Radiative corrections for semileptonic decays of hyperons: "Model-independent" part

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(Received 30 December 1985)

The "model-independent" part of the order-α radiative correction due to virtual-photon exchanges and inner bremsstrahlung is studied for semileptonic decays of hyperons. Numerical results of high accuracy are given for the relative correction to the branching ratio, the electron energy spectrum, and the (E_e,E_f) Dalitz distribution in cases of four different decays: $\Sigma^-\to ne\bar{\nu}$, $\Sigma^-\to\Lambda e\bar{\nu}$, $\Xi^-\to\Lambda e\bar{\nu}$, and $\Lambda\to pe\bar{\nu}$.

I. INTRODUCTION

In the last few years several high-statistics experiments were carried out to study semileptonic decays of hyperons. The most interesting question about these decays is whether the experimental results fit into the framework of the Cabibbo model.[1] At the level of quarks, and after the extension made by Kobayashi and Maskawa[2] this model has become an important ingredient of the standard Glashow-Salam-Weinberg theory of electroweak interactions.[3]

The improving precision of the measurements made it necessary to apply radiative corrections in the analysis of the experimental data. Several calculations exist in the literature for the corrections to the branching ratio and the electron energy spectrum,[4-6] all of them being descendant of the classical radiative correction calculations for neutron β decay.[7-9] We carried out a comprehensive calculation of the radiative corrections for the decays $\Sigma^-\to ne\bar{\nu}$, $\Sigma^-\to\Lambda e\bar{\nu}$, $\Xi^-\to\Lambda e\bar{\nu}$, and $\Lambda\to pe\bar{\nu}$ with the aim of obtaining coherent sets of results for the branching ratio, the electron energy spectrum, and the Dalitz distribution. In the course of this work we were in close contact with the WA2 experimental group at CERN. This group measured the above decay modes, and the main goal of our work was to supply the experimental analysis with the necessary radiative corrections. Our results concerning the branching ratio and electron energy spectrum are simply more accurate, in a sense to be explained later, than already existing results. The radiative corrections on the (electron energy, final baryon energy) Dalitz plot, which we are going to present here are so far unique in the literature. The obtained large variation of the latter correction is a warning, that, even if the integrated "theoretical" correction to the branching ratio is small, the experimentally observed radiative corrections can be relevant and quite different in various experiments, because of the acceptance properties of the experimental apparatus.

We mention that analytical formulas have already been published to give the radiative corrections for the decay distribution on the $(E_e,\cos\theta_{e\bar{\nu}})$ plane,[10,11] E_e and $\theta_{e\bar{\nu}}$ being the electron energy and the angle between the three-momenta of the electron and the antineutrino, respectively. However, this result is of very limited use from the point of view of analyzing experiments, since $\theta_{e\bar{\nu}}$ is never measured.[12]

In Sec. II we start with the presentation of the theoretical program for the calculation of radiative corrections to semileptonic decays. In Sec. III we describe the "model-independent" part of the virtual-photon corrections. In Sec. IV we discuss some characteristic properties of two-dimensional decay distributions in the presence of inner bremsstrahlung. Our results are presented in Sec. V, together with a detailed discussion of the inputs and tests of our numerical calculation. In an Appendix we shortly discuss the effect of some numerical approximations.

II. FORMULATION OF THE PROBLEM

The calculation of radiative corrections to semileptonic decays is an old and complicated theoretical problem. The weak interaction, responsible for these decays, is mixed up with the electromagnetic and strong interactions. Infrared and ultraviolet divergences spoil the calculation, which must be overcome in a reliable fashion.

The problem of infinities, at least to order α, the fine-structure constant, is now solved. The solution is simple in case of the infrared divergence, as the method familiar from QED works: one must add the decay probability of the bremsstrahlung process $B\to be\bar{\nu}\gamma$ to that of $B\to be\bar{\nu}$. The infrared-divergent parts for both processes are the same, as if the coupling between the (real, or virtual) photon and the charged baryon are pointlike.[7]

The problem of the cancellation of the ultraviolet infinities is much more difficult, the method of solution is rather complicated.[13] The result, however, can be expressed in a simple way. In a very general framework, which includes (1) the standard SU(2)⊗U(1) unified gauge theory of the weak and electromagnetic interactions, (2) generally accepted properties for the strong interactions, such as SU(3) color gauge group, asymptotic freedom, current-algebra relations, and (3) an appropriate choice of counterterms,[14] the $B\to be\bar{\nu}$ decay amplitude to order α can be written as

$$\mathcal{M}=\mathcal{M}_0\left[1-\frac{3\alpha}{8\pi}(1+2\bar{Q})\ln\cos^2\theta_W\right]+\mathcal{M}_\gamma\,, \qquad (2.1)$$

where θ_W is the Weinberg angle and $\bar{Q}$ is the average electric charge of the relevant weak isodoublet. (For quarks in hyperon decays $\overline{Q}=\frac{1}{6}$, for leptons in muon decay $\overline{Q}=-\frac{1}{2}$.) The notation $\mathcal{M}_0$ is used for the decay amplitude in lowest order:

$$\mathcal{M}_0 \approx \sqrt{2} G_F [\bar{u}_2 \gamma^\mu (1+\gamma^5) v_1] \langle f \,|\, J_W^\mu(0) \,|\, i \rangle \ . \tag{2.2}$$

The labels i, f, 1, and 2 in (2.2) refer to the decaying and final baryon, antineutrino, and electron, respectively. The coupling constant G_F is equal with that observed in muon decay, $G_F \equiv G_\mu$.

(We use the conventions of Ref. 15 for the Dirac γ matrices γ^μ, γ^5, and for the metric in the scalar product of four-vectors. We normalize the Dirac spinors as $\bar{u}u=-\bar{v}v=2m$.) The matrix element $\mathcal{M}$ in (2.1) is ultraviolet finite. The order-α part of the expression in the large parentheses in (2.1) is due to diagrams, which are infinite before renormalization. We do not go into the details here of how to eliminate the infinities, neither into the derivation of their finite remnant. The interested reader can study these problems in the excellent works of Sirlin.[13,16]

The term $\mathcal{M}_\gamma$ in (2.1) collects the contribution of three different types of diagrams:

$$\mathcal{M}_\gamma = \mathcal{M}_\gamma^{(1)} + \mathcal{M}_\gamma^{(2)} + \mathcal{M}_\gamma^{(3)} \ . \tag{2.3}$$

They are familiar from the literature, nevertheless, we write down the corresponding expressions in order to give explicitly the basis of our calculation.

