PHYSICAL REVIEW D VOLUME 33, NUMBER 11 1 JUNE 1986

# Baryon axial-vector couplings and SU(3)-symmetry breaking in chiral quark models

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(Received 14 November 1985)

SU(3)-symmetry breaking is studied in the framework of the chiral bag models. Comparisons are also made with the MIT bag model and the harmonic-oscillator quark model. An important clue for the nature of the symmetry breaking comes from the isoscalar axial-vector coupling constant $g_A^S$ which can be indirectly estimated from the Bjorken sum rules for deep-inelastic scattering. The chiral bag model with two radii reasonably well accounts for the empirical values of $g_A^S$ and of the axial-vector coupling constants measured in hyperon semileptonic decays.

## I. INTRODUCTION

In this paper we will study axial-vector coupling constants $g_A$ measured in the hyperon semileptonic decays[1-3] [and known there as $g_1(0)$ terms] together with the axial-vector isoscalar coupling constant $g_A^S$ which can be estimated from the Bjorken sum rules[4-6] for the deep-inelastic scattering of polarized electrons on a polarized proton. As will be shown in the following, a combination of those data can be used to disentangle and pinpoint the contributions which are responsible for the theoretical quark-model descriptions of the empirical coupling constants.

First, the estimated value of $g_A^S$ stresses the relativistic character of the internal quark motion, which has to be present in any successful quark model. In the static quark model SU(6) spin-flavor symmetry of the nucleon wave function predicts[7,8] $\frac{5}{3}$ for the isovector $g_A^I$ responsible for the decay $n \to p + \text{leptons}$. The same considerations give 1 for $g_A^S$ (Refs. 6 and 7). The relativistic effects reduce the static prediction for $g_A$ by a factor $\eta$:

$$g_A^I = \tfrac{5}{3}\eta \ . \tag{1.1}$$

The experimental value $g_A^I = 1.25$ (Refs. 1–3) is fitted with $\eta = 0.75$ (or $\frac{3}{4}$). This value can be easily reproduced in the so-called harmonic-oscillator (HO) quark model,[9,10] which will be, for the sake of completeness, described in Appendix A. In the simple MIT bag model[11,12] one finds $g_A \simeq 1$ (i.e., $\eta$ is too small) if one uses standard parameters. A nonstandard, and rather unconvincing approach, based on large model-quark masses[12] is discussed in Appendix B. In a version of the chiral bag model,[13,14] used in Sec. III, one finds too large a value: $g_A^I \simeq 1.8$. As will be shown in Sec. II a chiral bag model with two radii[15-17] can lead to a satisfactory value for $g_A$'s measured in the hyperon semileptonic decays.

As long as the flavor symmetry is unbroken, one finds a unique prediction valid for any quark model:

$$g_A^S = \eta \simeq 0.75 \ . \tag{1.2}$$

Using the existing experimental values for the deep-inelastic scattering[18] and with some theoretical extrapolation, Ioffe and collaborators[6] found the value

$$g_A^S \simeq 0.5 \ . \tag{1.3}$$

This value is in good agreement with the theoretical estimate

$$g_A^S = 0.5 \pm 0.2 \tag{1.4}$$

which was obtained[6] in a nonperturbative QCD approach. A comparable result

$$g_A^S = 0.68 \pm 0.02 \tag{1.5}$$

has been found[5] using SU(3) symmetry and parameters of hyperon semileptonic decays. As described in Sec. IV, this approach[5] involved a guess about an SU(3) scalar matrix element. The assumed value[5] is valid for any bag model as long as the flavor-SU(3) symmetry is not broken.

All direct estimates (1.3–1.5) of $g_A^S$ are in quantitative agreement with the rough quark-model prediction (1.2). This, as already stressed, can be explained only in terms of the relativistic internal motion of quarks. However, the estimates are always smaller than $\eta$, i.e.,

$$g_A^S(\text{est}) < \eta \ . \tag{1.6}$$

This can be understood as an SU(3)-symmetry-breaking effect. The HO model and the standard MIT bag model, discussed in Appendixes A and B, require a down-quark mass $m_d$ to be appreciably larger than the up-quark mass $m_u$, which is not a very attractive proposition. However, in the chiral bag models, discussed in Secs. II and III, the effect can be associated with the static meson fields, i.e., the soliton mesonlike degree of freedom.[13,15-17] Experimentally known meson mass differences break SU(3) symmetry in such a way that one finds

$$g_A^S / g_A^I \simeq 0.55 \tag{1.7}$$

already in a chiral bag model[13,19] (Sec. III) which gives absolute values $g_A^I$ and $g_A^S$ which are too large. A different situation is encountered in the cloudy bag model (see Ref. 20). This paper is based on a chiral bag model with a static "mesonic" phase,[15,19] which is, for the convenience of the reader, discussed in some detail in Appendix C. This compares nicely with the semiempirical range of values:[5,6]

$$0.24 \leq g_A^S/g_A^I \leq 0.56 . \tag{1.8}$$

The so-called[15] chiral bag model with skin explores the possibility that the quark confinement radius $R$ and the chiral radius $R_{ch}$ at which the mesonic phase couples to quarks are not equal. The model can to some extent account for both the ratio (1.8) and the values of the axial-vector coupling constant. As will be illustrated in the concluding section, experimental data on hyperon semileptonic decays might be in better agreement with the theoretical description of the SU(3)-symmetry breaking which is based on the mesonic degrees of freedom than with the description involving the differences in quark masses[21] (i.e., $m_s < m_{u,d}$) only. However, the mesonic degrees of freedom by themselves would not lead to the appearance of pseudotensor (i.e., $g_2$) form factors which must involve mass differences and/or other effects.[21–23] Center-of-mass (c.m.) corrections[23] do also influence the absolute value of $g_A$: the papers of Donoghue and Johnson and Tadić *et al.*, which are based on the MIT bag model, and which are in a full qualitative agreement, indicate that c.m. corrections can increase $g_A$ by about 10%. This problem requires further study from the chiral bag model point of view.

