RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 33, NUMBER 11 1 JUNE 1986

Bag formation in a chiral model

H. Gomm, P. Jain, R. Johnson, and J. Schechter
Physics Department, Syracuse University, Syracuse, New York 13244-1130
(Received 13 January 1986)

We show that a bag can be automatically formed in a model where the Skyrme Lagrangian is modified to possess the correct QCD scaling behavior.

The nonlinear chiral Lagrangian provides a succinct summary of the "current-algebra" results in low-energy pion physics which agree nicely with experiment. Since the algebra of currents is respected by QCD, this Lagrangian may be considered to be a basic building block for an effective low-energy QCD Lagrangian. The leading term[1] is

$$\mathscr{L}=-\frac{F_\pi^2}{8}\operatorname{Tr}(\partial_\mu U\partial_\mu U^\dagger)+\cdots,\quad U=\exp\left(\frac{2i\phi}{F_\pi}\right),\tag{1}$$

where $F_\pi \simeq 132$ MeV is the pion-decay constant and ϕ is the pseudoscalar field multiplet. A long-standing question of great importance has been to determine what additional terms are needed to fill in the three dots in (1). The whole question has been made more topical by the revival of Skyrme's model[2-6] for explaining the nucleon as a topological soliton of (1). To stabilize the soliton against collapse he added a term

$$\mathscr{L}_{\rm SK}=\frac{1}{32e_s^2}\operatorname{Tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2\}\tag{2}$$

(where e_s is a new dimensionless parameter), which might result from "integrating out" some of the heavier particles. It is remarkable that a model as economical as the sum of (1) and (2) can roughly explain the low-energy properties of the nucleon. However, there is no special reason why the effective Lagrangian should not also include reference to the other low-mass particles. Clearly, including them is a very complicated undertaking[7] since many new particles and couplings may be introduced without violating well-known principles. Thus it seems interesting to embark on a slightly different approach and to modify the Skyrme Lagrangian to take account of the anomalous symmetry structure of QCD. The most characteristic feature of QCD is its anomalous behavior under scale transformations. As it stands, the Skyrme model does not correctly display this behavior. In the present note we shall give a minimal extension of the Skyrme model to achieve the correct QCD scaling law[8] (with massless quarks):

$$\theta_{\mu\mu}=-\frac{\beta(g)}{g}\operatorname{Tr}(F_{\mu\nu}F_{\mu\nu})\equiv H\ ,\tag{3}$$

where $\theta_{\mu\mu}$, the trace of the energy-momentum tensor, is identical to the divergence of the scale current. We find that the resulting simple model provides a natural mechanism for the formation of a "bubble"[9] in the vacuum which has a higher-energy density.[10] The model introduces two new parameters: the vacuum value $\langle H\rangle$ and a scalar glueball mass.

First, let us consider a simple effective Lagrangian for QCD without matter. We assume that the degrees of freedom reduce to that of the single "order-parameter" field H defined in (3). Then, as has been discussed in detail elsewhere,[11,12] the unique Lagrangian (up to two derivatives) which satisfies the trace-anomaly equation (3) is

$$\mathscr{L}_H=-\tfrac{1}{2}aH^{-3/2}(\partial_\mu H)^2-\tfrac{1}{4}H\ln\left(\frac{H}{\Lambda^4}\right),\tag{4}$$

where a is a dimensionless parameter and Λ, which has mass dimension one, is the needed scale for QCD. The potential term in (4) has a minimum at $\langle H\rangle=\Lambda^4/e$. This yields a negative vacuum energy density $-\langle H\rangle/4$. A bubble of higher-energy density would increase the total energy and hence be unstable to collapse. We will see how solitonic "matter" stabilizes this bubble. Expanding H around its vacuum value gives a fluctuation field for a scalar glueball with squared mass

$$m_H{}^2=\frac{\langle H\rangle^{1/2}}{4a}\ .\tag{5}$$

Note that the Lagrangian in (4) is not well defined for $H\leqslant 0$. This, however, may not be a problem since the equation of motion for H has a solution for which H is always non-negative. We can therefore set[13] $H=\psi^4$ which is more convenient for the numerical treatment.

