 2. In the case of the solid curve (all data) the open circle marks the point of smallest variance but there is very little variation along the entire curve, so that in fact we can only impose the mutual constraint on $\alpha$ and $\lambda$ represented by the curve. In the case of the fit to data below 200 m there is a secondary minimum in the variance, the two minima being represented by the solid circles.

Figure 2 also indicates two constraints on $(\alpha,\lambda)$ imposed by other data. The lower bound was obtained in the laboratory experiment of Chen, Cook, and Metherell.[2] The upper bound is imposed by recognizing that at the Earth's surface gravity is given by Eq. (9) whereas at satellite altitudes there is no evidence of the second term. Thus referred to extrapolation from the satellite value of $(G_\infty M)$, there is a fractional anomaly in surface gravity

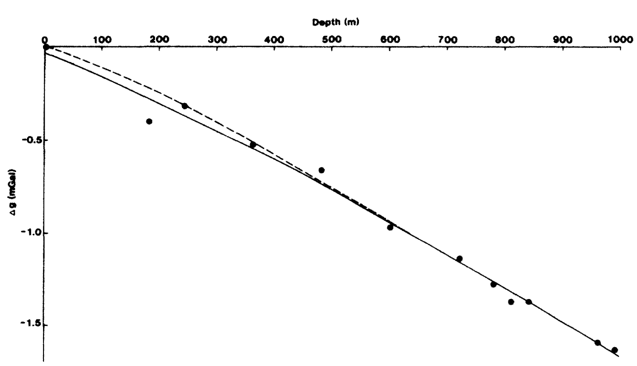


FIG. 1. Gravity residuals from P49 shaft at the Hilton mine. The curves are obtained from Eq. (10). The solid line is the best fit to all of the data, obtained with $\alpha=-0.010\,73$, $\lambda=1360$ m. The dashed line is the best fit to data below 200 m only and is given by $\alpha=-0.007\,89$, $\lambda=203$ m.

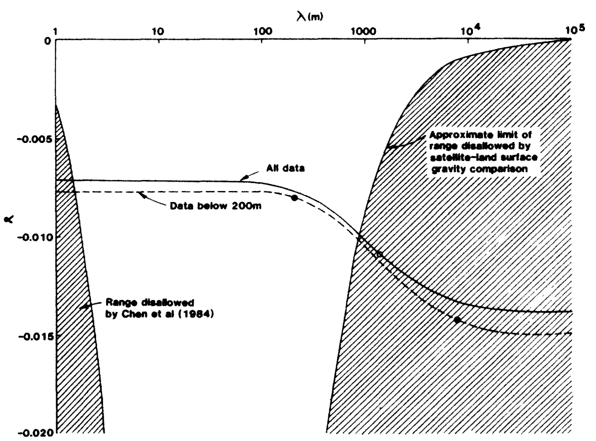


FIG. 2. Relationship between $\alpha$ and $\lambda$ [Eq. (1)] obtained by fitting the gravity residuals for the P49 shaft at the Hilton mine to Eq. (10).

given by

$$\frac{\Delta g_s}{g_0} = \frac{2\pi G_\infty \alpha \lambda \bar{\rho}}{g_0} = \frac{C\bar{\rho}\lambda\alpha}{1+\alpha} , \tag{13}$$

where $C = 4.28 \times 10^{-11}$ $\mathrm{m^2 kg^{-1}}$. According to Rapp, as quoted by Gibbons and Whiting,[1] $\Delta g_s / g_0 \not> 10^{-6}$, although this limit might be stretched a bit. Assuming this value and an effective land surface average density $\bar{\rho} = 2750$ $\mathrm{kg\,m^{-3}}$, we obtain the upper bound in Fig. 2. A closer scrutiny of this limit is obviously merited.

Nothing in our data compels interpretation in terms of Eq. (1), although this form has the natural advantage of permitting an inverse-square law to apply at both laboratory and planetary ranges. We note that Long[4,15] has advocated an alternative form

$$G = G_\infty \left[ 1 + \epsilon \ln \frac{r}{R_{\max}} \right] \quad \text{for } r \le R_{\max} . \tag{14}$$

Long favored the value $\epsilon = 0.002$, although several experiments now contradict this.[2,3] Fitting the P49 residuals to this relationship gives $\epsilon \approx 0.0007$, depending slightly upon the choice of value of $R_{\max}$, which is poorly constrained. If data below 200 m only are used, the fit is almost as good as that obtained by considering Eq. (1), but if the surface data point is included the fit becomes much poorer. On balance we can find no support for Eq. (14).