## II. CHIRAL BAG MODEL WITH SKIN

It has been shown[15,16] some time ago that a correct value for $g_A^I$ can be reproduced in a version of the chiral bag model in which the Wigner phase region does not coincide with the confinement region. This means that the pion field is not excluded from the bag at the confinement radius $R$ but at a smaller radius $R_{ch}$ corresponding to the length scale at which chiral symmetry is broken. In our calculation we have used the ratio

$$R_{ch}/R = \tfrac{2}{3} . \tag{2.1}$$

In general the axial-vector form factor has a quark and a meson contribution. The first one is of the form

$$g_A^Q(B_i \to B_f) = O(B_f, B_i) \int d^3r (u_i u_f - \tfrac{1}{3} v_i v_f) . \tag{2.2}$$

Here $B$ are baryons (hyperons) involved in the semileptonic transition

$$B_i \to B_f + \text{leptons}$$

and $O(B_i, B_f)$ is the SU(6)-symmetry factor given in Table I. The functions $u$ and $v$ are contained in an $S$-wave quark-model function

$$\psi^{\mu}(r) = \begin{bmatrix} u(r) \\ i\sigma\cdot\mathbf{r}_0 \;\; v(r) \end{bmatrix} \chi^{\mu} . \tag{2.3}$$

As outlined in Appendix C, this general form can be used for the chiral bag model[13] also. The quark wave functions are modified[19] by the interactions with static mesonlike fields. The functions $\psi$ depend on $u$-, $d$-, and $s$-quark masses, which can in principle be unequal. Here we have used $m_u \neq m_d \neq 0$ and $m_s = 0.2$, 0.3 GeV. (In Appendix B $m_u = m_d = 0$.) In the notation used in (2.2) one has $g_A^I = g_A(n \to p)$.

TABLE I. SU(3) and SU(6) symmetry. This table is in full agreement with Table 3 of Ref. 2. The third column follows from the second if $F = \frac{2}{3}$ and $D = 1$.

| Transition | SU(3) parametrization | $O(B_i \to B_f)$ |
|---|---|---|
| $\Sigma^- \to \Lambda e \nu$ | $(\frac{2}{3})^{1/2} D$ | $(\frac{2}{3})^{1/2}$ |
| $\Lambda \to p e \nu$ | $(-)(\frac{3}{2})^{1/2}(F + D/3)$ | $(-)(\frac{3}{2})^{1/2}$ |
| $\Xi^- \to \Sigma^0 e \nu$ | $\frac{1}{\sqrt{2}}(F+D)$ | $\frac{1}{\sqrt{2}}\frac{5}{3}$ |
| $\Xi^- \to \Lambda e \nu$ | $(\frac{3}{2})^{1/2}(F - D/3)$ | $\frac{1}{\sqrt{6}}$ |
| $\Sigma^- \to n e \nu$ | $-F + D$ | $\frac{1}{3}$ |
| $n \to p e \nu$ | $F + D$ | $\frac{5}{3}$ |

The meson contribution, described by a static (soliton) mesonlike field involves an SU(3) octet and an SU(3) singlet of mesons.[24]

The mesonic part of the axial-vector current is of the form[13–17,19]

$$A_{\mu}^{M}(x) = -f_M \partial_{\mu} \phi^M(x) . \tag{2.4}$$

Here $M$ denotes one of the nonet $(8 \oplus 1)$ of mesons. The static field $\phi^M$ is determined by the boundary condition at $r = R_{ch}$ (Refs. 15 and 16) and it satisfies Eqs. (C3), (C4), and (C5). These can be solved in terms of Green's functions. One finds for a baryon matrix element

$$f_M \langle B_f | \phi^M | B_i \rangle = \hat{g}_A(B_f, B_i) \Delta_1(r, R) \frac{3}{4\pi R} \times \langle B_f | \sigma\cdot\mathbf{r}_0 \frac{\lambda^M}{2} | B_i \rangle . \tag{2.5a}$$

Here, in the lowest approximation (see Appendix C)

$$g_A = \frac{1}{3}\frac{\omega_0}{\omega_0 - 1}, \quad \omega_0 = 2.04$$

and

$$\Delta_1^M(r, r') = \mu_M [j_1(i\mu_M r) + \gamma h_1^{(1)}(i\mu_M r_<)] \times h_1^{(1)}(i\mu_M r_>) . \tag{2.5b}$$

The symbols $j_1$ and $h_1^{(1)}$ denote spherical Bessel and Hankel functions. When $R_{ch} < R$ one has

$$\gamma = -[\partial_r j_1(i\mu_M r)/\partial_r h_1^{(1)}(i\mu_M r)]_{r=R_{ch}} = \frac{(2+\rho^2)\sinh\rho - 2\rho\cosh\rho}{(2+2\rho+\rho^2)e^{-\rho}} , \tag{2.5c}$$

where

$$\rho = \mu_M R_{ch} .$$

The final result

$$g_A^M = -\hat{g}\,{}_A^Q \frac{R_{\rm ch}{}^2}{R} \Delta_1^M(R_{\rm ch},R,\mu_M) O(B_i \to B_f) \quad (2.6)$$

and

$$\Delta_1^M(R_{\rm ch},R,\mu_M) = -\frac{R_{\rm ch}}{2R^2} \frac{(1+\mu_M R)e^{-\mu_M(R-R_{\rm ch})}}{1+\mu_M R_{\rm ch}+0.5(\mu_M R_{\rm ch})^2}$$

depends on the nonet meson mass $\mu_M$. In the case of the $\Delta S=0$, $\Delta I=1$ transition one encounters pions, while the $\Delta S \neq 0$ transitions obtain contributions from kaons. The isoscalar matrix elements ($\Delta S=0, \Delta I=0$) can have contributions from octet and singlet states $\eta_8$ and $\eta_1$.