The first term, $\mathcal{M}_\gamma^{(1)}$, in (2.3) is a contribution, which comes from the wave-function renormalization of the final-state electron due to the emission and reabsorption of a virtual photon:

$$\mathcal{M}_\gamma^{(1)} = \mathcal{M}_0 \delta Z_{(e)} \ , \tag{2.4}$$

where

$$\delta Z_{(e)} = \frac{i\alpha}{8\pi^3} \int dk\, D_{\mu\nu}^{<}(k) \frac{(2p_{2\mu}-k_\mu)(2p_{2\nu}-k_\nu)}{[(k-p_2)^2+m_e^2]^2} + \frac{\alpha}{32\pi^3} \frac{1}{m_e^2} \int dk\, D_{\mu\nu}^{<}(k) \frac{\bar{u}_2 [\gamma_\mu, \not{k}] \not{p}_2 \not{k} \gamma_\nu u_2}{[(k-p_2)^2+m_e^2]^2} \ . \tag{2.5}$$

A virtual photon can be emitted and reabsorbed also by the hadronic weak vertex. This is the origin of $\mathcal{M}_\gamma^{(2)}$,

$$\mathcal{M}_\gamma^{(2)} = -i\sqrt{2} G_F [\bar{u}_2 \gamma^\mu (1+\gamma^5) v_1] T^{\mu <} \ , \tag{2.6}$$

where

$$T^{\mu <} = \lim_{\bar{q} \to p_i - p_f} \left[\frac{\alpha}{8\pi^3} \int dk\, D_{\nu\rho}^{<}(k) \left[\int dy\, e^{-i\bar{q}y} \int dx\, e^{-ikx} \langle f \,|\, T[J_W^\mu(y) J_\gamma^\nu(x) J_\gamma^\rho(0)] \,|\, i \rangle - B^{\mu\nu\rho} \right] \right] \ . \tag{2.7}$$

We use the notation J_γ^μ for the hadronic electromagnetic current. $B^{\mu\nu\rho}$ is a counterterm to assure that the pole of the uncorrected and the $O(\alpha)$-corrected propagator for the i (or f) particle be at the same mass value: m_i (or m_f).

The last term $\mathcal{M}_\gamma^{(3)}$ in (2.3) corresponds to the exchange of a photon between the weak vertex and the electron:

$$\mathcal{M}_\gamma^{(3)} = \sqrt{2} G_F \frac{\alpha}{4\pi^3} \int dk\, D_{\mu\nu}(k) T^{\mu\rho}(k) \frac{M_W^2}{M_W^2+k^2} \times \bar{u}_2 \frac{2p_{2\nu}-k_\nu-\frac{1}{2}[\gamma^\nu, \not{k}]}{(k-p_2)^2+m_e^2} \times \gamma^\rho (1+\gamma^5) v_1 \ . \tag{2.8}$$

The tensor $T^{\mu\rho}(k)$ is defined as

$$T^{\mu\rho}(k) = \int dx\, e^{-ikx} \langle f \,|\, T[J_\gamma^\mu(x) J_W^\rho(0)] \,|\, i \rangle \ . \tag{2.9}$$

The symbol M_W stands for the mass of the charged weak vector boson, and $D_{\mu\nu}$ is the photon propagator in the Feynman gauge:

$$D_{\mu\nu}(k) = \frac{\delta_{\mu\nu}}{k^2+\lambda^2} \ .$$

A small photon mass λ is needed to regularize the infrared divergence of (2.5), (2.7), and (2.9). Finally, $D_{\mu\nu}^{<}$ denotes

$$D_{\mu\nu}^{<}(k) = \frac{M_W^2}{k^2+M_W^2} D_{\mu\nu}(k) \ .$$

When writing down expressions (2.5)—(2.9) we neglected the dependence of the W-boson propagator on $p_i - p_f$. This means the neglect of very small terms proportional to $G_F \alpha m_i^2 / M_W^2$ in the matrix element. As a result, we could write down the well-known formulas for the virtual photonic radiative corrections in the traditional current $\times$current theory of weak interactions.[7,8] Even an ultraviolet regularizing factor $M_W^2/(M_W^2+k^2)$, needed in this approach, is present in (2.5), (2.7), and (2.8). This is quite natural in the case of (2.8), since, in fact, our starting point is the Glashow-Weinberg-Salam theory of weak interactions. The situation is slightly different in the case of (2.5) and (2.7). In Sirlin's approach, which we follow here, these two types of diagrams are treated using the separation

$$D_{\mu\nu} = D_{\mu\nu}^{<} + D_{\mu\nu}^{>} \ ,$$

where

$$D_{\mu\nu}^{>}(k) = \frac{\delta_{\mu\nu}}{k^2+M_W^2} \ .$$

That part which contains $D_{\mu\nu}^{>}$ is ultraviolet divergent, and is treated together with the other UV-divergent diagrams arising in the SU(2)$\otimes$U(1) framework. Their finite remnant is included in the first term of (2.1). Since the factor

$M_W{}^2/(M_W{}^2+k^2)$ is not really a tool to make the order-α radiative correction UV finite, the "cutoff mass" M_W may survive even in the final results, and it was recently proved by Sirlin to indeed do so.[17] Some recent and old radiative correction calculations, which start with the current$\times$current theory, solve the problem of UV infinites by using momentum-transfer-dependent weak and electromagnetic form factors, the dependence being extrapolated from the low-q^2 region.[6,8] These calculations cannot account for the mentioned (logarithmic) dependence on M_W, and probably underestimate the large-k^2 part of the loop integrals.

Finally, the infrared problem requires us to deal with inner bremsstrahlung. The matrix element for the $B\to be\bar{\nu}\gamma$ process can be written as

$$\mathcal{M}'_\gamma=\mathcal{M}'^{(h)}_\gamma+\mathcal{M}'^{(l)}_\gamma\ , \tag{2.10}$$

$$\mathcal{M}'^{(h)}_\gamma=\sqrt{2}G_Fe[\bar{u}_2\gamma^\mu(1+\gamma^5)v_1]\,T^{\rho\mu}(k)e^{\rho*}(k,s)\ , \tag{2.11}$$

$$\mathcal{M}'^{(l)}_\gamma=\sqrt{2}G_Fe\{\bar{u}_2\not{e}^*(k,s)[i(\not{p}_2-\not{k})+m_e]^{-1}\times\gamma^\mu(1+\gamma^5)v_1\}\langle i\,|\,J^\mu_W(0)\,|\,f\rangle\ . \tag{2.12}$$

The purpose of this paper is to study the decay distribution

$$\Gamma(E_e,E_f)=\Gamma_0(E_e,E_f)+\Gamma_\alpha(E_e,E_f)\ , \tag{2.13}$$

where E_e and E_f are the energy of the electron and the final baryon, respectively, in the rest system of the decaying particle. The integral of $\Gamma(E_e,E_f)$ gives the order-α corrected branching ratio

$$\rho(B\to be\bar{\nu})=\frac{1}{\Gamma}\int\Gamma(E_e,E_f)dE_edE_f\ ,$$

where Γ is the total decay width of the particle B. The bremsstrahlung part of $\Gamma_\alpha(E_e,E_f)$ is obtained after integration over the whole kinematically allowed phase space for photons. Therefore our results are suitable for the purpose of experiments, which use no discrimination against hard photons at all.