Since (4) already satisfies the anomaly equation (3) we should take the rest of the Lagrangian to be scale invariant. It is convenient[14] to assign the pion field to scale with mass dimension zero: $\delta\phi(x)=-\rho(0+x\cdot\partial)\phi(x)$ for $\delta x_\mu=-\rho x_\mu$. Then we immediately observe that the Skyrme term (2) as well as any other similar four-derivative term, e.g., $\operatorname{Tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]_+\}^2$, is scale invariant as it stands. This is not true for (1), however, which should be multiplied by $\psi^2/\langle\psi\rangle^2$. Thus we have the following minimal extension[15] of the soliton Lagrangian which also satisfies the trace-anomaly constraint:

$$\mathscr{L}=-8a(\partial_\mu\psi)^2-\tfrac{1}{4}\psi^4\ln\left(\frac{\psi^4}{\Lambda^4}\right)-\frac{F_\pi{}^2\psi^2}{8\langle\psi\rangle^2}\operatorname{Tr}(\partial_\mu U\partial_\mu U^\dagger)+\mathscr{L}_{\rm SK}\ ,\tag{6}$$

where $\langle\psi\rangle=\Lambda/e^{1/4}$. Note that $\mathscr{L}$ obeys the large-N_c counting rules (see Ref. 5) with a, Λ^2, $F_\pi{}^2$, and $e_s{}^{-2}$ all behaving as N_c.

To investigate the solitons associated with (6) we make the Skyrme ansatz[2] $U_0=\exp[i\hat{\mathbf{x}}\cdot\boldsymbol{\tau}F(r)]$, $F(0)=\pi$ as well as the additional assumption $\psi=\psi(r)$ and obtain the formula for the static energy,

$$M=4\pi\int_0^\infty r^2dr\left[8a\psi'^2+\frac{\psi^4}{4}\ln\left(\frac{\psi}{\Lambda}\right)^4+\frac{\Lambda^4}{4e}+\frac{F_\pi^2}{4}\frac{e^{1/2}\psi^2}{\Lambda^2}\left(F'^2+\frac{2}{r^2}\sin^2F\right)+\frac{\sin^2F}{e_s^2r^2}\left(F'^2+\frac{\sin^2F}{2r^2}\right)\right] , \quad (7)$$

where the prime stands for a derivative with respect to r. Note that a term $\Lambda^4/4e$ is included in (7) in order to measure the soliton static energy with respect to the vacuum.

It is necessary to relate the parameters of the Lagrangian to the properties of the nucleon and its excitations. The nucleon-Δ mass splitting, expected to be due to rotational effects, is usually estimated with the aid of the collective-coordinate Lagrangian $\mathcal{L}_c$. Substituting $U=A(t)U_0A^\dagger(t)$, $A^\dagger=A^{-1}$ into (6) gives[6] $\mathcal{L}_c=\lambda\,\mathrm{Tr}(\dot{A}\dot{A}^{-1})-M$ with

$$\lambda=\frac{4\pi}{3}\int_0^\infty r^2\sin^2F\,dr\left[\frac{e^{1/2}F_\pi^2\psi^2}{\Lambda^2}+\frac{2}{e_s^2}\left(F'^2+\frac{\sin^2F}{r^2}\right)\right] . \quad (8)$$

While there has been some criticism[16] of the validity of this approach we may, in any case, regard (8) as a convenient measure of the soliton's "moment of inertia" and so as a quantity which should be at least roughly reproduced in a numerical fit. In the simplest interpretation[6]

$$m(N)=M+\frac{3}{8\lambda} ,$$
$$m(\Delta)=M+\frac{15}{8\lambda} , \quad (9)$$

which gives $M=0.87$ GeV and $\lambda=5.1$ GeV^{-1}. In this approach, in order to achieve a reasonable fit it seems to have been necessary[6] to take a value of F_π considerably less (by about 30%) than the experimental value. In the present model adjustment of the parameters a and Λ can be used to fit the N and Δ masses with the physical value of F_π.

