Brief Reports

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Bound on the W-boson electric dipole moment

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A W-boson electric dipole moment $\lambda_W e/2m_W$ can induce fermion electric dipole moments via quantum loop corrections. Updating an analysis by Salzman and Salzman, we estimate this effect and find that the current neutron-electric-dipole-moment bound $|d_n| < 6\times10^{-25}$ e cm implies (barring an unforeseen cancellation) $|\lambda_W| < 1\times10^{-3}$ or $|d_W| < 10^{-19}$ e cm. On the basis of this constraint, we conclude that electric-dipole-moment contributions to radiative W-boson processes must be very small and effectively undetectable.

Twenty years ago, Salzman and Salzman[1] conjectured that the W boson might have an intrinsic electric dipole moment:

$$d_W = \lambda_W e/2m_W \tag{1}$$

They speculated that for $\lambda_W \sim 1$, this moment could provide the source of CP violation then recently observed in neutral-kaon decays.[2] (Elementary-particle electric-dipole-moment interactions violate P and CP invariance.) Indeed, the magnitude of the CP-violation parameter $|\epsilon| \simeq 2.3\times10^{-3}$ is very close to α/π and thus suggestive of an electromagnetic loop correction to weak amplitudes.[3,4] They also realized that quantum loop effects involving virtual W bosons would induce fermion electric dipole moments nominally of $O(\lambda_W e G_F m_f/\pi^2)$, where $G_F = 1.166\,35\times10^{-5}$ GeV^{-2} is the Fermi constant. Employing the 1965 neutron-electric-dipole-moment bound $|d_n| < 5\times10^{-20}$ e cm, they (roughly) estimated[1] $\lambda_W < 4$. Since that bound did not contradict their model, they predicted $|d_n| \simeq 1\times10^{-20}$ e cm (for $\lambda_W \simeq 1$). Unfortunately, that proved not to be the case. Over the last 20 years, the bound on $|d_n|$ has been significantly reduced to[5]

$$|d_n| < 6\times10^{-25}\ e\,\text{cm}\ \ (\text{present bound})\ , \tag{2}$$

apparently ruling out the Salzman-Salzman theory of CP violation.

Although the original motivation for introducing λ_W has subsided and standard electroweak gauge theory predicts $\lambda_W = 0$ at tree level,[4,6,7] it is still interesting to ask the following: What is the present bound on λ_W? Such a question is timely, since we want to know as much as possible about the properties of the recently discovered $W^{\pm}$ bosons[8] to help unravel hints of new physics in future high-energy experiments. Furthermore, as yet a completely satisfactory understanding of CP violation is lacking; so we should examine potential sources of CP violation wherever they can occur. Finally, we are now in a better position to estimate the induced neutron electric dipole moment due to $\lambda_W \neq 0$. Whereas, Salzman and Salzman[1] used point-nucleon couplings in their calculation and were hampered by an ambiguous regularization of ultraviolet divergences, we have experience with the quark model and standard electroweak perturbation theory at our disposal. Short-distance ultraviolet divergences are still naively present, but we circumvent them by realizing that a W electric dipole moment only makes sense in moden day renormalizable field theories as an effective low-energy approximation. In the full theory, additional physics at mass scale Λ should cancel short-distance divergences and render loop effects finite.

With some of the motivations behind us, we now describe our calculation of induced fermion electric dipole moments. The starting point is an effective W-boson electric dipole interaction:[1]

$$L_{\text{int}} = ie\lambda_W W_\mu^\dagger W_\nu \tilde{F}^{\mu\nu}\ , \tag{3a}$$

$$\tilde{F}^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} = \tfrac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)\ , \tag{3b}$$

where W_μ is the field of a W^+ boson. Such a term reduces

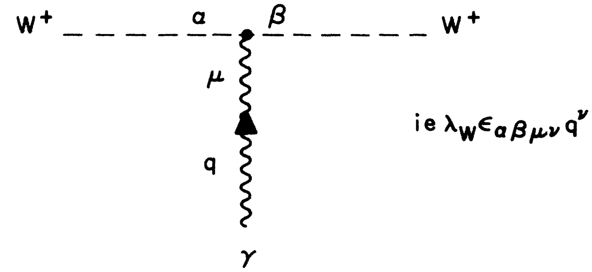

FIG. 1. $WW\gamma$ electric-dipole-moment interaction vertex.

