Light fermion bound states in two-particle relativistic quantum mechanics

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We study, in the framework of two-particle relativistic quantum mechanics, spin-$\frac{1}{2}$—spin-0 systems with general classes of interaction having the following properties: they preserve chiral symmetry, confine the two particles into bound states, and possess the short-distance behavior of vector interactions. The resulting spectrum displays as ground states an infinite number of light fermions with increasing spins, the masses of which vanish with the vanishing of the constituent-particle masses. In the absence of short-range interactions these fermions have the quantum numbers $j=l+\frac{1}{2}$, $l=0,1,\ldots$, $n=0$. A secondary interaction, here taken illustratively of the LS coupling type, is needed to give masses to the light high-spin fermions. This problem is relevant for the study of the dynamics of preonic systems.

I. INTRODUCTION

This paper is devoted to the construction of a general quantum-mechanical model of two-particle systems, composed of interacting one spin-$\frac{1}{2}$ fermion and one spin-0 boson and where the interaction is confining and chiral invariant. This problem might be relevant for the study of the dynamics of preonic systems. It is then expected that the ground state of the bound-state spectrum represents a light fermion, in the sense that it becomes massless when the masses of the constituent particles vanish, while the other particles of the spectrum remain massive.

The formalism used here is that of covariant relativistic quantum mechanics of two interacting particles and was presented by the author in a preceding article.[1] In this formalism the fermion-boson wave function satisfies two independent wave equations which are generalizations of the Dirac and Klein-Gordon equations, respectively. They are of the form

$$H_1\Psi \equiv (\gamma\cdot p_1 - m_1 - V)\Psi = 0\ , \tag{1.1a}$$

$$H_2\Psi \equiv [p_2{}^2 - m_2{}^2 - (\gamma\cdot p_1 + m_1)V]\Psi = 0\ , \tag{1.1b}$$

where the wave function Ψ is a four-component spinor function:

$$\Psi = \Psi_\alpha(x_1,x_2)\ \ (\alpha = 1,\ldots,4)\ . \tag{1.2}$$

The potential V is a Poincaré-invariant function of the coordinates, momenta, and Dirac matrices. The compatibility condition of the two wave equations requires that V depend on the relative coordinates x through the transverse components x^T, with respect to the total momentum p:

$$V = V(x^T, p_1, p_2, \gamma)\ , \tag{1.3}$$

with

$$\begin{aligned} &p = p_1 + p_2,\ \ v = \tfrac{1}{2}(p_1 - p_2),\ \ X = \tfrac{1}{2}(x_1 + x_2)\ ,\\ &x = x_1 - x_2,\ \ r_\mu \equiv x_\mu^T = x_\mu - (\hat{\mathbf{p}}\cdot x)\hat{\mathbf{p}}_\mu\ ,\\ &x_\mu^L = (\hat{\mathbf{p}}\cdot x)\hat{\mathbf{p}}_\mu,\ \ x_L = (\hat{\mathbf{p}}\cdot x)\ ,\\ &\hat{\mathbf{p}}_\mu = p_\mu/(p^2)^{1/2}\ \ (p^2 > 0)\ . \end{aligned} \tag{1.4}$$

Equations (1.1) completely determine the longitudinal-relative-coordinate (x_L) dependence of the wave function through the equation

$$(p_1{}^2 - p_2{}^2)\Psi = (m_1{}^2 - m_2{}^2)\Psi\ , \tag{1.5}$$

which is a consequence of Eqs. (1.1), and the solution of which is, for eigenfunctions of the total momentum p,

$$\Psi(x_1,x_2) = e^{-ip\cdot X} e^{-i(m_1{}^2 - m_2{}^2)p\cdot x/(2p^2)}\psi(x^T)\ . \tag{1.6}$$

The dynamics of the relative motion is therefore three dimensional through the coordinates x^T.

The wave function Ψ and the potential V are connected by definite relations to the wave function and the kernel of the Bethe-Salpeter equation.[1] This feature permits the classification of the potential V according to its tensor structure in the momenta and the Dirac matrices in a parallel way with that of an interaction Lagrangian.

The physical Hilbert space is defined by the subspace of solutions of Eqs. (1.1) which correspond to positive eigenvalues of both $\hat{\mathbf{p}}\cdot p_1$ and $\hat{\mathbf{p}}\cdot p_2$, the latter being also related by Eq. (1.5).

In the present work we consider potentials V which are local functions of x^T (not involving integral operators).