III. THE MODEL-INDEPENDENT CORRECTION

The first term $\Gamma_0(E_e,E_f)$ in (2.13) is the lowest-order distribution function for the process $B\to be\bar{\nu}$. It has been studied in detail by several authors; our basic reference is 18.

Following tradition we write the weak-current matrix element in (2.2) as

$$\langle f\,|\,J^\mu_W(0)\,|\,i\rangle=i(2\pi)^4\tfrac{1}{2}\bar{u}_fH^\mu u_i\ , \tag{3.1}$$

where

$$H^\mu=\gamma^\mu[f_1(q^2)-\gamma^5g_1(q^2)]-\frac{1}{m_i}q^\nu\sigma^{\mu\nu}f_2(q^2)\ ,\qquad q=p_i-p_f\ . \tag{3.2}$$

In its most general form H^μ contains three further form factors, f_3, g_2, and g_3, which we neglect in this paper. [The sign convention in (3.2) for the axial-vector form factor g_1 is the same as in Ref. 18.]

For the purpose of experimental analysis $\Gamma_\alpha(E_e,E_f)$ should be given in a form similar to $\Gamma_0(E_e,E_f)$, that is, as a bilinear combination of the unknown parameters f_1,f_2,g_1 with known functions of E_e and E_f as coefficients. At present, this task is too difficult to solve, since our knowledge about strong interactions is not sufficient to evaluate the matrix elements of the product of two or three hadronic currents, $T^{\mu\rho}(k)$ and $T^{\mu\nu\rho}(k)$.

In the bremsstrahlung case it seems reasonable to approximate $T^{\mu\rho}(k)$ as if the photon is coupled minimally to a pointlike baryon, since the photon energy cannot be large in the final state:

$$T^{\mu\rho}(k)\approx\frac{i}{2}(2\pi)^4\bar{u}_fH^\rho[i(\not{p}_i-\not{k})+m_i]^{-1}\gamma^\mu u_i\ , \tag{3.3a}$$

for the $\Sigma^-\to ne\bar{\nu}$ type of decays, and

$$T^{\mu\rho}(k)\approx-\frac{i}{2}(2\pi)^4\bar{u}_f\gamma^\mu[i(\not{p}_f+\not{k})+m_f]^{-1}H^\rho u_i \tag{3.3b}$$

for the $n\to pe\bar{\nu}$ type.

The situation is much more serious in the case of the virtual-photon corrections, since in (2.7) and in (2.8) k is an unbounded variable of integration. We shall follow the strategy of writing $\mathcal{M}_\gamma$ as the sum of a so-called model-independent and a model-dependent term. The idea of such a separation was originally invented by Sirlin in the case of neutron β decay.[7] Since the mathematical expression giving the model-independent part is quite general, it served later as a starting point of calculations also in the case of other semileptonic decays.[5,11] In this paper we use its model-independent part for $\mathcal{M}_\gamma$, and call, following tradition, the resulting $\Gamma_\alpha(E_e,E_f)$ the model-independent correction. By definition, this $\Gamma_\alpha(E_e,E_f)$ is a bilinear combination of the form factors f_1,f_2,g_1 and the coefficients (functions of E_e,E_f) are calculable. We want, however, to stress, that the radiative corrections are complete only, when also the model-dependent part of $\mathcal{M}_\gamma$ is included. Sirlin,[7] and later Garcia,[11] suggested, that, neglecting terms proportional to $G_F\alpha E_e/m_i$ in $\mathcal{M}_\gamma$, the only effect of the model-dependent part is that it changes f_1,g_1 to some "effective form factors" f'_1,g'_1 without changing the coefficient functions, already known from the calculation of the model-independent part. Perhaps this is true, but the notion of effective form factors is useless, when the ultimate purpose is to compare the experimental results with Cabibbo's predictions, which refer to the true form factors. We find it particularly disturbing that the model-independent—model-dependent separation is nonunique, and therefore the effective form factors are ill defined. We postpone the study of the problem of the model-dependent part to a subsequent paper.[19]

By the model-independent part of $\mathcal{M}_\gamma$ we mean the expressions given in Ref. 7 for (2.7) and (2.9). $\delta Z_{(e)}$ in (2.5) is well known from QED. In the Feynman gauge,

$$\delta Z_{(e)}=-\frac{\alpha}{2\pi}\left[\frac{1}{2}\ln\frac{M_W}{m_e}-\ln\frac{m_e}{\lambda}+\frac{9}{8}\right]\ . \tag{3.4}$$

Substituting the model-independent part for $T^{\mu<}$ in (2.6), $\mathcal{M}^{(2)}_\gamma$ takes the form

$$\mathcal{M}_\gamma^{(2)}=\mathcal{M}_0\delta Z\ , \tag{3.5}$$

where

$$\delta Z=\frac{i\alpha}{8\pi^3}\int dk\, D_{\mu\nu}^{<}(k)\frac{(2p_{\rm ch}^\mu-k^\mu)(2p_{\rm ch}^\nu-k^\nu)}{[(k-p_{\rm ch})^2+m_{\rm ch}{}^2]^2}\ . \tag{3.6}$$

This is part of the expression valid in QED for a pointlike, charged particle with $-p_{\rm ch}{}^2=m_{\rm ch}{}^2$ [cf. (2.5)]. Standard calculation gives

$$\delta Z=-\frac{\alpha}{2\pi}\left[\frac{1}{2}\ln\frac{M_W}{m_{\rm ch}}-\ln\frac{m_{\rm ch}}{\lambda}+\frac{3}{4}\right]\ . \tag{3.7}$$

The model-independent part of $\mathcal{M}_\gamma^{(3)}$ is obtained by writing in (2.8)

$$T^{\mu\rho}(k)=\tfrac{1}{2}(2\pi)^4\frac{2p_{\rm ch}^\mu-k^\mu}{(k-p_{\rm ch})^2+m_{\rm ch}{}^2}\bar{u}_fH^\rho u_i\ . \tag{3.8}$$

Then the contribution of $\mathcal{M}_\gamma^{(3)}$ is

$$\mathcal{M}_\gamma^{(3)}\approx\frac{\alpha}{2\pi}\mathcal{M}^{\rm Coulomb}+\frac{\alpha}{2\pi}(-d_0+d_1)\mathcal{M}_0 +\frac{\alpha}{2\pi}i(2\pi)^4\sqrt{2}G_F\left[\bar{u}_2\frac{\not p_{\rm ch}}{m_{\rm ch}}\gamma^\mu(1+\gamma^5)v_1\right] \times(\bar{u}_fH^\mu u_i)d_{11}\ . \tag{3.9}$$