In all calculations we have used empirical meson masses. Differences in their magnitudes lead to quite noticeable symmetry-breaking effects.

The valence-quark content of the isoscalar axial-vector current can be found from Refs. 5 and 6. According to Ref. 5 one has

$$\int (g_1^{en}+g_1^{ep})d\xi = \int \left[ \frac{2}{6\sqrt{3}} S_8^5(\xi) + 2(\tfrac{2}{27})^{1/2} S_0^5(\xi) \right] d\xi$$

$$\Longrightarrow \tfrac{10}{36}\langle \bar{u}u+\bar{d}d+\tfrac{2}{5}\bar{s}s\rangle \; . \quad (2.7a)$$

The last row in (2.7a) symbolizes the valence-quark content if $S_8$ contains $\lambda_8$ SU(3) matrix and $S_0$ contains $\lambda_0=(\frac{2}{3})^{1/2}\times 1$. From Ref. 5 one finds

$$\int (g_1^{en}+g_1^{ep})d\xi = \tfrac{10}{36} g_A^S \quad (2.7b)$$

so the valence-quark content is

$$g_A^S \sim \langle \bar{u}u+\bar{d}d+\tfrac{2}{5}\bar{s}s\rangle \; . \quad (2.7c)$$

This means that a mesonic contribution to $g_A^S$ would contain $\eta_1$ and $\eta_8$ static fields, i.e.,

$$(g_A^S)^M = -\frac{1}{3}\frac{\omega_0}{\omega_0-1}\frac{R_{\rm ch}{}^2}{R}(\tfrac{4}{5}\Delta_1^{\eta_1}+\tfrac{1}{5}\Delta_1^{\eta_8}) \; . \quad (2.8)$$

The valence-quark content $\bar{s}s$ appearing in (2.7c) was essential in obtaining a correct decomposition (2.8). It does not contribute to the quark piece $(g_A^S)^Q$ which can be found from the formula (2.2) with $O(B,B)=1$. Thus one has

$$g_A^S = (g_A^S)^Q + (g_A^S)^M \; . \quad (2.9)$$

The mesonic contributions to the semileptonic decay coupling constants $g_A$ is determined by the quark-valence content of the respective currents. This is either $(\bar{u}d)$ for $\Delta I=1$, $\Delta S=0$ meaning pion or $(\bar{u}s)$ for $\Delta S \neq 0$ meaning kaon.

The ratio (1.8) can be written in a very compact form

$$\frac{g_A^S}{g_A^I} = \frac{3}{5}\,\frac{1-\dfrac{R_{\rm ch}{}^2}{R}(\tfrac{1}{5}\Delta_1^{\eta_8}+\tfrac{4}{5}\Delta_1^{\eta_1})}{1-\dfrac{R_{\rm ch}{}^2}{R}\Delta_1^{\pi}} \quad (2.10)$$

which openly displays symmetry breaking. With all meson masses equal, one finds the old SU(6)-spin-flavor-symmetry value 0.6. With $\mu_8=0.549$ GeV and $\mu_1=0.958$ GeV one obtains confinement radius-$R$-dependent values as shown in Table II. It is gratifying that the $R$ dependence is very weak and that the SU(3)- [and/or U(3)-] symmetry breaking due to the meson masses determines the result.

The structure (2.9) holds quite generally:

$$g_A(B_i \to B_f) = O(B_i,B_f) g_f \left[ 1 - \frac{R_{\rm ch}{}^2}{R}\Delta_1^M(R_{\rm ch},R) \right] . \quad (2.11)$$

The factor $g_f$ is 0.653 for $\Delta S=0$ transitions and 0.7115 or 0.730 for $m_s=0.2$ or 0.3 GeV for $\Delta S \neq 0$ transitions. Obtained numerical values are displayed in Table III, showing that the chiral bag model with skin leads to the absolute values which are comparable to the experimental ones. This is quite different from the situation in the simple chiral bag model which will be discussed in the next section.

One can also comment on an assumption made by Ref. 5 in order to deduce the semiempirical result (1.5). As indicated above they have used $g_A^S$ with the following SU(3) structure:

$$g_A^S \sim \tfrac{4}{5}(\tfrac{3}{2})^{1/2}\langle \lambda_0\rangle + \tfrac{1}{5}\sqrt{3}\langle\lambda_8\rangle \; ,$$

$$\langle\lambda_0\rangle \sim (\tfrac{2}{3})^{1/2}\langle \bar{u}u+\bar{d}d+\bar{s}s\rangle \; ,$$

$$\langle\lambda_8\rangle \sim \frac{1}{\sqrt{3}}\langle \bar{u}u+\bar{d}d-2\bar{s}s\rangle \; .$$

Here we have also indicated the respective valence-quark content. The quark contributions which involve valence quarks in the nucleon states ignore $\bar{s}s$ pieces, thus

$$\langle\lambda_0\rangle^Q = \sqrt{2}\langle\lambda_8\rangle^Q$$

as used by Ref. 5. However, mesonic contributions introduce some differences:

TABLE II. $g_A^I, g_A^S$, and the ratio $g_A^S/g_A^I$.

| $R$ (GeV$^{-1}$) | $\Delta_1^\pi$ | $\Delta_1^{\eta_1}$ | $\Delta_1^{\eta_8}$ | $g_A^I$ | $g_A^S$ | $g_A^S/g_A^I$ |
|---|---|---|---|---|---|---|
| 3.50 | −0.087 | −0.024 | −0.047 | 1.236 | 0.682 | 0.552 |
| 4.96 | −0.058 | −0.009 | −0.023 | 1.226 | 0.669 | 0.546 |
| 5.00 | −0.057 | −0.008 | −0.022 | 1.226 | 0.669 | 0.546 |
| 5.65 | −0.049 | −0.006 | −0.017 | 1.221 | 0.665 | 0.545 |

TABLE III. $g_A$ for semileptonic decays calculated in the chiral bag model with skin. Experimental values correspond to the third column (fit with $g_1$ free) in Table VII.

| Transition | Without mesons $m_s=0.2$ | Without mesons $m_s=0.3$ | $R=4.96$ GeV$^{-1}$ $\mu=0$ $m_s=0.2$ | $\mu\neq0$ $m_s=0.2$ | $\mu=0$ $m_s=0.3$ | $\mu\neq0$ $m_3=0.3$ | $R=5.645$ GeV$^{-1}$ $\mu=0$ $m_s=0.2$ | $\mu\neq0$ $m_s=0.2$ | $\mu=0$ $m_s=0.3$ | $\mu\neq0$ $m_s=0.3$ | Expt. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Sigma\to\Lambda$ | 0.534 | 0.534 | 0.613 | 0.602 | 0.613 | 0.602 | 0.613 | 0.599 | 0.613 | 0.599 | 0.595 |
| $\Lambda\to p$ | 0.871 | 0.894 | 1.001 | 0.921 | 1.027 | 0.945 | 1.001 | 0.914 | 1.027 | 0.937 | 0.857 |
| $\Xi^-\to\Sigma^0$ | 0.839 | 0.860 | 0.963 | 0.886 | 0.988 | 0.909 | 0.963 | 0.879 | 0.988 | 0.902 | 0.891 |
| $\Xi^-\to\Lambda$ | 0.290 | 0.298 | 0.334 | 0.307 | 0.342 | 0.315 | 0.334 | 0.305 | 0.342 | 0.312 | 0.340 |
| $\Sigma^-\to n$ | 0.237 | 0.243 | 0.272 | 0.251 | 0.279 | 0.257 | 0.272 | 0.249 | 0.279 | 0.255 | 0.310 |
| $n\to p$ | 1.090 | 1.090 | 1.251 | 1.228 | 1.251 | 1.228 | 1.251 | 1.223 | 1.251 | 1.223 | 1.233 |

$$\langle\lambda_8\rangle^M=-\frac{1}{3}\left[\frac{2}{3}\right]^{1/2}\frac{\omega_0}{\omega_0-1}\frac{R_{\rm ch}{}^2}{R}\Delta_1^{\eta_1},$$

$$\langle\lambda_0\rangle^M=-\frac{1}{3\sqrt{3}}\frac{\omega_0}{\omega_0-1}\frac{R_{\rm ch}{}^2}{R}\Delta_1^{\eta_0},$$

$$\frac{\langle\lambda_0\rangle}{\langle\lambda_8\rangle}=\sqrt{2}\frac{1-\frac{R_{\rm ch}{}^2}{R}\Delta_1^{\eta_1}}{1-\frac{R_{\rm ch}{}^2}{R}\Delta_1^{\eta_8}}=0.97\sqrt{2}\ .$$

Thus even in the framework of our model the estimate used by Ref. 5 for the above ratio is quite good. Naturally, the absolute values, as can be also seen in Table III, can be also influenced by the symmetry breaking. The SU(3) estimate of the matrix element $\langle\lambda_8\rangle$ gives

$$\langle\lambda_8,{\rm SU}(3)\rangle=\frac{1}{\sqrt{3}}(3F-D)=\begin{cases}0.303, & \text{Ref. 1},\\ 0.383, & \text{Ref. 2}.\end{cases}\qquad(2.12\text{a})$$

Here

$$\left.\begin{array}{l}F=0.4434\,,\\ D=0.8056\,,\end{array}\right.\ \text{Ref. 1},$$

$$\left.\begin{array}{l}F=0.477\,,\\ D=0.756\,,\end{array}\right.\ \text{Ref. 2}.$$

The model calculation leads to (for $R=4.96$ GeV$^{-1}$)

$$\langle\lambda_8\rangle=\frac{1}{3\sqrt{3}}\frac{\omega_0}{\omega_0-1}\left[1-\frac{R_{\rm ch}{}^2}{R}\Delta_1^{\eta_8}\right]$$

$$=0.396\ .\qquad(2.12\text{b})$$

The difference between (2.12a) and (2.12b) is only about 3% if $F$ and $D$ values from Ref. 2 are used. With the values from (2.12a) one can also calculate

$$g_A^S[{\rm SU}(3)]=3F-D=\begin{cases}0.525, & \text{Ref. 1},\\ 0.675, & \text{Ref. 2}\end{cases}\qquad(2.13)$$

and compare it with values found in Table II. The model values are in good agreement with the SU(3) fit based on $F$ and $D$ values from Ref. 2. The model value for $R=4.96$ GeV$^{-1}$, for example,

$$g_A^S=0.669$$

is only about 1% smaller. Similar agreement can be found for other radii $R$.