We shall exhibit in this work the general class of confining and chiral-invariant potentials which might govern the dynamics of the fermion-boson system under consideration. We show that the ground states of the system are represented by an infinite number of light fermions with degenerate masses. In the classification scheme of quantum numbers j,l,n, these fermions have the quantum numbers $n=0, j=l+\frac{1}{2}, l=0,1,\ldots$. For the massive particles the spectrum displays parity doublets.

The inclusion in the above potentials of short-range vector interactions removes the mass degeneracies but leaves unchanged the qualitative feature of the existence of an infinite number of light fermions with increasing spins.

We also show that the inclusion of additional secondary interactions of the axial-vector type, acting as an LS coupling, can give masses to the high-spin light fermions, provided they exhibit a specific energy dependence.

It might also be that because of the singular behavior of the high-spin light-particle states near the massless limit,[2] radiative corrections in the underlying quantum field theory destabilize their spectrum and provide in a natural way large mass values to these high-spin light fermions, leaving only the spin-$\frac{1}{2}$ fermion light.

The appearance of an infinite number of light fermion bound states seems to be a general feature of relativistic quantum mechanics with local functions for the interaction potentials.

II. CONFINING POTENTIALS AND LIGHT FERMION BOUND STATES

Since the confining interaction must be chiral-symmetry preserving, then we have to choose the interaction potential among the vector interactions, the corresponding wave equations being explicitly invariant under chiral transformations, except for the mass term m_1.

The zero-mass limit of the light bound states being obtained by taking both the masses m_1 and m_2 of the constituent particles close to zero, then the reason of the smallness of the boson mass m_2 should also be justified. In the present problem the boson itself might represent an effective light bound state of two other light fermions and its small mass could then be related to the chiral-symmetry-breaking parameters. Therefore, we might view the present problem as a simplified version of a three-fermion bound-state problem. Another possibility is that the fermion and the boson are supersymmetric partners.

The vector interactions correspond to the case where the potential V represents a vector propagator (in its relativistic instantaneous approximation[1]) coupled to the fermion with the γ matrix and to the boson with the momentum p_2 of the latter:

$$V=\gamma_\mu[p_{2\nu},C^{\mu\nu}(r,p)]_+ \ , \tag{2.1}$$

where $[\ ,\ ]_+$ represents the anticommutator.

Among the three types of tensor functions $C_{\mu\nu}$ considered in Ref. 1 (Sec. VIII), the third one [case (c)] seems to be the only type which leads to confining systems. $C_{\mu\nu}$ has then the structure

$$C_{\mu\nu}=[r^2(g_{\mu\nu}-\hat{p}_\mu\hat{p}_\nu)-r_\mu r_\nu]C(r^2,p^2) \tag{2.2}$$

and the wave equations become

$$H_1\Psi\equiv[\gamma\cdot p_1-m_1+2\gamma_\mu C(r^2v^{T\mu}-r^\mu r\cdot v-i\hbar r^\mu)]\Psi=0 \ , \tag{2.3a}$$

$$H_2\Psi\equiv\left[p_2{}^2-m_2{}^2+2\hbar^2(3C+2r^2\dot{C}+2r^2C^2)-\frac{4}{p^2}C(1+r^2C)W_L{}^2-\frac{4}{p^2}W_L\cdot W_{1s}(3C+2r^2\dot{C}+2r^2C^2)\right]\Psi=0 \ , \tag{2.3b}$$

where

$$\dot{C}=\frac{\partial C}{\partial r^2} \tag{2.4}$$

and v^T is defined like x^T in Eq. (1.4). W_L and W_{1S} are the internal orbital angular momentum and spin operators, respectively:

$$\begin{aligned} &W_{L\mu}=\epsilon_{\mu\nu\alpha\beta}p^\nu r^\alpha v^\beta \quad (\epsilon_{0123}=+1)\ , \\ &W_{1s\mu}=-\frac{\hbar}{4}\epsilon_{\mu\nu\alpha\beta}p^\nu\sigma^{\alpha\beta}\ , \\ &W_{1s}{}^2=-\tfrac{3}{4}\hbar^2p^2\ , \\ &\sigma_{\alpha\beta}=\frac{1}{2i}[\gamma_\alpha,\gamma_\beta],\quad W_\mu=W_{L\mu}+W_{1S\mu}\ . \end{aligned} \tag{2.5}$$

Because of Eq. (1.5), the "square" of the operator H_1 is weakly equal to H_2:

$$\tilde{H}\equiv H_1(H_1+2m_1)\approx H_2 \ . \tag{2.6}$$

The particular feature of Eq. (2.3b) is that the operator H_2 (or equivalently $\tilde{H}$) commutes with the matrix $\gamma\cdot\hat{\mathbf{p}}$. This permits the resolution of the wave equations in the Foldy-Wouthuysen representation (see also Ref. 1, Sec. VII C).