In (3.9) the last term is included only for the sake of tradition. Let alone the very low end of the electron energy spectrum, it is negligibly small, the function d_{11} being

$$d_{11}=\frac{m_e}{2p_e'}\ln\frac{p_{e+}'}{m_e}\ ,$$

where $p_e'=(E_e'^2-m_e{}^2)^{1/2}$, $p_{e+}'=E_e'+p_e'$, and E_e' is the energy of the electron in the rest frame of the charged baryon. The first term on the right-hand side of (3.9) is the so-called Coulomb term. $\mathcal{M}^{\rm Coulomb}=0$, if the final baryon is neutral, and

$$\mathcal{M}^{\rm Coulomb}=\pi^2\frac{E_e'}{p_e'}\mathcal{M}_0\ , \tag{3.10}$$

if the final baryon is positively charged. Finally, the functions d_0 and d_1, neglecting terms proportional to $m_e/m_{\rm ch}$ and $(E_e'/m_{\rm ch})^2$, can be written as

$$d_1=\ln\frac{M_W}{m_e}+\frac{1}{2}+\frac{E_e'}{p_e'}\ln\frac{p_{e+}'}{m_e}\ , \tag{3.11}$$

$$d_0=\frac{2E_e'}{p_e'}\ln\frac{p_{e+}'}{m_e}\ln\frac{m_e}{\lambda}+\frac{E_e'}{p_e'}\ln^2\frac{p_{e+}'}{m_e} +\frac{E_e'}{p_e'}{\rm Sp}\left[\frac{2p_e'}{p_{e+}'}\right]+\frac{E_e'}{m_{\rm ch}}\left[\ln\frac{p_{e+}'^2}{m_{\rm ch}{}^2}-2\right]\ . \tag{3.12}$$

The mass $m_{\rm ch}$ is equal with m_i or $-m_f$, depending on whether the initial or final baryon is charged, respectively. For the definition of the Spence function in (3.12) we use the convention

$${\rm Sp}(x)=-\int_0^1 dt\frac{1}{t}\ln(1-xt)\ .$$

In summary, the model-independent part of $\mathcal{M}_\gamma$ is a multiple of $\mathcal{M}_0$ [neglecting now the term with d_{11} in (3.9)]:

$$\mathcal{M}_\gamma\approx\frac{\alpha}{2\pi}g_1(E_e')\mathcal{M}_0\ . \tag{3.13}$$

The order-$(\alpha/\pi)E_e'/m_{\rm ch}$ terms in the function $g_1(E_e')$ are of very little significance from the numerical point of view. The situation is different, when the $E_e'/m_{\rm ch}$ terms coming from $\mathcal{M}_0$ are considered. In hyperon decays they are not small enough to suppress large and remarkably varying terms of order α coming from $g_1(E_e')$. [Such a term is, i.e., $(E_e'/p_e')\ln^2(p_{e+}'/m_e)$.] We mention that in the case of the neutral-hyperon decays $(n,\Lambda\to pe\bar{\nu})$ an imaginary part should be added to the function $g_1(E_e')$. It gives, however, no contribution to any physical observable, if spin polarizations are not detected. Therefore, we omitted it in this paper.

IV. TWO-DIMENSIONAL DISTRIBUTIONS IN THE PRESENCE OF BREMSSTRAHLUNG

It is well known that in the case of the $B\to be\bar{\nu}$ process four-momentum conservation is very restrictive. Assuming that polarizations are not detected and the decaying particle is at rest, $E_i=m_i$, only two independent variables are available for the description of the final states. Several choices are possible for these two variables, the alternatives being easily related to each other. As a consequence, the quantities measured in an experiment can be freely transformed to other ones in order to obtain the wanted distribution. If radiative corrections are applied in the analysis such a possibility does not exist any more. This is a consequence of the presence of four particles, $be\bar{\nu}\gamma$, in the bremsstrahlung final states and of the integration over the three-momentum of the photon.

In order to illustrate what we mean we compare some properties of two distributions without and with radiative corrections.

A. Distributions in terms of (E_e,E_f)

If the $B\to be\bar{\nu}$ decay process alone is analyzed, the kinematically allowed region for these variables is

$$m_e\le E_e\le E_{e\max}\ , \tag{4.1}$$

$$E_{f\min}(E_e)\le E_f\le E_{f\max}(E_e)\ , \tag{4.2}$$

where

$$E_{e\max}=\frac{m_i{}^2-m_f{}^2+m_e{}^2}{2m_i}\ , \tag{4.3}$$

$$E_{f\max,f\min}=\frac{1}{2}\left[(m_i-E_e\pm p_e)+\frac{m_f{}^2}{m_i-E_e\pm p_e}\right]\ . \tag{4.4}$$

The variables E_e,E_f determine a decay event up to trivial rotations. The angle θ_{ef} between the three-vectors $\mathbf{p}_e$ and

$\mathbf{p}_f$ is uniquely fixed by the relation $m_i - E_e - E_f = |\mathbf{p}_e + \mathbf{p}_f|$:

$$\cos\theta_{ef} = \frac{m_i^2 + m_f^2 + m_e^2 - E_f(2m_i - E_e) - E_e(2m_i - E_f)}{2p_e p_f}, \tag{4.5}$$

where

$$p_f = (E_f^2 - m_f^2)^{1/2}, \quad p_e = (E_e^2 - m_e^2)^{1/2} .$$

If radiative corrections are taken into account the experimental analysis must cover an (E_e, E_f) region, which is larger than the one defined by (4.1) and (4.2). Because of inner bremsstrahlung extra events appear with

$$m_f \le E_f < E_{f\min}(E_e) , \tag{4.6}$$

if

$$m_e \le E_e < E'_{e\max} , \tag{4.7}$$

where

$$E'_{e\max} = \frac{1}{2}\left[(m_i - m_f) + \frac{m_e^2}{m_i - m_f}\right] . \tag{4.8}$$

For the set of events with given E_e, E_f the relation (4.5) is not true any more. This point can be conveniently discussed in terms of the variable

$$\hat{q} = |\mathbf{p}_e + \mathbf{p}_f| = (p_e^2 + p_f^2 + 2p_e p_f \cos\theta_{ef})^{1/2} . \tag{4.9}$$

Instead of the single value

$$\hat{q} = m_i - E_e - E_f \tag{4.10}$$

it is an interval, which is allowed for $\hat{q}$, and, therefore, for $\cos\theta_{ef}$ at each (E_e, E_f) point. Namely,

$$|p_e - p_f| \le \hat{q} \le m_i - E_e - E_f , \tag{4.11}$$

if (E_e, E_f) is in (4.1) and (4.2), and

$$|p_e - p_f| \le \hat{q} \le p_e + p_f , \tag{4.12}$$

if (E_e, E_f) is in (4.6) and (4.7). In the latter case

$$p_e + p_f < m_i - E_e - E_f . \tag{4.13}$$

The contribution of inner bremsstrahlung to $\Gamma_\alpha(E_e, E_f)$ is obtained by integration over $\hat{q}$. Another variable of integration is the energy E_γ of the bremsstrahlung photon. The range of the possible photon energies is a function of $\hat{q}$:

$$\tfrac{1}{2}(m_i - E_e - E_f - \hat{q}) \le E_\gamma \le \tfrac{1}{2}(m_i - E_e - E_f + \hat{q}) . \tag{4.14}$$