## RESULTS FROM MOUNT ISA MINE

In some important respects the situation at Mount Isa is less favorable for our experiment than that at Hilton. Terrain effects and corrections for excavations are more serious and, although the geological structure is essentially similar, there are recognized structural complications extending below the level of our measurements and not as well delineated by coring as the shallower levels. The mine openings are much more extensive than at Hilton and we have obtained four semi-independent gravity profiles, although all of them are in the ore-bearing Urquhart shale. Density sampling is very comprehensive and we have been able to use more than 12 000 bore-core density values obtained by mining company staff, but, although the Urquhart shale density doubt is less serious than at Hilton it is not as easily avoided. Another reason for interest in Mount Isa is that the refinery for both mines is sited there and the smelter flues are contained in a 260-m concrete chimney, allowing us to obtain a gravity profile to this height above ground level, as well as to a depth of 950 m.

The analysis procedure for the Mount Isa data is the same as for the Hilton data up to the point of obtaining values of $G$ by Eq. (2). However, we have not adjusted doubtful densities to make the several semi-independent values agree. This would be a questionable procedure in the case of the Mount Isa data because all of the available profiles penetrate the Urquhart shale, but is less necessary anyway because at Mount Isa this formation has much less incompetent rock. A simple comparison of the values of $G$ for Mount Isa with the corresponding value for Hilton suffices to demonstrate that the Mount Isa results support our conclusion from Hilton but are less accurate. Least-square fits to Eq. (2) with formal standard deviations arising from the scatter of the data give the results in Table II.

As for Hilton we have two extreme values for the mean density of Urquhart shale, but the range is only 1%. Not having a reliable average we assume the mean obtained from cores in making the calculations and allow an asymmetrical error bracket which recognizes that the systematic overestimate of density causes an underestimate of $G$. The results are listed in Table II. What we call the fitting errors are useful in indicating the relative errors of the determinations, but they underestimate the possible total error which is better represented by what we call the systematic error. Thus the effect of recognizing the systematic bias in Mount Isa densities is to bring the Mount Isa values into line with the Hilton result as nearly as we are able to determine. The greater accuracy and certainty of the Hilton result makes it more suitable for comparison with relationships such as Eq. (1).

TABLE II. Values of $G$ obtained by Eq. (2): a comparison of the Hilton result with four semi-independent values from Mount Isa. The unit is $10^{-11}$ $\mathrm{m^3\,kg^{-1}\,sec.^{-2}}$. The fitting error is the standard deviation of the data misfit to the equation. The listed possible systematic errors arise from the lack of precision in density determination, which recognizes the bias in Mount Isa densities.

| Mine | $G$ | Fitting error | Systematic error |
|---|---|---|---|
| Hilton | 6.720 | $\pm 0.002$ | $\pm 0.024$ |
| Mount Isa | 6.691 | $\pm 0.007$ | |
| | 6.693 | $\pm 0.010$ | $^{+0.089}_{-0.022}$ |
| | 6.729 | $\pm 0.009$ | |
| | 6.702 | $\pm 0.007$ | |

## FREE-AIR GRADIENT MEASUREMENTS

Inverting Eq. (2) to consider the variation of gravity with height above ground level, $X(z)$ becomes very small, being due only to the air density, and $U(z)$ is a very precisely known geometrical term, so that the "ideal" gravity variation is accurately known and any departure from this indicates an anomalous local gradient due to irregular distributions of mass within the Earth (or to a non-Newtonian effect). Since such an anomalous gradient must bias the subsurface profile as well, it is of interest to examine the free-air gradient for a bias that may account for the mine data. However, if the mine observations are to be explained as a non-Newtonian effect, represented by Eq. (1), then this will cause a gradient anomaly above ground as well as below and the two effects are not immediately distinguishable unless the height and depth of measurements both substantially exceed $\lambda$.

This problem is illustrated in Fig. 3, where the two fitted curves of Fig. 1 have been extended to a similar distance above ground. As is seen, the anomalous free-air gradient vanishes at $h \gg \lambda$, but the subterranean gradient reaches its maximum (steady) value at $z \gg \lambda$ and the anomalous gradient through the surface is half of this value. Thus the prospect of clearly recognizing a Yukawa term by its exponential range dependence requires identification of a characteristic residual curve.