## III. CHIRAL BAG MODEL

The chiral-bag-model[13–17,19] mesonic contributions to $g_A$ follow immediately from the formulas given in the previous section if one uses

$$R_{\rm ch}=R\ .\qquad(3.1)$$

This leads to a mesonic contribution in (2.11) that is too large. There is no way to match experimental values (see Table VII) of $g_A$ found in the hyperon semileptonic decays unless one uses an enormously large confinement radius $R$ which would lead to wrong predictions for hadron masses. The quark contribution $g_A^Q$ to the total $g_A^I$ given in (2.2) is changed due to the presence of the meson field ($\pi$-meson field in the case of $n\to p$ transition). The coupling of the meson field to quarks at the confinement radius $R$ changes the boundary condition. This leads to an $R$-dependent frequency $\omega$ given by a solution of the equation[19]

$$\tan[\omega^2-(mR)^2]^{1/2}=\frac{[\omega^2-(mR)^2]^{1/2}}{1-\frac{\omega+mR}{A}}\ ,\qquad(3.2\text{a})$$

where

$$A=1+(1/n)$$

and

$$\rho=\frac{\omega_0(1+\mu_\pi R)}{48\pi f_\pi{}^2R^2(\omega_0-1)(1+\mu_\pi R+0.5\mu_\pi{}^2R^2)}\tilde{\Sigma}\ .$$

Here $n$ is the number of the nonstrange quarks in the nucleon state $N$ and $\tilde{\Sigma}$ is determined from

$$\tilde{\Sigma}_N=\sum_{ij}\langle N\,|\,(\sigma_i\cdot\sigma_j)(\tau_i\cdot\tau_j)\,|\,N\rangle\ .\qquad(3.2\text{b})$$

The frequency $\omega$ has to be introduced in (2.3) in order to find $g_A^Q$. In order to obey the normalization condition of quark fields, quark wave functions are numerically normalized and the upper component becomes relatively larger with respect to the lower one. Some interesting values are displayed in Table IV. The decrease of $g_A$ with the increase of $R$ is connected with the fact that the

TABLE IV. Chiral-bag-model values.

| $R$ | $\omega$ [Eq. (3.2)] | $g_A^Q$ | $g_A^\pi$ | $g_A^I$ | $g_A^S$ | $g_A^S/g_A^I$ |
|---|---|---|---|---|---|---|
| 4.582 | 1.2182 | 1.4506 | 0.4838 | 1.934 | 0.998 | 0.516 |
| 5.645 | 1.4483 | 1.3640 | 0.4637 | 1.828 | 0.928 | 0.508 |
| 6.0 | 1.5158 | 1.3362 | 0.4569 | 1.793 | 0.906 | 0.505 |
| 6.5 | 1.6043 | 1.2982 | 0.4476 | 1.746 | 0.877 | 0.502 |
| 7.0 | 1.6862 | 1.2615 | 0.4383 | 1.699 | 0.849 | 0.500 |
| 7.5 | 1.7625 | 1.2262 | 0.4290 | 1.656 | 0.823 | 0.497 |

mesonic contributions are, roughly speaking, proportional to $\exp(-\mu R)$. The new boundary condition (3.2) increases also the calculated values of $g_A^Q$. If one just used the formula (2.11) with $\omega=\omega_0$ one would have ($R=5.645$ $\text{GeV}^{-1}$):

$$g_A^I=g_A^Q+g_A^M=1.088+0.301=1.389 .$$

The corresponding value in Table IV is $g_A^I=1.828$. The smaller number corresponds to the approximation used by Ref. 14 where quark wave functions were calculated without mesonic corrections contained in formula (3.2).

The mesonic contributions make, for example, the ratio

$$g_A^S/g_A^I=0.51, \quad R=5.6 \text{ GeV} \tag{3.3}$$

smaller than the SU(6) value 0.6. With $R_{\text{ch}}=R$ their influence is felt more than with $R_{\text{ch}}<R$ and the ratio (3.3) calculated in the chiral bag model is always smaller than the corresponding ratio shown in Table II.

## IV. COMPARISON WITH EXPERIMENTAL VALUES AND CONCLUSION

Among the considered quark models only the chiral bag model with skin[15,16] (CBS) does not run into obvious difficulties with empirical data. The chiral bag (CB) model[13,14,19] offers the same mechanism of the SU(3)-symmetry breaking as the CBS model but it leads to too large absolute values of the axial-vector coupling constants. In the chiral cloudy bag (CCB) model where $R_{\text{ch}}=0$ there are no mesonic contributions to the coupling constants $g_A$ so that the symmetry-breaking mechanism, which was discussed here, does not operate. In that model the ratio $g_A^S/g_A^I$ would be exactly 0.6, unless one introduces large differences among $u$-, $d$-, and $s$-quark masses. The situation in the CCB model is equal to the situation encountered in the MIT bag model, which is discussed in Appendix B. Very large quark masses, very much different, strain our credibility too much. A qualitatively similar conclusion follows from the HO model, as described in Appendix A. Obviously the ratio $g_A^S/g_A^I$ offers an important clue for the nature of the SU(3)-symmetry breaking and its description from the quark-model point of view. It is a pity that the coupling constant $g_A^S$ can be extracted from the experimental data only via some theoretical extrapolation,[6] or some SU(3)-symmetry-based[5] analysis. (The last approach was to some extent examined in Sec. II.) The value $g_A^S/g_A^I<0.6$ is also supported by the theoretical calculation in a nonperturbative QCD approach which found the value (1.2) Ref. 6). As this calculation[6] was based on a sum rule involving empirical quantities some symmetry-breaking effects must have entered in the theoretical estimate. The same must hold, to a lesser extent, for the result (1.5). Although it is based on the $F$ and $D$ values obtained by analyzing hyperon semileptonic decays, they are average parameters for a number of decays, which have to exhibit SU(3)-breaking effects.