Let $\tilde{\Psi}$ be a solution of Eq. (2.3b) with eigenvalue $+1$ of $\gamma\cdot\hat{\mathbf{p}}$ corresponding to the positive values of $\hat{\mathbf{p}}\cdot p_1$:

$$\tilde{H}\tilde{\Psi}=0,\quad \gamma\cdot\hat{\mathbf{p}}\tilde{\Psi}=\tilde{\Psi} \ . \tag{2.7}$$

The solution of Eq. (2.3a) is given by

$$\Psi=(H_1+2m_1)\tilde{\Psi}/(2m_1) \ . \tag{2.8}$$

Defining the internal wave function $\tilde{\psi}(r)$ as in (1.6), Eq. (2.3b) becomes, in terms of the latter,

$$\left[\tfrac{1}{4}p^2-\tfrac{1}{2}(m_1{}^2+m_2{}^2)+\frac{(m_1{}^2-m_2{}^2)^2}{4p^2}+v^{T^2}+2\hbar^2(3C+2r^2\dot{C}+2r^2C^2)-\frac{4}{p^2}C(1+r^2C)W_L{}^2 -\frac{4}{p^2}W_L\cdot W_{1S}(3C+2r^2\dot{C}+2r^2C^2)\right]\tilde{\psi}(r)=0 \ . \tag{2.9}$$

The solutions of this equation are eigenfunctions of the operators $W_{1S}{}^2, W_L{}^2, W^2, \mathbf{W}\cdot\mathbf{p}/|\mathbf{p}|$, defined in Eqs. (2.5), with eigenvalues $-\frac{3}{4}\hbar^2p^2$; $-\hbar^2p^2l(l+1)$, $l=0,1,\ldots$; $-\hbar^2p^2j(j+1)$, $j=l\pm\frac{1}{2}$; $\hbar(p^2)^{1/2}m$, $m=-j,-j+1,\ldots,j$. We designate the corresponding (r^2-independent) eigenfunctions by $\tilde{\mathscr{Y}}^{m+}_{l(1/2)j}$. Then the wave function $\tilde{\psi}$ can be written as

$$\tilde{\psi}^{m}_{nl(1/2)j}=F_{nlj}(r^2)\tilde{\mathscr{Y}}^{m+}_{l(1/2)j} \quad (n=0,1,\ldots)\,, \tag{2.10}$$

$$\gamma\cdot\hat{\mathbf{p}}\tilde{\mathscr{Y}}^{m+}_{l(1/2)j}=\tilde{\mathscr{Y}}^{m+}_{l(1/2)j}\,. \tag{2.11}$$

Equation (2.9) becomes, in terms of the radial wave function F,

$$\left[\tfrac{1}{4}p^2-\tfrac{1}{2}(m_1{}^2+m_2{}^2)+\frac{(m_1{}^2-m_2{}^2)^2}{4p^2}-\hbar^2\left[6\frac{d}{dr^2}+4r^2\frac{d^2}{(dr^2)^2}\right]+\hbar^2\frac{l(l+1)}{r^2}+2\hbar^2(3C+2r^2\dot{C}+2r^2C^2)\right.$$
$$\left.+4\hbar^2 l(l+1)C(1+r^2C)+2\hbar^2[j(j+1)-l(l+1)-\tfrac{3}{4}](3C+2r^2\dot{C}+2r^2C^2)\right]F_{nlj}(r^2)=0\,. \tag{2.12}$$

For confining interactions it is the term C^2r^2 which dominates at large spacelike distances in the potential of Eq. (2.12) [its coefficient is proportional to $(j+\frac{1}{2})^2$] and the condition on C is that

$$-C^2r^2 \underset{-r^2\to\infty}{\longrightarrow} \infty\,. \tag{2.13}$$

We now show that Eq. (2.12) possesses among its solutions an infinite number of light fermions with the quantum numbers $n=0$ (no nodes), $j=l+\frac{1}{2}, l=0,1,\ldots$. It can be checked that the functions

$$F_{0,l,l+1/2}=a_{0lj}(-r^2)^{l/2}\exp\left[(j+\tfrac{1}{2})\int^{r^2}C\,dr^2\right]\,, \tag{2.14}$$

where a_{0lj} are constants and the asymptotic sign of C has been chosen positive, are solutions of Eq. (2.12) with the mass eigenvalue

$$(p^2)^{1/2}=m_1+m_2\,, \tag{2.15}$$

which goes to zero when m_1 and m_2 vanish.