Now let us discuss the main result of this model—the formation of a "bag." We say that a bag is formed, if, in the small-r region, ψ is suppressed from its asymptotic value $\Lambda/e^{1/4}$. Our procedure will involve the numerical minimization of (7) but the basic feature can be understood from the following qualitative argument. Consider the "potential" terms for the field ψ with, as seems to be roughly the case in our numerical work, *fixed* soliton shape function $F(r)$:

$$V(\psi)=\frac{\psi^4}{4}\ln\left(\frac{\psi}{\Lambda}\right)^4+\psi^2G(r)\left(\frac{F_\pi^2}{4}\frac{e^{1/2}}{\Lambda^2}\right) , \quad (10)$$

where $G(r)=(F'^2+2\sin^2F/r^2)$. For large r the quantity $G(r)$ will be very small and the first term of (10) dominates. This yields the "outside" vacuum value $\langle\psi\rangle=\Lambda/e^{1/4}$. On the other hand, for small r, and suitable values of the parameters, the second term will dominate, which yields $\psi=0$ inside the bag. The exact step-function shape for $\psi(r)$ indicated by this argument is smoothed out by the ψ kinetic term in (7). In fact, increasing the value of the parameter a sufficiently will make the bag very shallow. From (5) this is seen to correspond to a very light glueball.

It is useful to have a criterion from (10) for the existence of a bag. Consider a fixed small value of r. Figure 1(a) shows a sketch of $V(\psi)$ in the case when no bag is formed while Fig. 1(b) shows the case when a bag does exist. The critical point is achieved when the second dip reaches the $V(\psi)=0$ axis. In terms of our parameters this occurs when

$$G(r)\geq\frac{2\Lambda^4}{F_\pi^2e^{3/2}} . \quad (11)$$

Notice that the right-hand side (RHS) behaves as N_c for $N_c\to\infty$ while the left-hand side behaves as N_c^0. Thus no bag should form for $N_c\to\infty$, where, however, the nucleon mass $\to\infty$. Physically this is due to the dominance of the second term in (6) for large N_c. For a finite nucleon mass (physical values of the parameters), the present model suggests the possibility of a bag forming in the gluon condensate. In the $N_c\to\infty$ limit we recover the usual Skyrme soliton whose size scales as $(e_sF_\pi)^{-1}\sim N_c^0$. Using an approximate analytic form[17] for $F(r)$ we may evaluate $G(r)$ and, by taking the equality sign in (11), find a rough expression for the bag size R:

$$R^2\approx\frac{1}{e_s^2F_\pi^2}\left(\frac{\sqrt{6}e_sF_\pi^2e^{3/4}}{\Lambda^2}-1\right) . \quad (12)$$

The RHS must clearly be positive; this shows that there exists a maximal value of Λ for bag formation.

We next discuss the results of our numerical analysis. We first assume that F_π is fixed at its experimental value. The correct values of e_s and a are not known while the value of Λ is variously estimated as

$$\Lambda\approx0.26\text{ GeV [bag model (Ref. 9)] },$$
$$\Lambda\approx0.44\text{ GeV [QCD sum rules (Ref. 10)] }. \quad (13)$$

With the requirement that the values of M and λ given by (9) be reproduced, we have been able to find fits only when Λ is less than about 0.26 GeV. A typical solution occurs for $\Lambda=0.22$ GeV, $e_s=4.5$, and $a=7\times10^{-4}$ (glueball mass ≈3.2 GeV). The corresponding $F(r)$ is plotted as curve 1 in Fig. 2(a) while $\psi(r)$ is curve 1 in Fig. 2(b). The radius of the bag is about 1 fm in agreement with the estimate in (12). To see the effects of the ψ kinetic term we have varied a keeping Λ and e_s fixed. Curves 2 correspond to a glueball mass of 0.86 GeV ($a=0.01$) yielding $M=1.06$ GeV, $\lambda=4.6$ GeV^{-1} while curves 3 correspond to a glueball mass of 0.38 GeV ($a=0.05$) giving $M=1.24$ GeV, $\lambda=5.0$ GeV. Note that as the glueball mass decreases, the "vacuum energy density" within the bag {i.e., $\psi^4(0)\ln[\psi(0)/\Lambda]$} becomes less negative. Although the shape of the bag depends sensitively on the glueball mass, the chiral field U_0 remains roughly the same. However, compared to the original Skyrme Lagrangian with parameters chosen as in Ref. 6 [see dotted curve in Fig. 2(a)], the present fit does have a significantly smaller tail.