Those SU(3)-symmetry-breaking effects are very nicely illustrated in the review paper, Ref. 2, and they are reproduced here as Table VII (Appendix D). In connection with experimental analyses one should also mention the induced pseudotensor whose existence might influence all fits to the experimental data.[2,21,25] In our framework its appearance is connected with the existence of the quark-mass difference (in general some fermion mass difference[25]) but its magnitude can depend on mesonic terms. As the induced pseudotensor term is proportional to the momentum transfer $q$ its value will also depend on the recoil and on the center-of-mass corrections.[10,22,23] All this requires detailed study which is beyond the scope of the present paper.

The $g_A$ values are not influenced by the recoil corrections and do not crucially depend on the details of the "relativistic" structure of the model. However, some relativistic structure is needed in order to produce reasonable absolute $g_A$ values. In the case of the chiral bag models the mesonic term is also important. In the CBS model one finds pretty good agreement with the experimental values, as one sees by comparing Tables III and VII. The worst disagreement is for $g_A(\Lambda\to p)$ and $g_A(\Sigma\to\Lambda)$, about 9%, and for $g_A(\Sigma\to n)$ about 22%. Without SU(3)-symmetry breaking the analysis[2] (Table VII) finds $g_A(\Xi^-\to\Lambda)/g_A(\Sigma^-\to n)\leq 1$, while the SU(3)-symmetry breaking leads to $g_A(\Xi^-\to\Lambda)/g_A(\Sigma^-\to n)>1$. Our SU(3)-symmetry-breaking mechanism supports the second alternative (see Table III) while an alternative SU(3)-breaking theoretical scheme[21] (see Table VII) would support the first possibility. However, all SU(3)-breaking schemes are in trouble with the ratio

$$\frac{g_A(\Lambda\to p)}{g_A(\Xi^-\to\Sigma^0)}=A . \tag{4.1a}$$

The fit 5 in Table VII gives

$$A=1.02 , \tag{4.1b}$$

while two fits with $g_1$ free give

$$A=0.96 \tag{4.1c}$$

and

$$A = 0.957 , \tag{4.1d}$$

respectively. Our analysis predicts

$$1.04 \geq A \geq 1.02 \tag{4.1e}$$

and the fit with SU(3) breaking in Table VII gives

$$A = 1.02 . \tag{4.1f}$$

The discrepancy (4.1) seems to be the utmost quantitative difficulty with our CBS model-based calculations. However, this ratio is only about 8% wrong, while the calculated absolute values in question are too large by about 9% for $g_A(\Lambda \to p)$ and by about 1–2% for $g_A(\Xi^- \to \Sigma^0)$. All other $g_A$ values are relatively speaking in agreement with empirical values.[2] For example, all $g_A$'s are smaller than $g_A(n \to p)$, while the smallest $g_A$'s are $g_A(\Sigma^- \to \Lambda)$ and $g_A(\Sigma^- \to n)$. Naturally, these relative magnitudes are determined by the underlying spin-flavor SU(6) symmetry of the baryon states, irrespectively of its being broken by mesonic terms and by quark masses.

The absolute magnitude of the mesonic terms (2.6) decreases with the increase of the meson mass $\mu$. This was the mechanism which made the ratio (2.10) smaller than 0.6 as it should be. The usefulness of the same mechanism can be also seen from Table III. The constants $g_A$ corresponding to the $\Delta S \neq 0$ hyperon decays are smaller relative to $\Delta S = 0$ coupling constants when $\mu \neq 0$. That effect, which is due to $\mu_K > \mu_\pi$, improves the agreement with the empirical data.[2] An important test for the model is the determination of the isoscalar $g_A^S$ coupling constant. Taking into account that fits to the experimental data depend on the theoretical assumptions (induced pseudotensor, $q$ dependence, etc.) the chiral bag model with skin seems to work reasonably well in the description of the hyperon semileptonic decays.

## ACKNOWLEDGMENTS

This work has been partially supported by the joint Yugoslav-USA programs JFP-683, NSF and YOR 84/078.

## APPENDIX A: HARMONIC-OSCILLATOR QUARK MODEL

In the harmonic-oscillator quark model of Refs. 9 and 10 one finds

$$g_A^I = \tfrac{5}{3} \int d^3p_\rho d^3p_\lambda G^2(p_\rho,p_\lambda) \times \frac{1}{6} \frac{(E_u+m_u)(E_d+m_d)}{4E_uE_d} \left[ 6 - \frac{E_u-m_u}{E_d-m_d} - \frac{E_d-m_d}{E_u-m_u} \right] \tag{A1}$$

and

$$g_A^S = \int d^3p_\rho d^3p_\lambda G^2(p_\rho,p_\lambda)\left(\tfrac{4}{3}C_u - \tfrac{1}{3}C_d\right) , \quad C_i = \frac{2m_i+E_i}{3E_i}, \quad i = u,d . \tag{A2}$$

TABLE V. Coupling constants $g_A$ and $g_A^S$ in the HO model.

| $\alpha$ (MeV) | $m_u$ (MeV) | $m_d$ (MeV) | $g_A^S$ | $g_A$ |
|---|---|---|---|---|
| 226 | 177.5 | 137.5 | 0.702 | 1.250 |
| 280 | 165 | 225 | 0.692 | 1.250 |
| 240 | 130 | 210 | 0.660 | 1.248 |
| 234 | 82.5 | 342.5 | 0.504 | 1.255 |
| 234 | 72.5 | 352.5 | 0.468 | 1.239 |