One can also see that these states are the ground states of the spectrum. First one notices that the state with quantum numbers $n=0,l,j=l+\frac{1}{2}$ is the ground state of the sector $(n,l,j=l+\frac{1}{2})$. Then one compares the masses of the states with quantum numbers $n',l'=l+1,j=l'-\frac{1}{2}$ to that of the previous state. The positivity of the additional potential terms ensure that these states are heavier than the former one. [For this a sufficient condition is that $C+2r^2\dot{C}>0$, with C satisfying (2.13).]

The solutions $\tilde{\Psi}$ of Eqs. (2.6)–(2.12) can be written in the Dirac representation by means of the inverse Foldy-Wouthuysen transformation (2.8). It turns out that the ground-state wave functions are still eigenfunctions of the matrix $\gamma\cdot\hat{\mathbf{p}}$ in the Dirac representation and have the same expressions (2.10), (2.11), and (2.14) as in the Foldy-Wouthuysen representation. This can be checked directly on Eq. (2.3a) by using the relation

$$r^2\gamma\cdot v^T=\gamma\cdot r\left[r\cdot v+\frac{2i}{\hbar p^2}W_L\cdot W_{1s}\right]\,. \tag{2.16}$$

III. PARITY DOUBLING

The spectrum of p^2 can be exactly calculated in the particular case where C is a constant λ. Equation (2.12) becomes

$$\left[\tfrac{1}{4}p^2-\tfrac{1}{2}(m_1{}^2+m_2{}^2)+\frac{(m_1{}^2-m_2{}^2)^2}{4p^2}-\hbar^2\left[6\frac{d}{dr^2}+4\frac{d^2}{(dr^2)^2}\right]+\hbar^2\frac{l(l+1)}{r^2}+4\hbar^2\lambda^2r^2(j+\tfrac{1}{2})^2\right.$$
$$\left.+2\hbar\lambda[3(j+\tfrac{1}{2})^2-l(l+1)]\right]F_{nlj}(r^2)=0\,, \tag{3.1}$$

which corresponds to the harmonic-oscillator case. The spectrum is

$$p^2=(m_1{}^2+m_2{}^2)+4\hbar\lambda[(j+\tfrac{1}{2})(2l+4n-3j+\tfrac{3}{2})+l(l+1)]$$
$$+(\{(m_1{}^2+m_2{}^2)+4\hbar\lambda[(j+\tfrac{1}{2})(2l+4n-3j+\tfrac{3}{2})+l(l+1)]\}^2-(m_1{}^2-m_2{}^2)^2)^{1/2}\,. \tag{3.2}$$

On replacing j by its two possible values $j=l\pm\frac{1}{2}$, one also gets

$$p^2=(m_1{}^2+m_2{}^2)+16\hbar\lambda(l+1)n+\{[(m_1{}^2+m_2{}^2)+16\hbar\lambda(l+1)n]^2-(m_1{}^2-m_2{}^2)^2\}^{1/2}\quad (j=l+\tfrac{1}{2},l\ge 0)\,;$$
$$p^2=(m_1{}^2+m_2{}^2)+16\hbar\lambda l(n+1)+\{[(m_1{}^2+m_2{}^2)+16\hbar\lambda l(n+1)]^2-(m_1{}^2-m_2{}^2)^2\}^{1/2}\quad (j=l-\tfrac{1}{2},l\ge 1)\,. \tag{3.3}$$

The spectrum displays the parity-doubling phenomenon for the massive states. The state with quantum numbers $(n+1,l,j=l+\frac{1}{2})$ is degenerate with the state with the quantum numbers $(n'=n,l'=l+1,j'=l'-\frac{1}{2}=j)$. This phenomenon has occurred in spite of the fact that chiral symmetry is explicitly broken by the mass term m_1.