The above fit nicely illustrates the mechanism of bag formation. However, it is also interesting to investigate the situation when one chooses $\Lambda\approx0.44$ GeV to agree with the QCD sum-rule determination.[10] For definiteness we choose $e_s=5.45$ and $F_\pi=0.091$ GeV ≈0.69 $(F_\pi)_{\text{expt}}$ as in Ref. 6.

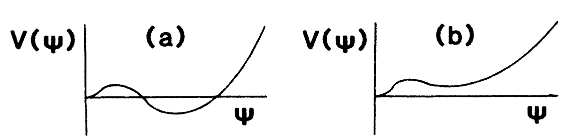

FIG. 1. Sketch of $V(\psi)$ vs ψ for the case of (a) no bag formation and (b) bag formation.

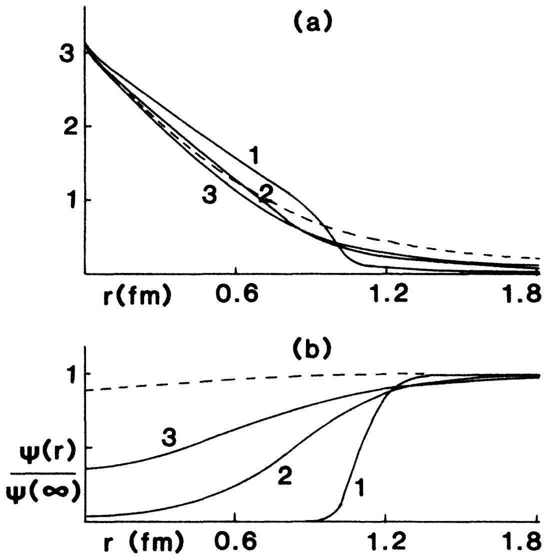

FIG. 2. (a) The soliton shape function $F(r)$ and (b) the bag shape function $\psi(r)$ obtained by minimizing (7). The solid curves correspond to $F_\pi=0.132$ GeV, $\Lambda=0.22$ GeV, $e_s=4.5$, and $a=7\times10^{-4}$ (curves 1), 0.01 (curves 2), 0.05 (curves 3). The dashed curves correspond to $F_\pi=0.091$ GeV, $\Lambda=0.44$ GeV, $e_s=5.45$, and $a=0.01$.

Then Eq. (12) shows that one is just on the verge of bag formation. This is borne out by the numerical analysis leading to the dotted curve for ψ in Fig. 2(b) (normalized to the appropriate outside value). In this situation the soliton parameters M and λ are very slightly changed from their previous values. One has a very shallow bag. Interestingly enough, it permits one to implement the idea of Ref. 10 to reconcile the sum-rule and bag determinations of Λ by interpreting the bag value (now Λ_B) as corresponding to the difference of inside and outside energy densities, i.e.,

$$\Lambda_B{}^4/4e=\epsilon_{\text{inside}}-\epsilon_{\text{outside}} \ .$$

In our notation this yields

$$\Lambda_B\approx\left[\Lambda^4-4e\left|\psi^4(0)\ln\left(\frac{\psi(0)}{\Lambda}\right)\right|\right]^{1/4} , \tag{14}$$

where for simplicity we have approximated the inside energy density by its value at the origin. The shallow bag shown in Fig. 2(b) corresponds to a glueball mass of 1.7 GeV ($a=0.01$) and $\psi(0)=0.31$ GeV. From (14) this gives an expected bag model $\Lambda_B=0.21$ GeV with a "true" outside value $\Lambda=0.44$ GeV.

To sum up, we have presented a fairly simple "toy" model which provides a mechanism for bag formation in the effective Lagrangian approach to QCD. It has the amusing feature that the pion is treated as a Goldstone boson with no need for a bag while the bag develops automatically in the case of the nucleon, where the bag model is on a stronger footing. Although a good fit to the soliton parameters is obtained only for the "deep" bag, we would imagine that making the model more complete (especially taking mixing in the scalar-singlet channel into account[15]) would push the favored solution more in the direction of the "shallow bag." Finally, we remark that the present model seems to share some conceptual features with the "nontopological bag,"[18] the "cloudy bag,"[19] the Skyrmion-quark mixing model,[20] and the one-loop Lagrangian approaches[21] to similar problems. In particular, we may note that the radius of the bag in the fit corresponding to curve 1 is similar to that of the MIT bag model, but the conceptual foundation is, because of conservation of the chiral current, more similar to the cloudy bag.