Here $G$ is the HO model Gaussian

$$G(p_\rho,p_\lambda) = \frac{1}{(\pi^{1/2}\alpha)^3} \exp\left[ -\frac{1}{\alpha^2}({p_\rho}^2+{p_\lambda}^2) \right] \tag{A3}$$

and

$$E_i = (\tfrac{2}{3}{p_\lambda}^2 + {m_i}^2)^{1/2}, \quad i = u,d . \tag{A4}$$

In Table V we show the values of $g_A^S$ for various combinations of model parameters for which $g_A^I$ is close to its experimental value. In order to obtain relatively small $g_A^S$ one needs remarkably strong isospin-symmetry breaking, i.e., $m_d/m_u \sim 4.9$ for $g_A^S \sim 0.5$. This is not very convincing. One should obviously search for some additional different effects. However, results for $g_A^I$ definitely speak for the importance of the relativistic internal momentum-dependent effects.[9,10]

## APPENDIX B: MIT BAG MODEL

The quark-model calculation of form factors is usually based on two assumptions which can be only approximately valid.

(i) Baryon (nucleon, hadron) states contain valence quarks only.

(ii) State vectors have SU(6)-spin-flavor symmetry.

Consequences of the relaxation of the first assumption are explored in the main text. The assumption (ii) can be modified by allowing for nonequal quark masses. This has been already explored in the study of the SU(3)-symmetry-breaking effects.[21] When applied to the calculation of $g_A^I$ and $g_A^S$, as will be illustrated here, this means the isospin symmetry breaking.

From the existing quark-model-based theoretical expressions [see also (B3) below] one can conclude that a smaller up-quark mass, i.e., $m_u/m_d < 1$ leads to $g_A^S < \eta$. Such a requirement seems to be satisfied by the current-quark masses $\hat{m}_q$. One of the recent analyses,[26] for example, gives

$$\hat{m}_u = 12 \text{ MeV}, \quad \hat{m}_d = 22 \text{ MeV} . \tag{B1}$$

These values are in reasonable agreement with QCD running masses (modified minimal subtraction scheme) given in Ref. 27. It is not obvious how this should be connected with the quark-model masses $m_q$, but in the MIT bag model those masses are usually small and comparable with (B1) (Ref. 13). However, in order to obtain $g_A^I$ comparable with experimental value, one has to use rather larger quark-model masses.[12]

In the MIT bag model with unequal quark masses one finds

$$g_\Lambda^I=\frac{5}{3}\frac{1}{(2\omega_u{}^2-2\omega_u+m_uR)^{1/2}(2\omega_d{}^2-2\omega_d+m_dR)^{1/2}}$$
$$\times\Bigg[\tfrac{1}{2}[(k_uR)(k_dR)-\tfrac{1}{3}(\omega_u-m_uR)(\omega_d-m_dR)]$$
$$\times\left[\frac{(k_dR)\omega_u-(k_uR)\omega_d+(k_dR)(m_uR)-(k_uR)(m_dR)}{k_uR-k_dR}+1\right]\frac{1}{(k_uR)(k_dR)}$$
$$+\tfrac{1}{2}[(k_uR)(k_dR)+\tfrac{1}{3}(\omega_u-m_uR)(\omega_d-m_dR)]$$
$$\times\left[\frac{(k_dR)\omega_u+(k_uR)\omega_d+(k_dR)(m_uR)+(k_uR)(m_dR)}{k_uR+k_dR}-1\right]\frac{1}{(k_uR)(k_dR)}$$
$$+\frac{1}{3}\frac{(\omega_u-m_uR)(\omega_d-m_dR)}{(k_uR)(k_dR)}\Bigg],\quad k_iR=[\omega_i{}^2-(m_iR)^2]^{1/2},\quad i=u,d\ , \qquad \text{(B2)}$$

and

$$g_A^S=\tfrac{4}{3}\Omega_u-\tfrac{1}{3}\Omega_d\ ,$$
$$\Omega_i=\frac{1}{3}\frac{2\omega_i+4(m_iR)\omega_i-3(m_iR)}{2\omega_i{}^2-2\omega_i+m_iR}\ . \qquad \text{(B3)}$$

Here $R$ is the confinement radius and $\omega_i$ is the ground-state frequency for a quark of mass $m$.

The values of $g_A^S$, for various combinations of quark masses for which $g_A^I$ is close to its experimental value, are shown in Table VI. Unusually large quark masses are needed. In order to obtain $g_A^S=0.6$ one needs $m_d/m_u=18$. All this makes the exposed model quite unconvincing and suggests a search for a different symmetry-breaking mechanism.

## APPENDIX C: MODELS WITH STATIC MESON PHASE

In the model in which the confinement region and the region in which chiral symmetry is spontaneously broken intersect there is a region in which quark phase and mesonic phase coexist. A suitable generic name for that region is "skin." Such a model is defined by the following equations of motion and boundary conditions:[15,28–30]

$$\{\theta(R-r)[i\partial\!\!\!/-m\theta(r-R_{\rm ch})]-e^{i\gamma_5\lambda^a\phi_a/f_M}\delta(r-R_{\rm ch})\}\psi=0\ , \qquad \text{(C1)}$$

TABLE VI. Coupling constants $g_A$ and $g_A^S$ in the MIT bag model.