in the nonrelativistic limit to the interaction of a W^+-boson electric dipole moment $d_W = e\lambda_W/2m_W$, with the electric-field component of $F_{\alpha\beta}$. (W^- has the opposite sign d_W.) From Eq. (3), one finds that the $WW\gamma$ vertex illustrated in Fig. 1 is given by

$$ie\lambda_W \epsilon_{\alpha\beta\mu\nu} q^\nu \ , \tag{4}$$

where q is the external photon momentum. Next, using the standard-$SU(2)_L \times U(1)$-model Wff' coupling[9,10] $-i(g/\sqrt{2})\gamma_\mu(1-\gamma_5)/2$ and unitary-gauge propagators, we find that the induced CP-violating amplitude in Fig. 2 is given by

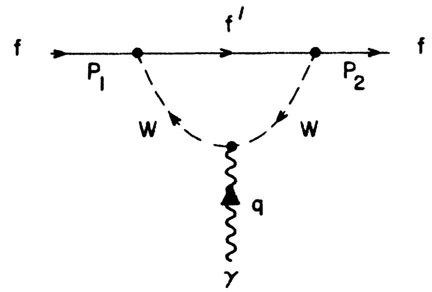

FIG. 2. Fermion-electric-dipole amplitude corresponding to Eq. (5).

$$D_\mu = eg^2 T_{3L} \int \frac{d^4k}{(2\pi)^4} \lambda_W \epsilon_{\alpha\beta\mu\nu} q^\nu \bar{u}(p_2) \frac{\gamma_\rho[\not{k} + (\not{p}_1 + \not{p}_2)/2]\gamma_\delta}{k^2 + k\cdot(p_1+p_2) + m_f^2 - m_{f'}^2} \frac{1-\gamma_5}{2} u(p_1)$$
$$\times \frac{[g^{\alpha\rho} - (k-q/2)^\alpha (k-q/2)^\rho/m_W^2]}{k^2 - k\cdot q - m_W^2} \frac{[g^{\beta\delta} - (k+q/2)^\beta (k+q/2)^\delta/m_W^2]}{k^2 + k\cdot q - m_W^2} \tag{5}$$

where T_{3L} is the third component of weak $SU(2)_L$ isospin for the external fermion (e.g., $T_{3L} = -\frac{1}{2}$ for e^- and d, $T_{3L} = +\frac{1}{2}$ for u). The above integral is logarithmically divergent; therefore, we account for the additional physics at mass scale Λ which cancel the divergence by replacing λ_W with a form factor,

$$\lambda_W \to \lambda_W \frac{(\Lambda^2 - m_W^2)^2}{(k^2 - k\cdot q - \Lambda^2)(k^2 + k\cdot q - \Lambda^2)} \ . \tag{6}$$

This form factor is constructed such that it reduces to λ_W when both virtual W's are on mass shell, but is damped to zero at very high k^2, i.e., short distances. Note that for $\Lambda^2 = m_W^2$ it vanishes (an artifact of our parametrization); therefore, in the following analysis we will consider mainly $\Lambda^2 \gg m_W^2$ scenarios. Inserting this expression in the integrand of Eq. (5), combining denominators by Feynman parametrization, and carrying out the loop integration we find (setting $m_f^2 = m_{f'}^2$ and neglecting m_f^2/m_W^2 effects)

$$D_\mu = \frac{-ieg^2 T_{3L}}{64\pi^2} \frac{\lambda_W}{m_W^2} \bar{u}(p_2) q^\rho (\not{p}_2 \sigma_{\mu\rho}\gamma_5 + \sigma_{\mu\rho}\gamma_5 \not{p}_1) u(p_1)$$
$$\times \left[\ln\frac{\Lambda^2}{m_W^2} - 3 - \frac{m_W^2}{\Lambda^2} + 4\frac{m_W^2}{\Lambda^2 - m_W^2} \ln\frac{\Lambda^2}{m_W^2} \right] \ , \tag{7}$$

where $\sigma_{\mu\rho} = (i/2)[\gamma_\mu, \gamma_\rho]$. The short-distance logarithm $\ln\Lambda^2/m_W^2$ is the dominant contribution for $\Lambda^2 \gg m_W^2$ and unlike the other terms, it is insensitive to the specific form factor employed. We, therefore, denote the factor on the right-hand side of Eq. (7) as $\ln(\Lambda^2/m_W^2) + O(1)$ in our following discussion, where $O(1)$ represents nonleading model-dependent terms not under our control.

