

## Exotic fermions in $E_6$ and the anomalous magnetic moments of leptons

Thomas G. Rizzo

*Ames Laboratory and Department of Physics, Iowa State University, Ames, Iowa 50011*

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The mixing between the exotic fermions and the usual fermions (in  $E_6$  theories) can lead to flavor-changing neutral currents. Such couplings can produce new and potentially large contributions to the anomalous magnetic moments of the electron and muon. We analyze these possibilities and comment on other processes (such as  $\mu \rightarrow e\gamma$ ) which can be used to constrain  $E_6$  theories.

The recent advent of superstring theories<sup>1</sup> has renewed interest in grand unified theories (GUT's) based on the gauge group  $E_6$  (Ref. 2). Superstrings have the promise of being complete and finite theories of all of the fundamental forces including gravity. Freedom from anomalies is also possible if the gauge group is chosen to be  $SO(32)$  or  $E_8 \times E_8$  in ten dimensions.<sup>3</sup> The latter gauge group has attracted the most attention since it leads to chiral fermions. In particular, upon compactification,  $E_8$  breaks to  $E_6$  with local  $N=1$  supersymmetry below the Planck scale. The second group  $E_8$  remains unbroken and is associated with a new form of matter (shadow matter).<sup>4</sup> The fermions then transform as (at least) three 27 representations of  $E_6$  with the possibility that "mirror" fermions<sup>5</sup> in the  $\bar{27}$  representation could also be present. They are all singlets with respect to  $E_6$ .

Under the  $SU(5)$  subgroup of  $E_6$  the 27 decomposes as

$$27 = (1_1 + \bar{5}_1 + 10) + (5 + \bar{5}_2) + 1_2, \quad (1)$$

where explicitly for the first generation we have

$$10 = \begin{pmatrix} u \\ d \end{pmatrix}_L + u_L^c + e_L^c, \quad 5 = \begin{pmatrix} N \\ E \end{pmatrix}_L + D_L, \\ \bar{5}_1 = \begin{pmatrix} \nu \\ e \end{pmatrix}_L + d_L^c, \quad \bar{5}_2 = \begin{pmatrix} N \\ E \end{pmatrix}_L + D_L^c, \quad (2) \\ 1_1 = \nu_L^c, \quad 1_2 = N_L^c.$$

Investigations<sup>6</sup> of symmetry-breaking patterns in the context of  $E_6$  have shown that the low-energy electroweak gauge group is larger than the standard model by (at least) an additional  $U(1)$  factor [which we will call  $U(1)_x$ ]. We have recently considered a large class of such theories with only an additional  $U(1)_x$  factor<sup>7</sup> and we limit ourselves to this class in our analysis although our results can be extended to a much larger set of theories.

An important point to notice is that the above fermions do not satisfy the Glashow-Weinberg-Paschos<sup>8</sup> conditions for the natural absence of flavor-changing neutral currents (FCNC's); i.e., any mixing among the states  $(e, E)$ ,  $(d, D)$ ,  $(\nu, N, N')$  leads to FCNC's.<sup>9</sup> These FCNC interactions result not only from the usual  $Z$  (now called  $Z_1$ ) exchange, but are also due to exchange of the gauge boson  $Z_2$  (which may be light) associated with the second  $U(1)_x$  factor. (The phenomenology of this possibility has recently been discussed by Robinett.<sup>9</sup>) The existence of

nonchiral FCNC couplings can, as we will see, produce large contributions to the electron and muon anomalous magnetic moments. (We will neglect any mixing between the two  $Z$  bosons in this analysis since the relevant mixing angle has been shown to be quite small.<sup>7</sup>)