Interesting properties of $\Gamma_\alpha(E_e, E_f)$ follow from (4.11)–(4.14). When (E_e, E_f) is in (4.6) and (4.7) the "correction" $\Gamma_\alpha(E_e, E_f)$ comes from bremsstrahlung alone. It is finite, since $\min(E_\gamma) > 0$. But, when the curve $E_{f\min}(E_e)$ is approached, $\min(E_\gamma) \to 0$, and $\Gamma_\alpha(E_e, E_f)$ grows logarithmically:

$$\Gamma_\alpha(E_e, E_f) \sim -\frac{\alpha}{\pi}\ln\left[1 - \frac{m_i - E_e - E_f}{p_e + p_f}\right] \to +\infty . \tag{4.15}$$

On the curve $E_{f\min}(E_e)$ (and, for $E_e < E'_{e\max}$) $\Gamma_\alpha(E_e, E_f)$ is finite, because here the infrared-divergent bremsstrahlung and virtual-photon corrections sum up to give a finite result.

Another interesting case is when E_e and E_f are on the curve $E_{f\max}(E_e)$ or on $E_{f\min}(E_e)$ (and, in the latter case, $E_e > E'_{e\max}$). In both cases $m_i - E_e - E_f = |p_e - p_f|$, and the two-dimensional region of integration over $(E_\gamma, \hat{q})$ degenerates to a line:

$$\hat{q} = |p_e - p_f| ,$$

$$0 \le E_\gamma \lesssim |p_e - p_f| .$$

It is straightforward to verify, that, as a result of this degeneracy,

$$\Gamma_\alpha(E_e, E_f) \sim \frac{\alpha}{\pi}\ln\left[1 - \frac{|p_e - p_f|}{m_i - E_e - E_f}\right] \to -\infty , \tag{4.16}$$

when the above-mentioned boundary curves are approached. As the bremsstrahlung contribution to $\Gamma_\alpha(E_e, E_f)$ becomes finite in this limit, the infrared divergence of the virtual-photon part reappears.

B. Distributions in terms of $(E_e, \cos\theta_{ef})$.

Whether radiative corrections are considered or not, possible values of $\cos\theta_{ef}$ are

$$-1 \le \cos\theta_{ef} \le 1 , \tag{4.17}$$

if

$$m_e \le E_e \le E'_{e\max} ,$$

and

$$\cos\theta_{ef} \le 0 , \quad 0 \le \sin\theta_{ef} \le \frac{m_i}{m_f}\frac{E_{e\max} - E_e}{p_e} , \tag{4.18}$$

if $E'_{e\max} < E_e \le E_{e\max}$. For those events which have the same E_e and $\cos\theta_{ef}$, the possible energies for the final baryon are different depending on whether it comes from a $B \to be\bar{\nu}$ or $B \to be\bar{\nu}\gamma$ event, but these events are not distinguished from each other. Let us denote by $E_f^{(\pm)}$ the quantities

$$E_f^{(\pm)} = \frac{1}{a}\{(m_i - E_e)[m_i(E_{e\max} - E_e) + m_f^2] \pm p_e\cos\theta_{ef}[m_i^2(E_{e\max} - E_e)^2 - m_f^2 p_e^2 \sin^2\theta_{ef}]^{1/2}\} , \tag{4.19}$$

where $a = (m_i - E_e)^2 - p_e^2\cos^2\theta_{ef}$. In the case of $B \to be\bar{\nu}$ events E_f is uniquely determined by E_e and $\cos\theta_{ef}$ if $E_e < E'_{e\max}$:

$$E_f = E_f^{(-)} . \tag{4.20}$$

The relation is two to one if $E_e \ge E'_{e\max}$:

$$E_f = E_f^{(\pm)} . \tag{4.21}$$

For $B \to be\bar{\nu}\gamma$ events these relations change to

$$m_f \leq E_f \leq E_f^{(-)} , \qquad (4.22)$$

when $E_e < E'_{e\max}$, and to

$$E_f^{(+)} \leq E_f \leq E_f^{(-)} , \qquad (4.23)$$

when $E_e \geq E'_{e\max}$. In order to evaluate the bremsstrahlung contribution to $\Gamma_\alpha(E_e,\cos\theta_{ef})$, one must integrate over E_f and E_γ. The allowed range for E_γ is given by (4.14) and (4.9). Unbounded behavior of $\Gamma_\alpha(E_e,\cos\theta_{ef})$ emerges only, when $E_e > E'_{e\max}$, and

$$\sin\theta_{ef} \to \frac{m_i}{m_f}\frac{E_{e\max}-E_e}{p_e} .$$

Along this boundary curve $E_f^{(+)}=E_f^{(-)}$, and

$$\Gamma_\alpha(E_e,\cos\theta_{ef}) \sim \frac{\alpha}{\pi}\ln\left[1-\frac{E_f^{(+)}}{E_f^{(-)}}\right] \to -\infty . \qquad (4.24)$$

This is another example of the recovery of the infrared divergence coming from the virtual-photon corrections.

Further illustrative examples could be brought to stress that kinematical relations, which are commonly known for $B \to be\bar{\nu}$ decay, might be incorrect to use in experimental analysis if radiative corrections are relevant. Nonetheless it is difficult to tell in a given context whether or not the use of $B \to be\bar{\nu}$ kinematics is an acceptable approximation. Intuitively one expects that this approximation can be used, if in most of the $be\bar{\nu}\gamma$ final states the photon and the antineutrino move parallel to each other. This is, however, not the case, because the smallness of the electron mass results in a sharp maximum of the bremsstrahlung matrix element when the photon and the electron have parallel momenta. A theoretical calculation of radiative corrections must be designed very carefully in order not to confuse theoretical and experimentally measured quantities. The best example is $\cos\theta_{e\bar{\nu}}$, $\theta_{e\bar{\nu}}$ being the angle between the momenta of the electron and the antineutrino. In experiments $\theta_{e\bar{\nu}}$ is an indirectly obtained quantity, since the antineutrino is not seen. A radiative correction calculation must use the "experimental" definition of $\theta_{e\bar{\nu}}$, if it is destined for the purpose of analyzing experiments. The radiative correction to the $(E_e,\cos\theta_{e\bar{\nu}})$ distribution given in Refs. 10 and 11 is only of theoretical value, since in this paper $\theta_{e\bar{\nu}}$ means the actual angle between the momenta of the electron and the antineutrino. For a similar reason, any result known to us in the literature concerning the radiative correction to the asymmetry parameter $\alpha_{e\bar{\nu}}$ is inadequate to apply to the experimentally measured $\alpha_{e\bar{\nu}}$ (Ref. 12).