For an elementary fermion such as the electron, we can use the Dirac equation $\not{p}_1 u(p_1) = m_f u(p_1)$ to simplify the expression in Eq. (7) and read off the fermion electric dipole moment [the coefficient of $-i\bar{u}(p_2)\sigma_{\mu\rho}q^\rho\gamma_5 u(p_1)$]. In that way we find

$$d_f = \frac{eT_{3L} G_F m_f \lambda_W}{4\sqrt{2}\pi^2} \left[\ln\frac{\Lambda^2}{m_W^2} + O(1) \right] \ , \tag{8}$$

where $G_F = g^2/4\sqrt{2}m_W^2$, or in more standard units,

$$d_f = (4.2 \times 10^{-21}\ e\,\mathrm{cm}) \times \frac{T_{3L} m_f \lambda_W}{(1\ \mathrm{GeV})} \left[\ln\frac{\Lambda^2}{m_W^2} + O(1) \right] \ . \tag{9}$$

Employing the present bound on the electron electric dipole moment,[5]

$$|d_e| < 2 \times 10^{-24}\ e\,\mathrm{cm}, \tag{10}$$

we obtain from Eq. (9)

$$\left| \lambda_W \left[\ln\frac{\Lambda^2}{m_W^2} + O(1) \right] \right| < 2 \ \ (\text{from } d_e \text{ bound}) \ . \tag{11}$$

At best, one concludes $|\lambda_W| \lesssim 1$ from this constraint. We can do orders of magnitude better by using the experimental bound on the neutron electric dipole moment in Eq. (2), but that requires an estimate of d_n in terms of elementary quark moments which we now describe.

To estimate d_n, we follow two somewhat different approaches. First, we simply interpret Eq. (7) as a short-distance quark amplitude. Then, following the usual (and successful) approach for dealing with nucleon magnetic dipole moments, we incorporate strong-interaction effects by including constituent-quark masses $m_u = m_d \simeq m_N/3$ (m_N = nucleon mass) in the effective Lagrangian.[11] The Dirac equation $\not{p}_1 u(p_1) \simeq (m_N/3) u(p_1)$ then leads to effective constitutent-quark electric dipole moments:

$$d_{u,d} = \pm \frac{eG_F m_N}{24\sqrt{2}\pi^2} \lambda_W \left[\ln\frac{\Lambda^2}{m_W^2} + O(1) \right] \ . \tag{12}$$

(Note that these constituent moments are much larger than the bare-quark electric dipole moments in which $m_N/3$ is replaced by current-quark masses[12] $m_u \simeq 5$ MeV and $m_d \simeq 9$ MeV.) Employing SU(6) wave functions, one finds

$$d_n = \tfrac{4}{3} d_d - \tfrac{1}{3} d_u \tag{13}$$

or

$$d_n = -\frac{5}{3} \frac{eG_F m_N}{24\sqrt{2}\pi^2} \lambda_W \left[\ln\frac{\Lambda^2}{m_W^2} + O(1) \right] \ \ (\text{first method}) \ . \tag{14}$$

In our second approach, we rewrite the amplitude in Eq. (7) using

$$\bar{u}(p_2) q^\rho (\not{p}_2 \sigma_{\mu\rho}\gamma_5 + \sigma_{\mu\rho}\gamma_5 \not{p}_1) u(p_1) = i(p_1 + p_2)_\mu \bar{u}(p_2) \not{q} \gamma_5 u(p_1) \ . \tag{15}$$

Then recognizing that this expression corresponds to the divergence of an axial-vector neutral current (for u and d quarks), we relate nucleon matrix elements of this quantity to β-decay axial-vector charged-current matrix elements (by isospin) and find

$$d_n = -g_A \frac{eG_F m_N}{8\sqrt{2}\pi^2}\xi\lambda_W\left[\ln\frac{\Lambda^2}{m_W^2}+O(1)\right] , \tag{16}$$

where $g_A=1.26$ (measured in neutron decay) and ξ represents the average fraction of the neutron momentum carried by a valence quark. If we naively take $\xi=\frac{1}{3}$, then Eqs. (16) and (14) are equal under the replacement $g_A \to \frac{5}{3}$, which is to be expected, since SU(6) predicts $g_A=\frac{5}{3}$. A more phenomenological prescription for determining ξ is to use measured quark momentum distribution functions to determine the average quark momentum in the nucleon. That procedure gives $\xi \simeq 0.2$; so we finally have

$$d_n = -g_A \frac{eG_F m_N}{40\sqrt{2}\pi^2}\lambda_W\left[\ln\frac{\Lambda^2}{m_W^2}+O(1)\right] \quad \text{(second method)} \tag{17}$$

Equation (17) is about a factor of 2 smaller than Eq. (12). In the following comparison with experiment we use the estimate in Eq. (17) because we believe it is more reliable. Also, since we are after a bound on λ_W, it is appropriate to use the smaller of the two estimates. (In any case, it is reassuring that the two estimates agree as well as they do.)