The neutral-current interactions for any fermion  $f$  can be described by

$$L = \frac{g}{2c_w} \bar{f} \gamma_\mu [(T_{3L} + T_{3R} - 2x_w Q) - (T_{3L} - T_{3R}) \gamma_5] f Z_1^\mu \\ + \frac{g_x}{2} \bar{f} \gamma_\mu [(x_L + x_R) - (x_L - x_R) \gamma_5] f Z_2^\mu, \quad (3)$$

where  $Q$  is the fermion charge,  $T_{3L}$  ( $T_{3R}$ ) is the third component of weak isospin for the left-handed (right-handed) fermion.  $x_L$  ( $x_R$ ) is the  $U(1)_x$  quantum number for the left- (right-) handed fermion and  $g_x$  is the associated coupling constant. We limit our discussion in what follows to the charged leptons  $e$  and  $E$ , but we can easily extend the analysis beyond the first generation. We now imagine a mixing between the  $e$  and  $E$  fields which, in general may be different for their left- and right-handed components:

$$\begin{pmatrix} e \\ E \end{pmatrix}_{L,R} \rightarrow U_{L,R}^e \begin{pmatrix} e \\ E \end{pmatrix}_{L,R}, \quad (4)$$

where  $U_{L,R}^e$  are some general  $2 \times 2$  unitary matrices which we will take to be purely real and orthogonal for simplicity. Note that if the  $(e, E)$  mass matrix is Hermitian then  $U_L^e = U_R^e$  so that  $\theta_L^e = \theta_R^e$ . Let us first examine the  $Z_1$  couplings after this rotation; we see that the combination  $T_{3L} - 2x_w Q$  has the same value for both  $e$  and  $E$ . This implies that the resulting FCNC connecting  $e$  and  $E$  will be purely right handed and, thus, chiral. Writing

$$L_{Z_1} = \frac{g}{2c_w} \bar{f} \alpha \gamma_\mu (v_1 - a_1 \gamma_5) f_\beta Z_1^\mu \quad (5)$$

we find (with  $s_{L(R)}^e = \sin \theta_{L(R)}^e$ ,  $c_{L(R)}^e = \cos \theta_{L(R)}^e$ ) the couplings

$$f_\alpha = f_\beta = e, \quad v_1 = -\frac{1}{2} + 2x_w - \frac{1}{2}(s_R^e)^2, \quad a_1 = -\frac{1}{2}(c_R^e)^2, \\ f_\alpha = f_\beta = E, \quad v_1 = -\frac{1}{2} + 2x_w - \frac{1}{2}(c_R^e)^2, \quad a_1 = -\frac{1}{2}(s_R^e)^2, \quad (6) \\ f_\alpha = e, \quad f_\beta = E, \\ f_\alpha = E, \quad f_\beta = e, \quad v_1 = -\frac{1}{2}s_R^e c_R^e, \quad a_1 = -\frac{1}{2}s_R^e c_R^e.$$

A similar, but more complex, situation occurs for  $Z_2$  couplings since  $x_{L,R}^e$  and  $x_{L,R}^E$  are not in general equal. Writing

$$L_{Z_2} = \frac{g_x}{2} \bar{f} \alpha \gamma_\mu (v_2 - a_2 \gamma_5) f \beta Z_2^\mu, \quad (7)$$

we obtain

$$f_\alpha = f_\beta = e, \quad \left. \begin{array}{l} v_2 \\ a_2 \end{array} \right\} = [x_L^e (c_L^e)^2 + x_L^E (s_L^e)^2] \\ \pm [x_R^e (c_R^e)^2 + x_R^E (s_R^e)^2], \quad (8)$$

$$f_\alpha = f_\beta = E, \quad \left. \begin{array}{l} v_2 \\ a_2 \end{array} \right\} = [x_L^E (c_L^e)^2 + x_L^e (s_L^e)^2] \\ \pm [x_R^E (c_R^e)^2 + x_R^e (s_R^e)^2],$$

$$\left. \begin{array}{l} f_\alpha = e, \quad f_\beta = E, \quad v_2 \\ f_\alpha = E, \quad f_\beta = e, \quad a_2 \end{array} \right\} = (x_L^e - x_L^E) s_L^e c_L^e \\ \pm (x_R^e - x_R^E) s_R^e c_R^e, \quad (9)$$

so that unless either  $(x_L^e - x_L^E)$  or  $(x_R^e - x_R^E) = 0$  our FCNC's are nonchiral even if  $\theta_L^e = \theta_R^e$ . It should be noted that the mixing angles for the other fermions such as  $(d, D)$  could be different from  $\theta_L^e$  and  $\theta_R^e$  and different FCNC's will be present for those fermions. In the  $(e, E)$  sector we know the angle  $\theta_R^e$  must be small from data on  $e^+ e^- \rightarrow \mu^+ \mu^-$  (Ref. 9).