It turns out that this feature is not peculiar to the harmonic oscillator but is a general phenomenon valid for the solutions of Eq. (2.9) for any kind of potential C. The reason for this is that, because of the particular structure of the vector interaction (2.2) we are considering (magnetic-type forces), there is still a chiral operator which commutes with the total energy operator or the equations of motion. In the Foldy-Wouthuysen representation it is

$$\widetilde{Q}_5=\gamma\cdot\hat{\mathbf{p}}\frac{W_{1S}\cdot p_1'}{p\cdot p_1}=-\hbar\frac{(p^2)^{1/2}}{p\cdot p_1}\widetilde{\gamma}^{\,T}\cdot p_1' , \tag{3.4}$$

where we have defined

$$p_{1\mu}'=p_{1\mu}+2C(r^2v_\mu^T-r_\mu r\cdot v-i\hbar r_\mu) , \tag{3.5}$$

and $\widetilde{\gamma}^{\,T}$ is defined like x^T in Eq. (1.4), with $\widetilde{\gamma}_\mu=\gamma_\mu\gamma_5$. Notice that H_1 in Eq. (2.3a) can be written with p_1' as

$$H_1=\gamma\cdot p_1'-m_1 . \tag{3.6}$$

The commutation of $\widetilde{Q}_5$ with the "square" operator $\widetilde{H}$ in Eq. (2.5) can be checked by observing that

$$\widetilde{H}\approx(\gamma\cdot p_1')^2-m_1^{\,2} \tag{3.7}$$

and

$$p\cdot p_1'=p\cdot p_1 . \tag{3.8}$$

In the Dirac representation the chiral operator (3.4) is

$$Q_5=\frac{\hbar}{2}\gamma_5\left[1-m_1\frac{\gamma\cdot p}{p\cdot p_1}\right] \tag{3.9}$$

and it commutes weakly with the constraint operator H_1 (3.6).

Under the action of this charge the ground states behave as singlets.

IV. INCLUSION OF SHORT-DISTANCE VECTOR INTERACTIONS

We next study the effect on the preceding results of the presence of short-range vector interactions. The prototype of this kind of interaction is provided by case (a) of Sec. VIII of Ref. 1, which possesses the structure of Coulomb-type interactions. The corresponding tensor function $C_{\mu\nu}$ (2.1) has the form

$$C_{\mu\nu}=g_{\mu\nu}D(r^2,p^2) . \tag{4.1}$$

After adding the two contributions (2.2) and (4.1) the wave equation for the spin-$\frac{1}{2}$ particle becomes

$$H_1\Psi\equiv\left[\gamma\cdot p_1-m_1-2\gamma_\mu\left[Dp_2^\mu-\frac{2i\hbar\dot{D}}{1-2D}r^\mu\right]\right.$$
$$\left.+2\gamma_\mu C(r^2v^{T\mu}-r^\mu r\cdot v-i\hbar r^\mu)\right]\Psi=0 . \tag{4.2}$$

[$\dot{D}$ is defined as in (2.4).] The coefficient of the $\dot{D}$ term has been fixed in such a way as to have the total energy operator Hermitian in the norm (in the c.m. frame):

$$(\psi,\psi)=\int d^3\mathbf{x}\,\psi^\dagger\psi . \tag{4.3}$$

Furthermore the wave equation of the spin-0 particle, or equivalently the square of the operator H_1 in Eq. (2.6) does not provide us with more results than what is contained in H_1 and the constraint (1.5). The total interaction contained in the wave equation (4.2) still remains confining provided the potential C satisfies condition (2.13) as before.

Contrary to the purely confining case of Sec. II, the square (2.6) of the operator H_1 does no longer commute with the matrix $\gamma\cdot\hat{\mathbf{p}}$ and therefore the passage to the Foldy-Wouthuysen representation is less in order here. Furthermore Eq. (4.2) cannot be solved explicitly for the ground states for arbitrary potentials D. However, we can expand this equation in the masses m_1 and m_2 (with respect to its explicit dependences on these parameters, but not for the kinematic term p^2 present in r_μ) and calculate the ground-state solutions to lowest order in m and the corresponding masses to first order in m. Because of the dissymmetry between the properties of the fermion and boson terms in the wave equations (1.1) it is more indicated to proceed here via two steps: first by considering the equal-mass case $m_1=m_2=m$ and then by introducing the mass difference as a new perturbation.