The data that we have been able to obtain at Mount Isa are plotted in Fig. 3 but, as is obvious from the figure, they are seriously disturbed by terrain and near-surface excavation features that have not been adequately corrected. (This is much more problematical close to the refinery than in more remote parts of the mine.) The corrections are much less serious over the top half of the chimney, where the residual gradient comes closer to the ideal value (zero), but it is evident that to be useful we would need results above 500 m.

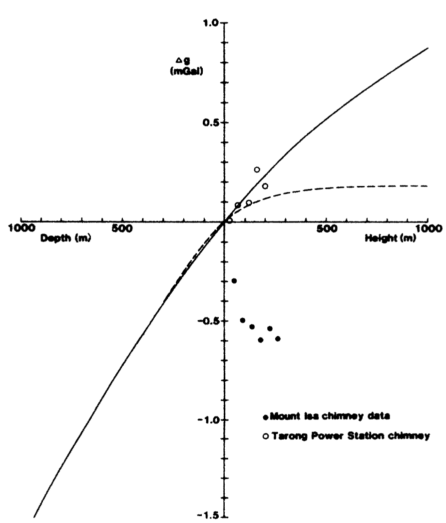


FIG. 3. Comparison of the extrapolations above ground level of the curves of Fig. 1 with gravity residuals from chimneys of the Mount Isa refinery and the power station at Tarong in southeast Queensland.

We have also obtained free-air gradient data in a simpler and more homogeneous environment at the site of a new power station at Tarong, in southeast Queensland. These are represented by open circles in the figure, but again the scatter precludes any thought of seeking a characteristic curvature.

## DISCUSSION

The evidence for a defect in Newtonian gravity at kilometer range has improved with each new data set, but it still remains less than completely conclusive. Originally it seemed that the most serious doubts arose from inadequacy of the sampling of rock densities in the vicinities of mines and boreholes where gravity measurements were made and the possibility that laboratory-measured core densities systematically underestimated the *in situ* densities of rocks. We now claim to have overcome these doubts by extensive sampling in areas where the geological structures are well known and by the fact that, except near to the surface, the rock porosities were too low ($<0.3\%$) for dilation by release of overburden pressure to have affected the densities materially. The remaining uncertainty arises from the possibility of a regional or extensive local bias in gravity gradient by deep-seated mass irregularities that have not been recognized.

In principle, the presence of an anomalous vertical gradient is identifiable from a surface-gravity survey. The rigorous method would be to Fourier analyze the surface data in two dimensions so that each harmonic term can be extrapolated upward independently and then summed. In practice the available data never appear adequate for this to be effective, partly because a very wide range of spatial frequencies is required and therefore both very close spacing of data (100 m) and a very extensive range (100 km). In two dimensions this becomes prohibitive, and although we have investigated the possibility that the Fourier-Bessel method is less demanding of data, this has not yet worked. In any case there is is a problem that over terrain that is not perfectly flat there are terrain (Bouguer) corrections that assume knowledge of surface densities and so introduce errors that appear as noise in the Fourier spectrum.

Since the geological structure of the area is nearly two dimensional, with features trending almost north-south, we have used a simpler approach, linear Fourier analyses of east-west gravity profiles across the strike of the geological structure. Correction of the gravity gradient by this means gives a slight *increase* in the estimate of $G$. However, to be convincing much more extensive data would be needed.

Even if such an analysis were completely satisfying it could not answer all of the questions that have to be asked. As we see in the Hilton data, which clearly give the best mine profile available, assuming the non-Newtonian effect to be real, we cannot separately identify the parameters $\alpha$ and $\lambda$. An effort must be made to conduct experiments over a range of several kilometers at least, both above the surface and below. This means making measurements in the deep ocean and the atmosphere.

In neither case can we expect to construct platforms with sufficient positional stability to make absolute gravity measurements and so we propose to measure gravity gradient directly. Gravity gradiometers of various designs are under development in several laboratories, including our own, and it appears that any of them would probably suffice. In the direct measurement of gravity a crucial observation will be the identification of a change in gravity gradient with the form indicated in Fig. 3.