This same mixing results in a modification of the charged-current (CC) structure as well. Limiting ourselves to the electroweak group  $SU(2)_L \times U(1)_Y \times U(1)_X$  we still have only a single  $W$  boson so that CC remains left handed. The two left-handed doublets (we ignore  $N'$  for now)

$$\left[ \begin{array}{c} \nu \\ e \end{array} \right]_L, \quad \left[ \begin{array}{c} N \\ E \end{array} \right]_L \quad (10)$$

are then mixed by the rotation  $U_L^e$  as well as the corresponding  $(\nu, N)$  rotation  $U_Y^e$ . Thus the CC coupling to  $W$  takes the form

$$\frac{g}{2\sqrt{2}} [\bar{\nu} \gamma_\mu (1 - \gamma_5) e + \bar{N} \gamma_\mu (1 - \gamma_5) E] \cos(\theta_L^e - \theta_L^e) W^\mu \\ + \frac{g}{\sqrt{2}} [\bar{\nu} \gamma_\mu (1 - \gamma_5) E + \bar{N} \gamma_\mu (1 - \gamma_5) e] \sin(\theta_L^e - \theta_L^e) W^\mu. \quad (11)$$

Clearly, universality constrains the difference in angles  $\theta_L^e - \theta_L^e$  to be quite small.<sup>9</sup> Note that these couplings are all still chiral.

Thus present CC and NC data tell us that  $\theta_R^e$  is small and the difference  $\theta_L^e - \theta_L^e$  is small; the individual values of  $\theta_L^e, \nu$  may still be large.

Figure 1 shows the two generic contributions of gauge bosons  $X$  and fermions  $F$  to the muon anomalous magnetic moment. The various contributions depend, of course, on the nature of the particles  $X$  and  $F$ . For the models under consideration here these are several distinct sets of identifications possible. For diagram (A) we have the possibilities

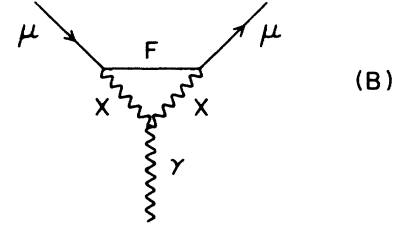
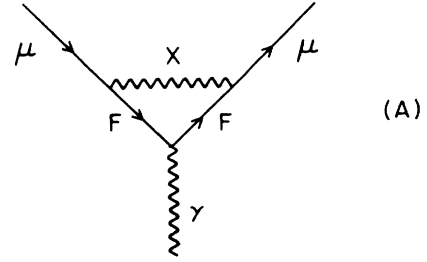


FIG. 1. Feynman diagrams contributing to  $a_\mu$  in a general gauge theory.

$$\begin{aligned} A_0: X = Z_1, \quad F = \mu, \\ A_1: X = Z_1, \quad F = M, \\ A_2: X = Z_2, \quad F = \mu, \\ A_3: X = Z_2, \quad F = M, \end{aligned} \quad (12)$$

where  $M$  is the new muon-type heavy charged lepton corresponding to  $E$ . For diagram (B) we find two possibilities

$$\begin{aligned} B_0: X = W, \quad F = \nu_\mu, \\ B_1: X = W, \quad F = N_\mu. \end{aligned} \quad (13)$$

The standard-model (SM) contributions are already well known<sup>10</sup> and are given by the sum of  $A_0$  and  $B_0$ ; this leads to the SM prediction

$$(a_\mu)^{\text{SM}} = 20 \times 10^{-10} \quad (14)$$

and thus<sup>11</sup>

$$(a_\mu)^{\text{QED}} + (a_\mu)^{\text{SM}} - (a_\mu)^{\text{expt}} = (27 \pm 69) \times 10^{-10}, \quad (15)$$

so that any additional contributions beyond the SM are quite constrained. We now turn to the new contributions  $A_{1-3}$  and  $B_1$ .