V. RESULTS

Using the model-independent expressions of Sec. III for the virtual-photon corrections and the "electromagnetically pointlike" baryon approximation [(3.3a) and (3.3b)] for the description of inner bremsstrahlung we have calculated radiative corrections to the branching ratio, the electron energy spectrum, and the (E_e,E_f) Dalitz distribution for four different semileptonic baryon decays, $\Sigma^- \to ne\bar{\nu}$, $\Sigma^- \to \Lambda e\bar{\nu}$, $\Xi^- \to \Lambda e\bar{\nu}$, $\Lambda \to pe\bar{\nu}$, assuming that the decaying particle is at rest. In this calculation we have needed the weak form factors as input. We have used the zero-momentum-transfer values of f_1, f_2, and g_1 obtained by the WA2 group at CERN from a first fitting of the experimental data without applying radiative corrections. We have checked that our results do not change under the influence of a few percent change (which is allowed by the experimental errors) in the value of these parameters. We have put equal with zero the form factors f_3, g_2, and g_3. Exact SU(3) and the conserved-vector-current hypothesis justifies this in the case of f_3 and g_2. The term with g_3 in the matrix element of the weak current is very much suppressed in the lowest-order decay matrix element; therefore, it is usually not included in experimental analysis. The suppression is resolved in the order-α corrections, but, unless one expects an unreasonably large value for g_3, its contribution cannot be more than 0.1–0.2 %. We have also ignored the momentum-transfer dependence of the form factors, its effect being of the order of $(\alpha/\pi)[(m_i-m_f)^2/m_i^2]$. In fact, to calculate the relative corrections given in this paper one needs only the form factors divided by f_1 (by the cosine of the Cabibbo angle in the case of $\Sigma^- \to \Lambda e\bar{\nu}$). These input numbers are summarized in Table I, together with the corresponding ones for $n \to pe\bar{\nu}$ decay, as they have been used in the Cabibbo analysis of the WA2 group.[19]

TABLE I. The form-factor values used in the present calculation.

	f_1	f_2	g_1
$\Sigma^- \to ne\bar{\nu}$	1	−1.139	0.310
$\Sigma^- \to \Lambda e\bar{\nu}$	0	1.213	−0.588
$\Xi^- \to \Lambda e\bar{\nu}$	1	−0.065	−0.249
$\Lambda \to pe\bar{\nu}$	1	0.974	−0.699
$n \to pe\bar{\nu}$	1	1.974	−1.239

We have obtained our results from computer calculation. We have used REDUCE algebraic programs to calculate traces of complicated products of Dirac γ matrices.[20] To evaluate the three-, four-, and five-dimensional integrals required by the bremsstrahlung part of the correction to the Dalitz distribution, electron energy spectrum and branching ratio, respectively, we have used the DIVON

TABLE II. Relative correction to the semileptonic decay rates in %.

	This calculation	Sirlin	Garcia[a]
$\Sigma^- \to ne\bar{\nu}$	−0.41	−0.25	−0.81
$\Sigma^- \to \Lambda e\bar{\nu}$	0.14	0.12	−0.23
$\Xi^- \to \Lambda e\bar{\nu}$	−0.20	−0.15	−0.50
$\Lambda \to pe\bar{\nu}$[b]	−0.57	−0.22	−0.89
$n \to pe\bar{\nu}$[c]	1.53	1.50	1.50

[a]See the Appendix.
[b] + 2.29% Coulomb correction.
[c] + 3.5% Coulomb correction.

TABLE III. Relative correction to the electron energy spectrum in %. At each x the upper number gives the value of Sirlin's $(\alpha/2\pi)g(E_e,E_{e\max})$, the lower one is our result. (Coulomb correction is not included.)

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\Sigma^-\to ne\bar{\nu}$	14.9	5.84	2.71	0.76	−0.80	−2.23	−3.70	−5.41	−7.83
	18.2	7.2	3.5	1.3	−0.4	−2.0	−3.8	−5.6	−8.6
$\Sigma^-\to\Lambda e\bar{\nu}$	11.4	4.90	2.54	1.01	−0.24	−1.40	−2.61	−4.01	−6.01
	13.4	5.5	3.0	1.3	−0.2	−1.3	−2.6	−4.1	−6.2
$\Xi^-\to\Lambda e\bar{\nu}$	14.3	5.70	2.71	0.84	−0.66	−2.05	−3.47	−5.12	−7.47
	16.2	6.5	3.2	1.2	−0.4	−1.8	−3.4	−5.0	−7.6
$\Lambda\to pe\bar{\nu}$	13.7	5.45	2.57	0.75	−0.71	−2.05	−3.44	−5.04	−7.34
	18.0	7.0	3.4	1.3	−0.4	−2.0	−3.7	−5.7	−8.5
$n\to pe\bar{\nu}$				1.88	1.77	1.60	1.39	1.13	0.74
				2.0	1.8	1.7	1.5	1.1	0.7

general-purpose routine for numerical integration.[21] We have had to subtract the infrared-divergent part of the square of the bremsstrahlung matrix element [(3.10)–(3.12)]:

$$|\mathscr{M}'_\gamma|_{\rm IR}{}^2=\frac{\alpha}{2\pi^2}\left[\frac{p^\mu_{i,f}}{k\cdot p_{i,f}}-\frac{p^\mu_2}{k\cdot p_2}\right]^2|\mathscr{M}_0|^2 . \qquad (5.1)$$

For the three-dimensional integration of (5.1) over the photon momenta we have used standard methods described, e.g., in Ref. 22. The integration of the remaining finite part is, in principle, straightforward. Convergence problems arise, however, due to the electron propagator in (3.12). In order to make the numerical integration convergent, we have had to smooth the large variations of the integrand by means of appropriately chosen variables of integration. For the Spence function we have used power-series expansion.

To check our programs and to study the convergence properties of the DIVON routine in the case of our specific problem we have made the following tests.

(1) We have computed the model-independent radiative correction to the $n\to pe\bar{\nu}$ decay rate. This number, 1.5%, is well known from the literature.[7] Our result by computer is 1.54%.