In the equal-mass case the ground-state solutions are

$$\psi_{0l(1/2)j}^m=F_{0lj}(r^2)\widetilde{\mathscr{Y}}_{l,1/2,j=l+1/2}^{m+}+O(m),\quad j=l+\tfrac{1}{2} , \tag{4.3a}$$

$$F_{0lj}=a_{0lj}(-r^2)^{l/2}\left[\frac{1-2D}{1+2D}\right]^{1/2}\exp\left[(j+\tfrac{1}{2})\int^{r^2}\left[\frac{C}{1+2D}\right]dr^2\right] , \tag{4.3b}$$

$$[(p^2)_{0,l,l+1/2}]^{1/2}=2m\left[\frac{\int(-r^2)^lF_{0,l,l+1/2}^{\;2}\left[\frac{1}{1-2D}\right]d^3x^T}{\int(-r^2)^lF_{0,l,l+1/2}^{\;2}d^3x^T}\right]+O(m^2) , \tag{4.3c}$$

where $\widetilde{\mathscr{Y}}$ has been defined in Sec. II [Eq. (2.11) and the paragraph before] and a_{0lj} are constants.

We again find an infinite number of light fermions, the masses of which vanish with m. The main modification with respect to the preceding result is that the short-distance vector interaction, which is essentially of the electric type, removes the mass degeneracies of the light fermions. It will also remove, in the presence of the mass parameters m, the parity-doubling degeneracies of the massive states.

Notice also that in order to have normalizable solutions, it is necessary that the vector potential D remain bounded by $\frac{1}{2}$. A similar condition was also met in the case of fermion-antifermion systems.[1]

As a second step one can introduce the mass difference (m_1-m_2) as a new perturbation, by keeping, for instance, $m_1+m_2=2m$ fixed. It can be easily seen that the new terms which appear in the wave equation (4.2) are all of first order in (m_1-m_2). Therefore the masses (4.3c) still remain globally of first order in the mass parameters m_1 and m_2.

In case one replaces the potential D by a constant mean value $D=D_0$, then the ground-state solutions can be calculated exactly. The wave function ψ_{0lj} is given by the first term of the right-hand side of Eq. (4.3a) [without the $O(m)$ term], F_{0lj} is defined by (4.3b) and the masses are

$$[(p^2)_{0,l,l+1/2}]^{1/2}=\frac{m_1+[m_2{}^2+4D_0{}^2(m_1{}^2-m_2{}^2)]^{1/2}}{1-2D_0}\ , \tag{4.4}$$

which are now degenerate.

In conclusion, the introduction of short-distance vector interactions in the confining vector potential does not modify the qualitative result about the existence of an infinite number of light fermions with increasing spins.

V. AXIAL-VECTOR-TYPE INTERACTIONS

The persistence in the spectrum of an infinite number of light fermions with increasing spins appears to be a drawback on both phenomenological and theoretical grounds.

Phenomenologically, quarks and leptons, considered as bound states of preons, do not seem to exhibit such a feature. Although quarks, which are themselves confined and are "observed" only through their bound states, might escape the phenomenological difficulty if the bound states made of light high-spin quarks appear, for some dynamical reason, at high-mass scales, the same is not true for leptons which are directly observed.

Theoretically it is known that massless high-spin particles do not interact at zero frequencies with the electromagnetic and gravitational fields by means of Lorentz-covariant conserved currents.[2] This feature means that the zero mass (or light mass) limit of a massive high-spin particle-field theory has rather physically unstable properties.

It is therefore desirable to search in the present framework for an appropriate mechanism giving masses to the high-spin light fermions. Such a mechanism should respect the chiral-invariance properties of the interaction and act selectively on the light fermions by maintaining the spin-$\frac{1}{2}$ fermion light.

It appears that axial-vector-type interactions, considered in Ref. 1 [Eq. (8.18)], which act as an LS coupling, possess the required properties, if some additional conditions are satisfied. These potentials cannot arise in the ladder approximation of parity-conserving interactions. However, they can arise from a local approximation of fourth-order irreducible diagrams in vector interactions in the Bethe-Salpeter kernel.[1] The corresponding potential has the form

$$V=-\hbar\tilde{\gamma}\cdot W_L A(r^2,p^2)=-\frac{2}{p^2}\gamma\cdot pW_L\cdot W_{1S}A(r^2,p^2)\ , \tag{5.1}$$

where W_L and W_{1S} are the internal orbital angular momentum and the fermion spin operators, respectively, defined in Eqs. (2.5) and $\tilde{\gamma}_\mu=\gamma_\mu\gamma_5$.