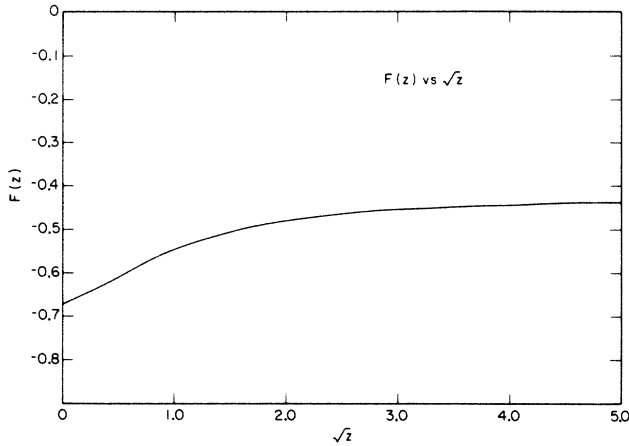
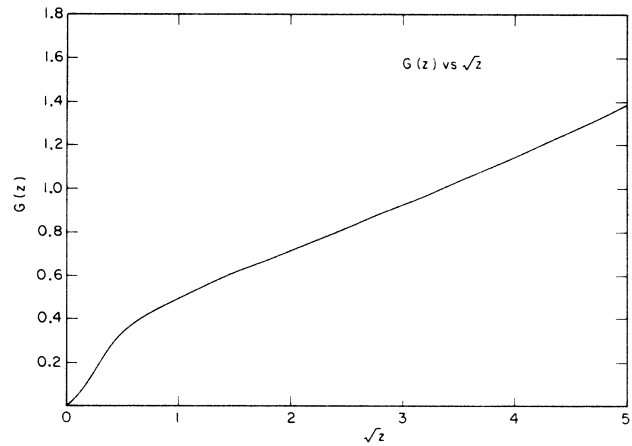
The  $A_1$  contribution<sup>10</sup> can be written as

$$A_1 = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} (s_R^\mu c_R^\mu)^2 F(z) \quad (16)$$

with  $z = (M_M/M_1)^2$ ,  $r = (1-z)^{-1}$ , and

$$F(z) = r \left[ \frac{5}{6} - \frac{5}{2} + r^2 + r(r^2 - 3r + 2) \ln \frac{r-1}{r} \right] \\ + \frac{r}{2} z \left[ \frac{5}{6} + \frac{3r}{2} + r^2 + r^2(1+r) \ln \frac{r-1}{r} \right]. \quad (17)$$

Note that  $F(z)$  is a slowly varying function of  $z$ . Figure 2

FIG. 2. A plot of the function  $F(z)$  vs  $\sqrt{z}$ .FIG. 3. A plot of the function  $G(z)$  vs  $\sqrt{z}$ .

shows a plot of the function  $F(z)$ . As  $z$  varies from 0 to 5,  $F(z)$  varies from  $-0.67$  to  $-0.43$  so that with  $|s_R^\mu c_R^\mu|^2 \leq 0.05$  we find a very small contribution to  $a^\mu$

$$A_1 \leq (-0.50 \sim -0.73) \times 10^{-10}. \quad (18)$$

The second contribution is  $A_2$ ; for  $m_\mu^2/M_2^2 \ll 1$  we find that

$$A_2 = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_2^2} \frac{g_x^2}{12} [(x_L^\mu + x_R^\mu)^2 - 5(x_L^\mu - x_R^\mu)^2] \\ = \frac{\alpha}{\pi} \frac{m_\mu^2}{M_1^2} \left[ \frac{\alpha_x}{\alpha} \right] \left[ \frac{M_1}{M_2} \right]^2 \Delta, \quad (19)$$

where  $\alpha_x \equiv g_x^2/4\pi$  and  $\Delta \equiv x_L^\mu x_R^\mu - (x_L^{\mu 2} + x_R^{\mu 2})/3$ . Note  $x_{L(R)}^\mu = x_{L(R)}^f$ ,  $x_{L(R)}^M = x_{L(R)}^E$ ; these values can be found in Table I for the four models discussed in Ref. 7. In this same reference we saw that, roughly speaking,  $(\alpha_x/\alpha)(M_1/M_2)^2$  can be in the range  $(6-50) \times 10^{-3}$  and the value of  $\Delta$  is given by Table I. Thus  $A_2$  is quite small:

$$|A_2| = (1.6-268) \times 10^{-12} \quad (20)$$

with the sign depending on the sign of  $\Delta$ ; three out of the four cases lead to  $A_2 < 0$ . This contribution is quite small.