(2) We have computed the model-independent radiative correction to the electron energy spectrum in $n\to pe\bar{\nu}$ decay. In Table II we present our results together with the corresponding values of the famous $g(E_e,E_{e\max})$ function of Sirlin.[7]

(3) We have computed the total order-α photonic radiative correction to the $\mu^-\to e\nu_\mu\bar{\nu}$ decay rate. The classic

TABLE IV. Radiative correction to the Dalitz distribution in %. In the case of $\Lambda\to pe\bar{\nu}$ uniformly 2.3% must be added for the Coulomb correction.

		x_1	x_2	x_3	x_4	x_5	x_6
$\Sigma^-\to ne\bar{\nu}$	ξ_5	5.4	1.7	−0.7	−2.9	−5.7	−9.0
$\Sigma^-\to\Lambda e\bar{\nu}$		3.9	1.6	0.4	−1.9	−5.0	−6.5
$\Xi^-\to\Lambda e\bar{\nu}$		1.4	2.3	−1.0	−3.4	−5.2	−8.2
$\Lambda\to pe\bar{\nu}$		4.4	1.1	−1.2	−3.2	−5.9	−8.7
$\Sigma^-\to ne\bar{\nu}$	ξ_4	10.9	3.4	0.3	−2.2	−5.6	−11.6
$\Sigma^-\to\Lambda e\bar{\nu}$		5.4	2.8	0.8	−1.0	−4.1	
$\Xi^-\to\Lambda e\bar{\nu}$		6.1	3.7	0.5	−2.0	−5.4	−15.0
$\Lambda\to pe\bar{\nu}$		9.5	3.0	0.2	−2.1	−5.2	−13.0
$\Sigma^-\to ne\bar{\nu}$	ξ_3		4.5	0.5	−2.3	−6.6	
$\Sigma^-\to\Lambda e\bar{\nu}$			2.9	0.9	−1.0	−3.6	
$\Xi^-\to\Lambda e\bar{\nu}$			4.6	0.6	−2.1	−6.5	
$\Lambda\to pe\bar{\nu}$			4.1	0.5	−2.0	−6.1	
$\Sigma^-\to ne\bar{\nu}$	ξ_2		6.8	0.7	−2.5	−9.8	
$\Sigma^-\to\Lambda e\bar{\nu}$			2.9	0.8	−1.4		
$\Xi^-\to\Lambda e\bar{\nu}$			6.7	0.6	−2.5	−12.0	
$\Lambda\to pe\bar{\nu}$			5.5	0.8	−2.1	−19.8	
$\Sigma^-\to ne\bar{\nu}$	ξ_1			1.0	−3.0		
$\Sigma^-\to\Lambda e\bar{\nu}$				0.4	−3.9		
$\Xi^-\to\Lambda e\bar{\nu}$				0.4	−5.0		
$\Lambda\to pe\bar{\nu}$				0.8	−2.4		

TABLE V. The (x,ξ) coordinate values belonging to the points, in which the radiative correction to the Dalitz contribution is calculated $(x=E_e/E_{e\max}, \xi=E_f/m_i)$. $x_i=0.1, 0.25, 0.45, 0.65, 0.85, 0.95$.

	ξ_1	ξ_2	ξ_3	ξ_4	ξ_5
$\Sigma^-\to ne\bar{\nu}$	0.7925	0.7965	0.8005	0.8045	0.8075
$\Sigma^-\to \Lambda e\bar{\nu}$	0.9320	0.9325	0.9330	0.9335	0.9340
$\Xi^-\to \Lambda e\bar{\nu}$	0.8460	0.8500	0.8520	0.8540	0.8560
$\Lambda\to pe\bar{\nu}$	0.8440	0.8465	0.8490	0.8515	0.8535

result for it is[23,24,8]

$$-\frac{\alpha}{2\pi}(\pi^2-\tfrac{25}{4})=-0.42\% \; .$$

This is an example, in which $(m_i-m_f)/m_i$ is not negligible. (We have assumed $m_f\equiv m_{\nu_\mu}=0$.) For the purposes of this calculation we have had to keep the complete "pointlike" expressions for $T^{\mu<}$ in (2.6) and for $T^{\mu p}$ in (2.8). In our computer program there has been a formal dependence on M_W. It is well known that in $V-A$ theory and to order α, the radiative correction to μ decay is independent of the cutoff mass. We have put 80 GeV for M_W, but our results have not changed, when this number had been changed to 800 and to 80 000. We have obtained -0.45% for the correction to the $\mu\to e\nu_\mu\bar{\nu}$ decay rate.

(4) We have calculated the correction to all of the branching ratios in two different ways. First, we have taken the complete order-α expression for the radiative correction, and computed its five-dimensional integral. In the second case, we have decomposed the weak current matrix element (3.1) in terms of the form factors F_1, F_2, F_3, and G_1 instead of the ones f_1, f_2, and g_1 (see Ref. 18). Then, we have separated and integrated the kinematical coefficients for $F_1{}^2$, F_1F_2, $F_2{}^2$, etc., and have obtained the correction to the branching ratio as a combination of these terms. This procedure is extremely sensitive to numerical inaccuracies, because in most cases large terms with opposite sign sum up to give small result. In this way we have been able to excellently reproduce the results obtained by the first method. (In an early report we studied the Dalitz distribution for $\Sigma^-\to ne\bar{\nu}$ decay following the second method. In that calculation we used the complete pointlike expression for the Σ^--photon coupling.[25])

On the basis of these investigations we can say that the percentage values we give here for the relative corrections (RC%) have a numerical accuracy $\sim$0.1% [(RC$\pm$0.1)%] for all branching ratios, and for the electron energy spectrum and the Dalitz distribution in the case of the $\Sigma^-\to ne\bar{\nu}$ and $\Lambda\to pe\bar{\nu}$ decays. In some points of the energy spectrum and the Dalitz distribution for $\Sigma^-\to\Lambda e\bar{\nu}$ and $\Xi^-\to\Lambda e\bar{\nu}$ decays this accuracy is worse, (RC$\pm$0.5)%. The reason for this is that we saved computer time.

Of course, we do not think, that our present theoretical knowledge allows us to produce the complete radiative correction, i.e., including the model-dependent part, with the above accuracy. However, we wanted to avoid numerical uncertainties in the calculation of the model-independent part, which are possibly comparable with the theoretical uncertainties. We have been very careful about terms proportional to $(\alpha/\pi)E_e/m_i$ or $(\alpha/\pi)(m_i-m_f)/m_i$, particularly, because large factors, such as $\ln(E_e/m_e)$ and $\ln^2(E_e/m_e)$, can make them significant.

We present our results in Tables II–V. Tables II and III contain the relative model-independent correction for the branching ratios and the electron energy spectra. For comparison, we give our numbers together with the ones which can be obtained by using Sirlin's well-known results derived originally for $n\to pe\bar{\nu}$ decay, neglecting consistently all the terms with E_e/m_i or $(m_i-m_f)/m_i$. With the exception of $\Lambda\to pe\bar{\nu}$ decay there is no difference between the two sets of numbers in the case of the branching ratios. (Table II does not contain the Coulomb part of the correction, which is $+3.5\%$ for $n\to pe\bar{\nu}$, and $+2.3\%$ for $\Lambda\to pe\bar{\nu}$.) The situation is different for the electron energy spectra. In comparison with the limiting curve $g(E_e,E_{e\max})$ of Sirlin we have obtained a steeper function for the relative corrections. The difference is best visible in the lower third of the curve.