This work was supported by the Department of Energy under Contract No. 02-85ER40231.

[1]See, for example, J. A. Cronin, Phys. Rev. **161**, 1483 (1967).

[2]T. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); Nucl. Phys. **31**, 556 (1962); J. Math. Phys. **12**, 1735 (1971).

[3]J. G. Williams, J. Math. Phys. **11**, 2611 (1970); N. K. Pak and H. C. Tze, Ann. Phys. (N.Y.) **117**, 164 (1979).

[4]A. P. Balachandran, V. P. Nair, S. G. Rajeev, and A. Stern, Phys. Rev. D **27**, 1153 (1983).

[5]E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983).

[6]G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[7]A recent discussion with further references is given by I. J. R. Aitchison, C. M. Fraser, and P. J. Miron, Phys. Rev. D **33**, 1994 (1986).

[8]N. K. Nielsen, Nucl. Phys. **B210**, 212 (1977); J. C. Collins, A. Duncan, and S. D. Joglekar, Phys. Rev. D **16**, 438 (1977); see also, S. L. Adler, J. C. Collins, and A. Duncan, *ibid.* **15**, 1712 (1977); M. S. Chanowitz and J. Ellis, *ibid.* **7**, 2490 (1973); R. J. Crewther, Phys. Rev. Lett. **28**, 1421 (1972).

[9]See, for example, J. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D **12**, 2066 (1975).

[10]The situation where the inside has nonzero vacuum energy density has been advocated by M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[11]J. Schechter, Phys. Rev. D **21**, 3393 (1980); A. Salomone, J. Schechter, and T. Tudron, *ibid.* **23**, 1143 (1981).

[12]See, also, A. A. Migdal and M. A. Shifman, Phys. Lett. **114B**, 445 (1982); J. M. Cornwall and A. Soni, Phys. Rev. D **29**, 1424 (1984).

[13]An alternate approach is to set $H=\langle H\rangle e^{\psi}$.

[14]J. Ellis, Nucl. Phys. **B22**, 478 (1970); C. Isham, A. Salam, and J. Strathdee, Phys. Rev. D **2**, 658 (1970).

[15]A simple calculation shows that the width predicted for $H\rightarrow 2\pi(\Gamma=3m_H/2048\pi a^2)$ is typically much larger than the mass of H. This situation is analogous to that for the σ particle in the linear σ model and suggests that the H particle would not be a true resonance. For our present purpose it, of course, is irrelevant whether or not H is a true resonance. Actually we have recently shown [Phys. Rev. D **33**, 801 (1986)] that the effect of H mixing with a quarkonium scalar system could naturally reduce the width drastically. See, also, J. Ellis and J. Lanik, Phys. Lett. **150B**, 289 (1985); H. Gomm and J. Schechter, *ibid.* **158B**, 449 (1985).

[16]E. Braaten and J. P. Ralston, Phys. Rev. D **31**, 598 (1985). Actually the only self-consistent solution of the Skyrme model in the collective-coordinate approximation for $m_\pi=0$ gives $m_N=m_\Delta$ since the moment of inertia diverges.

[17]We take (see Ref. 4) $F=2\,\text{arccot}(e_sF_\pi r)$ which implies

$$G=12e_s{}^2F_\pi{}^2/(1+e_s{}^2F_\pi{}^2r^2)^2 \ .$$

[18]R. Friedberg and T. D. Lee, Phys. Rev. D **18**, 2623 (1978).

[19]References may be traced from P. J. Mulders, Phys. Rev. D **30**, 1073 (1984).

[20]V. P. Nair and V. G. J. Rodgers, Institute for Advanced Study report, 1984 (unpublished).

[21]S. L. Adler and T. Piran, Rev. Mod. Phys. **56**, 1 (1984), and references therein.