| $R$ (MeV$^{-1}$) | $m_u$ (MeV) | $m_d$ (MeV) | $g_A^S$ | $g_A$ |
|---|---|---|---|---|
| $7.331\times10^{-3}$ | 80 | 210 | 0.685 | 1.2465 |
| $7.331\times10^{-3}$ | 70 | 230 | 0.5724 | 1.2485 |
| $7.331\times10^{-3}$ | 72.5 | 352.5 | 0.6576 | 1.2842 |
| $7.331\times10^{-3}$ | 30 | 360 | 0.6157 | 1.2540 |
| $7.331\times10^{-3}$ | 20 | 360 | 0.6057 | 1.2457 |

$$(-i\hat{n}\psi+e^{i\gamma_5\lambda^a\phi_a/f_M}\psi)\delta(r-R)=0\ , \qquad \text{(C2)}$$

$$D^\mu D_\mu\phi_a-\mu^2\phi_a=\frac{-i}{2f_M}\bar\psi e^{i\gamma_5\lambda^a\phi_a/f_M}\lambda_a\psi\theta(R-r)\ ,\quad r>R_{\rm ch}\ , \qquad \text{(C3)}$$

$$n_\mu D^\mu\phi^a=0,\quad r=R_{\rm ch}\ , \qquad \text{(C4)}$$

$$n_\mu D^\mu\phi^a=\frac{1}{2f_M}\bar\psi e^{i\gamma_5\lambda^a\phi_a/f_M}\lambda^a\psi,\quad r=R\ , \qquad \text{(C5)}$$

$$D_\mu\phi^a=\left[\frac{\sinh(\lambda^a\phi_a)}{\lambda^a\phi_a}\partial_\mu\phi^a\right]\ . \qquad \text{(C6)}$$

Here $\psi$ is a quark field, which is in our case $(u,d,s)$ triplet in the flavor space. The symbol $\lambda_a$ stands for SU(3)-flavor matrices. The octet of meson fields is denoted by $\phi_a$, $a=1,\dots,8$. The whole formalism can be trivially generalized to include the SU(3)-singlet meson field $\phi_0$. The meson fields appearing in (C1)—(C6) are not quantized in this model. The equations are solved in the leading order in meson fields.

In Sec. II the $\phi$'s were neglected in (C1) and (C2) and retained only in (C3), (C4), and (C5). This crude approximation is, roughly speaking, equivalent to neglecting both the wave-function renormalization and the vertex correction in the cloudy-bag-model[28] picture. Such an approximation was, in our case, in keeping with our main interest, which was to study the SU(3)-symmetry breaking. That breaking is, in the present model, mostly due to its mesonic phase. The model parameters: confinement radius $R$ and the chiral-symmetry-breaking radius $R_{\rm ch}$ can always be changed in such a way as to effectively compensate for the quark wave-function corrections. The static mesonic phase is of the form

$$\phi_a(r)=\sum_{\substack{f,f'\\mm'}}[2Ru(R)v(R)]\frac{1}{f_\pi}\Delta_1(r,R)\times b_f^\dagger\chi_m^{\dagger f}(\tfrac{1}{2}\lambda_a\sigma\cdot\hat{\mathbf{r}})\chi_{m'}^{f'}b_{f'}\ . \qquad \text{(C7)}$$

TABLE VII. Values of $g_1/f_1$ in various fits.

| | Fit 5 | Fit with $g_1$ free | Fit with $g_1$ free and $\sin\theta_c$ constrained | Fit with SU(3) breaking |
|---|---|---|---|---|
| $\Sigma^- \to \Lambda e\nu$ | 0.62 | 0.595 | 0.594 | 0.62 |
| $\Lambda \to pe\nu$ | 0.893 | 0.857 | 0.893 | 0.98 |
| $\Xi^- \to \Sigma^0 e\nu$ | 0.872 | 0.891 | 0.933 | 0.962 |
| $\Xi^- \to \Lambda e\nu$ | 0.27 | 0.34 | 0.38 | 0.29 |
| $\Sigma^- \to ne\nu$ | −0.28 | −0.31 | −0.35 | −0.33 |
| $n \to pe\nu$ | 1.233 | | | 1.227 |

Here $b^\dagger$ ($b$) are creation (annihilation) operators of quarks while $\chi^f_m$ are spin-flavor states.

In the case $R = R_{\rm ch}$ corresponding to the chiral bag model of, for example, Refs. 13, 14, and 19 the leading mesonic terms in Eq. (C2) were also included. The boundary condition (C2) with

$$e^{i\gamma_5\lambda^a\phi_a/f_M} \simeq 1 + i\gamma_5\frac{1}{f_M}\lambda^a\phi_a \qquad \text{(C8)}$$

leads to Eq. (3.2) in the main text in which the quantity $\rho$ determines mesonic corrections to the quark wave function. The functions $u(r)$ and $v(r)$ in (2.3) are now calculated and normalized using $\omega$ from (3.2).

The expressions for $\rho$ and $\tilde{\Sigma}$ are related to the diagrams shown in Fig. 1 of Ref. 28.

## APPENDIX D: EXPERIMENTAL VALUES FOR $g_A$

For the sake of completeness let us first reproduce Table 12 from Ref. 2. The values of our Table VII have been obtained from the Ref. 2 values by multiplying them by appropriate factors like $1/\sqrt{2}$ and $\sqrt{2/3}$. Reference 2 has tabulated values $g_1(0)/f_1(0)$ while we give here values $g_1(0)$ to simplify the comparison with our Table I.

As discussed in Ref. 2 and in Ref. 25 in particular, the extracted values of $g_A$ (we use $g_A \equiv g_1/f_1$) can depend on the value of the induced pseudotensor. It seems that the value for $g_A(\Sigma \to n)$ quoted in Table VII can easily be in agreement with the solutions labeled (7) and (9) in Ref. 25.

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