We now turn to the potentially large contribution  $A_3$ ; in this case we have ( $z = m_M^2/M_2^2$ )

$$A_3 = -\frac{\alpha_x}{4\pi} \frac{m_\mu^2}{M_2^2} \left[ (v_2^2 + a_2^2)F(z) + (v_2^2 - a_2^2)G(z) \left[ \frac{M_2}{m_\mu} \right] \right], \quad (21)$$

$v_2$  and  $a_2$  are given by Eq. (9) and  $F(z)$  is given by Eq. (17).  $G(z)$  is  $[r=(1-z)^{-1}]$

$$G(z) = rz^{1/2} \left[ \left[ 2r-1 + 2r(r-1) \ln \frac{r-1}{r} \right] - \frac{1}{2}z \left[ \frac{1}{2} + r + r^2 \ln \frac{r-1}{r} \right] \right]. \quad (22)$$

Note that the term proportional to  $(v_2^2 - a_2^2)$  is enhanced by an extremely large factor  $M_2/m_\mu \gtrsim 2000$ . This term vanishes identically, however, in the limit of chiral couplings  $v_2^2 - a_2^2 \rightarrow 0$ . The function  $G(z)$  is shown in Fig. 3. Equation (21) can be recast in the form

$$A_3 = -\frac{\alpha_x}{2\pi} \frac{m_\mu^2}{M_2^2} \left[ [(x_L^\mu - x_L^M)^2 (s_L^\mu c_L^\mu)^2 + (x_R^\mu - x_R^M)^2 (s_R^\mu c_R^\mu)^2] F(z) + 2(x_L^\mu - x_L^M)(x_R^\mu - x_R^M) s_L^\mu s_R^\mu c_L^\mu c_R^\mu \times G(z) \left[ \frac{M_2}{m_\mu} \right] \right]. \quad (23)$$

TABLE I. Values of the various parameters for the four models discussed in Ref. 7.

Model	$x_L^f$	$x_R^f$	$x_L^E$	$x_R^E$	$\Delta$	$\chi$	$\rho$
A	1	-1	-2	2	$-\frac{5}{3}$	-9	4
B	3	1	-2	-2	$-\frac{1}{3}$	15	4
C	-1	-2	-1	4	$\frac{1}{3}$	-12	1
D	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{12}$	0	$\frac{1}{4}$

Now  $(s_R^\mu c_R^\mu)^2 \lesssim 0.05$  suppresses the  $(x_R^\mu - x_R^M)^2$  term while  $(s_L^\mu c_L^\mu)^2$  can, in principle, be quite large with  $\rho \equiv (x_L^\mu - x_L^M)^2$  of order unity (see Table I for values of  $\rho$  in the four models under consideration). With both  $F(z)$  and  $G(z)$  of order unity,  $A_3$  can be approximated by

$$A_3 = -32.1 \times 10^{-10} \left[ \frac{\alpha_x}{\alpha} \frac{M_1^2}{M_2^2} \right] \chi s_L^\mu c_L^\mu s_R^\mu c_R^\mu G(z) \frac{M_2}{m_\mu}, \quad (24)$$

where  $\chi \equiv (x_L^\mu - x_L^M)(x_R^\mu - x_R^M)$ . As a numerical example we take  $M_2 = 200$  GeV and  $M_M = 180$  GeV with  $(\alpha_x/\alpha)M_1^2/M_2^2$  given by the above range. This yields

$$A_3 = (-173 \text{ to } -1440) \times 10^{-10} \chi s_L^\mu c_L^\mu s_R^\mu c_R^\mu. \quad (25)$$