We now study the effect on the spectrum of the presence of this type of interaction. For simplicity we shall ignore the short-range vector interactions. The wave equation of the fermion becomes

$$H_1\Psi\equiv\left[\gamma\cdot p_1-m_1+2\gamma_\mu C(r^2v^{T\mu}-r^\mu r\cdot v-i\hbar r^\mu)+\frac{2}{p^2}\gamma\cdot pW_L\cdot W_{1S}A\right]\Psi=0\ . \tag{5.2}$$

The new interaction does not affect the ground-state spin-$\frac{1}{2}$ fermion. Its wave function and mass are given by the same expressions as in the initial case:

$$\psi^m_{00(1/2)(1/2)}=a_{00(1/2)}\tilde{\mathscr{Y}}^{m+}_{0(1/2)(1/2)}\exp\left[\int^{r^2}C\,dr^2\right]\ ,$$
$$[(p^2)_{0(1/2)(1/2)}]^{1/2}=m_1+m_2\ . \tag{5.3}$$

The high-spin light fermions are now affected by this interaction. We consider the particular case in which A is constant in $r^2, A=A_0(p^2)$. Then the solutions are

$$\psi_{0l(1/2)j}=a_{0lj}\tilde{\mathscr{Y}}^{m+}_{l(1/2)j}(-r^2)^{l/2}\exp\left[(j+\tfrac{1}{2})\int^{r^2}C\,dr^2\right]\quad(j=l+\tfrac{1}{2})\ ,$$
$$[(p^2)_{0,l,l+1/2}]^{1/2}=m_1+\hbar^2(p^2)^{1/2}A_0l+[m_2{}^2+2m_1\hbar^2(p^2)^{1/2}A_0l+\hbar^4p^2A_0{}^2l^2]^{1/2}\ . \tag{5.4}$$

In order that the masses remain different from zero when m_1 and m_2 go to zero, it is necessary that the potential A_0 behave as $(p^2)^{-1/2}$ when $p^2\to 0$:

$$A_0=\frac{\tilde{A}_0}{(p^2)^{1/2}}\ , \tag{5.5}$$

where $\tilde{A}_0$ is finite (different from zero in the above limit).

The spectrum of the light fermions becomes, for $\tilde{A}_0$ independent of p^2,

$$[(p^2)_{0,l,l+1/2}]^{1/2}=m_1+\hbar^2\tilde{A}_0l+(m_2{}^2+2m_1\hbar^2\tilde{A}_0l+\hbar^4\tilde{A}_0{}^2l^2)^{1/2}\ . \tag{5.6}$$

If the order of magnitude of the potential $\widetilde{A}_0$ is smaller than that of the scale of the confining potential C, then the mass of the spin-$\frac{3}{2}$ fermion ($l=1,n=0$) will still lie below the masses of the heavy spin-$\frac{1}{2}$ fermions ($l=0,n\geq 1$; $l=1,n\geq 0$).

The above mechanism is an illustration of the possibility of giving masses to the light-high-spin fermions in the present framework of relativistic quantum mechanics. There remains to see whether the p^2 dependence of the potential A as given by Eq. (5.5) can be theoretically justified. This demands, however, a detailed analysis of the local approximation of the fourth-order diagram which led to formula (5.1) (Ref. 1). We shall not examine this aspect of the question here.

A second type of mechanism giving masses to the light-high-spin fermions might also exist outside the present simple framework of relativistic quantum mechanics. It is possible that nonlocal effects of radiative corrections of the corresponding quantum field theory destabilize the spectrum of these particles and provide them in a natural way with large mass values, leaving only the spin-$\frac{1}{2}$ fermion light.

VI. CONCLUDING REMARKS

The present work was devoted, in the framework of manifestly covariant two-particle relativistic quantum mechanics, to the study of fermionic bound states produced by chiral-symmetry-preserving confining interactions. A general property of this class of interactions is the appearance, in the spectrum, of light fermion bound states. This result matches the main qualitative feature of preonic dynamics, as far as the light spin-$\frac{1}{2}$ fermion bound state is concerned.

Except for the harmonic-oscillator case, we did not investigate throughout this work the detailed aspect of the resulting bound-state spectrum. It requires a particular choice for the forms of the interaction potentials belonging to the general class of interactions relevant for the present problem. This would lead to a more quantitative study of preonic or supersymmetric bound states. This problem is left for future work.

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