Table IV contains the relative correction to the two-dimensional distributions in some points of the (E_e,E_f) Dalitz plot. (The points were specifically chosen to meet the needs of the WA2 experiment. Table V gives the dimensionless coordinate values $x=E_e/E_{e\max}$ and $\xi=E_f/m_i$ for the various decays.) As discussed in Sec. IV, part of the (E_e,E_f) distribution is due to bremsstrahlung events alone. Here the "relative correction" would, of course, be infinite. Therefore, in Table VI we separately present the contribution of these events to the electron energy spectrum.

TABLE VI. Radiative correction in % to the electron energy spectrum, caused by bremsstrahlung events, which fall outside the three-body Dalitz plot (see Sec. IV).

x	0.1	0.2	0.3	0.4	0.5
$\Sigma^-\to ne\bar{\nu}$	7.8	1.5	0.5	0.1	0.02
$\Sigma^-\to\Lambda e\bar{\nu}$	8.4	2.4	0.9	0.2	0.01
$\Xi^-\to\Lambda e\bar{\nu}$	6.5	1.2	0.3	0.1	0.01
$\Lambda^-\to pe\bar{\nu}$	9.5	2.3	0.8	0.25	0.02

ACKNOWLEDGMENTS

The authors are indebted to CERN, both the EP and TH Divisions, for hospitality and for generously providing them with computer time. They are thankful to Dr. D. Froidevaux, Dr. P. Igo-Kemenes and, in particular, to J. M. Gaillard for invaluable discussions and encouragement.

APPENDIX

In this paper we gave an account of our calculation of the order-α radiative corrections to the (E_e,E_f) Dalitz distribution, the electron energy spectrum, and the branching ratio for semileptonic hyperon decays. The numbers given in the tables refer to the model-independent part of the corrections. Since there now exist several calculations of the model-independent corrections to the electron energy spectrum and the branching ratio, we find it necessary to clearly state the differences.

We have carried out our calculations without approximations in the lowest-order expression for the decay matrix element,[18] and keeping all the terms proportional to E_e/m_i, $(m_i-m_f)/m_i$ in the order-α virtual- and real-photonic expressions. The tables give our results for the corrections in the percentage of the precisely calculated lowest-order quantity.

In the tables we marked another set of numbers by the name of Sirlin. These numbers come from calculations, which were originally designed to describe $n\rightarrow pe\bar{\nu}$ decay, and, in which all the E_e/m_i, $(m_i-m_f)/m_i$ terms are neglected. That is, the formulas for the corrected electron energy spectrum and the branching ratio are

$$\Gamma_{B\rightarrow be\bar{\nu}}(E_e)=\frac{G_F{}^2}{2\pi^3}(f_1{}^2+3g_1{}^2)E_e{}^2(E_{e\max}-E_e)^2 \times\left[1+\frac{\alpha}{2\pi}g(E_e,E_{e\max})\right], \quad \text{(A1)}$$

and

$$\Gamma_{B\rightarrow be\bar{\nu}}=\frac{G_F{}^2}{60\pi^3}E_{e\max}{}^5(f_1{}^2+3g_1{}^2) \times\left[1+\frac{\alpha}{2\pi}\bar{g}(E_{e\max})\right], \quad \text{(A2)}$$

where

$$\bar{g}(E_{e\max})=\frac{3}{2}\ln\frac{m_{\text{ch}}{}^2}{4E_{e\max}{}^2}+\tfrac{81}{10}-\tfrac{4}{3}\pi^2 . \quad \text{(A3)}$$

In hyperon decay m_i-m_f is not small enough; therefore, the approximate lowest-order quantities in (A1) and (A2) are not suitable for the purpose of the present experiments.

Garcia has attempted to cure this problem in Ref. 11, and he has given a general expression for the $(E_e,\cos\theta_{e\bar{\nu}})$ distributions which is valid also when polarizations are detected. This result is not suitable for application in experimental analysis, because $\cos\theta_{e\bar{\nu}}$ is not a good variable.[12] One can, however, integrate Garcia's result over $\cos\theta_{e\bar{\nu}}$ and, e.g., perform summation over the polarization to obtain for the electron energy spectrum:

$$\Gamma_{B\rightarrow be\bar{\nu}}(E_e)=\Gamma_{0B\rightarrow be\bar{\nu}}(E_e)\left[1+\frac{\alpha}{2\pi}g(E_e,E_{e\max})\right], \quad \text{(A4)}$$

where $\Gamma_{0B\rightarrow be\bar{\nu}}(E_e)$ is the lowest-order function for the electron energy spectrum without approximations.[18] [In the notations of Ref. 11: $g(E_e,E_{e\max})=2(\Phi_1+\theta_1)$.] The relative correction is the same, as in (A1)]. An unaesthetic point about (A4) is that it follows from a result in Ref. 11, which is obtained after *ad hoc* manipulations with $\alpha/\pi(E_e/m_i)$, $(\alpha/\pi)(m_i-m_f)/m_i$ terms in the inner bremsstrahlung contributions. The purpose of these manipulations is to obtain a result, which contains the precise lowest-order quantities. The problem is, that large logarithmic factors multiply $\alpha/\pi(E_e/m_i)$ and $\alpha/\pi(m_i-m_f)/m_i$ and, therefore, they are not really small in hyperon decays. An illustration of this is the correction to the branching ratio. Garcia gives

$$\Gamma_{B\rightarrow be\bar{\nu}}=\Gamma_{0B\rightarrow be\bar{\nu}}\left[1+\frac{\alpha}{2\pi}\bar{g}(E_{e\max})\right]. \quad \text{(A5)}$$

[It is better to say the numerical values of $(\alpha/2\pi)\bar{g}(E_{e\max})$ are given in Ref. 11 for several hyperon decays.] The relative correction is again the same, as in Sirlin's case, but in (A5) $\Gamma_{0B\rightarrow be\bar{\nu}}$ is the lowest-order decay rate without approximation. In contrast with (A5) the actual relative correction, which follows from (A4) is

$$\frac{\alpha}{2\pi}\frac{1}{\Gamma_{0B\rightarrow be\bar{\nu}}}\int\Gamma_{0B\rightarrow be\bar{\nu}}(E_e)g(E_e,E_{e\max})dE_e .$$

This quantity is given in our Table II under the name of Garcia. (In Ref. 19, Table IV has just the opposite heading.) These numbers are definitely different from the relative correction in (A5). In the case of $\Lambda\rightarrow pe\bar{\nu}$ decay the difference, 0.7%, is not even small in comparison with the error, 2%, of the currently best experiment.[26]

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