Since  $|\chi| \sim 10$  in all but one model (where it vanishes identically) the value of  $A_3$  can be quite large. For further demonstration purposes taking model B with  $(s_R^\mu c_R^\mu)^2 \simeq 0.05$  we find

$$A_3 = (-130 \text{ to } -1080) \times 10^{-10} (s_L^\mu c_L^\mu / s_R^\mu c_R^\mu) \quad (26)$$

which can be very sizable. From this we conclude that either  $M_2$  is much heavier than expected ( $\gtrsim 200$  GeV or so), the mixing angles  $\theta_L^\mu$ ,  $\theta_R^\mu$  are both very tiny, or that  $\chi$  is naturally small (or zero as in model D).

To see how (26) when combined with (15) can constrain  $E_6$  model parameters let us again consider model B. With  $\alpha_x \simeq \frac{1}{3000}$  we find in general that

$$A_3 = -18083 \times 10^{-10} \frac{M_1}{M_2} (s_L^\mu c_L^\mu) (s_R^\mu c_R^\mu) G(z). \quad (27)$$

We take  $G(z) \simeq \frac{1}{2}$  and  $\theta_L^\mu = \theta_R^\mu$  (the case of an Hermitian mass matrix) and find that at the  $1\sigma$  ( $2\sigma$ ) level

$$\frac{M_1}{M_2} (s_L^\mu c_L^\mu)^2 \leq 4.65 \times 10^{-3} (1.23 \times 10^{-2}). \quad (28)$$

Figure 4 shows the allowed region in the  $M_2/M_1 - (s_L^\mu c_L^\mu)^2$  plane from the  $a^\mu$  constraint as well as universality and the  $\rho$  neutral-current parameter.<sup>7</sup> Similar results hold for the other models.

We now turn to the last contribution to  $a^\mu$ , that coming from  $B_1$ . We find ( $z = M_N^2/M_W^2$ )

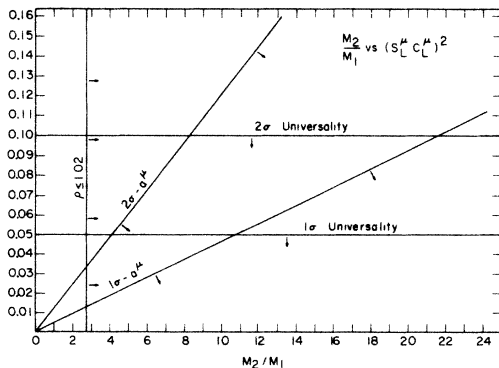


FIG. 4. The allowed region in the  $M_2/M_1 - (s_L^\mu c_L^\mu)^2$  plane constrained by the  $\rho$  parameter, universality, and the value of  $a^\mu$ . Both  $1\sigma$  and  $2\sigma$  limits are shown.

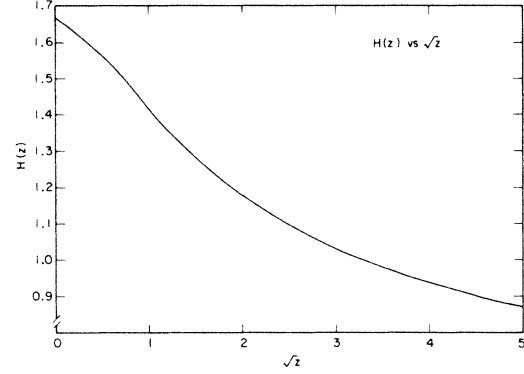


FIG. 5. A plot of the function  $H(z)$  vs  $\sqrt{z}$ .

$$B_1 = \frac{1}{4\pi^2} \frac{m_\mu^2}{M_W^2} \left[ \frac{g}{2\sqrt{2}} \sin(\theta_L^\mu - \theta_L^\nu) \right]^2 H(z). \quad (29)$$

With  $r = (1-z)^{-1}$  and  $t = rz$  we find

$$H(z) \equiv 2r \left[ \frac{5}{6} - \frac{3}{2}t + t^2 + t^2(1-t) \ln \frac{1+t}{t} \right] + zr \left[ \frac{5}{6} + \frac{5}{2}t + t^2 - t(2+3t+t^2) \ln \frac{1+t}{t} \right]. \quad (30)$$