Comparing the estimate in Eq. (17) with the bound in Eq. (2), we find

$$\left|\lambda_W\left[\ln\frac{\Lambda^2}{m_W^2}+O(1)\right]\right| < 1\times 10^{-3} . \tag{18}$$

Barring a subtle cancellation, we assume $|\ln\Lambda^2/m_W^2+O(1)|>1$ and obtain the bounds

$$|\lambda_W| < 1\times 10^{-3} , \tag{19a}$$

$$|\lambda_W e/2m_W| < 1\times 10^{-19}\ e\,\text{cm} \tag{19b}$$

quoted in the abstract. Of course, one might expect λ_W and Λ to be correlated. Indeed, since the $SU(2)_L\times U(1)$ gauge symmetry requires $\lambda_W=0$ at the effective-field theory tree level in the decoupling limit $\Lambda\to\infty$, a natural expectation is[13]

$$\lambda_W = C\left[\frac{m_W}{\Lambda}\right]^n , \quad n=1 \text{ or } 2 , \tag{20}$$

with C a model-dependent constant. For example, in some scenarios with a heavy new particle of mass Λ with CP-violating couplings to the W, one might expect $C \simeq O(\alpha/\pi)$ from one-loop corrections. For that case, the λ_W bound is essentially given by Eq. (19), but Λ is not very constrained. A more restrictive bound is obtained in some composite models with a strong source of CP violation at the composite mass scale Λ, such that $C\simeq O(1)$. In that type of scheme, we find from Eqs. (18) and (20) with $n=2$

$$\frac{m_W^2}{\Lambda^2}\ln\frac{\Lambda^2}{m_W^2} < 1\times 10^{-3} , \tag{21a}$$

or

$$\Lambda > 100 m_W \simeq 8\ \text{TeV} , \tag{21b}$$

$$|\lambda_W| < 1\times 10^{-4} . \tag{21c}$$

(For $n=1$, one finds $|\lambda_W|<2.5\times10^{-4}$ or $\Lambda>330$ TeV.) The bound in Eq. (21c) is a factor of 10 more restrictive than Eq. (19a), but also more model dependent; so we stick with $|\lambda_W|<1\times10^{-3}$ in our subsequent discussion.

The bound in Eq. (19) is already quite restrictive. (In the future it may be lowered several orders of magnitude by cold-neutron experiments.[5]) It essentially guarantees that W-electric-dipole-moment contributions to radiative W-boson processes will be experimentally unobservable, since they are generally suppressed by $|\lambda_W|^2<10^{-6}$. For example, the effect of λ_W on radiative W decay $W\to f_1+\bar{f}_2+\gamma$ is to increase it by $\Delta\Gamma(W\to f_1\bar{f}_2\gamma)$, where[14]

$$\Delta\Gamma(W\to f_1\bar{f}_2\gamma) = \frac{\alpha}{36\pi}|\lambda_W|^2\Gamma(W\to f_1\bar{f}_2) . \tag{22}$$

Such a shift is unobservably small for $|\lambda_W|^2<10^{-6}$. One can, in principle, do better by trying to measure a CP-violating correlation asymmetry due to the interference of ordinary radiative W amplitudes and the electric-dipole-moment amplitude which is only suppressed by $|\lambda_W|$, but that would still seem to be extremely difficult (for $|\lambda_W|<10^{-3}$) at presently contemplated high-energy facilities. Of course, some day in the unforeseeable future a very clean copious source of W bosons may make such measurements possible.

In conclusion, we have found that (barring a subtle cancellation) the present neutron-electric-dipole-moment bound severely restricts the allowed magnitude of the W-boson electric dipole moment. Therefore, if sizeable CP violation is observed in high-energy W-boson experiments, we can rather safely assume that it is coming from some other source.[15] Similarly, if deviations from the standard-model predictions for radiative W-production cross sections[16] or decay rates[14] are detected, they can perhaps be attributed to a W anomalous magnetic dipole moment or electric quadrupole moment for which the bounds are not very restrictive,[17] or other even more exotic phenomenon, but a W electric moment can be discounted. A large W electric dipole moment would have been interesting, but it seems that was not part of nature's plan.

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