Meanwhile, independent evidence of another kind has been presented by Fischbach, Sudarsky, Szafer, and Talmadge,[16] who reanalyzed early measurements[17] of relative gravitational accelerations of chemically different materials, finding evidence of a dependence on nuclear mass defect. If, as they suggest, this is interpreted directly in terms of Eq. (1), with $(G_\infty \alpha)$ determined not by mass but by fundamental particle count, then the geometry envisaged for the original experiment by von Eötvös, Pekar, and Fekete[17] is only appropriate if the density structure within several $\lambda$ of the experimental site is perfectly layered, so that the normal (long-range) gravity and the short-range force are precisely parallel. If we make this assumption and apply a sign correction[18] to the analysis of Fischbach *et al.*,[16] taking the mean density of the Earth in the vicinity of the Eötvös balance to be 2750 $\mathrm{kg\,m^{-3}}$, the analysis gives $\alpha\lambda \approx +48$ m. This is opposite to the sign of the effect that we find and several times the magnitude limit imposed by the satellite-surface gravity comparison (shown in Fig. 2 for negative $\alpha$), which requires $|\alpha\lambda| < \sim 10$ m. However, we cannot rely upon the layered earth assumption. Not knowing how serious the topographic and density irregularities were in the vicinity of the Eötvös experiment, we must allow the possibility that the postulated short-range force was misaligned with the normal gravity, in which case the geometry that led to the $(\alpha\lambda)$ estimate above is irrelevant.

The geometry of the Eötvös experiment that must now be considered is of the superposition of a short-range force on a combination of "normal" (long-range) gravity and the centrifugal component of observed gravity that are both independent of material, having combined magnitude $g \approx 9.8\ \mathrm{m\,sec^{-2}}$, and directed very close to the ideal theoretical gravity vector. In particular it has the same direction exactly for all attracted materials at the same point. The short-range force $\Delta g$, may make quite a large angle $\theta$ with the normal gravity vector, due to local geological structure. Then if $\Delta g$ differs by a small amount $\delta(\Delta g)$ for two different materials the angle between the total gravity vectors for these materials is $[\delta(\Delta g)/g]\sin\theta$. On this basis the value of $|\alpha\lambda|$ inferred from the plot of Fischback *et al.*, of the Eötvös data would be less than the limit of 10 m if $\theta > 3.0$ degree. This is not merely possible, but highly likely.

We can now see the possibility of explaining several discrepancies. First, no problem arises from the magnitude of the effect reported in the analysis of Fischbach *et al.*[16] because the value of $(\alpha\lambda)$ must be determined from the local geological and topographic structure, which is unknown in this case. The problem of the sign disappears similarly because the misalignment of $\Delta g$ and $g$ could have any orientation with respect to the centrifugal component of $g$. Further, the repetition of the Eötvös experiment by Renner, criticized by Roll, Krotkov, and Dicke[19] for the unsatisfactory error analysis and discounted by Fischback *et al.*,[16] on that account, would not be expected to give the same results as the original Eötvös experiments unless they happened to be carried out at precisely the same site. We should express some concern that the effects reported by Renner are as small as they are. We may suppose that he happened to choose a site where $\theta$ is particularly small, or that the orientation of this misalignment with respect to his reference direction (determined by the centrifugal component of gravity) happened to be unfavorable to a positive result.

At this stage no conflict can be claimed between our results and the inference of Fischbach *et al.*,[16] from the Eötvös data, essentially because no specific inference about $(\alpha,\lambda)$ can be drawn from the Eötvös results without detailed knowledge of geological and topographic structure. New Eötvös-type experiments are urgently needed and they should be carried out at sites with strong topographic relief that will maximize the misalignment $\theta$ between $\Delta g$ and $g$. It does not appear impossible to make $\theta = 30°$, in which case, according to the hypothesis of Fischbach *et al.*,[16] orientation of the vertical would differ by about $\alpha\lambda \times 1.3 \times 10^{-8}$ degree (for $\lambda$ in meters) for materials with nuclear mass defects differing by $4 \times 10^{-3}$ (copper and lithium hydride).

Meanwhile we can examine critically the constraints that our own data impose on these arguments. Referring to Fig. 2, the band of values of $(\alpha,\lambda)$ that are acceptable within the maximum uncertainties of the data extend $\pm 50\%$ either side of the preferred value of $\alpha$ and between the limits imposed by the shaded areas, so that

$$0.004 \le -\alpha\lambda \le 10\ \mathrm{m}$$

where

$$0.035 \le -\alpha \le 0.15 \text{ and } 1 \le \lambda \le 1000\ \mathrm{m}\ .$$

We expect to soon have data up to $\alpha\lambda \sim 0.4$ m from a hydroelectric lake experiment[20,21] that has just become operational, but experiments that will improve constraints in the range $50 < \lambda < 1000$ m are still at the developmental stage.

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