$H(z)$  is shown in Fig. 5.  $B_1$  can be rewritten as

$$B_1 = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} H(z) \sin^2(\theta_L^\mu - \theta_L^\nu) = 32.1 \times 10^{-10} H(z) \sin^2(\theta_L^\mu - \theta_L^\nu). \quad (31)$$

Note that the sign of  $B_1$  is always positive unlike the model-dependent sign of the  $A_i$ .  $B_1$  is constrained to be small by universality; if  $\sin^2(\theta_L^\mu - \theta_L^\nu) \lesssim 0.05$ , then

$$B_1 \lesssim 2.7 \times 10^{-10} \quad (32)$$

which is a small but perhaps observable contribution.

What about  $a_e$ ? Including the latest results from Kinoshita and co-workers<sup>12</sup> we find that

$$(a_e)^{\text{th}} - (a_e)^{\text{expt}} = (109 \pm 111) \times 10^{-12}. \quad (33)$$

The new  $E_6$  contributions for  $a_e$  can be obtained from those for  $a_\mu$  by letting  $m_\mu \rightarrow m_e$  (with  $\mu \rightarrow e$  in all the angles as well). As before, the only major contribution comes from  $A_3$ ; we find

$$A_3(e) = -29393 \times 10^{-12} G(z) \left[ \frac{\alpha_x}{\alpha} \frac{M_1^2}{M_2^2} \right] \times \chi s_L^e c_L^e s_R^e c_R^e \left[ \frac{M_2}{200 \text{ GeV}} \right]. \quad (34)$$

With  $\alpha_x \simeq \frac{1}{3000}$  and  $\theta_L^e = \theta_R^e$  we find

$$A_3(e) = -583\chi \times 10^{-12} \frac{M_1}{M_2} (s_L^e c_L^e)^2 G(z) \quad (35)$$

and for model B with  $G(z) \simeq \frac{1}{2}$  the constraint at the  $1\sigma$  ( $2\sigma$ ) level becomes

$$\frac{M_1}{M_2} (s_L^e c_L^e)^2 \lesssim 2.29 \times 10^{-6} (2.58 \times 10^{-2}). \quad (36)$$

If the  $1\sigma$  limit is taken seriously the improvement over (28) is better than a factor of 2000. The  $2\sigma$  limit from (36) is about twice that from (28). Similar constraints hold for the other models as well.

As a last remark, we consider the radiative decays  $\mu \rightarrow e\gamma$  and  $\nu \rightarrow \nu'\gamma$  which are highly suppressed in the SM and slight extensions thereof. Since virtually the same kinds of diagrams as shown in Fig. 1 are also responsible for these decays we might expect large enhancements in these radiative rates in  $E_6$  theories (although further intergenerational mixing angles are also present). As is well known<sup>13</sup> the existence of fermions with masses comparable or greater than that of the  $W$  or  $Z$  tend to negate the Glashow-Iliopoulos-Maiani mechanism responsible for suppressing these processes. It may be possible to strengthen our constraints on the  $M_1/M_2$  ratio and the various mixing angles from further examination of these processes. An explicit calculation<sup>14</sup> indicates no large contribution in radiative  $\nu$  decay because of chiral cou-

plings, whereas large contributions can occur in two out of the four models for  $\mu \rightarrow e\gamma$ .

In conclusion, we have examined the new fermion and gauge-boson contributions to the muon and electron anomalous magnetic moments present in  $E_6$  theories. Because of the existence of nonchiral flavor-changing neutral-current couplings present in such theories we can constrain the mixing between the ordinary and exotic fermions and/or the ratio  $M_1/M_2$ . Both  $a_e$  and  $a_\mu$  give comparable limits at present, but new experiments could show evidence of non-SM contributions. The amplitude for  $\mu \rightarrow e\gamma$  may also be enhanced in these models. (Our analysis has ignored Higgs-boson contributions which may also be potentially large in such theories but are very model dependent.) Much more work needs to be done in examining the phenomenology of  $E_6$  theories.

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